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DIFFERENT DISPOSITIONS ON THE CHESSBOARD

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PROBLEM

How many different dispositions (for rotation or reflection) I have in an order-N ChessBoard, if I place K-Queens -- or in general K-Pieces ?

KEY by *Timothy CHOW, Mario VELUCCHI*

The answer is in the "Polya's Theorem", the wanted number is $\frac{1}{8}$ of the coefficient of $a^K \cdot b^{(N^2-K)}$ in the polynomials:

for N even
$$p(a,b,N)=(a+b)^{N^2}+2\cdot(a+b)^N\cdot\left(a^2+b^2\right)^{\frac{(N^2-N)}{2}}+3\cdot\left(a^2+b^2\right)^{\frac{N^2}{2}}+2\cdot\left(a^4+b^4\right)^{\frac{N^2}{4}}$$

for N odd
$$p(a,b,N)=(a+b)^{N^2}+2\cdot(a+b)\cdot\left(a^4+b^4\right)^{\frac{(N^2-1)}{4}}+(a+b)\cdot\left(a^2+b^2\right)^{\frac{(N^2-1)}{2}}+4\cdot(a+b)^N\cdot\left(a^2+b^2\right)^{\frac{(N^2-N)}{2}}$$

◆ ***first sample table (K = N)***

N	Total Dispositions	Different Dispositions
1	1	1
2	6	2
3	84	16
4	1820	252
5	53130	6814
6	1947792	244344
7	85900584	10746377
8	4426165368	553319048
9	260887834350	32611596056
10	17310309456440	2163792255680
11	1276749965026536	159593799888052
12	103619293824707388	12952412056879996
13	9176358300744339432	1147044793316531040
14	880530516383349192480	110066314584030859544
15	91005567811177478095440	11375695977099383509351
16	10078751602022313874633200	1259843950257390597789296
17	1190739044344491048895397910	148842380543159458506703546
18	149482492334195165714038760136	18685311541775061906510072648

19	198708670535437560041332476954OO 2483858381692984848273972297368
20	27883609836708967378728510729940 80 3485451229588622001224017714633 28
.	.
30	980334816737458556637693014290725536123011020542738448 00 1225418520921823195797116272382374454697050830389597 6184
.	.
40	1095649347418889634624623399657932138331156966895574519919117287283862137288869 20 136956168427361204328077924957241517292381124327152438917818437166469426555875 76
.	.
50	15838085607502492086871896388315564653360003299598838392035094537626522076602329592023328582402432582862 00 19797607009378115108589870485394455816700004124498620562234055645868942342336340442782363408868305482 7800
.	.

◆ *second sample table (N = 8; 0 ≤ K ≤ 64)*

K	Total Dispositions	Different Dispositions
0	1	1
1	64	10
2	2016	278
3	41664	5278
4	635376	79920
5	7624512	954226
6	74974368	9377618
7	621216192	77664262
8	4426165368	553319048
9	27540584512	3442665226
10	151473214816	18934455246
11	743595781824	92950002558
12	3284214703056	410528341328
13	13136858812224	1642109774130
14	47855699958816	5981968554282
15	159518999862720	19939884042150
16	488526937079580	61065887431100
17	1379370175283520	172421300174130
18	3601688791018080	450211156261470
19	8719878125622720	1089984840360150
20	19619725782651120	2452465861427952
21	41107996877935680	5138499778566330
22	80347448443237920	10043431343942730
23	146721427591999680	18340178778584670
24	250649105469666120	31331138705087032
25	401038568751465792	50129821652952594
26	601557853127198688	75194732462864406
27	846636978475316672	105829623137088214
28	1118770292985239888	139846287758051216
29	1388818294740297792	173602287915813594
30	1620288010530347424	202536002692174818
31	1777090076065542336	222136260729743262
32	1832624140942590534	229078019084673798
33 = 31	1777090076065542336	222136260729743262
34 = 30	1620288010530347424	202536002692174818
...		

◆ *Note:*

The total number of dispositions is $\binom{N^2}{K}$ and the number of different dispositions is approximately $\frac{1}{8}$ of the precedent binomial number.

◆ references:

◆ *Internet/Web*

i. Neil SLOANE “WWW Encyclopedia of Integer Sequences” http://www.research.att.com:80/~njas/sequences/eisonline.html

ii. Eric WEISSTEIN “Web Eric's Treasure Trove of Mathematics“ http://www.gps.caltech.edu/~eww/math/math.html

iii. from my page: http://www.bigfoot.com/~velucchi/papers.html

◆ Comment

This computation is a math-compendium (the <u>solution domain</u>, <u>computation domain dimension</u>) for my main project about the Non-Dominating Queen problem (‘Placing N Queens on an order-N board to leave a maximum number of unattacked vacant cells’), for further -- research and bibliographical -- details see my paper: Mario VELUCCHI “For me, this is the best Chess-Puzzle!” (apart available).

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