Small Ramsey Numbers

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ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but concentrate on their specific values.

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Contents

1. Scope and Notation

There is a vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey papers [GrRö, Neš, ChGra2]), but the progress on evaluating the basic numbers themselves has been very unsatisfactory for a long time. In the last two decades, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color **Ramsey number** for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments.

If $s = 2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k, then we can write k instead of G_i, and if $G_i = G$ for all i we can use the abbreviation $R_m(G; s)$ or $R_m(G)$. For $s = 2$, $K_k - e$ denotes a K_k without one edge, and for $s = 3$, $K_k - t$ denotes a K_k without one triangle (hyperedge). P_i is a **path** on i vertices, C_i is a **cycle** of length i, and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of some cycle C_{i-1}. $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \bar{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. The **fan** graph F_n is defined by $F_n = K_1 + nK_2$. For a graph G, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively. Finally, let $\chi(G)$ be the chromatic number of G, and let nG denote n disjoint copies of G.

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, and section 3 for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete. Section 4 lists the most studied two color cases for other graphs. The multicolor and hypergraph cases are gathered in sections 5 and 6, respectively. Finally, section 7 gives pointers to cumulative data and to most of the previous surveys.

2. Classical Two Color Ramsey Numbers

2.1. Upper and lower bounds on $R(k, l)$

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	14	18	23	28	36	40 43	46 51	52 59	59 69	66 78	73 88
4		18	25	35 41	49 61	56 84	69 115	92 149	97 191	128 238	133 291	141 349	153 417
5			43 49	58 87	80 143	101 216	121 316	141 442	157	181	205	233	261
6				102 165	111 298	127 495	169 780	178 1171	253	262	317		401
7					205 540	216 1031	232 1713	 2826	405	416	511		
8						282 1870	317 3583	 6090			817		861
9							565 6588	580 12677					
10								798 23556					1265

Table I. Known nontrivial values and bounds for two color Ramsey numbers $R(k, l) = R(k, l; 2)$.

k \ l	4	5	6	7	8	9	10	11	12	13	14	15
3	GG	GG	Kéry	Ka2 GY	GR MZ	Ka2 GR	Ex5 RK2	Ka2 RK2	Ex12 Les	Piw1 RK2	Ex8 RK2	WW Les
4	GG	Ka1 MR4	Ex9 MR5	Ex3 Mac	Ex15 Mac	RK1 Mac	HaKr Mac	2.3.e Spe3	SLL2 Spe3	2.3.e Spe3	XXR Spe3	XXR Spe3
5		Ex4 MR5	Ex9 HZ1	CET Spe3	HaKr Spe3	Haa Mac	Ex12 Mac	XXER	Ex12	XXER	XXER	XXER
6			Ka1 Mac	2.3.e Mac	XXR Mac	XXER Mac	2.3.e Mac	XXR	2.3.e	XXER		2.3.h
7				She1 Mac	2.3.e Mac	2.3.g HZ1	 Mac	XXER	2.3.e	XXR		
8					BR Mac	XXER Ea1	 HZ1			XXER		2.3.h
9						She1 ShZ1	2.3.e Ea1					
10							She1 Shi2					2.3.h

References for Table I.

We split the data into the table of values and a table with corresponding references. In Table I, known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries.

The task of proving $R(3,3) \leq 6$ was the second problem in Part I of the William Lowell Putnam Mathematical Competition held in March 1953 [Bush].

All the critical graphs for the numbers $R(k,l)$ (graphs on $R(k,l)-1$ vertices without K_k and without K_l in the complement) are known for $k=3$ and $l=3$, 4, 5 [Kéry], 6 [Ka2], 7 [RK3, MZ], and there are 1, 3, 1, 7 and 191 of them, respectively. All $(3,k)$-graphs, for $k \leq 6$, were enumerated in [RK3], and all (4,4)-graphs in [MR2]. There exists a unique critical graph for $R(4,4)$ [Ka2]. There are 430215 such graphs known for $R(3,8)$ [McK], 1 for $R(3,9)$ [Ka2] and 350904 for $R(4,5)$ [MR4], but there might be more of them. In [MR5] evidence is given for the conjecture that $R(5,5) = 43$ and that there exist 656 critical graphs on 42 vertices. The graphs constructed by Exoo in [Ex9, Ex12, Ex13, Ex14, Ex15, Ex16], and some others, are available electronically from http://ginger.indstate.edu/ge/RAMSEY.

The construction by Mathon [Mat] and Shearer [She1] (see also sections 2.3.i, 5.2.h and 5.2.i), using data obtained by Shearer [She1], gives the following lower bounds for higher diagonal numbers: $R(11,11) \geq 1597$, $R(13,13) \geq 2557$, $R(14,14) \geq 2989$, $R(15,15) \geq 5485$, and $R(16,16) \geq 5605$. Similarly, $R(17,17) \geq 8917$, $R(18,18) \geq 11005$ and $R(19,19) \geq 17885$ were obtained in [LSL]. The same approach does not improve on an easy bound $R(12,12) \geq 1637$ [XXR], which can be obtained by applying twice 2.3.e. Only some of the higher bounds implied by 2.3.* are shown, and more similar bounds could be easily derived. In general, we show bounds beyond the contiguous small values if they improve on results previously reported in this survey or published elsewhere. Some easy upper bounds implied by 2.3.a are marked as [Ea1].

Cyclic (or *circular*) graphs are often used for Ramsey graph constructions. Several cyclic graphs establishing lower bounds were given in the Ph.D. dissertation by J.G. Kalbfleisch in 1966, and many others were published in the next few decades. Only recently Harborth and Krause [HaKr] presented all best lower bounds up to 102 from cyclic graphs avoiding complete graphs. In particular, no lower bound in Table I can be improved with a cyclic graph on less than 102 vertices. See also item 2.3.k and section 4.16 [HaKr].

The claim that $R(5,5) = 50$ posted on the web [Stone] is in error, and despite being shown so more than once, this incorrect value is being cited by some authors. The bound $R(3,13) \geq 60$ [XZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw1]. Another incorrect construction for $R(3,10) \geq 41$ was described in [DuHu].

There are really only two general upper bound inequalities useful for small parameters, namely 2.3.a and 2.3.b. Stronger upper bounds for specific parameters were difficult to obtain, and they often involved massive computations, like those for the cases of (3,8) [MZ], (4,5) [MR4], (4,6) and (5,5) [MR5]. The bound $R(6,6) \leq 166$, only 1 more than the best known [Mac], is an easy consequence of a theorem in [Walk] (2.3.b) and $R(4,6) \leq 41$. T. Spencer [Spe3], Mackey [Mac], and Huang and Zhang [HZ1], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs listed in [MR3, MR5], were able to establish new upper bounds for several higher Ramsey numbers, improving on all of the previous longstanding results by Giraud [Gi3, Gi5, Gi6].

We have recomputed the upper bounds in Table I marked [HZ1] using the method from the paper [HZ1], because the bounds there relied on an overly optimistic personal communication from T. Spencer. Further refinements of this method are studied in [HZ2, ShZ1, Shi2]. The paper [Shi2] subsumes the main results of the manuscripts [ShZ1, Shi2].

2.2. Lower bounds on $R(k, l)$, higher parameters

The lower bounds marked [XXR], [XXER], 2.3.e and 2.3.h need not to be cyclic, while all other lower bounds listed in Table II were obtained by construction of cyclic graphs.

k \ l	15	16	17	18	19	20	21	22	23
3	73 WW	79 WW	92 WWY1	98 WWY1	106 WWY1	109 WWY1	122 WWY1	125 WWY1	136 WWY1
4	153 XXR		182 LSS	187 2.3.e	198 LSZL	230 SLZL	242 SLZL	282 SL	
5	261 XXER	289 2.3.h	313 2.3.h	365 2.3.h	389 2.3.h	421 2.3.h	433 2.3.h	485 2.3.h	509 2.3.h
6	401 2.3.h	434 SLLL	548 SLLL	614 SLLL	710 SLLL	878 SLLL		1070 SLLL	
7			673 2.3.h	725 2.3.h	908 SLLL		1214 SLLL		
8	861 2.3.h		925 2.3.h		1054 XXR	1094 SLLL	1617 2.3.h		

Table II. Known nontrivial lower bounds for higher two color Ramsey numbers $R(k, l)$, with references.

Exoo in [Ex15] gives the bounds $R(3, 27) \geq 158$ and $R(3, 31) \geq 198$. The constructions establishing $R(3, 26) \geq 150$, $R(3, 29) \geq 174$, $R(3, 31) \geq 198$ and $R(3, 32) \geq 212$ are presented in [SLL1], [SLL3], [LSS] and [LSZL], respectively. Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing several lower bounds for $R(3, k)$, for $k \geq 61$. Only two of these bounds, $R(3, 61) \geq 479$ and $R(3,103) \geq 955$, cannot be easily improved by the inequality $R(3, 4k+1) \geq 6R(3, k+1) - 5$ from [CCD] (2.3.c) and data from Tables I and II. Finally, for higher parameters we mention two more cases which improve on bounds listed in earlier revisions: $R(9, 17) \geq 1411$ is given in [XXR] and $R(10, 15) \geq 1265$ can be obtained by using 2.3.h.

In general, one can expect that the lower bounds in Table II are weaker than those in Table I, in the sense that with some work many of them should not be hard to improve, in contrast to the bounds in Table I, especially smaller ones.

2.3. Other results on $R(k,l)$

(a) $R(k,l) \le R(k-1,l) + R(k,l-1)$, with strict inequality when both terms on the right hand side are even [GG]. There are obvious generalizations of this inequality for avoiding graphs other than complete.

(b) $R(k,k) \le 4R(k,k-2)+2$ [Walk].

(c) Explicit construction for $R(3,4k+1) \ge 6R(3,k+1)-5$, for all $k \ge 1$ [CCD].

(d) Constructive results on triangle-free graphs in relation to the case of $R(3,k)$ [BBH1, BBH2, Fra1, Fra2, FrLo, Gri, KM1, Loc, RK3, RK4, Stat, Yu1].

(e) Bounds for the difference between consecutive Ramsey numbers, in particular the bound $R(k,l) \ge R(k,l-1) + 2k - 3$ for $k,l \ge 3$ [BEFS].

(f) By taking a disjoint union of two critical graphs one can easily see that $R(k,p) \ge s$ and $R(k,q) \ge t$ imply $R(k,p+q-1) \ge s+t-1$. Xu and Xie [XX1] improved this construction to yield better general lower bounds, in particular $R(k,p+q-1) \ge s+t+k-3$.

(g) For $2 \le p \le q$ and $3 \le k$, if (k,p)-graph G and (k,q)-graph H have a common induced subgraph on m vertices without K_{k-1}, then $R(k,p+q-1) > n(G) + n(H) + m$. In particular, this implies the bounds $R(k,p+q-1) \ge R(k,p) + R(k,q) + k - 3$ and $R(k,p+q-1) \ge R(k,p) + R(k,q) + p - 2$ [XX1, XXR].

(h) $R(2k-1,l) \ge 4R(k,l-1) - 3$ for $l \ge 5$ and $k \ge 2$, and in particular for $k = 3$ we obtain $R(5,l) \ge 4R(3,l-1) - 3$ [XXER].

(i) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(k,k) \ge p+1$ and $R(k+1,k+1) \ge 2p+3$ [She1, Mat]. Data for larger p was obtained in [LSL]. See also items 5.2.h and 5.2.i for similar multicolor results.

(j) Study of Ramsey numbers for large disjoint unions of graphs [Bu1, Bu9], in particular $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$, for n large enough [Bu8].

(k) $R(k,l) \ge L(k,l) + 1$, where $L(k,l)$ is the maximal order of any cyclic (k,l)-graph. A compilation of many best cyclic bounds was presented in [HaKr].

(l) Two-color lower bounds can be obtained by using items 5.2.k, 5.2.l and 5.2.m with $r = 2$. Some generalizations of these were obtained in [ZLLS].

In the last six items of this section we only briefly mention some pointers to the literature dealing with asymptotics of Ramsey numbers. This survey was designed mostly for small, finite, and combinatorial results, but still we wish to give the reader some useful and representative references to more traditional papers looking first of all at the infinite.

(m) In a 1995 breakthrough Kim proved that $R(3,k) = \Theta(k^2/\log k)$ [Kim].

(n) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR].

(o) Other general and asymptotic results on triangle-free graphs in relation to the case of $R(3,k)$ [AKS, Alon2, CCD, CPR, Gri, FrLo, Loc, She2].

(p) In 1947, Erdös gave an amazingly simple probabilistic proof that $R(k,k) \geq c \cdot k 2^{k/2}$ [Erd1]. Spencer [Spe1] improved the constant in the last result. More probabilistic asymptotic lower bounds for other Ramsey numbers were obtained in [Spe1, Spe2, AlPu].

(q) Other asymptotic bounds for $R(k,k)$ can be found, for example, in [Chu3, McS] (lower bound) and [Tho] (upper bound), and for many other bounds in the general case of $R(k,l)$ consult [Spe2, GRS, GrRö, Chu4, ChGra2, LRZ, AlPu, Kriv].

(r) Explicit construction of a graph with clique and independence k on $2^{c \log^2 k / \log\log k}$ vertices by Frankl and Wilson [FraWi]. Further constructions by Chung [Chu3] and Grolmusz [Grol1, Grol2]. Explicit constructions like these are usually weaker than known probabilistic results.

3. Two Colors - Dropping One Edge from Complete Graph

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3-e	3	5	7	9	11	13	15	17	19
K_3	5	7	11	17	21	25	31	37 38	42 47
K_4-e	5	10	13	17	28	29 38	34	41	
K_4	7	11	19	27 36	37 52				
K_5-e	7	13	22	31 39	40 66				
K_5	9	16	30 34	43 67	112				
K_6-e	9	17	31 39	45 70	59 135				
K_6	11	21	37 55	119	205				
K_7-e	11	28	40 66	59 135	251				
K_7	13	28 34	51 88	204					

Table III. Two types of Ramsey numbers $R(G,H)$,
includes all known nontrivial values.

The exact values in Table III involving K_3-e are trivial, since one can easily see that $R(K_3-e, K_k) = R(K_3-e, K_{k+1}-e) = 2k-1$, for all $k \geq 2$. Other bounds (not shown in Table III) can be obtained by using Table I, an obvious generalization of the inequality

$R(k,l) \le R(k-1,l) + R(k,l-1)$, and by monotonicity of Ramsey numbers, in this case $R(K_{k-1}, G) \le R(K_k - e, G) \le R(K_k, G)$. The upper bounds from the manuscripts [ShZ1, ShZ2] are subsumed by a later article [Shi2].

G \ H	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3	CH2	Clan	FRS1	GH	Ra1	Ra1	MPR MPR	WWY2 MPR
K_4-e	CH1	FRS2	McR	McR	Ea1 HZ2	Ex14	Ex14	
K_4	CH2	EHM1	Ex11 Ea1	Ex14 HZ2				
K_5-e	FRS2	CEHMS	Ex14 Ea1	Ex14 HZ2				
K_5	BH	Ex8 Ex8	Ea1 HZ2	HZ2				
K_6-e	McR	Ex14 Ea1	Ex14 HZ2	Ex14 HZ2				
K_6	McN	Ex14 Ea1	ShZ2	ShZ2				
K_7-e	McR	Ex14 HZ2	Ex14 HZ2	ShZ1				
K_7	Ea1 Ea1	Ex14 ShZ2	ShZ2					

References for Table III.

All $(K_3, K_l - e)$-graphs for $l \le 6$ have been enumerated [Ra1]. For the following numbers it was established that the critical graphs are unique: $R(K_3, K_l - e)$ for $l = 3$ [Tr], 6 and 7 [Ra1], $R(K_4 - e, K_4 - e)$ [FRS2], $R(K_5 - e, K_5 - e)$ [Ra3] and $R(K_4 - e, K_7 - e)$ [McR]. The number of $R(K_3, K_l - e)$-critical graphs for $l = 4, 5$ and 8 is 4, 2 and 9, respectively [MPR], and there are at least 6 such graphs for $R(K_3, K_9 - e)$ [Ra1]. The bound $R(K_3, K_{12} - e) \ge 46$ is given in [MPR]. Wang, Wang and Yan in [WWY2] constructed cyclic graphs showing $R(K_3, K_{13} - e) \ge 54$, $R(K_3, K_{14} - e) \ge 59$ and $R(K_3, K_{15} - e) \ge 69$.

The upper bounds in [HZ2] were obtained by a reasoning generalizing the bounds for classical numbers in [HZ1]. Several other results from section 2.3 apply, though checking in which situation they do may require looking inside the proofs whether they still hold for $K_n - e$.

4. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results (or references to them), but only those out of general bounds and other results which, in our opinion, have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 7, or be a special case of a general result listed in this section. Note that $B_1 = F_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

4.1. Paths

$R(P_n, P_m) = n + \lfloor m/2 \rfloor - 1$ for all $n \ge m \ge 2$ [GeGy]

4.2. Cycles

$R(C_3, C_3) = 6$ [GG]
$R(C_4, C_4) = 6$ [CH1]

Result obtained independently in [Ros] and [FS1], new simple proof in [KáRos]:

$$R(C_n, C_m) = \begin{cases} 2n-1 & \text{for } 3 \le m \le n,\ m \text{ odd, } (n,m) \ne (3,3) \\ n-1+m/2 & \text{for } 4 \le m \le n,\ m \text{ and } n \text{ even, } (n,m) \ne (4,4) \\ \max\{n-1+m/2, 2m-1\} & \text{for } 4 \le m < n,\ m \text{ even and } n \text{ odd} \end{cases}$$

Unions of cycles, formulas and bounds for $R(nC_p, mC_q)$ [MS, Den]
$R(nC_3, mC_3) = 3n + 2m$ for $n \ge m \ge 1$, $n \ge 2$ [BES]
$R(nC_4, mC_4) = 2n + 4m - 1$ for $m \ge n \ge 1$, $(n,m) \ne (1,1)$ [LiWa1]
Formulas for $R(nC_4, mC_5)$ [LiWa2]

4.3. Wheels

$R(W_3, W_5) = 11$ [Clan]
$R(W_3, W_n) = 2n - 1$ for all $n \ge 6$ [BE2]
All critical colorings for $R(W_3, W_n)$ for all $n \ge 3$ [RaJi]
$R(W_4, W_5) = 17$ [He3]
$R(W_5, W_5) = 15$ [HaMe2, He2]
$R(W_4, W_6) = 19$, $R(W_5, W_6) = 17$ and $R(W_6, W_6) = 17$, and all critical colorings (2, 1 and 2) for these numbers [FM]. $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$ gives a counterexample $G = W_6$ to the Erdös conjecture (see [GRS]) $R(G,G) \ge R(K_{\chi(G)}, K_{\chi(G)})$.

4.4. Books

$R(B_1, B_n) = 2n + 3$ for all $n > 1$ [RS1]
$R(B_3, B_3) = 14$ [RS1, HaMe2]
$R(B_2, B_5) = 16$, $R(B_3, B_5) = 17$, $R(B_5, B_5) = 21$,
$R(B_4, B_4) = 18$, $R(B_4, B_6) = 22$, $R(B_6, B_6) = 26$ [RS1]
$254 \le R(B_{37}, B_{88}) \le 255$ [Par6]
$R(B_n, B_m) = 2n + 3$ for all $n \ge cm$ for some c [NiRo1, NiRo2]
$R(B_n, B_n) = (4 + o(1))n$ [RS1, NiRS]

In general, $R(B_n, B_n) = 4n + 2$ for $4n + 1$ a prime power, and several other general equalities and bounds for $R(B_n, B_m)$ [RS1, FRS7, Par6, NiRS].

4.5. Complete bipartite graphs

HINT: This section gathers information on Ramsey numbers where specific bipartite graphs are avoided in a coloring of K_n (as everywhere in this survey), in contrast to often studied bipartite Ramsey numbers (not covered in this survey) where the initial coloring is of a bipartite graph $K_{n,m}$.

$R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ if both n and m are even and $\varepsilon = 0$ otherwise [Har1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

$R(nK_{1,3}, mK_{1,3}) = 4n + m - 1$ for $n \ge m \ge 1$, $n \ge 2$ [BES]

$R(K_{2,3}, K_{2,3}) = 10$ [Bu4]
$R(K_{2,3}, K_{2,4}) = 12$ [ExRe]
$R(K_{2,3}, K_{1,7}) = 13$ [Par4]
$R(K_{2,3}, K_{3,3}) = 13$ and $R(K_{3,3}, K_{3,3}) = 18$ [HaMe3]
$R(K_{2,2}, K_{2,8}) = 15$ and $R(K_{2,2}, K_{2,11}) = 18$ [HaMe4]
$R(K_{2,2}, K_{1,15}) = 20$ [La2]

$R(K_{2,n}, K_{2,n}) \le 4n - 2$ for all $n \ge 2$, exact values 6, 10, 14, 18, 21, 26, 30, 33, 38, 42, 46, 50, 54, 57 and 62 of $R(K_{2,n}, K_{2,n})$ for $2 \le n \le 16$, respectively.

The first open diagonal case is $65 \le R(K_{2,17}, K_{2,17}) \le 66$ [EHM2].

Conjecture that $4n - 3 \le R(K_{2,n}, K_{2,n}) \le 4n - 2$ for $n \ge 2$ [LorMe1].

Bounds and some values for the numbers of the form $R(K_{k,n}, K_{k,m})$ [LorMe1], and $R(K_{2,n-1}, K_{2,n})$ and $R(K_{2,n}, K_{2,n})$ [LorMe2].

The values of $R(K_{2,n}, K_{2,m})$ for all $2 \le n, m \le 10$ are gathered in [LorMe3] except 8 cases, for which lower and upper bounds are given. Several theorems giving exact formulas and bounds assuming special dependencies between n and m [LorMe3].

Asymptotics for $K_{2,m}$ versus K_n [CLRZ]

Upper bound asymptotics for $K_{k,m}$ versus K_n [LZ]
See section 4.10 for stars versus various bipartite graphs

4.6. Triangle versus other graphs

$R(3,k) = \Theta(k^2/\log k)$ [Kim]
Explicit construction for $R(3,4k+1) \geq 6R(3,k+1)-5$, for all $k \geq 1$ [CCD]
Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CPR]
$R(K_3, K_7 - 2P_2) = R(K_3, K_7 - 3P_2) = 18$ [SchSch2]
$R(K_3, K_3 + \overline{K}_m) = R(K_3, K_3 + \overline{C}_m) = 2m+5$ for $m \geq 212$ [Zhou1]

$R(K_3, G) = 2n(G) - 1$ for any connected G on at least 4 vertices and with at most $(17n(G)+1)/15$ edges, in particular for $G = P_i$ and $G = C_i$, for all $i \geq 4$ [BEFRS1]

$R(K_3, G) \leq 2e(G) + 1$ for any graph G without isolated vertices [Sid3, GK]
$R(K_3, G) \leq n(G) + e(G)$ for all G, a conjecture [Sid2]
$R(K_3, G)$ for all connected G up to 9 vertices BBH1, BBH2], see also section 7.1
$R(K_3, K_n)$, see section 2
$R(K_3, K_n - e)$, see section 3
Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates [Zeng]
Since $B_1 = F_1 = C_3 = W_3 = K_3$, other sections apply
See also [AKS, BBH1, BBH2, FrLo, Fra1, Fra2, Gri, Loc, KM1, LZ, RK3, RK4, She2, Spe2, Stat, Yu1]

4.7. Paths versus other graphs

P_3 versus special graphs G [CH2]
Paths versus stars [Par2, BEFRS2]
Paths versus trees [FS4]
Paths versus books [RS2]
Paths versus cycles [FLPS, BEFRS2]
Paths versus K_n [Par1]
Paths versus $K_{n,m}$ [Häg]
Paths versus W_5 and W_6 [SuBa1]
Paths versus W_7 and W_8 [Bas]
Paths versus wheels [BaSu, ChenZZ1]
Paths and cycles versus trees [FSS1]
Sparse graphs versus paths and cycles [BEFRS2]
Graphs with long tails [Bu2, BG]
Unions of paths [BuRo2]

4.8. Cycles versus complete graphs

	C_3	C_4	C_5	C_6	C_7	C_8	...	C_n for $n \geq m$
K_3	6 GG	7 CS	9 CS	11 FS1	13 FS1	15 FS1	...	$2n-1$ FS1
K_4	9 GG	10 CH2	13 He2/JR4	16 JR2	19 YHZ1	22 YHZ1	...	$3n-2$ YHZ1
K_5	14 GG	14 Clan	17 He2/JR4	21 JR2	25 YHZ2	29 BJYHRZ	...	$4n-3$ BJYHRZ
K_6	18 Kéry	18 Ex2/RoJa1	21 JR5	26 Schi1	31 Schi1	36 Schi1	...	$5n-4$ Schi1
K_7	23 Ka2/GY	22 RT/JR1	25 Schi2		37 conj.	43 conj.	...	$6n-5$ conj.
K_8	28 GR/MZ	26 RT				50 conj.	...	$7n-6$ conj.
K_9	36 Ka2/GR	≥ 30 RT					...	$8n-7$ conj.
K_{10}	40 - 43 Ex5/RK2	≥ 34 RT					...	$9n-8$ conj.

Table IV. Known Ramsey numbers $R(C_n, K_m)$.

- The first column in Table IV gives data from the first row in Table I.

- Joint credit [He2/JR4] in Table IV refers to two cases in which Hendry [He2] announced the values without presenting the proofs, which later were given in [JR4]. For other joint credits in Table IV, the first reference is for the lower bound and the second for the upper bound. The special cases of $R(C_6, K_5) = 21$ [JR2] and $R(C_7, K_5) = 25$ were also solved independently in [YHZ2] and [BJYHRZ].

- Since 1976, it was conjectured that $R(C_n, K_m) = (n-1)(m-1)+1$ for all $n \geq m \geq 3$, except $n = m = 3$ [FS4, EFRS2]. The parts of this conjecture were proved as follows: for $n \geq m^2 - 2$ [BoEr], for $n > 3 = m$ [FS1], for $n \geq 4 = m$ [YHZ1], for $n \geq 5 = m$ [BJYHRZ], for $n \geq 6 = m$ [Schi1], for $n \geq m \geq 7$ with $n \geq m(m-2)$ [Schi1], and for $n \geq 4m+2$, $m \geq 3$ [Nik]. Still open conjectured cases are marked in Table IV by "conj."

- General study of cycles versus K_n numbers, including asymptotics [BoEr, Spe2, FS4, EFRS2, CLRZ, Sud1, ZaLi, AlRö].

4.9. Cycles versus other graphs

C_4 versus stars [Par3, Par5, BEFRS5, Chen, ChenJ, GoMC]
C_4 versus trees [EFRS4, Bu7, BEFRS5, Chen]
C_4 versus $K_{m,n}$ [HaMe4] and $K_{2,n}$ [LorMe3]
C_4 versus all graphs on six vertices [JR3]
$R(C_4, B_n) = 7, 9, 11, 12, 13$ and 16, for $2 \le n \le 7$, respectively [FRS6]
$R(C_4, B_n) = 17, 18, 19, 20$ and 21, for $8 \le n \le 12$, respectively [Tse1]
$R(C_4, B_{13}) = 22$ and $R(C_4, B_{14}) = 24$ [Tse2]
$R(C_4, W_n) = 10, 9, 10, 9, 11, 12, 13, 14, 16$ and 17, for $4 \le n \le 13$, respectively [Tse1]
$R(C_4, G) \le 2q + 1$ for any isolate-free graph G with q edges [RoJa2]
$R(C_4, G) \le p + q - 1$ for any connected graph G on p vertices and q edges [RoJa2]

$R(C_5, W_6) = 13$ [ChvS]
$R(C_5, K_6 - e) = 17$ [JR4]
$R(C_5, B_1) = R(C_5, B_2) = 9$ [CRSPS]
$R(C_5, B_3) = 10$, and in general $R(C_5, B_n) = 2n + 3$ for $n \ge 4$ [FRS8]
C_5 versus all graphs on six vertices [JR4]
$R(C_6, K_5 - e) = 17$ [JR2]
C_6 versus all graphs on five vertices [JR2]

$R(C_n, G) \le 2q + \lfloor n/2 \rfloor - 1$, for $3 \le n \le 5$, for any isolate-free graph G with $q > 3$ edges. It is conjectured that it also holds for other n [RoJa2].

Cycles versus paths [FLPS, BEFRS2]
Cycles versus stars [La1, Clark, see Par6]
Cycles versus trees [FSS1]
Cycles versus books [FRS6, FRS8, Zhou1]
Cycles versus $K_{n,m}$ [BoEr]
Cycles versus W_5 and W_6 [SuBB2]
Cycles versus wheels [Zhou2]
See also bipartite graphs for $K_{2,2} = C_4$

4.10. Stars versus other graphs

Stars versus C_4 [Par3, Par5, Chen, ChenJ, GoMC]
Stars versus W_5 and W_6 [SuBa1]
Stars versus wheels [ChenZZ2]
Stars versus paths [Par2, BEFRS2]
Stars versus cycles [La1, Clark, see Par6]
Stars versus books [CRSPS, RS2]
Stars versus $K_{2,n}$ [Par4, GoMC]
Stars versus $K_{n,m}$ [Stev, Par3]
Stars versus bipartite graphs [Par4, Stev]
Stars versus trees [Bu1, Coc, GV, ZZ]

Stars versus stripes [CL, Lor]
Stars versus $K_n - tK_2$ [Hua1, Hua2]
Stars versus $2K_2$ [MO]
Union of two stars [Gros2]

4.11. Books versus other graphs

$R(B_3, K_4) = 14$ [He3]
$R(B_3, K_5) = 20$ [He2][BaRT]
Books versus paths [RS2]
Books versus trees [EFRS7]
Books versus stars [CRSPS, RS2]
Books versus cycles [FRS6, FRS8, Zhou1, Tse1, Tse2]
Books versus K_n [LR1, Sud2]
Books versus wheels [Zhou3]
Books versus $K_2 + C_n$ [Zhou3]
Books and $(K_1 + tree)$ versus K_n [LR1]
Generalized books $K_r + qK_1$ versus K_n [NiRo3]

4.12. Wheels versus other graphs

$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(W_5, K_5) = 27$ [He2][RST]
W_5 and W_6 versus stars and paths [SuBa1]
Wheels versus stars [ChenZZ2]
W_5 and W_6 versus trees [BSNM]
W_5 and W_6 versus cycles [SuBB2]
$R(W_6, C_5) = 13$ [ChvS]
W_7 and W_8 versus paths [Bas]
W_7 versus trees T with $\Delta(n(T)) \geq n(T) - 3$ [ChenZZ3]
Wheels versus paths [BaSu, ChenZZ1]
Odd wheels versus star-like trees [SuBB1]
Wheels versus C_4 [Tse1]
Wheels versus cycles [Zhou2]
Wheels versus books [Zhou3]
Wheels versus linear forests [SuBa2]

4.13. Trees and Forests

Trees, forests [Bu1, Bu7, CsKo, EFRS3, EG, FSS1, GeGy, GHK, GRS, GV, HaŁT]
Trees versus K_n [Chv]
Trees versus C_4 [EFRS4, Bu7, Chen]
Trees versus paths [FS4]
Trees versus paths and cycles [FSS1]
Trees versus stars [Bu1, Coc, GV, ZZ]

Trees versus books [EFRS7]
Trees versus W_5 and W_6 [BSNM]
Trees T with $\Delta(n(T)) \geq n(T) - 3$ versus W_7 [ChenZZ3]
Star-like trees versus odd wheels [SuBB1, ChenZZ3]
Trees versus $K_n + \overline{K}_m$ [RS2, FSR]
Trees versus bipartite graphs [BEFRS5, EFRS6]
Trees versus almost complete graphs [GJ2]
Trees versus small ($n(G) \leq 5$) connected G [FRS4]
Trees versus multipartite complete graphs [EFRS8, BEFRSGJ]
Linear forests, forests [BuRo2, FS3, CsKo]
Linear forests versus wheels [SuBa2]
Forests versus K_n [Stahl]
Forests versus almost complete graphs [CGP]

4.14. Mixed special cases:

$R(C_5 + e, K_5) = 17$ [He5]
$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(B_3, K_5) = 20$ [He2][BaRT]
$R(W_5, K_5) = 27$ [He2][RST]
$25 \leq R(K_5 - P_3, K_5) \leq 28$ [He2]
$26 \leq R(K_{2,2,2}, K_{2,2,2})$, $K_{2,2,2}$ is an octahedron [Ex8]

4.15. Mixed general cases

Unicyclic graphs [Gros1, Köh, KrRod]
$K_{2,m}$ and C_{2m} versus K_n [CLRZ]
$K_{2,n}$ versus any graph [RoJa2]
nK_3 versus mK_3, in particular $R(nK_3, nK_3) = 5n$ for $n \geq 2$ [BES]
nK_3 versus mK_4 [LorMu]
$R(nK_4, nK_4) = 7n + 4$ for large n [Bu8]
$2K_2$ versus K_n and general graphs G [CH2]
Variety of results on numbers $R(nG, mH)$ [Bu1]
Stripes [CL, Lor]
Union of two stars [Gros2]
Double stars* [GHK]
Graphs with bridge versus K_n [Li]
Fans $F_n = K_1 + nK_2$ versus K_m [LR2]
$R(F_1, F_n) = R(K_3, F_n) = 4n + 1$ for $n \geq 2$, and bounds for $R(F_m, F_n)$ [GGS]
Multipartite complete graphs [BEFRS3, EFRS4, FRS3, Stev]

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* double star is a union of two stars with their centers joined by an edge

Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
Disconnected graphs versus any graph [GJ1]
Graphs with long tails [Bu2, BG]
Brooms+ [EFRS3]

4.16. Other general results

[Chv] $R(K_n, T_m) = (n-1)(m-1)+1$ for any tree T on m vertices.

[CH2] $R(G,H) \geq (\chi(G)-1)(c(H)-1)+1$, where $\chi(G)$ is the chromatic number of G, and $c(H)$ is the size of the largest connected component of H.

[BE1] $R(G,G) \geq \lfloor (4n(G)-1)/3 \rfloor$ for any connected G, and $R(G,G) \geq 2n-1$ for any connected nonbipartite G.

[BE2] Graphs yielding $R(K_n, G) = (n-1)(n(G)-1)+1$ and related results (see also [EFRS5]).

[Bu2] Graphs H yielding $R(G,H) = (\chi(G)-1)(n(H)-1)+s(G)$, where $s(G)$ is a chromatic surplus of G, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H's are called G-good). This idea, initiated in [Bu2], is a basis of a number of exact results for $R(G,H)$ for large and sparse graphs H [BG, BEFRS2, BEFRS4, Bu5, FS, EFRS4, FRS3, BEFSRGJ, BF, LR4]. A survey of this area appeared in [FRS5].

[BaLS] Graph G is Ramsey saturated if $R(G+e, G+e) > R(G,G)$ for every edge e in $\overline{G}$. Several theorems on Ramsey saturated and unsaturated graphs. A conjecture that almost all graphs are Ramsey unsaturated.

[Par3] Relations between some Ramsey graphs and block designs. See also [Par4].

[Bra3] $R(G,H) > h(G,d)\,n(H)$ for all nonbipartite G and almost every d-regular H, for some h unbounded in d.

[LZ] Lower bound asymptotics of $R(G,H)$ for large dense H [LZ].

[CSRT] $R(G,G) \leq c_d n(G)$ for all G, where constant c_d depends only on the maximum degree d in G. The constant was improved in [GRR1]. Tight lower and upper bounds for bipartite G [GRR2].

[ChenS] $R(G,G) \leq c_d n$ for all d-arrangeable graphs G on n vertices, in particular with the same constant for all planar graphs. The constant c_d was improved in [Eaton]. An extension to graphs not containing a subdivision of K_d [RöTh]. Progress towards a conjecture that the same inequality holds for all d-degenerate graphs G [KoRö1, KoRö2, KoSu].

[EFRS9] Study of graphs G, called *Ramsey size linear*, for which there exists a constant c_G such that for all H with no isolates $R(G,H) \leq c_G e(H)$. An overview and

+ broom is a star with a path attached to its center further results were given in [BaSS].

[LRS] $R(G,G) < 6n$ for all n-vertex graphs G, in which no two vertices of degree at least 3 are adjacent. This improves the result $R(G,G) \le 12n$ in [Alon1].

[AlKS] Discussion of a conjecture by Erdös that there exists a constant c such that $R(G,G) \le 2^{c\sqrt{e(G)}}$. Proof for bipartite graphs G and progress towards the conjecture in other cases.

[Kriv] Lower bound on $R(G,K_n)$ depending on the density of subgraphs of G. This construction for $G = K_m$ produces a bound similar to the best known probabilistic lower bound by Spencer [Spe2].

[NiRo3] $R(K_{p+1}, B_q^r) = p(q+r-1)+1$ for generalized books $B_q^r = K_r + qK_1$, for all sufficiently large q.

[Shi1] $R(Q_n, Q_n) \le 2^{(3+\sqrt{5})n/2+o(n)}$, for the n-dimensional cube Q_n with 2^n vertices. This bound can also be derived from a theorem in [KoRö1].

[Gros1] Conjecture that $R(G,G) = 2n(G)-1$ if G is unicyclic of odd girth. Further support for the conjecture was given in [Köh, KrRod].

[RoJa2] $R(K_{2,k}, G) \le kq+1$, for $k \ge 2$, for isolate-free graphs G with $q \ge 2$ edges.

[FSS1] Discussion of the conjecture that $R(T_1, T_2) \le n(T_1)+n(T_2)-2$ holds for all trees T_1, T_2. See also [Bu1, Bu7, CsKo, EFRS3, EG, GeGy, GHK, GRS, GV].

[HaŁT] If tree T is viewed as a bipartite graph with parts t_1 and t_2, $t_2 \ge t_1$, let $b(T) = \max(2t_1+t_2-1, 2t_2-1)$. Then the bound $R(T,T) \ge b(T)$ holds always, and $R(T,T) = b(T)$ holds for many classes of trees, and asymptotically.

[FM] $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$. This gives a counterexample $G = W_6$ to the Erdös conjecture (see [GRS]) $R(G,G) \ge R(K_{\chi(G)}, K_{\chi(G)})$.

[LR3] Bounds on $R(H+\overline{K}_n, K_n)$ for general H. Also, for fixed k and m, as $n \to \infty$, $R(K_k+\overline{K}_m, K_n) \le (m+o(1))\, n^k/(\log n)^{k-1}$ [LRZ].

[Zeng] Formulas for $R(nK_3, mG)$ for all isolate-free graphs G on 4 vertices.

[BES] Study of Ramsey numbers for multiple copies of graphs.
See also [Bu1, Bu8, Bu9, LorMu].

[HaKr] Study of cyclic graphs yielding lower bounds for Ramsey numbers. Exact formulas for paths and cycles, small complete graphs and for graphs with up to five vertices.

[Bu6] Given integer m and graphs G and H, determining whether $R(G,H) \le m$ holds is NP-hard.

[-] Special cases of multicolor results listed in section 5.

[-] See also surveys listed in section 7.

5. Multicolor Graph Numbers

The only known value of a multicolor classical Ramsey number:

$R_3(3) = R(3,3,3) = R(3,3,3\,;2) = 17$	[GG]
2 critical colorings (on 16 vertices)	[KaSt, LayMa]
2 colorings on 15 vertices	[Hein]
115 colorings on 14 vertices	[PR1]

General upper bound, implicit in [GG]:

$$R(k_1, \dots, k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1, \dots, k_{i-1}, k_i - 1, k_{i+1}, \dots, k_r) \qquad \text{(a)}$$

Inequality in (a) is strict if the right hand side is even, and at least one of the terms in the summation is even. It is suspected that this upper bound is never tight for $r \ge 3$ and $k_i \ge 3$, except for $r = k_1 = k_2 = k_3 = 3$. However, only two cases are known to improve over (a), namely $R_4(3) \le 62$ [FKR] and $R(3,3,4) \le 31$ [PR1, PR2], for which (a) produces only the bounds of 66 and 34, respectively.

5.1. Bounds for multicolor classical numbers

Diagonal Cases

r \ m	3	4	5	6	7	8	9
3	17 GG	128 HiIr	415 XXER	1070 Mat	3214 Xu	5384 XX2	13761 XXER
4	51 Chu1	634 XXER	3049 Xu	15202 XXER	62017 XXER		
5	162 Ex10	3416 XXER	26912 Xu				
6	538 FreSw						
7	1682 FreSw						

Table V. Known nontrivial lower bounds for diagonal multicolor Ramsey numbers $R_r(m)$, with references.

The best published bounds corresponding to the entries in Table V marked by personal communication [Xu] are: $3211 \le R_3(7)$ [Mat], $2721 \le R_4(5)$ [XXER] and $26082 \le R_5(5)$ [XXER].

The most studied and intriguing open case is

$$\text{[Chu1]} \quad 51 \le R_4(3) = R(3,3,3,3) \le 62 \quad \text{[FKR]}$$

The inequality 5.a implies $R_4(3) \le 66$, Folkman [Fo] in 1974 improved this bound to 65, and Sánchez-Flores [San] in 1995 proved $R_4(3) \le 64$. The upper bounds in $162 \le R_5(3) \le 307$, $538 \le R_6(3) \le 1838$, $1682 \le R_7(3) \le 12861$, and $128 \le R(4,4,4) \le 236$ are implied by 5.(a) (we repeat lower bounds from Table V just to see easily the ranges).

Off-Diagonal Cases

Three colors:

k \ m	4	5	6	7	8	9	10	11	12	13	14
3	30 Ka2	45 Ex2	60 Rob3	79 Ex16	98 ZSL	110 SLZL	141 5.2.c	157 5.2.c	181 5.2.c	205 5.2.c	233 5.2.c
4	55 KLR	80 Ex12	99 5.2.g								
5	80 Ex12	123 5.2.g									

Table VI. Known nontrivial lower bounds for 3-color
Ramsey numbers of the form $R(3, k, m)$, with references.

In addition, the bounds $303 \le R(3,6,6)$, $609 \le R(3,7,7)$ and $1689 \le R(3,9,9)$ were derived in [XXER] (used there for building other lower bounds for some diagonal cases).

The other most studied, and perhaps the only open case of a classical multicolor Ramsey number, for which we can anticipate exact evaluation in the not-too-distance future is

$$\text{[Ka2]} \quad 30 \le R(3,3,4) \le 31 \quad \text{[PR1, PR2]}$$

In [PR1] it is conjectured that $R(3,3,4) = 30$, and the results in [PR2] eliminate some cases which could give $R(3,3,4) = 31$. The upper bounds in $45 \le R(3,3,5) \le 57$, $55 \le R(3,4,4) \le 79$, and $80 \le R(3,4,5) \le 160$ are implied by 5.(a) (we repeat lower bounds from the Table VI to show explicitly the current ranges).

Four colors:

$93 \le R(3,3,3,4) \le 153$	[Ex16, XXER], 5.(a)
$162 \le R(3,3,3,5)$	[XXER]
$171 \le R(3,3,4,4)$	[Ex16, XXER]
$561 \le R(3,3,3,11)$	[XX2, XXER]

Lower bounds for higher numbers can be obtained by using general constructive results from section 5.2 below. For example, the bounds $193 \le R(3,4,8)$, $261 \le R(3,3,15)$ and $241 \le R(3,3,3,7)$ were not published explicitly but are implied by 5.2.(c), 5.2.(c) and 5.2.(d), respectively.

5.2. General multicolor results for complete graphs

(b) $R_r(3) \ge 3R_{r-1}(3) + R_{r-3}(3) - 3$ [Chu1]

(c) $R(3,k,l) \ge 4R(k,l-1) - 3$, and in general for $r \ge 2$ and $k_i \ge 2$
$R(3,k_1,\ldots,k_r) \ge 4R(k_1-1,k_2,\ldots,k_r) - 3$ for $k_1 \ge 5$, and
$R(k_1,2k_2-1,k_3,\ldots,k_r) \ge 4R(k_1-1,k_2,\ldots,k_r) - 3$ for $k_1 \ge 5$ [XX2, XXER]

(d) $R(3,3,3,k_1,\ldots,k_r) \ge 3R(3,3,k_1,\ldots,k_r) + R(k_1,\ldots,k_r) - 3$ [Rob2]

(e) Bounds for $R_k(3)$ [AbbH, Fre, Chu2, ChGri, GrRö, Wan]

(f) $R(k_1,\ldots,k_r) \ge S(k_1,\ldots,k_r) + 2$, where $S(k_1,\ldots,k_r)$ is the generalized Schur number [AbbH, Gi1, Gi2]. In particular, the special case $k_1 = \ldots = k_r = 3$ has been widely studied [Fre, FreSw, Ex10, Rob3].

(g) $R(k_1,\ldots,k_r) \ge L(k_1,\ldots,k_r) + 1$, where $L(k_1,\ldots,k_r)$ is the maximal order of any cyclic $(k_1,\ldots,k_r)$-coloring, which can be considered a special case of Schur partitions defining (symmetric) Schur numbers. Many lower bounds for Ramsey numbers were established by cyclic colorings. The following recurrence can be used to derive lower bounds for higher parameters. For $k_i \ge 3$

$L(k_1,\ldots,k_r,k_{r+1}) \ge (2k_{r+1}-3)L(k_1,\ldots,k_r) - k_{r+1} + 2$ [Gi2]

(h) $R_r(m) \ge p+1$ and $R_r(m+1) \ge r(p+1)+1$ if there exists a K_m-free cyclotomic $r-$class association scheme of order p [Mat].

(i) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(s,k+1,k+1) \ge 4ps - 6p + 3$ [XXER].

(j) $R_r(m) \ge c_m(2m-3)^r$, and some slight improvements of this bound for small values of m [AbbH, Gi1, Gi2, Song2].

(k) $R_r(pq+1) > (R_r(p+1)-1)(R_r(q+1)-1)$ [Abb1]

(l) $R_r(pq+1) > R_r(p+1)(R_r(q+1)-1)$ for $p \ge q$ [XXER]

(m) $R(p_1q_1+1,\ldots,p_rq_r+1) > (R(p_1+1,\ldots,p_r+1)-1)(R(q_1+1,\ldots,q_r+1)-1)$ [Song3]

(n) $R_{r+s}(m) > (R_r(m)-1)(R_s(m)-1)$ [Song2]

(o) $R(k_1,k_2,\ldots,k_r) > (R(k_1,\ldots,k_i) - 1)(R(k_{i+1},\ldots,k_r) - 1)$ in [Song1], see [XXER].

(p) $R(k_1,k_2,\ldots,k_r) > (k_1 + 1)(R(k_2 - k_1 + 1,k_3,\ldots,k_r) - 1)$ [Rob4]

(q) Further lower bound constructions, though with more complicated assumptions, were presented in [XX2, XXER].

(r) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (section 2.3.r) to more colors and to hypergraphs [Grol3] (section 6).

All lower bounds in (b) through (r) above are constructive. (d) generalizes (b), (m) generalizes both (k) and (o), and (o) generalizes (n). (l) is stronger than (k). Finally observe that the construction (m) with $q_1 = ... = q_i = 1 = p_{i+1} = ... = p_r$ is the same as (o).

5.3. Special multicolor cases

$R_3(C_4) = 11$	[BS, see also Clap]
$R_3(C_5) = 17$	[YR1]
$R_3(C_6) = 12$	[YR2]
$R_3(C_7) = 25$	[FSS2]
$18 \le R_4(C_4) \le 19$	[Ex2] [Eng]
$27 \le R_5(C_4) \le 29$	[LaWo1]
$R(C_4, C_4, K_3) = 12$	[Schu]
$R(C_4, K_3, K_3) = 17$	[ExRe]
$13 \le R(C_3, C_4, C_5)$	[Rao]
$R(K_{1,3}, C_4, K_4) = 16$	[KM2]
$R(P_4, P_4, C_3) = 9$	[AKM]
$R(P_4, P_4, C_4) = 7$	[AKM]
$R(P_4, P_4, C_5) = 9$	[DzKu]
$R(K_4 - e, K_4 - e, P_3) = 11$	[Ex7]
$28 \le R_3(K_4 - e) \le 30$	[Ex7] [Piw2]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ExRe]
$27 \le R(K_3, K_3, C_4, C_4)$	[Eng]
$86 \le R(K_4, K_4, C_4, C_4)$	[Bev], 5.2.(o)+

All colorings for $(K_4 - e, K_4 - e, P_3)$ were found in [Piw2].

5.4. General multicolor results for cycles and paths

- $R(C_n, C_n, C_n) \le (4 + o(1))n$, with equality for odd n [Łuc]. It was conjectured by Bondy and Erdös, see [Erd2], that $R(C_n, C_n, C_n) \le 4n - 3$ for $n \ge 4$. If true, then for all odd $n \ge 5$ we have $R(C_n, C_n, C_n) = 4n - 3$.
- Formulas for $R(C_n, C_m, C_k)$ and $R(C_n, C_m, C_k, C_l)$ for n sufficiently large [EFRS1].

- $R_k(C_4) \le k^2 + k + 1$ for all $k \ge 1$, $R_k(C_4) \ge k^2 - k + 2$ for all $k - 1$ which is a prime power [Ir, Chu2, ChGra1], and $R_k(C_4) \ge k^2 + 2$ for odd prime power k [LaWo1]. The latter was extended to any prime power k in [Ling, LaMu].
- Bounds for $R_k(C_n)$ [Bu1, GRS].
- $R(P_3, C_n, C_n) = 2n - 1$ $(= R(C_n, C_n))$ for odd $n \ge 5$ [DzKu].
- $R(P_4, P_4, C_n) = n + 2$ for $n \ge 6$, and $R(P_3, P_5, C_n) = n + 1$ for $n \ge 8$ [DzKu].
- Formulas for $R_k(P_3)$ for all k, and for $R_k(P_4)$ if k is not divisible by 3 [Ir]. Wallis [Wall] showed $R_6(P_4) = 13$, which already implied $R_{3t}(P_4) = 6t + 1$, for all $t \ge 2$. Independently, the case $R_k(P_4)$ for $k \ne 3^m$ was completed by Lindström in [Lind], and later Bierbrauer proved $R_{3^m}(P_4) = 2 \cdot 3^m + 1$ for all $m \ge 1$.
- Monotone paths and cycles [Lef].
- Formulas for $R(P_{n_1}, \dots, P_{n_k})$, except few cases [FS2].
- Formulas for $R(n_1 P_2, \dots, n_k P_2)$ [CL1].
- Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].
- See also sections 5.3 and 7.2, especially [AKM] for a number of small cases in three colors similar to those listed in section 5.3.
- Study of asymptotics for $R(C_m, \dots, C_m, K_n)$ [AlRö].
- Study of asymptotics for $R(C_{2m}, C_{2m}, K_n)$ for fixed m [ShiuLL, AlRö].

5.5. Other general multicolor results

- General bounds for $R_k(G)$ [CH3, Par6].
- Formulas for $R_k(G)$ for G being one of P_3, $2K_2$ and $K_{1,3}$ for all k, and for P_4 if k is not divisible by 3 [Ir].
- Bounds on $R_k(K_{s,t})$, in particular for $K_{2,2} = C_4$ and $K_{2,t}$ [ChGra1, AFM].
- $tk^2 + 1 \le R_k(K_{2,t+1}) \le tk^2 + k + 2$, where the upper bound is general, and the lower bound holds when both t and k are prime powers [ChGra1, LaMu].
- Bounds on $R_k(G)$ for unicyclic graphs G of odd girth. Some exact values for special graphs G, for $k = 3$ and $k = 4$ [KrRod].
- Formulas for $R(S_1, \dots, S_k)$, where S_i's are arbitrary stars [BuRo1].
- Formulas for $R(S_1, \dots, S_k, K_n)$, where S_i's are arbitrary stars [Jac].
- Formulas for $R(S_1, \dots, S_k, nP_2)$, where S_i's are arbitrary stars [CL2].
- Formulas for $R(S_1, \dots, S_k, T)$, where S_i's are stars and T is a tree [ZZ].
- Study of $R(G_1, \dots, G_k, G)$ for large sparse G [EFRS1, Bu3].
- Study of asymptotics for $R(C_n, \dots, C_n, K_m)$ [AlRö].
- Cockayne and Lorimer [CL1] found the exact formula for $R(n_1 P_2, \dots, n_k P_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1 P_2, \dots, n_k P_2)$.

Still more general cases of the latter, with multiple copies of the complete graph and forests, were studied in [Stahl, LorSe, LorSo].

- If G is connected and $R(K_k, G) = (k-1)(n(G)-1)+1$, in particular if G is any tree, then $R(K_{k_1}, \dots, K_{k_r}, G) = (R(k_1, \dots, k_r)-1)(n(G)-1)+1$ [BE2]. A generalization for connected $G_1, \dots, G_n$ in place of G appeared in [Jac].
- If F, G, H are connected graphs then $R(F, G, H) \geq (R(F, G)-1)(\chi(H)-1) + \min\{R(F, G), s(H)\}$, where $s(G)$ is the chromatic surplus of G (see item [Bu2] in section 4.16). This leads to several formulas and bounds for F and G being stars and/or trees when $H = K_n$ [ShiuLL].
- $R(K_{k_1}, \dots, K_{k_r}, G_1, \dots, G_s) \geq (R(k_1, \dots, k_r)-1)(R(G_1, \dots, G_s)-1)$ for arbitrary graphs $G_1, \dots, G_s$ [Bev]. This generalizes 5.2.(o).
- Constructive bound $R(G_1, ..., G_{t^{n-1}}) \geq t^n + 1$ for some families of decompositions of K_{t^n} [LaWo1, LaWo2].
- Bounds for trees $R_k(T)$ and forests $R_k(F)$ [EG, GRS, BB, GT, Bra1, Bra2, SwPr].
- Bounds on $R_k(G)$ for trees, forests, stars and cycles [Bu1].
- See also surveys listed in section 7.

6. Hypergraph Numbers

The only known value of a classical Ramsey number for hypergraphs:

$R(4,4;3) = 13$	[MR1]
more than 200000 critical colorings	

Other hypergraph cases:

$33 \leq R(4, 5; 3)$	[Ex13]
$63 \leq R(5, 5; 3)$	[Ea1]
$56 \leq R(4,4,4; 3)$	[Ex8]
$34 \leq R(5, 5; 4)$	[Ex11]
$R(K_4 - t, K_4 - t; 3) = 7$	[Ea2]
$R(K_4 - t, K_4; 3) = 8$	[Sob, Ex1, MR1]
$14 \leq R(K_4 - t, K_5; 3)$	[Ex1]
$13 \leq R(K_4 - t, K_4 - t, K_4 - t; 3) \leq 17$	[Ex1] [Ea1]

The computer evaluation of $R(4,4;3)$ in [MR1] consisted of an improvement of the upper bound from 15 to 13, which followed an extensive theoretical study of this number in

[Gi4, Is1, Sid1]. Exoo in [Ex1] announced the bounds $R(4,5;3) \geq 30$ and $R(5,5;4) \geq 27$ without presenting the constructions. The bound of $R(4,5;3) \geq 24$ was obtained by Isbell [Is2]. Shastri in [Sha] shows a weak bound $R(5,5;4) \geq 19$ (now 34 in [Ex11]), nevertheless his lemmas and those in [Ka3, Abb2, GRS, HuSo] can be used to derive other lower bounds for higher numbers.

General hypergraph results:

- Several lower bound constructions for 3-uniform hypergraphs were presented in [HuSo]. Study of lower bounds on $R(p,q;4)$ can be found in [Song3] and [SYL, Song4] (the latter two papers are almost the same in contents). Most lower bounds in these papers can be easily improved by using the same techniques, but starting with better constructions for small parameters listed above.
- Let $H^{(r)}(s,t)$ be the complete r-partite r-uniform hypergraph with $r-2$ parts of size 1, one part of size s, and one part of size t (for example, for $r=2$ it is the same as $K_{s,t}$). For the multicolor numbers, Lazebnik and Mubayi [LaMu] proved that

$$tk^2 - k + 1 \leq R_k(H^{(r)}(2,t+1)) \leq tk^2 + k + r,$$

  where the lower bound holds when both t and k are prime powers. For the general case of $H^{(r)}(s,t)$, more bounds are presented in [LaMu].
- Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (section 2.3.r) to more colors and to hypergraphs [Grol3].
- Lower bounds on $R_m(k;s)$ are discussed in [DLR, AbbW]. In [AbbS], it is shown that for some values of a, b the numbers $R(m,a,b;3)$ are at least exponential in m.
- General lower bounds for large number of colors were given in an early paper by Hirschfeld [Hir], and some of them were later improved in [AbbL].
- Other theoretical results on hypergraph numbers are gathered in [GrRö, GRS].

7. Cumulative Data and Surveys

7.1. Cumulative data for two colors

[CH1] $R(G,G)$ for all graphs G without isolates on at most 4 vertices.

[CH2] $R(G,H)$ for all graphs G and H without isolates on at most 4 vertices.

[Clan] $R(G,H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries (now all solved).

[He4] All critical colorings for $R(G,H)$, for isolate-free graphs G and H as in [Clan] above.

[Bu4] $R(G,G)$ for all graphs G without isolates and with at most 6 edges.

[He1] $R(G,G)$ for all graphs G without isolates and with at most 7 edges.

[HaMe2] $R(G,G)$ for all graphs G on 5 vertices and with 7 or 8 edges.

[He2] $R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries (3 still open, see the paragraph at the end of this section).

[HoMe] $R(G,H)$ for $G=K_{1,3}+e$ and $G=K_4-e$ versus all connected graphs H on 6 vertices, except $R(K_4-e,K_6)$. The result $R(K_4-e,K_6)=21$ was claimed by McNamara [McN, unpublished].

[FRS4] $R(G,T)$ for all connected graphs G on at most 5 vertices and all (except some cases) trees T.

[FRS1] $R(K_3,G)$ for all connected graphs G on 6 vertices.

[Jin] $R(K_3,G)$ for all connected graphs G on 7 vertices. Some errors in [Jin] were found by [SchSch1].

[Brin] $R(K_3,G)$ for all connected graphs G on at most 8 vertices. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KM1].

[BBH1] $R(K_3,G)$ for all connected graphs G on 9 vertices. See also [BBH2].

[JR3] $R(C_4,G)$ for all graphs G on at most 6 vertices.

[JR4] $R(C_5,G)$ for all graphs G on at most 6 vertices.

[JR2] $R(C_6,G)$ for all graphs G on at most 5 vertices.

[LorMe3] $R(K_{2,n},K_{2,m})$ for all $2 \le n,m \le 10$ except 8 cases, for which lower and upper bounds are given.

[HaKr] All best lower bounds up to 102 from cyclic graphs. Formulas for best cyclic lower bounds for paths and cycles, small complete graphs and for graphs with up to five vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved. Out of 7 open cases in [He2] 4 have been solved, namely $R(4,5)=R(G_{19},G_{23})=25$ and the items 2, 3 and 4 in section 4.14. The still open 3 cases are for K_5 versus the graphs K_5 (section 2.1), K_5-e (section 3), and K_5-P_3 (section 4.14).

7.2. Cumulative data for three colors

[YR3] $R_3(G)$ for all graphs G with at most 4 edges and no isolates.

[YR1] $R_3(G)$ for all graphs G with 5 edges and no isolates, except K_4-e.
The case of $R_3(K_4-e)$ remains open (see section 5.3).

[YY] $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases.

[AKM] $R(F,G,H)$ for most triples of isolate-free graphs with at most 4 vertices. Some of the missing cases completed in [KM2].

7.3. Surveys

- [Bu1] A general survey of results in Ramsey graph theory by S. A. Burr (1974)
- [Par6] A general survey of results in Ramsey graph theory by T. D. Parsons (1978)
- [Har2] Summary of progress by Frank Harary (1981)
- [ChGri] A general survey of bounds and values by F. R. K. Chung and C. M. Grinstead (1983)
- [JGT] Special volume of the *Journal of Graph Theory* (1983)
- [Rob1] A review of Ramsey graph theory for newcomers by F. S. Roberts (1984)
- [Bu7] What can we hope to accomplish in generalized Ramsey Theory ? (1987)
- [GrRö] Survey of asymptotic problems by R. L. Graham and V. Rödl (1987)
- [GRS] An excellent book by R. L. Graham, B. L. Rothschild and J. H. Spencer, second edition (1990)
- [FRS5] Survey by Faudree, Rousseau and Schelp of graph goodness results, i.e. conditions for the formula $R(G,H) = (\chi(G)-1)(n(H)-1)+s(G)$ (1991)
- [Neš] A chapter in *Handbook of Combinatorics* by J. Nešetřil (1996)
- [Caro] Survey of zero-sum Ramsey theory by Y. Caro (1996)
- [Chu4] Among 114 open problems and conjectures of Paul Erdös, presented and commented by F. R. K. Chung, 31 are concerned directly with Ramsey numbers. 216 references are given (1997). An extended version of this work was prepared jointly with R. L. Graham [ChGra2]. (1998)
- [CoPC] Special issue of *Combinatorics, Probability and Computing* (2003)

The surveys by S. A. Burr [Bu1] and T. D. Parsons [Par6] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Har2], where he also gathered many references including seven to other early surveys of this area. More than two decades ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this work, but included a broad discussion of different methods used in Ramsey computations in the classical case. S. A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu7] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS5, Neš, Chu4, ChGra2], though these focus on asymptotic theory not on the numbers themselves. Finally, this compilation could not pretend to be complete without mentioning special volumes of the *Journal of Graph Theory* [JGT, 1983] and *Combinatorics, Probability and Computing* [CoPC, 2003], dedicated entirely to Ramsey theory. Besides a number of research papers, they include historical notes and present to us Frank P. Ramsey (1903-1930) as a person.

8. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, induced Ramsey numbers, local Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested reader can find such information in the surveys listed in section 7 here.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections or additions will be greatly appreciated and considered for inclusion in the next revision of this survey.

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References

We mark the papers containing results obtained with the help of computer algorithms with stars. We identify two categories of such papers: marked with * involving some use of computers, where the results are easily verifiable with some computations, and those marked with **, where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special classes of graphs.

The references are ordered alphabetically by the last name of the first author, for the same first author by the last name of the second author, etc. We preferred that all work by the same author be in consecutive positions. Unfortunately, this causes that some of the abbreviations are not in alphabetical order, for example [BaRT] is earlier on the list than [BaLS].

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