A Proof of Primality of 15 Integers

by Rocke Verser

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This report contains a proof of the primality of the 15 integers:

$1,632,373,745,527,558,118,201 = 1,632,373,745,527,558,118,190 + 11$
$1,632,373,745,527,558,118,203 = 1,632,373,745,527,558,118,190 + 13$
$1,632,373,745,527,558,118,207 = 1,632,373,745,527,558,118,190 + 17$
$1,632,373,745,527,558,118,209 = 1,632,373,745,527,558,118,190 + 19$
$1,632,373,745,527,558,118,213 = 1,632,373,745,527,558,118,190 + 23$
$1,632,373,745,527,558,118,219 = 1,632,373,745,527,558,118,190 + 29$
$1,632,373,745,527,558,118,221 = 1,632,373,745,527,558,118,190 + 31$
$1,632,373,745,527,558,118,227 = 1,632,373,745,527,558,118,190 + 37$
$1,632,373,745,527,558,118,231 = 1,632,373,745,527,558,118,190 + 41$
$1,632,373,745,527,558,118,233 = 1,632,373,745,527,558,118,190 + 43$
$1,632,373,745,527,558,118,237 = 1,632,373,745,527,558,118,190 + 47$
$1,632,373,745,527,558,118,243 = 1,632,373,745,527,558,118,190 + 53$
$1,632,373,745,527,558,118,249 = 1,632,373,745,527,558,118,190 + 59$
$1,632,373,745,527,558,118,251 = 1,632,373,745,527,558,118,190 + 61$
$1,632,373,745,527,558,118,257 = 1,632,373,745,527,558,118,190 + 67$

The importance of these integers is described by K. Conrow and J. J. Devore [see *Prime n-tet* $(8 \leq n \leq 15)$ *Homologs of* $\{11, 13, \ldots, 67\}$*; Discovery of a Prime Pentadecet Homolog of* $\{11, 13, \ldots, 67\}$, to appear].

The method used herein is described more completely by D. E. Knuth [see *The Art of Computer Programming, 2nd ed.*, **v. 2, Seminumerical algorithms**, 374–378, 395].

Fermat's Theorem states that $x^{p-1} \bmod p = 1$ whenever p is prime and $x < p$.

Using Fermat's Theorem, the cited reference proves the following: *If, for each prime divisor, p of $n-1$, there is a number x_p such that $x_p^{(n-1)/p} \bmod n \neq 1$ and $x_p^{n-1} \bmod n = 1$, then n is prime.*

We have constructed a table containing primes from 2 to 3571.

Any number that is below 3571^2 (12752041) can be factored (or tested for primality) by performing a trial division by each of these small primes.

In the following proofs, purported primes less than 12752041 have been verified as prime by trial division. Purported primes greater than 12752041 have been verified as prime by recursive application of this proof.

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Proof of primality of $n = 16323737455275558118201$:

x	p	$x^{(n-1)/p}$ mod n	x^{n-1} mod n
3	2	16323737455275558118200	1
2	5	38049387584683440686	1
2	61	1560405861445376980642	1
2	2065799	54855588127647024932	1
2	64769673469	1443938588232124372222	1

$n - 1 = 16323737455275558118200 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 61 \cdot 2065799 \cdot 64769673469$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 64769673469$:

x	p	$x^{(n-1)/p}$ mod n	x^{n-1} mod n
2	2	64769673468	1
5	3	48133253321	1
2	43	59889985682	1
2	125522623	10056936445	1

$n - 1 = 64769673468 = 2 \cdot 2 \cdot 3 \cdot 43 \cdot 125522623$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 125522623$:

x	p	$x^{(n-1)/p}$ mod n	x^{n-1} mod n
3	2	125522622	1
2	3	106751650	1
2	258277	99913324	1

$n - 1 = 125522622 = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 258277$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 16323737455275558118203$:

x	p	$x^{(n-1)/p}$ mod n	x^{n-1} mod n
2	2	16323737455275558118202	1
2	3	1157629553861929182351	1
2	13	82153379897185456852	1
2	43	346983830548255800110	1
2	486694617032664913	292795922756066183111	1

$n - 1 = 16323737455275558118202 = 2 \cdot 3 \cdot 13 \cdot 43 \cdot 486694617032664913$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 486694617032664913$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
5	2	486694617032664912	1
2	3	23886473473792386	1
2	23	288032345099138730	1
2	290993	164002969369680294	1
2	1514973121	93049660354601166	1

$n - 1 = 486694617032664912 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 23 \cdot 290993 \cdot 1514973121$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 1514973121$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
11	2	1514973120	1
3	3	331238927	1
2	5	1488112402	1
2	541	1485231067	1
2	2917	1362794393	1

$n - 1 = 1514973120 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 541 \cdot 2917$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118207$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
5	2	1632373745527558118206	1
2	11	925228422648597060066	1
2	23	487283231056135834790	1
2	32260350702125654 51	883279733803064824671	1

$n - 1 = 1632373745527558118206 = 2 \cdot 11 \cdot 23 \cdot 3226035070212565451$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 3226035070212565451$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	3226035070212565450	1
2	5	103856204157404837	1
2	7	1191063932455672780	1
2	463	1332109714264222180	1
2	2835589	1722258521379017821	1
2	7020641	569464593403725216	1

$n - 1 = 3226035070212565450 = 2 \cdot 5 \cdot 5 \cdot 7 \cdot 463 \cdot 2835589 \cdot 7020641$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 16323737455275581182 09$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
7	2	163237374552755811 8208	1
2	3	82583476922494930188	1
2	83	100353847866491694 9349	1
2	7951	61714340896122347 7331	1
2	12883045083803	26863983029213222 3035	1

$n - 1 = 163237374552755811 8208 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 83 \cdot 7951 \cdot 12883045083803$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 12883045083803$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	12883045083802	1
2	160403	10742367613119	1
2	40158367	8588525519614	1

$n - 1 = 12883045083802 = 2 \cdot 160403 \cdot 40158367$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 40158367$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
3	2	40158366	1
2	3	10311324	1
2	6693061	64	1

$n - 1 = 40158366 = 2 \cdot 3 \cdot 6693061$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 163237374552755811 8213$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	163237374552755811 8212	1
2	729941	115661294932421196 1181	1
2	1061897	182307342453721451 809	1
2	526489189	765831002680706634 644	1

$n - 1 = 163237374552755811 8212 = 2 \cdot 2 \cdot 729941 \cdot 1061897 \cdot 526489189$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 526489189$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	526489188	1
2	3	295346624	1
2	139	496763582	1
2	439	315089220	1
2	719	456974216	1

$n - 1 = 526489188 = 2 \cdot 2 \cdot 3 \cdot 139 \cdot 439 \cdot 719$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 163237374552755811 8219$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1632373745527558118218	1
2	7	481618587760701934 29	1
2	47	878992965292950284888	1
2	118747	407360900410909762139	1
2	2984509880449	603893278006707065082	1

$n - 1 = 1632373745527558118218 = 2 \cdot 7 \cdot 7 \cdot 47 \cdot 118747 \cdot 2984509880449$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 2984509880449$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
7	2	2984509880448	1
2	3	1898793133444	1
2	19	1094612102848	1
2	1709	1273904395911	1
2	239357	944920234445	1

$n - 1 = 2984509880448 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 19 \cdot 1709 \cdot 239357$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118221$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1632373745527558118220	1
2	3	1014343704629074655278	1
2	5	660872507529762113248	1
2	187347971	1131876146326739168612	1
2	145217634047	554242107163670614467	1

$n - 1 = 1632373745527558118220 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 187347971 \cdot 145217634047$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 187347971$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	187347970	1
3	5	19886980	1
2	2393	123094640	1
2	7829	136239209	1

$n - 1 = 187347970 = 2 \cdot 5 \cdot 2393 \cdot 7829$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 145217634047$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
5	2	145217634046	1
2	19559	35980410639	1
2	3712297	142818369174	1

$n - 1 = 145217634046 = 2 \cdot 19559 \cdot 3712297$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118227$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1632373745527558118226	1
2	3	1524612703268808677970	1
2	79	434912840839564159639	1
2	3041	663032936404719040375	1
2	1132465132310989	1492221602751993569939	1

$n - 1 = 1632373745527558118226 = 2 \cdot 3 \cdot 79 \cdot 3041 \cdot 1132465132310989$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 1132465132310989$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1132465132310988	1
3	3	1069491229797005	1
3	7	95176829014440	1
2	739	19589970924927	1
2	14747	355594500358081	1
2	1237079	1050167094344911	1

$n - 1 = 1132465132310988 = 2 \cdot 2 \cdot 3 \cdot 7 \cdot 739 \cdot 14747 \cdot 1237079$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 16323737455275581182\,31$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
11	2	1632373745527558118230	1
2	5	868250725370239225858	1
2	577	399899819355563423295	1
2	215197	1613474830011936555112	1
2	1314642219067	1591872545687937024879	1

$n - 1 = 1632373745527558118230 = 2 \cdot 5 \cdot 577 \cdot 215197 \cdot 1314642219067$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 1314642219067$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1314642219066	1
2	3	486855802013	1
2	11	970980734281	1
2	43	898823565653	1
2	51469823	593971176810	1

$n - 1 = 1314642219066 = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 11 \cdot 43 \cdot 51469823$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 51469823$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
5	2	51469822	1
2	19	14302640	1
2	349	48388863	1
2	3881	29427255	1

$n - 1 = 51469822 = 2 \cdot 19 \cdot 349 \cdot 3881$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118233$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
5	2	1632373745527558118232	1
2	3	399871936603518688126	1
2	7	713555181665635704853	1
2	6719	75239390124705133465	1
2	4679809	486284308970851644030	1
2	11444947	1319755810736794318925	1

$n - 1 = 1632373745527558118232 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 6719 \cdot 4679809 \cdot 11444947$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118237$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1632373745527558118236	1
2	3769	1392156352980113568946	1
2	108276316365584911	76207261487123901290	1

$n - 1 = 1632373745527558118236 = 2 \cdot 2 \cdot 3769 \cdot 108276316365584911$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 108276316365584911$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
3	2	108276316365584910	1
3	3	36279535992922884	1
2	5	93024865030239805	1
2	839	81643202524514297	1
2	4301800411823	55726973768490162	1

$n - 1 = 108276316365584910 = 2 \cdot 3 \cdot 5 \cdot 839 \cdot 4301800411823$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 4301800411823$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
5	2	4301800411822	1
2	23	3044834055321	1
2	137	1695193564170	1
2	7417	4226007422913	1
2	92033	392278402315	1

$n - 1 = 4301800411822 = 2 \cdot 23 \cdot 137 \cdot 7417 \cdot 92033$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118243$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1632373745527558118242	1
2	19	1113707223532399582735	1
2	443	4667300190448562162	1
2	2454791	1354741322806388454080	1
2	39501879143	304878051553867872766	1

$n - 1 = 1632373745527558118242 = 2 \cdot 19 \cdot 443 \cdot 2454791 \cdot 39501879143$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 39501879143$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
5	2	39501879142	1
2	11	12122340322	1
2	1051	16715895016	1
2	1708411	13442451670	1

$n - 1 = 39501879142 = 2 \cdot 11 \cdot 1051 \cdot 1708411$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118249$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
3	2	1632373745527558118248	1
2	607432409	903130515720704536021	1
2	335916745909	1447540358367062977204	1

$n - 1 = 1632373745527558118248 = 2 \cdot 2 \cdot 2 \cdot 607432409 \cdot 335916745909$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 607432409$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
3	2	607432408	1
2	11	40035690	1
2	271	285077621	1
2	25471	260921304	1

$n - 1 = 607432408 = 2 \cdot 2 \cdot 2 \cdot 11 \cdot 271 \cdot 25471$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 335916745909$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	335916745908	1
3	3	82281131376	1
2	19	310994247247	1
2	1473319061	166100530504	1

$n - 1 = 335916745908 = 2 \cdot 2 \cdot 3 \cdot 19 \cdot 1473319061$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 1473319061$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1473319060	1
2	5	67071836	1
2	8539	213148514	1
2	8627	549661506	1

$n - 1 = 1473319060 = 2 \cdot 2 \cdot 5 \cdot 8539 \cdot 8627$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118251$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	1632373745527558118250	1
2	3	866896303503275516156	1
2	5	651383157099524530377	1
2	11	310065666490544502882	1
2	139	1098874400831246674935	1
2	179	1579558293992718176564	1
2	2650797587267	1448000408130058378970	1

$n - 1 = 1632373745527558118250 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 11 \cdot 139 \cdot 179 \cdot 2650797587267$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 2650797587267$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	2650797587266	1
2	29	2198836675456	1
2	97	487734700777	1
2	101	1802097630023	1
2	131	2267398911182	1
2	149	355999588895	1
2	239	2141258375571	1

$n - 1 = 2650797587266 = 2 \cdot 29 \cdot 97 \cdot 101 \cdot 131 \cdot 149 \cdot 239$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 1632373745527558118257$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
5	2	1632373745527558118256	1
2	3	341735867558890225982	1
2	491833	1052456454696489885063	1
2	69144986947109	1606558348878640804693	1

$n - 1 = 1632373745527558118256 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 491833 \cdot 69144986947109$

Since these are the prime factors of $n-1$, we have proven that n is prime.

Proof of primality of $n = 69144986947109$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	69144986947108	1
2	693757	2024794915065	1
2	24916861	29368197746107	1

$n - 1 = 69144986947108 = 2 \cdot 2 \cdot 693757 \cdot 24916861$

Since these are the prime factors of $n - 1$, we have proven that n is prime.

Proof of primality of $n = 24916861$:

x	p	$x^{(n-1)/p} \bmod n$	$x^{n-1} \bmod n$
2	2	24916860	1
2	3	21636993	1
2	5	16014854	1
2	138427	21039780	1

$n - 1 = 24916860 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 138427$

Since these are the prime factors of $n - 1$, we have proven that n is prime.