

BII Journal Club (27 December 2002)

The Saga of Fermat's Last Theorem

Presented by:
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What is it about ?

- A brief history of (pure) mathematics ...
... through the story of Fermat's Last Theorem ...
... the most baffling, tantalizing, intriguing problem of mathematics
- Based on the book *Fermat's Last Theorem* by Simon Singh

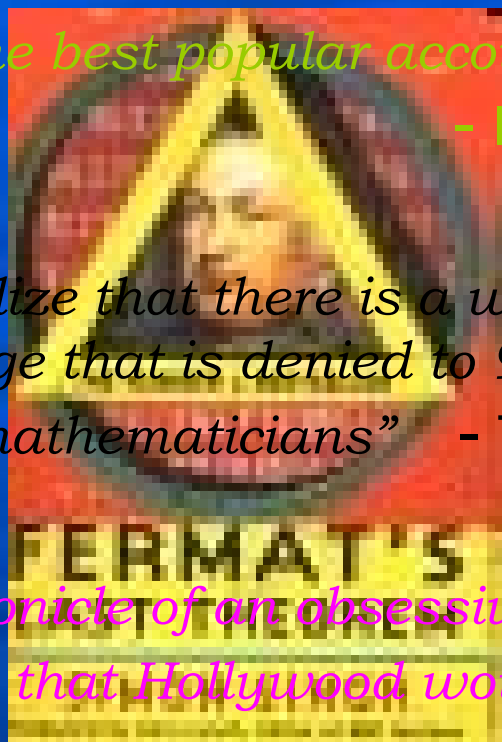
Why this book ?

- *“This is probably the best popular account of a scientific topic I have ever read”* - IRISH TIMES

- *“To read it is to realize that there is a world of beauty and intellectual challenge that is denied to 99.9 per cent of us who are not high-level mathematicians”* - THE TIMES

- *“Reads like the chronicle of an obsessive love affair. It has the classic ingredients that Hollywood would recognise”*

- DAILY MAIL



What's the problem ?!

- Question Time!!

- $x + y = z$ *infinite integer solutions*
 $\{(x,y,z)\} = \{(1,2,3); (10,40,50) \dots\}$

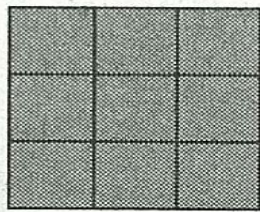
- $x^2 + y^2 = z^2$ *infinite integer solutions*
 $\{(x,y,z)\} = \{(3,4,5); (5,12,13) \dots\}$

- $x^3 + y^3 = z^3$ *NO integer solutions*

⋮

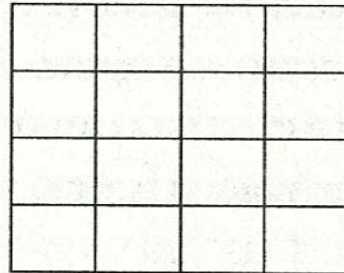
- $x^n + y^n = z^n$ *NO integer solutions for all $n > 2$*

.... the problem



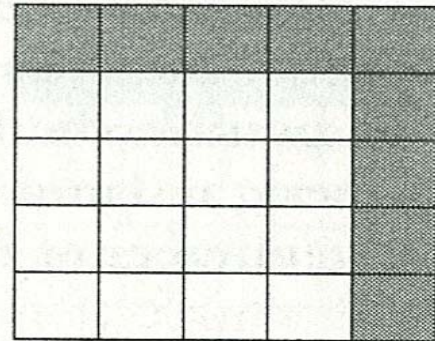
$$3^2 \\ 9$$

+



$$4^2 \\ 16$$

=



$$5^2 \\ 25$$

.... the problem

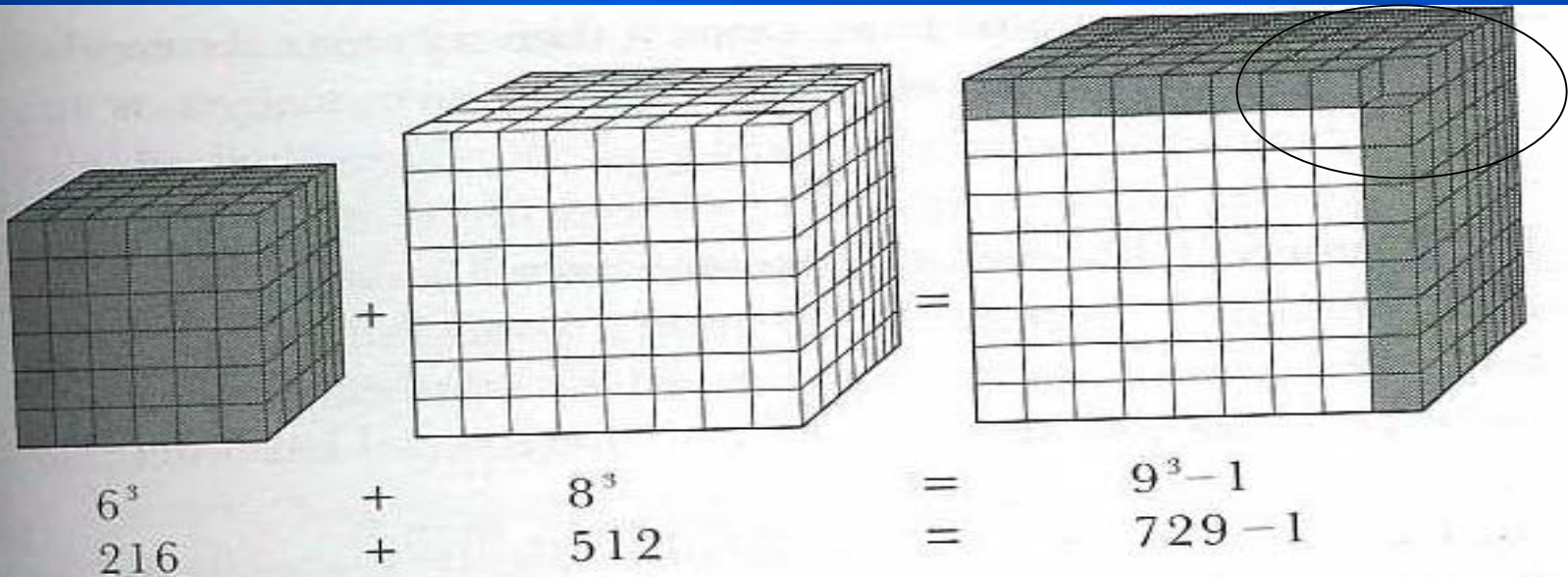
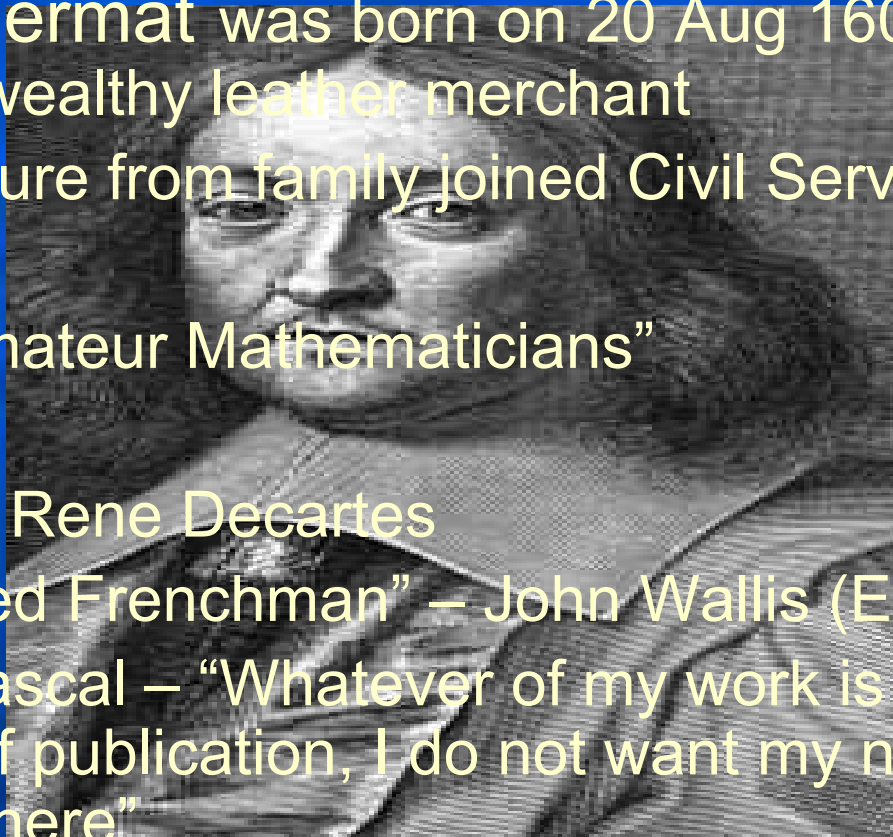


Figure 5. Is it possible to add the building blocks from one cube to another cube, to form a third, larger, cube? In this case a $6 \times 6 \times 6$ cube added to an $8 \times 8 \times 8$ cube does not have quite enough building blocks to form a $9 \times 9 \times 9$ cube. There are 216 (6^3) building blocks in the first cube, and 512 (8^3) in the second. The total is 728 building blocks, which is 1 short of 9^3 .

Who was Fermat ?!

- Pierre de Fermat was born on 20 Aug 1601 in S/W France to a wealthy leather merchant
- Due to pressure from family joined Civil Service
- “Prince of Amateur Mathematicians”
- Notorious!
 - “Braggart” – Rene Decartes
 - “That damned Frenchman” – John Wallis (Englishman)
 - To Blaise Pascal – “Whatever of my work is judged worthy of publication, I do not want my name to appear there”



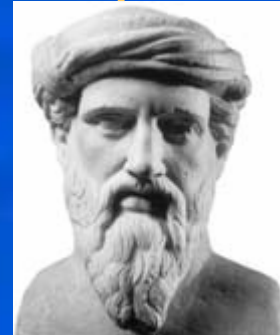
What did he do ?!

- Developed Probability Theory with Pascal
- Calculus
- Biggest Contribution was Number Theory
 - proved 26 is the only number sandwiched between a square and cube
 - Friendly Numbers .. discovered second pair (17296,18416) after Pythagoras (220,284)

Origin of the problem (some history) ..

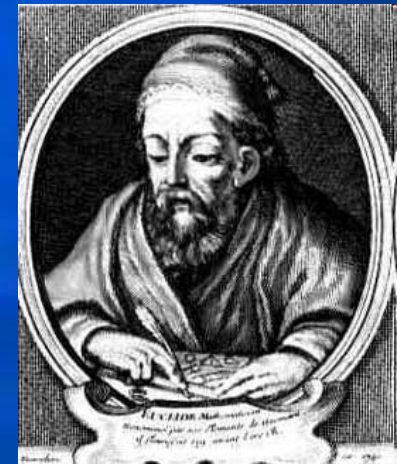
600 BC

Pythagorean Brotherhood
Proof, counting numbers



332 BC

Alexandria Library
Euclid
Elements (second best seller after bible!)
Irrational numbers, *reductio ad absurdum*



250 AD

Diophantus wrote *Arithmetica* (13 vols)

389 AD

First blow to Alexandrian library by Christian rulers

642 AD

Second and final blow by Moslem ruler Omar Caliph

~600 AD

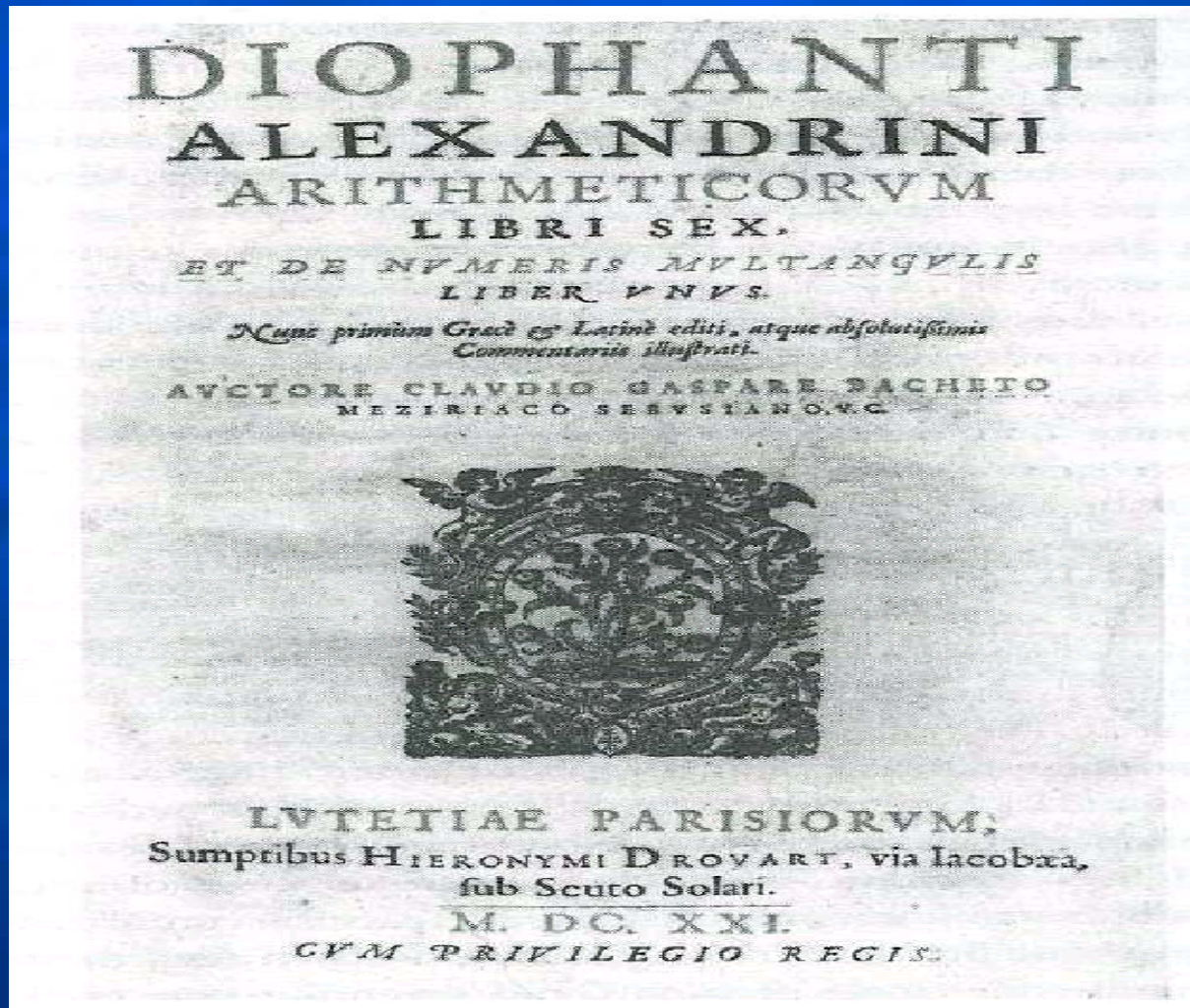
Zero was discovered by Indians and Indo-Arabic notation became prevalent ie 155/601 instead of CLV/DCI !!

1601 AD

Fermat was born

6 volumes of *Arithmetica* survived and they were destined for the desk of Fermat

Origin of the problem ...



Luckily,
The text
had generous
margins!

Bachet did a Latin translation of *Arithmetica* titled "Problèmes plaisans et delectables qui se font par les nombres" Having 100 problems in number theory with detailed solutions

...Fermat commented it !

DIOPHANTII
ALEXANDRINI
ARITHMETICORVM
LIBRI SEX,
ET DE NVMERIS MVLTVGLIS
LIBER VNVS.

*CVM COMMENTARIIS C. G. BACHETI V. C.
& observationibus D. P. de FERMAT Senatoris Tolofani.*

Accessit Doctrinae Analiticae, quae hactenus in Epistolis
ex varijs eiusdem Auctoris Epistolis collectum.



TOLOSE,
Excudebat BERNARDVS BOSCH, & Regione Collegij Societatis Iesuit.
M. DC. LXX.

Why is it called the 'Last Theorem' ?

There were many such 'propositions' by Fermat

For eg:

All primes can be written as $4p+1$ or $4p - 1$

Type 1 ie $4p+1 = x^2 + y^2$ but not $4p-1$

Euler proved this in 1749 after 7 years of work !

BUT

The "last" theorem could not be proved

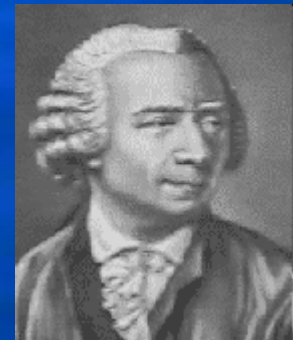
... and the ordeal begins ...

1665

Death of Fermat

1753

Euler proved for $n=3$ based of Fermat's hint for $n=4$ case using method of infinite descent and imaginary numbers



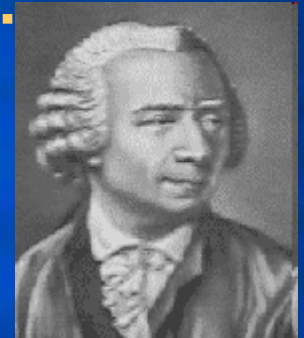
... and the ordeal continues ...

1665

Death of Fermat

1753

Euler proved for $n=3$ based of Fermat's hint for $n=4$ case using method of infinite descent



1776

Birth of **Sophie Germain** (aka Monsieur Le Blanc)
Considered the most brilliant French woman ever

Story time!

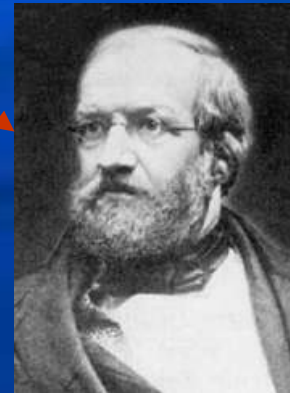
Germain Primes, (1794) Ecole Polytechnique, (1806) Gauss, Lagrange

Pg 112



1825

Gustav Lejune Dirichlet
&
Adrien-Marie Legendre



Independently showed $n = 5$



Note: Only Prime n need to be considered

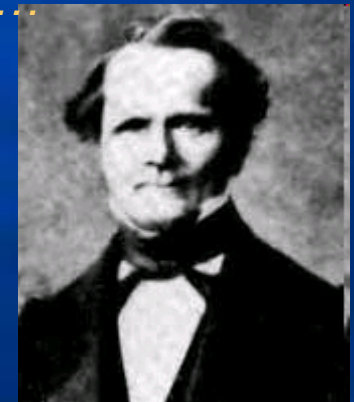
... the ordeal continues ...

- 1665 Death of Fermat
- 1753 Euler proved for $n=3$ based of Fermat's hint for $n=4$
- 1776 Birth of Sophie Germain
- 1825 Dirichlet & Adrien-Marie Legendre ($n = 5$)
- 1839 Another French **Gabriel Lamé** did $n = 7$
- 1847 **Cauchy** vs Lamé



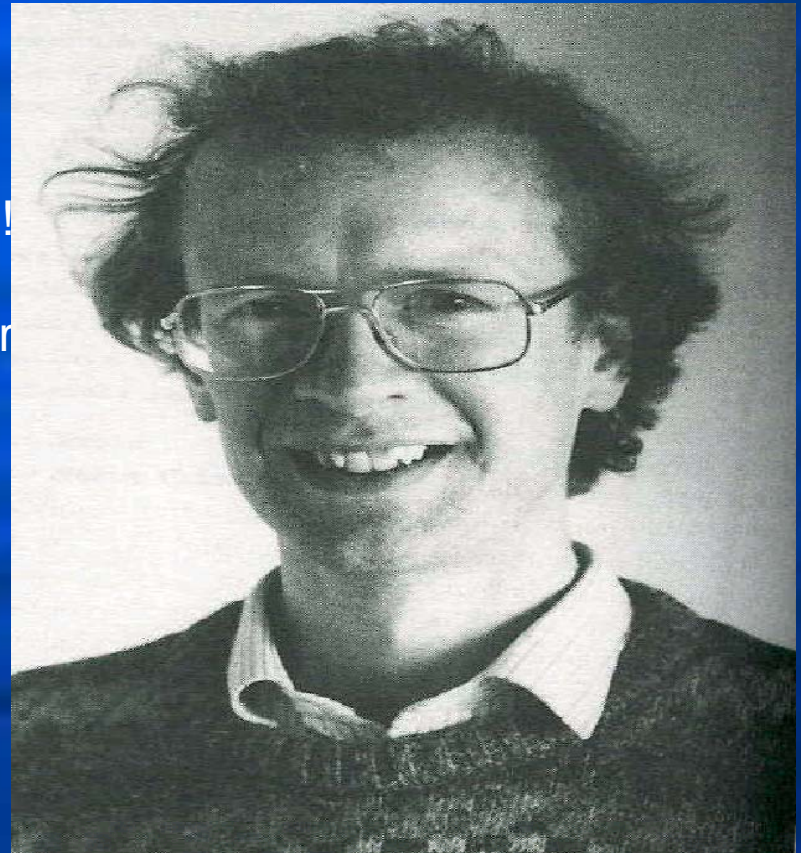
Story time!

German **Ernst Kummer** ... Unique factorization (by Euclid) ...
irregular primes (37,59,67) were the problem
A BIG BLOW (pg 128)!



Fast Forward >>

- *Andrew Wiles*, was born on 11 April, 1953 at Cambridge, UK
- Stumbled upon Fermat's Last Theorem when he was 10 and tried to prove it then !
- Studied Euler, Germain, Cauchy, Lamé', Kummer and hit the same wall as Kummer
- Phd in Elliptic Equations
 $y^2 = x^3 + ax^2 + bx + c$
 $a = 0, b = 0, c = -2$ case was solved by Fermat
- Migrated to Princeton in 1980s
- In **1993** (almost) presented the solution to Fermat's Equation at Newton Institute, Cambridge



Back to our story ...

1847

Cauchy vs Lamé and Kummer's Wall

A long gap of indifference

1908

Paul Wolfskehl, A German Industrialist rejevunated the quest

Story Time!!

Love, Suicide, Award, Landau (1909-34)..
Univ of Gottingen (Pg 144)

the story continues ...

1847

Cauchy vs Lamé and Kummer's Wall

1908

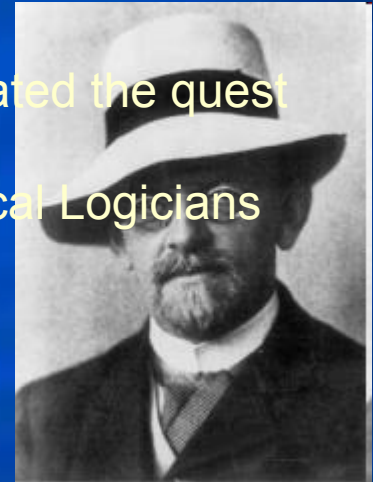
Paul Wolfskehl, A German Industrialist rejevunated the quest

~1900

Hilbert Program was started by the Mathematical Logicians

Proving laws like *Law of Trichotomy*

and defining things like 'threeness' (pg 150)



Story Time!!

Defn of Logic

Bertrand Russel's paradox (pg 154)

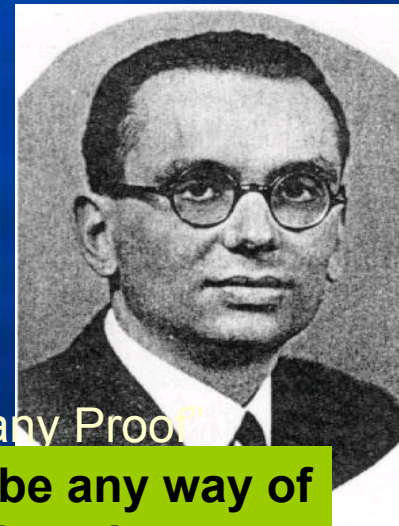
1931

Kurt Godel, a Czech mathematician gave his

Theorem's of Indecidability !

something like: "I am a liar"

"This statement does not have any Proof"



IMPLICATION: Certain theorems might be true but there may not be any way of proving them..... and Fermat's Last Theorem might be one of them !

The quest continues ...

1847

Cauchy vs Lamé and Kummer's Wall

1908

Paul Wolfskehl, A German Industrialist rejevunated the quest

1931

Kurt Godel, a Czech mathematician gave his

~1950

After Second world war computers were used to implement
Brute Force Method

Alan Turing was involved in it

Demonstrated for $n < 10000$

~1980 At Univ of Illinois $n < 25000$

more recently $n < 4$ million !!

Interesting facts:

•31,331,3331, ... , 33 333 331 are all primes

BUT $333\ 333\ 331 = 17 \times 19607843$!!

•Euler Conjecture

$x^4 + y^4 + z^4 = w^4$ has no integer solution

BUT in 1988 Naom Elkies at Harward found one (pg 178) !!

The quest continues ...

1847

Cauchy vs Lamé and Kummer's Wall

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~1950

Brute Force Method

1955

Taniyama-Shimura Conjecture

stated the Equivalence of Modular forms and Elliptic curves (whatever that means!)

This was a mammoth idea (pg 211)

Grand Unification Scheme by Langlands

1984

Gerhard Frey claimed:

“Proof of T-S will immediately prove Fermat's Last theorem”

Also

“If Fermat's theorem is wrong so is T-S and that would be disastrous”



The most important problem in mathematics of 17th century got linked to most important problem of 20th century!

Finally ... the quest concludes !

1931
~1950

Kurt Godel, a Czech mathematician gave his
Brute Force Method

1955

Taniyama-Shimura Conjecture

1984

Gerhard Frey

1986

Andrew Wiles started working to prove T-S conjecture

pg 223

Ken Ribet, Barry Mazur, Evariste Galois

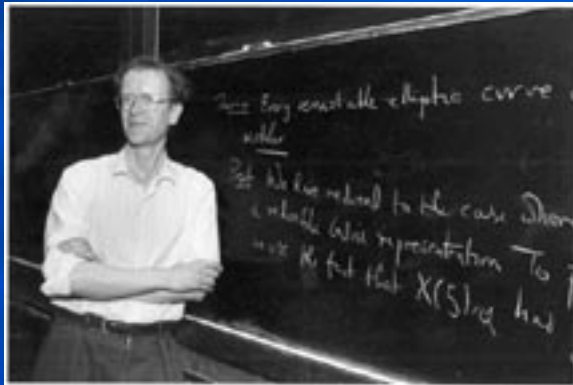
1993

Andrew Wiles presented the proof of T-S conjecture ... Almost!!

1995

Andrew Wiles completed the proof of T-S conjecture ... Conclusively

"I think I will stop here"



- 3 Lectures 21-23 June 1993 at the Newton Institute
- The Lecture of the Century
- 180 page proof in the *Annals of Mathematics* (May 1995)

My Comments !

“I think I’ll stop here” ;)

Wish You a Happy New Year !

Questions ?!