Polynomials
for 'same game'

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In N.J.A. Sloane's On-Line Encyclopedia of Integer Sequences the sequence A035615 deals with the winning n-digit binary strings in 'same game'. Erich Friedman describes this game as: Strings that can be reduced to null string by repeatly removing an entire run of two or more consecutive digits. Example: 11011001 is a winning string since 110(11)001→11(000)1→(111)→null.

The first values of this sequence are : 0, 2, 2, 6, 12, 26, 58, 126, 278, 602, 1300, 2774, 5878, 12350, 25778, 53470, 110332, 226610, 463602, 945214, 1921550, 3896642, 7885092, 15927086, 32121582, 64697726.

Algorithm:

win(string s, int links, int rechts)

  if rechts-links==0 return *false*

  if all digits in s a equal return *true*

  for all entire runs of two or more consecutive digits r do

    if *win*(s-r,links',rechts')==*true* return *true*

  return *false*

main()

  solutions=0;

  for all binary strings s with length n do

    if $win(s, 1, r) = true$ solutions++

In A035617 the 'same game' is treated for ternary strings. More general let a(n,b) denote the number of winning n-digit b-ary strings in 'same game'. The above algorithm can obviously used for the general problem. Becaus of the fact, that a winnig n-digit b-ary string can only have $\lfloor \frac{n}{2} \rfloor$ different digits there exist for a(n,b) a polynomial with maximal degree $\lfloor \frac{n}{2} \rfloor$.

The first few polynomials are:

$a(1,b) = 0$
$a(2,b) = b$
$a(3,b) = b$
$a(4,b) = 2b^2 - b$
$a(5,b) = 5b^2 - 4b$
$a(6,b) = 5b^3 - 3b^2 - b$
$a(7,b) = 21b^3 - 35b^2 + 15b$
$a(8,b) = 14b^4 - 36b^2 + 23b$
$a(9,b) = 84b^4 - 204b^3 + 162b^2 - 41b$
$a(10,b) = 42b^5 + 60b^4 - 405b^3 + 465b^2 - 161b$
$a(11,b) = 330b^5 - 990b^4 + 990b^3 - 341b^2 + 12b$

Literatur:

N.J.A. Sloane (2001), The On-Line Encyclopedia of Integer Sequences, published electronically at http://www.research.att.com/ njas/sequences/.