Formula for the number of covers of an unlabeled *n*-set such that every point of the set is covered by exactly *m* subsets of the cover and that intersection of every *m* subsets of the cover contains at most one point

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Let $Z(S_m^{(n)};x_1,x_2,...)$ be the cycle index of (restricted disordered) *n*-ary symmetric group S_m of degree *m* (i.e. the group of permutations of all *n*-subsets of an *m*-set, induced by S_m), which can be calculated in the following way:

$$Z(S_m^{(n)};x_1,x_2,...)=\frac{1}{m!}\sum_{\pi(m)}\frac{m!}{k_1!1^{k_1}\,k_2!2^{k_2}...k_m!m^{k_m}}\cdot\prod_{i|k}x_i^{e_i}\ ,$$

where $\pi(m)$ runs through all partitions of *m* (i.e. nonnegative solutions of $k_1+2k_2+...+mk_m=m$);

$k=lcm\{i\mid k_i\neq 0\}$;

$$e_i=e_i(\pi,n)=\frac{1}{i}\sum_{d|i}\mu(\frac{i}{d})\cdot\sum\prod_{l=1}^{n}\binom{(l,d)\,k_l}{t_l},$$

where μ is Mobius function and the last sum is taken over all nonnegative solutions of

$$t_1\frac{1}{(1,d)}+t_2\frac{2}{(2,d)}+...+t_n\frac{n}{(n,d)}=n.$$

Let $Z(S_m^{(n)};1+x)=Z(S_m^{(n)};1+x,1+x^2,1+x^3,...)$, i. e. $Z(S_m^{(n)};1+x)$ is obtained if we replace x_i by $1+x^i$, *i*=1,2,…, in the cycle index $Z(S_m^{(n)};x_1,x_2,...)$.

Then the number of covers of an unlabeled *n*-set such that every point of the set is covered by exactly *m* subsets of the cover and that intersection of every *m* subsets of the cover contains at most one point is the coefficient of x^n in $Z(S_{nm}^{(m)};1+x)$.