First occurrences of square-free gaps and an algorithm for their computation

Louis Marmet

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1 Abstract

This paper reports the results of a search for first occurrences of square-free gaps using an algorithm based on the sieve of Eratosthenes. Using $Qgap(L)$ to denote the starting number of the first gap having exactly the length L, the following values were found since August 1999: $Qgap(10) = 262\,315\,467$, $Qgap(12) = 47\,255\,689\,915$, $Qgap(13) = 82\,462\,576\,220$, $Qgap(14) = 1\,043\,460\,553\,364$, $Qgap(15) = 79\,180\,770\,078\,548$, $Qgap(16) = 3\,215\,226\,335\,143\,218$, $Qgap(17) = 23\,742\,453\,640\,900\,972$ and $Qgap(18) = 125\,781\,000\,834\,058\,568$. No gaps longer than 18 were found up to $N = 125\,870\,000\,000\,000\,000$.

2 Introduction

2.1 Square-free numbers

A number is said to be square-free if its prime decomposition contains no repeated factors. For example, 30 is square-free since its prime decomposition $2 \times 3 \times 5$ contains no repeated factors. However, 18 is not square-free since the factor 3 appears twice in its prime decomposition $2 \times 3 \times 3$.

The first few square-free numbers give the sequence: 1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 26, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47, 51, 53, 55, 57, 58, 59, 61, 62, 65, 66, 67, 69, 70, 71, 73, 74, 77, 78, 79, 82, 83, 85, 86, 87, 89, 91, 93, 94, 95, 97, etc. (Sloane's A005117 [1].)

More details on square-free numbers can be found at mathworld.wolfram.com/Squarefree.html. If a square-free number is used as the argument of the Möbius function[2], a non-zero value ($+1$ or -1) is obtained.

2.2 Gaps between square-free numbers

A square-free gap is a series of L consecutive numbers missing from the sequence of square-free numbers. The first square-free gap in the sequence of square-free numbers starts at $N = 4$ and has a length of one. The next gap starts at $N = 8$ and has a length $L = 2$ (since 8 and 9 are non-square-free). The following table lists the first few gaps and their lengths.

Table 1: First few square-free gaps and their lengths

Gap starts at N	4	8	12	16	18	20	24	27	32	36	40	44	48	52	54	56
Length of gap L	1	2	1	1	1	1	2	2	1	1	1	2	3	1	1	1

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[1] oeis.org/A005117

[2] mathworld.wolfram.com/MoebiusFunction.html

For any L, it can be shown that there exist infinitely many gaps of length greater than L in the sequence of square-free numbers. Longer lists of square-free gaps and recent results are available in Appendices B, C and D. These gaps are series of consecutive squareful[3] numbers (Sloane's A013929 [4]).

Note that the term "squarefull" sometimes denotes a positive integer n such that if p is a prime dividing n, then p^2 divides n.[5] (See the selected Preprints of Michael Filaseta[6].)

The smallest integer of the first gap having exactly the length L is denoted here as $Qgap(L)$ (Q for "Quadratfrei"[7], or squarefree[8]). Thus $Qgap(1) = 4$, $Qgap(2) = 8$, $Qgap(3) = 48$, etc.

2.3 Upper limits for $Qgap(16)$ to $Qgap(24)$

Erick Bryce Wong[9] found upper limits for $Qgap(L)$ for $L = 16$ to 24. His idea was to find them by prescribing a repeated prime factors for each term and using the Chinese Remainder Theorem[10] to obtain a number. More precisely, he prescribed all but five of the moduli and then tested the last moduli, up to the first 1000 primes, to check if the number is squarefree. He tried this over millions of permutations. His impressive results are:

	Upper limit	Found on
$Qgap(16) \leq$	46 717 595 829 767 167	Feb. 17th 2000
$Qgap(17) \leq$	23 742 453 640 900 972	Feb. 21st 2000
$Qgap(18) \leq$	125 781 000 834 058 568	June 26th 2000
$Qgap(19) \leq$	31 310 794 237 768 728 712	July 18th 2000
$Qgap(20) \leq$	148 372 453 443 663 297 638 331	July 10th 2000
$Qgap(21) \leq$	321 362 101 382 225 854 472	Feb. 17th 2000
$Qgap(22) \leq$	213 922 449 434 979 698 424 416	Aug. 4th 2000
$Qgap(23) \leq$	687 445 369 966 391 012 821 156 868	July 18th 2000
$Qgap(24) \leq$	28 548 715 276 566 524 078 226 797 585 011	Sept. 4th 2000

2.4 First occurrence of a gap of length L

$Qgap(L)$ is listed in the following table for L up to 18. The third column gives the prime factors that are repeated for each number in the gap. The values of $Qgap(L < 10)$ and $Qgap(11)$ have been confirmed by different sources. (See for example, "Sloane's On-Line Encyclopedia of Integer Sequences"[11], sequence A045882 [12].) The sequence $Qgap(L)$ is listed under A051681 [13] in the Encyclopedia of Integer sequences.

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[3] mathworld.wolfram.com/Squareful.html

[4] oeis.org/A013929

[5] M. Filaseta, O. Trifonov, "The distribution of squarefull numbers in short intervals," Acta Arith. **67**, pp. 323-333, 1994.

[6] www.math.sc.edu/~filaseta/paperindex.html

[7] mathworld.wolfram.com/Quadratfrei.html

[8] mathworld.wolfram.com/Squarefree.html

[9] erick at sfu.ca.

[10] www.cut-the-knot.org/blue/chinese.shtml and mathworld.wolfram.com/ChineseRemainderTheorem.html

[11] oeis.org/

[12] oeis.org/A045882.

[13] oeis.org/A051681

L	$Qgap(L)$	Repeated prime factors of each number in the gap	Gap reported by
1	4	2	E. Friedman
2	8	2, 3	E. Friedman
3	48	2, 7, 5	E. Friedman
4	242	11, 3, 2, 7	E. Friedman
5	844	2, 13, 3, 11, 2	E. Friedman
6	22 020	2, 19, 11, 3, 2, 5	E. Friedman
7	217 070	7, 3, 2, 113, 11, 5, 2	E. Friedman
8	1 092 747	19, 2, 7, 5, 11, 2, 3, 13	E. Friedman
9	8 870 024	2, 5, 11, 29, 2, 7, 31, 3, 2	P. De Geest
10	262 315 467	3, 2, 29, 2957, 79, 2, 7, 17, 5, 2 × 3	D. Bernier
11	221 167 422	3, 31, 2, 5, 37, 13, 2, 7, 11, 3, 2	P. De Geest
12	47 255 689 915	7, 2, 3, 103, 43, 2, 29, 17, 13, 2, 5, 3	L. Marmet
13	82 462 576 220	2, 3, 13, 23, 2, 5, 17, 41, 2, 19, 3, 7, 2	L. Marmet
14	1 043 460 553 364	2, 3, 7, 19, 2, 13 × 59, 67, 43, 2, 181, 3, 5, 2, 11	L. Marmet
15	79 180 770 078 548	2, 3, 5, 29, 2, 13, 17, 53, 2, 19, 3, 41, 2, 31, 67	L. Marmet
16	3 215 226 335 143 218	11, 23, 2, 3, 269, 53, 2, 5, 17, 163, 2, 101, 3, 19, 2, 137	Z. McGregor-Dorsey
17	23 742 453 640 900 972	2, 11 × 23, 127, 5, 2, 3, 53, 37, 2, 7, 13, 17, 2, 19, 3, 29, 2	E. Wong
18	125 781 000 834 058 568	2, 3, 37, 31, 2, 19, 29, 5, 2, 7 × 23, 3, 139, 2, 11, 17, 13, 2, 199	L. Marmet

The first gaps reported in this work were found on the following dates.

	First gap	Found on	Found by
$Qgap(10) =$	262 315 467	August 1999	D. Bernier,
$Qgap(12) =$	47 255 689 915	October 19th 1999	L. Marmet,
$Qgap(13) =$	82 462 576 220	October 20th 1999	L. Marmet,
$Qgap(14) =$	1 043 460 553 364	October 25th 1999	L. Marmet,
$Qgap(15) =$	79 180 770 078 548	November 29th 1999	L. Marmet,
$Qgap(16) =$	3 215 226 335 143 218	July 22nd 2000	Z. McGregor-Dorsey *et al.*,
$Qgap(17) =$	23 742 453 640 900 972	July 8th 2001	E. Wong *et al.*,
$Qgap(18) =$	125 781 000 834 058 568	September 9th 2005	L. Marmet *et al.*

2.4.1 Basic algorithm: the sieve of Eratosthenes

The square-free gaps can be calculated by finding consecutive numbers that are not square-free. A simple method to show that N is not squarefree is to find a prime factor of N whose square divides N. By trying every prime up to the square root of N, one can establish whether N is square-free or not. However, this is a very inefficient way to test billions of numbers.

A faster algorithm is used by "Mathematica" to determine if a number is square-free. The method is quite interesting[14].

However, to determine which of many consecutive numbers are square-free, an algorithm based on to the sieve of Eratosthenes[15] is much faster. It uses a list of numbers from which each composite number is removed. Once the process is finished, only the prime numbers are in the list.

To find the square-free numbers using a sieve, a similar technique is used but the algorithm eliminates numbers that are not square-free. Starting with a list of integers, first cross out the multiples of 4:

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[14] reference.wolfram.com/mathematica/ref/SquareFreeQ.html

[15] mathworld.wolfram.com/SieveofEratosthenes.html

```
1  2  3  X  5  6  7  X  9 10 11  X 13 14 15  X 17 18 19  X 21 22 23  X 25 26 ...
```

then the multiples of 9, 25, etc., up to the last number in the list:

```
1  2  3  X  5  6  7  X  X 10 11  X 13 14 15  X 17  X 19  X 21 22 23  X  X 26 ...
```

The remaining numbers are square-free numbers; the gaps are indicated by the series of consecutive "X".

2.4.2 Improvements of the algorithm

The following improvements were implemented in a computer program and are presented in the same order they were added to the program.

2.4.3 Improvement I

"Lists of squared-primes and the next non-square-free number use less memory."

To implement this algorithm on a computer, it is not necessary to keep the entire list of integers in memory. An improvement of the algorithm uses instead two shorter arrays to calculate the next non-square-free number after N:

- the first array, called `p2`, gives the squares of the prime numbers up to the largest number to be tested N_{max},
- the second array, called `nsqf`, gives for each `p2[i]` the next non-square-free number, that is, the smallest number larger than N that is a multiple of `p2[i]`. This array can easily be calculated using modulo arithmetic.

These arrays will have approximately $2\sqrt{N_{max}}/\ln N_{max}$ elements[16].

To find square-free gaps, one finds sequences of non-square-free numbers. The following example shows the arrays used to find gaps starting from $N = 20$ [17].

Index	`i`	0	1	2	3	4	5	6	7	8	...
Squared prime	`p2[i]`	4	9	25	49	121	169	289	361	529	...
Next non-square-free	`nsqf[i]`	24	27	25	49	121	169	289	361	529	...

Using this table, it is easy to find the next non-square-free number: it is the smallest number in the array `nsqf`, that is, 24. We set $N = 24$ and recalculate the array. This is easy since the only needed operation is to add the corresponding squared-prime to the multiple: $24 + 4 = 28$. The following table is obtained:

Index	`i`	0	1	2	3	4	5	6	7	8	...
Squared prime	`p2[i]`	4	9	25	49	121	169	289	361	529	...
Next non-square-free	`nsqf[i]`	28	27	25	49	121	169	289	361	529	...

Again, the next non-square-free number is the smallest `nsqf[i]`. By repeating this procedure, N takes the values of all the non-square-free numbers. It is advantageous to sort `nsqf` in increasing order at each step. This way, the smallest is always `nsqf[0]`. We set $N = 25$ and recalculate the array to obtain:

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[16] www.utm.edu/research/primes/howmany.shtml

[17] The notation used in the programming language C is used here, where the index of an array starts at 0.

Index	`i`	0	1	2	3	4	5	6	7	8	...
Squared prime	`p2[i]`	9	4	49	25	121	169	289	361	529	...
Next non-square-free	`nsqf[i]`	27	28	49	50	121	169	289	361	529	...

The order of the array `p2` has also been changed so that each number `p2[i]` always corresponds to its multiple `nsqf[i]`. Repeating the procedure will generate the non-square-free numbers $N = 27$, 28, 32, 36, 40, etc. Note that special care has to be taken when some numbers in `nsqf` are equal - each of these has to be increased by the value of its corresponding `p2[i]`.

The sort is relatively efficient since after `nsqf[0]` is given its new value, `nsqf[1]`, `nsqf[2]` and the following elements are still in increasing order. The new value is moved up the array until its proper place is found.

2.4.4 Improvement II

"Many non-square-free numbers can be skipped."

If gaps of a given length L_{min} or more are searched, some `nsqf[i]` can be skipped. To show this, one finds first the minimum number of squared-primes $\mathtt{NP2}_{min}$ [18] required in a gap of length L:

Gap length L	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	...
$\mathtt{NP2}_{min}\mathtt{[}L\mathtt{]}$	1	2	3	4	4	5	6	7	7	7	8	9	9	10	11	12	12	...

For example if we choose $L = 7$, $\mathtt{NP2}_{min}\mathtt{[}L\mathtt{]}= 6$ prime factors are required for the gap starting at $N = 217070$ (in this case, the prime factors are 2, 3, 5, 7, 11 and 113).

Continuing with the example given above, we now specifically search gaps with length $L_{min} = 7$ or longer. There is no gap of length $L_{min} = 7$ in the interval starting at `nsqf[0]` and ending at `nsqf[5]`, if `nsqf[5]`$>$`nsqf[0]`$+7$. (In general, there is no gap of length L_{min} in the interval starting at `nsqf[0]` and ending at $\mathtt{nsqf[NP2}_{min}\mathtt{[}L_{min}-1\mathtt{]]}$, if $\mathtt{nsqf[NP2}_{min}\mathtt{[}L_{min}-1\mathtt{]]}>\mathtt{nsqf[0]}+L_{min}$.)

With $N = 40$, we have:

Index	`i`	0	1	2	3	4	5	6	7	8	...
Squared prime	`p2[i]`	4	9	49	25	121	169	289	361	529	...
Next non-square-free	`nsqf[i]`	44	45	49	50	121	169	289	361	529	...

Since $\mathtt{nsqf[5]} = 169 > \mathtt{nsqf[0]}+7 = 51$, there is no gap of length 7 in the interval starting at 44 and ending at 169. We can therefore safely set $N = \mathtt{nsqf[5]} - L_{min} + 1 = 163$ and recalculate the following table:

Index	`i`	0	1	2	3	4	5	6	7	8	...
Squared prime	`p2[i]`	4	169	9	25	49	121	289	361	529	...
Next non-square-free	`nsqf[i]`	164	169	171	175	196	242	289	361	529	...

This cuts down on the number of non-square-free that have to be tested and the speed of the calculation is increased. A factor five in speed was obtained when this was implemented in the program which was used to find $Qgap(14)$ and $Qgap(15)$.

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[18] http://oeis.org/A107079

2.4.5 Improvement III

“The smallest squared-primes are not needed to calculate the sieve.”

This variation on Improvement II was suggested by Joseph Wetherell. It turns out to be more efficient when it is combined with Improvements IV and V. The trick is to consider the smallest squared-primes separately from the large squared-primes. Most of the time in the algorithm is spent on the process of taking the smallest elements off of `nsqf` and sorting them back into the array. If one can reduce the number of non-square-free numbers which are tested, the speed of the algorithm will improve.

This is actually possible since the smallest squared-primes are not needed to calculate the sieve. If we have a gap of, say, $L = L_{min} = 7$ non-square-free numbers, then at least $\texttt{NP2}_{min}\texttt{[7]} = 6$ different primes are found in the gap. This means that the smallest $k1 = \texttt{NP2}_{min}\texttt{[}L_{min}\texttt{]} - 1 = 5$ squared-primes can be left out of the calculation. If we search for the 6^{th} squared-prime, every gap of length $L = 7$ (or more) will be found.

This method therefore separates the small squared-primes from the large ones, creating a base with $k1$ elements. The table for $N = 163$ would now look like this:

Index	`i`	0	1	2	3	4	$k1$	6	7	8	...
Squared prime	`p2[i]`	4	9	25	49	121	169	289	361	529	...
Next non-square-free	`nsqf[i]`	$--$	$--$	$--$	$--$	$--$	169	289	361	529	...

with the base shown as $--$ for the values of `nsqf[i]`. From the table, one sees that one must test for a gap having $L_{min} = 7$ around $N = 169$. If none is found, then `nsqf[`$k1$`]` is increased by 169 and sorted back into the array of large squared-primes to get:

Index	`i`	0	1	2	3	4	$k1$	6	7	8	...
Squared prime	`p2[i]`	4	9	25	49	121	289	169	361	529	...
Next non-square-free	`nsqf[i]`	$--$	$--$	$--$	$--$	$--$	289	338	361	529	...

The sort is faster since it is only done on the large squared-primes. The process is then continued at $N = 289$.

2.4.6 Improvement IV

“The values of the small modulos can be computed ahead of time.”

To test if there is a gap of L_{min} around N, one must still know about multiples of the $k1$ small squared-primes near N. As suggested by Joseph Wetherell, this can be done by trial division; even if trial division is slow, it is faster than resorting the base array. One can also optimize the trial divisions, because the trial divisions for, say, $N + 1$ and $N + 2$ are related to each other. For each small prime p, compute $N\%$`p2` and store it in a list `mod` (the “%” symbol is the modulo function in the C language). Now to see if `p2` divides $N + 1$, we just test if this stored value is -1 (mod `p2`). To see if `p2` divides $N + 2$, we test if this stored value is -2 (mod `p2`). (We also precompute the value of -1 (mod `p2`), -2 (mod `p2`), etc., for the small set of values which we will possibly need to test.) Note that it is also necessary to test $N - 1$, $N - 2$, etc. For $N = 289$, we have the following arrays:

Index	`i`	0	1	2	3	4	$k1$	6	7	8	...
Squared prime	`p2[i]`	4	9	25	49	121	289	169	361	529	...
Next non-square-free	`nsqf[i]`	$--$	$--$	$--$	$--$	$--$	289	338	361	529	...
$N\%$`p2[i]`	`mod[i]`	1	1	14	44	47					

We see that `p2[0]` and `p2[1]` divide $N - 1 = 288$, but this is the only other non-square-free number for this gap of length 2. We can therefore increase `nsqf[`$k1$`]` by 289, sort it back into the array (reordering `p2` accordingly) and continue with $N = 338$:

Index	`i`	0	1	2	3	4	$k1$	6	7	8	...
Squared prime	`p2[i]`	4	9	25	49	121	169	361	529	289	...
Next non-square-free	`nsqf[i]`	−−	−−	−−	−−	−−	338	361	529	578	...
N%`p2[i]`	`mod[i]`	2	5	13	44	96					

2.4.7 Improvement V

"Two large primes-squared that are too far cannot result in a gap."

If we include `p2[4]` in the array of large squared-primes, (so there are only $k2 = \texttt{NP2}_{min}[L_{min}]-2 = 4$ elements in the base), then we know that a number $N =$ `nsqf[`$k2$`]` we are testing can be part of a gap only if the next number in `nsqf` is close to N, that is, if `nsqf[k2+1]` is not larger than $N + L_{min} - 1$. Based on this suggestion by Joseph Wetherell, the arrays become with $N = 338$:

Index	`i`	0	1	2	3	$k2$	5	6	7	8	...
Squared prime	`p2[i]`	4	9	25	49	169	361	121	529	289	...
Next non-square-free	`nsqf[i]`	−−	−−	−−	−−	338	361	363	529	578	...
N%`p2[i]`	`mod[i]`	−−	−−	−−	−−						

Since `nsqf[`$k2+1$`]`$= 361$ is larger than $N + L_{min} - 1 = 344$, we can skip to $N = 361$. Since N does not pass the closeness test, we do not have to do any computations with the base, saving us a lot of time.

2.4.8 Improvement VI

"A chained list is faster for the sort."

A chained list can be built with a set of numbers that specify an order for the elements of an array, as suggested by Joseph Wetherell. If we use a chained list represented by the array called `next` such that `nsqf[next[i]]`$\geq$`nsqf[i]`, we can sort the array `nsqf` without moving any data within the arrays `nsqf` or `p2`. For a reason that will become obvious later, we choose to have `p2` sorted in increasing order. With $N = 361$, the arrays would be:

Index	`i`	0	1	2	3	$k2$	5	6	7	8	...
Chained list	`next[i]`	−−	−−	−−	$k2$	5	8	9	**4**	6	...
Squared prime	`p2[i]`	4	9	25	49	121	169	289	361	529	...
Next non-square-free	`nsqf[i]`	−−	−−	−−	−−	363	507	578	361	529	...
N%`p2[i]`	`mod[i]`	1	1	11	18						

We use an additional variable, "`head`", which points to the smallest item in the array `nsqf`. For the table above, we have `head` $= 7$ (`next[head]` is highlighted). Since we only need to change two values in the array `next` to perform a sort, this method is faster. Searching through the array `nsqf` now consists of going through the data in the following order:

```
for (i=head; tempnsqf>=nsqf[next[i]]; i=next[i])  ...
```

There is another advantage to this method: since the array `p2` is sorted in increasing order, we know that if we have to sort item `nsqf[i=head]`, then `nsqf[i-1]`$\leq$`nsqf[i]`+`p2[i]`. This means we can jump immediately to `i=head-1` and start searching from there. In general, this cuts the search in half! Searching through the list now consists of:

```
for (i=head-1; tempnsqf>=nsqf[i2=next[i]]; i=i2)  ...
```

We continue the example with the above table. Since `nsqf[next[head]]`$= 363$ is not larger than $N+L_{min}-1=367$, we calculated the modulos. Clearly, there is only a gap of length $L=2$ starting at 360. We therefore set $N=363$ and sort `tempnsqf`=`nsqf[head]`=`nsqf[head]`+`p2[head]`$= 722$. We start with `i=6` and the sort requires only one comparison! The following arrays are obtained:

Index	i	0	1	2	3	$k2$	5	6	7	8	...
Chained list	`next[i]`	−−	−−	−−	$k2$	$\underline{\mathbf{5}}$	8	7	9	6	...
Squared prime	`p2[i]`	4	9	25	49	121	169	289	361	529	...
Next non-square-free	`nsqf[i]`	−−	−−	−−	−−	363	507	578	722	529	...
N%`p2[i]`	`mod[i]`	−−	−−	−−	−−						

with `head` now equal to 4.

A factor three in speed was obtained when this algorithm was implemented in the program.

2.4.9 Improvement VII

"Look for the largest spacing between two of three large squared-primes."

Instead of having `k2` elements in the base and look for two large squared-primes that are not too far apart, we can use a base with `k3`=`NP2`$_{min}$`[`L_{min}`]`$-3=3$ squared-primes, but look to see if `nsqf[head]` and `nsqf[next[next[head]]]` are not more than L_{min} apart.

Index	i	0	1	2	$k3$	4	5	6	7	8	...
Chained list	`next[i]`	−−	−−	$k3$	5	$\underline{\mathbf{3}}$	8	7	9	6	...
Squared prime	`p2[i]`	4	9	25	49	121	169	289	361	529	...
Next non-square-free	`nsqf[i]`	−−	−−	−−	392	363	507	578	722	529	...
N%`p2[i]`	`mod[i]`	−−	−−	−−							

In this example, we look at the difference between `nsqf[head]`$= 363$ and `nsqf[next[next[head]]]`$= 507$. Since the difference between the two values is larger than $L_{min}-1$, there is no gap of length L_{min} or longer.

This improvement gives the program a 37% speed increase with $L_{min}=14$.

The program becomes slower if four or more large squared-primes are considered (this was confirmed in tests for $L>13$, $N=10^{14}$ to $10^{14}+10^{9}$).

2.4.10 Evaluation of order of algorithm

This evaluation applies to the first improvement of the algorithm. It was found empirically that for a given value of L_{min}, the other improvements increased the speed of the calculation by a constant factor.

To calculate if N is a square-free number, the algorithm takes advantage of the known remainders for $N-1$. Each time N is tested, a new non-square-free number is calculated using `nsqf[0]=nsqf[0]+p2[0];`. This new value has to be moved up the list to keep `nsqf` in increasing order. It is that operation that requires most of the computation time. To evaluate the speed of the algorithm, it is necessary to find the average number of moves m that will be required to bring the new value `nsqf[0]` to its correct position in the list, above the number `nsqf[`m`]`. This is done by evaluating, for every `i`, the probability that `nsqf[0]`>`nsqf[i]`, and then summing over `i`.

First, consider the case when $p2[0] = 4$ which occurs with a probability of 1/4. Since `nsqf[0]` has been increased by 4, there is a probability of 4/9 that it will have to be moved above `nsqf[j]` (if `p2[j]`= 9). There is an additional probability of 4/25 that `nsqf[0]` will have to be moved above `nsqf[k]` (if `p2[k]`= 25), etc. We therefore get the average number of steps required to place the new `nsqf[0]` to its correct position:

$$S(1) = 1/4 \times (4/9 + 4/25 + 4/49 + 4/121 + ...) = \sum_{i=2} \frac{1}{p^2(i)}$$

where $p(i)$ is the ith prime number ($p(1) = 2$, $p(2) = 3$, $p(3) = 5$, etc.) and $p^2(i) = p(i) \times p(i)$.

In the case when `p2[0]`= 9 (which occurs with a probability of 1/9), one move is always necessary to bring `nsqf[0]` above `nsqf[j]` (if `p2[j]`= 4). There is a probability of 9/25 that `nsqf[0]` will have to be moved above `nsqf[k]` (if p2[k]=25), etc. We therefore get:

$$S(2) = 1/9 \times (1 + 9/25 + 9/49 + 9/121 + ...) = \frac{1}{p^2(2)} + \sum_{i=3} \frac{1}{p^2(i)}$$

In general, when `p2[0]`= $p^2(m)$, the average number of moves is:

$$S(m) = \frac{m-1}{p^2(m)} + \sum_{i=m+1} \frac{1}{p^2(i)}$$

The sum over all the $S(m)$ gives the average number of moves required to place `nsqf[0]` to its correct position in the list:

$$\sum_{m=1} S(m) = 2 \times \sum_{i=2} \frac{i-1}{p^2(i)}$$

This series converges, as determined with a convergence test[19]. It converges very slowly to approximately 1.30... Therefore, given the remainders for $N-1$, the number of operations required to find out if N is square-free is independent of the value of N.

2.5 Acknowledgements

Contributed to find $Qgap(10)$: David Bernier.

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[19] "Suppose that $f(x)$ is a positive decreasing function and that

$$lim_{k\to\infty} \frac{e^k f(e^k)}{f(k)} = q$$

for natural k. If $q < 1$, the series $\sum_{k=1}^{\infty} f(k)$ converges. If $q > 1$, this series diverges. (Ermakov)" Equation 0.224, "Table of Integrals, Series, and Products," Gradshteyn and Ryzhik (Academic Press, Inc., p. 5)

Contributed to find *Qgap*(17): E. Wong, Z. McGregor-Dorsey, L. Marmet, Jean-Pierre Bernier, D. Bernier, Nancy Robertson, N. Marmet, Charles Ward and G. Engebreth.

Contributed to find *Qgap*(18): D. Bernier, L. Marmet, E. Wong, J. Wetherell, Z. McGregor-Dorsey, G. Engebreth, A. Simpson, N. Marmet, N. Robertson, J.-P. Bernier, C.R. Ward, Bruno Le Tual and Horand Gassmann.

This project started from an idea that was initially suggested to me by David Bernier.

2.6 Related web pages and references

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M. Filaseta, "On the distribution of gaps between squarefree numbers," Mathematika, 40 (1993), 88-101.

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A translation of this paper is available in Belorussian at
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This article was first published at www.marmet.org/louis/sqfgap/index.html.

Appendix A

The following graph shows an estimation of how large we can expect $Qgap(L)$ to be.

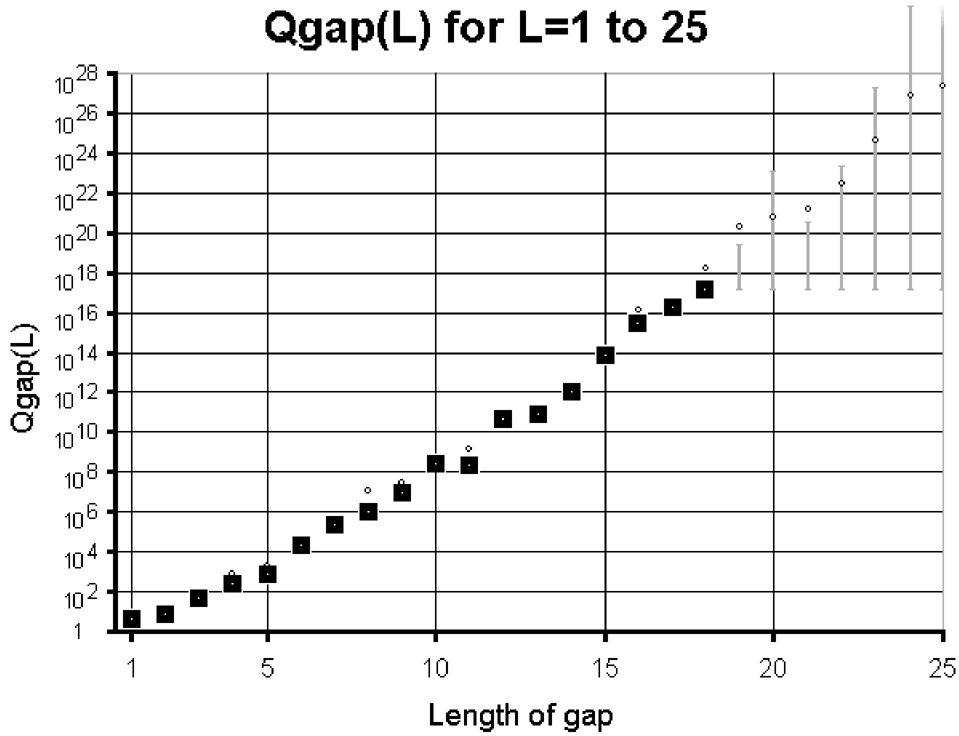

Figure 1: Known values of $Qgap(L)$ (squares) and estimated values (empty circles).

The empirical estimation, based on the calculated values for $L < 16$, uses an approximation of the probability of obtaining the minimum number of primes required to produce a gap with length L. The upper limits for $Qgap(L > 16)$ were obtained by E. Wong. The values of $Qgap(L)$ lie within the ranges indicated by the gray lines.

Appendix B

Square-free gaps and their length ≥ 14, up to $5\,000\,000\,000\,000\,000$

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Appendix C

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Appendix D

Square-free gaps and their length ≥ 16, up to 125 870 000 000 000 000

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Appendix E

Completed ranges computed on 44 different processors by 14 different users.

User Name	(Computer Name)	*Range*	*DateCompleted*
L. Marmet	(neurone5 Linux .8GHz)	125.7×10^{15} to 125.8×10^{15}	September 12th, 2005
H. Gassmann	(Gus's beast)	125.3×10^{15} to 125.7×10^{15}	August 19th, 2005
L. Marmet	(neurone3 .8GHz)	125.1×10^{15} to 125.3×10^{15}	November 28th, 2005
N. Marmet	(Poisson.51)	124.7×10^{15} to 125.1×10^{15}	August 19th, 2005
N. Marmet	(Poisson.35)	124.4×10^{15} to 124.7×10^{15}	August 19th, 2005
N. Marmet	(Poisson.141)	124.1×10^{15} to 124.4×10^{15}	August 19th, 2005
N. Marmet	(Poisson.142.Brain1)	123.8×10^{15} to 124.1×10^{15}	August 19th, 2005
N. Robertson	(Laika.Droica)	123.4×10^{15} to 123.8×10^{15}	August 20th, 2005
L. Marmet	(neurone2 .8GHz)	122.7×10^{15} to 123.4×10^{15}	August 10th, 2005
L. Marmet	(neurone1 .8GHz)	122.1×10^{15} to 122.7×10^{15}	August 7th, 2005
N. Marmet	(Poisson.94)	121.6×10^{15} to 122.1×10^{15}	August 19th, 2005
N. Marmet	(Poisson.63)	121.3×10^{15} to 121.6×10^{15}	August 19th, 2005
L. Marmet	(neurone5 Linux .8GHz)	120.9×10^{15} to 121.3×10^{15}	July 26th, 2005
N. Robertson	(Laika.Droica)	120.5×10^{15} to 120.9×10^{15}	July 16th, 2005
L. Marmet	(neurone4 .8GHz)	119.3×10^{15} to 120.5×10^{15}	August 13th, 2005
N. Marmet	(Poisson.142.Brain2)	118.0×10^{15} to 119.3×10^{15}	September 8th, 2005
N. Marmet	(Poisson.107)	116.3×10^{15} to 118.0×10^{15}	September 2nd, 2005
N. Marmet	(Poisson.31)	115.7×10^{15} to 116.3×10^{15}	August 29th, 2005
N. Marmet	(Poisson.30)	114.9×10^{15} to 115.7×10^{15}	August 28th, 2005
J.-P. Bernier	(Pentium 600MHz)	114.0×10^{15} to 114.9×10^{15}	August 24th, 2005
L. Marmet	(neurone0 W95 .233GHz)	113.5×10^{15} to 114.0×10^{15}	August 16th, 2005
L. Marmet	(neurone5 Linux .8GHz)	113.1×10^{15} to 113.5×10^{15}	June 30th, 2005
N. Robertson	(Laika.Droica)	112.7×10^{15} to 113.1×10^{15}	June 24th, 2005
N. Marmet	(Poisson.29)	111.5×10^{15} to 112.7×10^{15}	August 28th, 2005
H. Gassmann	(Gus's beast)	111.0×10^{15} to 111.5×10^{15}	July 27th, 2005
N. Marmet	(Poisson.44)	110.4×10^{15} to 111.0×10^{15}	August 24th, 2005
N. Marmet	(Poisson.37)	109.6×10^{15} to 110.4×10^{15}	September 1st, 2005
N. Marmet	(Poisson.34)	108.8×10^{15} to 109.6×10^{15}	September 3rd, 2005
J.-P. Bernier	(Athlon 2000)	106.3×10^{15} to 108.8×10^{15}	August 5th, 2005
B. Le Tual	(Celeron 2.4GHz)	105.3×10^{15} to 106.3×10^{15}	July 16th, 2005

User Name	(Computer Name)	*Range*	*DateCompleted*
B. Le Tual	(Celeron .9GHz)	104.9×10^{15} to 105.3×10^{15}	July 16^{th}, 2005
L. Marmet	(neurone2 .8GHz)	103.2×10^{15} to 104.9×10^{15}	July 8^{th}, 2005
B. Le Tual	(Celeron 2.4GHz)	102.7×10^{15} to 103.2×10^{15}	May 13^{th}, 2005
L. Marmet	(neurone4 .8GHz)	101.1×10^{15} to 102.7×10^{15}	June 25^{th}, 2005
L. Marmet	(neurone1 .8GHz)	99.5×10^{15} to 101.1×10^{15}	July 2^{nd}, 2005
L. Marmet	(Poisson.63)	98.8×10^{15} to 99.5×10^{15}	July 12^{th}, 2005
H. Gassmann	(Gus's beast)	98.4×10^{15} to 98.8×10^{15}	June 1^{st}, 2005
N. Robertson	(Laika.Droica)	98.0×10^{15} to 98.4×10^{15}	May 9^{th}, 2005
J.-P. Bernier	(Athlon 2000)	96.2×10^{15} to 98.0×10^{15}	May 24^{th}, 2005
L. Marmet	(neurone0 W95 .233GHz)	95.7×10^{15} to 96.2×10^{15}	June 10^{th}, 2005
N. Marmet	(Poisson.142.Brain2)	94.2×10^{15} to 95.7×10^{15}	June 13^{th}, 2005
N. Marmet	(Poisson.94)	92.7×10^{15} to 94.2×10^{15}	July 12^{th}, 2005
J.-P. Bernier	(Pentium 600MHz)	91.7×10^{15} to 92.7×10^{15}	June 12^{th}, 2005
N. Marmet	(Poisson.142.Brain1)	89.7×10^{15} to 91.7×10^{15}	July 21^{st}, 2005
N. Marmet	(Poisson.34)	88.9×10^{15} to 89.7×10^{15}	May 30^{th}, 2005
N. Marmet	(Poisson.107)	86.8×10^{15} to 88.9×10^{15}	June 13^{th}, 2005
N. Marmet	(Poisson.29)	85.5×10^{15} to 86.8×10^{15}	June 2^{nd}, 2005
N. Marmet	(Poisson.31)	84.7×10^{15} to 85.5×10^{15}	June 13^{th}, 2005
N. Marmet	(Poisson.51)	82.4×10^{15} to 84.7×10^{15}	July 25^{th}, 2005
N. Marmet	(Poisson.30)	81.4×10^{15} to 82.4×10^{15}	May 13^{th}, 2005
N. Marmet	(Poisson.45)	80.8×10^{15} to 81.4×10^{15}	May 30^{th}, 2005
N. Marmet	(Poisson.37)	80.0×10^{15} to 80.8×10^{15}	May 30^{th}, 2005
L. Marmet	(neurone5 Linux .8GHz)	79.2×10^{15} to 80.0×10^{15}	June 9^{th}, 2005
L. Marmet	(neurone5 Linux .8GHz)	78.8×10^{15} to 79.2×10^{15}	April 21^{st}, 2005
J.-P. Bernier	(Athlon 2000)	78.1×10^{15} to 78.8×10^{15}	April 9^{th}, 2005
N. Marmet	(Poisson.94)	77.6×10^{15} to 78.1×10^{15}	April 5^{th}, 2005
N. Marmet	(Poisson.142.Brain2)	77.1×10^{15} to 77.6×10^{15}	April 5^{th}, 2005
N. Marmet	(Poisson.142.Brain1)	76.6×10^{15} to 77.1×10^{15}	March 31^{st}, 2005
N. Marmet	(Poisson.107)	76.1×10^{15} to 76.6×10^{15}	March 30^{th}, 2005
N. Marmet	(Poisson.51)	75.6×10^{15} to 76.1×10^{15}	March 29^{th}, 2005
N. Marmet	(Poisson.63)	75.1×10^{15} to 75.6×10^{15}	April 26^{th}, 2005
N. Marmet	(Poisson.30)	74.9×10^{15} to 75.1×10^{15}	March 28^{th}, 2005
N. Marmet	(Poisson.31)	74.7×10^{15} to 74.9×10^{15}	March 30^{th}, 2005
N. Marmet	(Poisson.34)	74.5×10^{15} to 74.7×10^{15}	March 30^{th}, 2005
N. Marmet	(Poisson.37)	74.4×10^{15} to 74.5×10^{15}	March 23^{rd}, 2005
N. Marmet	(Poisson.45)	74.3×10^{15} to 74.4×10^{15}	March 25^{th}, 2005
N. Robertson	(Laika.Droica)	73.8×10^{15} to 74.3×10^{15}	April 5^{th}, 2005
H. Gassmann	(Gus's beast)	73.5×10^{15} to 73.8×10^{15}	April 27^{th}, 2005
L. Marmet	(neurone4 .8GHz)	71.9×10^{15} to 73.5×10^{15}	April 30^{th}, 2005
L. Marmet	(neurone2 .8GHz)	70.2×10^{15} to 71.9×10^{15}	May 4^{th}, 2005

User Name	(Computer Name)	*Range*	*DateCompleted*
L. Marmet	(neurone1 .8GHz)	68.7×10^{15} to 70.2×10^{15}	April 28^{th}, 2005
J.-P. Bernier	(Pentium 600MHz)	68.4×10^{15} to 68.7×10^{15}	April 3^{rd}, 2005
B. Le Tual	(Celeron .9GHz)	68.0×10^{15} to 68.4×10^{15}	May 15^{th}, 2005
B. Le Tual	(Celeron 2.4GHz)	67.6×10^{15} to 68.0×10^{15}	May 1^{st}, 2005
J.-P. Bernier	(Athlon 2000)	67.2×10^{15} to 67.6×10^{15}	March 17^{th}, 2005
N. Marmet	(Poisson.29)	66.9×10^{15} to 67.2×10^{15}	March 30^{th}, 2005
J.-P. Bernier	(Athlon 2000)	66.5×10^{15} to 66.9×10^{15}	January 1^{st}, 2004
J.-P. Bernier	(Pentium 600MHz)	66.2×10^{15} to 66.5×10^{15}	January 12^{th}, 2004
B. Le Tual	(Celeron 2.4GHz)	65.9×10^{15} to 66.2×10^{15}	January 4^{th}, 2004
B. Le Tual	(Celeron .9GHz)	65.5×10^{15} to 65.9×10^{15}	January 9^{th}, 2004
J.-P. Bernier	(Athlon 2000)	65.1×10^{15} to 65.5×10^{15}	December 19^{th}, 2003
J.-P. Bernier	(Pentium 600MHz)	64.8×10^{15} to 65.1×10^{15}	December 14^{th}, 2003
J.-P. Bernier	(Athlon 2000)	64.4×10^{15} to 64.8×10^{15}	December 4^{th}, 2003
B. Le Tual	(Celeron 2.4GHz)	64.1×10^{15} to 64.4×10^{15}	December 9^{th}, 2003
L. Marmet	(neurone0 W95 .233GHz)	63.8×10^{15} to 64.1×10^{15}	April 11^{th}, 2005
J.-P. Bernier	(Athlon 2000)	63.4×10^{15} to 63.8×10^{15}	November 13^{th}, 2003
B. Le Tual	(Celeron .9GHz)	63.0×10^{15} to 63.4×10^{15}	December 5^{th}, 2003
J.-P. Bernier	(Pentium 600MHz)	62.7×10^{15} to 63.0×10^{15}	November 24^{th}, 2003
D. Bernier	(Gecko)	62.4×10^{15} to 62.7×10^{15}	December 27^{th}, 2003
J.-P. Bernier	(Athlon 2000)	62.0×10^{15} to 62.4×10^{15}	November 1^{st}, 2003
N. Robertson	(Laika.Droica)	61.6×10^{15} to 62.0×10^{15}	March 9^{th}, 2005
L. Marmet	(neurone5 Linux .8GHz)	61.2×10^{15} to 61.6×10^{15}	March 27^{th}, 2005
J.-P. Bernier	(Athlon 2000)	60.9×10^{15} to 61.2×10^{15}	October 15^{th}, 2003
L. Marmet	(Riyadh)	60.6×10^{15} to 60.9×10^{15}	November 9^{th}, 2003
J.-P. Bernier	(Pentium 600MHz)	60.3×10^{15} to 60.6×10^{15}	October 31^{st}, 2003
B. Le Tual	(Celeron 2.4GHz)	60.0×10^{15} to 60.3×10^{15}	November 12^{th}, 2003
B. Le Tual	(Celeron .9GHz)	59.6×10^{15} to 60.0×10^{15}	November 1^{st}, 2003
N. Robertson	(Laika.Droica)	59.2×10^{15} to 59.6×10^{15}	October 1^{st}, 2003
B. Le Tual	(Celeron .9GHz)	58.9×10^{15} to 59.2×10^{15}	September 28^{th}, 2003
B. Le Tual	(Celeron 2.4GHz)	58.6×10^{15} to 58.9×10^{15}	October 5^{th}, 2003
J.-P. Bernier	(Pentium 600MHz)	58.3×10^{15} to 58.6×10^{15}	October 7^{th}, 2003
L. Marmet	(Riyadh)	58.0×10^{15} to 58.3×10^{15}	October 7^{th}, 2003
D. Bernier	(Gecko)	57.7×10^{15} to 58.0×10^{15}	October 29^{th}, 2003
N. Robertson	(Maya)	57.4×10^{15} to 57.7×10^{15}	September 19^{th}, 2003
N. Robertson	(Laika.Droica)	57.1×10^{15} to 57.4×10^{15}	September 10^{th}, 2003
J.-P. Bernier	(Pentium 600MHz)	56.8×10^{15} to 57.1×10^{15}	September 10^{th}, 2003
N. Robertson	(Maya)	56.5×10^{15} to 56.8×10^{15}	August 4^{th}, 2003
N. Robertson	(Laika.Droica)	56.2×10^{15} to 56.5×10^{15}	July 29^{th}, 2003
L. Marmet	(Riyadh)	55.9×10^{15} to 56.2×10^{15}	August 13^{th}, 2003
J.-P. Bernier	(Pentium 600MHz)	55.6×10^{15} to 55.9×10^{15}	August 4^{th}, 2003

User Name	(Computer Name)	*Range*	*DateCompleted*
J.-P. Bernier	(Pentium 600MHz)	55.3×10^{15} to 55.6×10^{15}	July 10th, 2003
J.-P. Bernier	(Pentium 600MHz)	55.0×10^{15} to 55.3×10^{15}	June 15th, 2003
L. Marmet	(Riyadh)	54.7×10^{15} to 55.0×10^{15}	June 28th, 2003
N. Robertson	(Maya)	54.4×10^{15} to 54.7×10^{15}	June 10th, 2003
N. Robertson	(Laika.Droica)	54.1×10^{15} to 54.4×10^{15}	July 9th, 2003
J.-P. Bernier	(Pentium 600MHz)	53.8×10^{15} to 54.1×10^{15}	May 14th, 2003
D. Bernier	(Gecko)	53.3×10^{15} to 53.8×10^{15}	August 23rd, 2003
N. Robertson	(Maya)	53.0×10^{15} to 53.3×10^{15}	May 12th, 2003
N. Robertson	(Laika.Droica)	52.7×10^{15} to 53.0×10^{15}	May 7th, 2003
L. Marmet	(Riyadh)	52.4×10^{15} to 52.7×10^{15}	May 16th, 2003
J.-P. Bernier	(Pentium 600MHz)	52.1×10^{15} to 52.4×10^{15}	April 23rd, 2003
N. Robertson	(Maya)	51.8×10^{15} to 52.1×10^{15}	March 31st, 2003
L. Marmet	(Riyadh)	51.5×10^{15} to 51.8×10^{15}	April 6th, 2003
N. Robertson	(Laika.Droica)	51.2×10^{15} to 51.5×10^{15}	March 22nd, 2003
D. Bernier	(Gecko)	51.1×10^{15} to 51.2×10^{15}	April 20th, 2003
D. Bernier	(Gecko)	50.7×10^{15} to 51.1×10^{15}	April 2nd, 2003
J.-P. Bernier	(Pentium 600MHz)	50.4×10^{15} to 50.7×10^{15}	March 24th, 2003
D. Bernier	(Gecko)	50.1×10^{15} to 50.4×10^{15}	February 26th, 2003
L. Marmet	(Riyadh)	49.8×10^{15} to 50.1×10^{15}	March 9th, 2003
J.-P. Bernier	(Pentium 600MHz)	49.5×10^{15} to 49.8×10^{15}	February 25th, 2003
L. Marmet	(Riyadh)	49.2×10^{15} to 49.5×10^{15}	January 31st, 2003
J.-P. Bernier	(Pentium 600MHz)	49.0×10^{15} to 49.2×10^{15}	January 25th, 2003
J.-P. Bernier	(Pentium 600MHz)	48.8×10^{15} to 49.0×10^{15}	December 26th, 2002
L. Marmet	(Riyadh)	48.5×10^{15} to 48.8×10^{15}	December 22nd, 2002
N. Robertson	(Maya)	48.2×10^{15} to 48.5×10^{15}	November 28th, 2002
N. Robertson	(Laika.Droica)	47.9×10^{15} to 48.2×10^{15}	January 11th, 2003
N. Robertson	(Arthurus)	47.8×10^{15} to 47.9×10^{15}	February 13th, 2003
J.-P. Bernier	(Pentium 600MHz)	47.5×10^{15} to 47.8×10^{15}	December 6th, 2002
D. Bernier	(Pentium 500MHz)	47.2×10^{15} to 47.5×10^{15}	November 6th, 2002
J.-P. Bernier	(Pentium 600MHz)	46.9×10^{15} to 47.2×10^{15}	November 7th, 2002
D. Bernier	(Pentium 500MHz)	46.6×10^{15} to 46.9×10^{15}	October 14th, 2002
J.-P. Bernier	(Pentium 600MHz)	46.3×10^{15} to 46.6×10^{15}	October 13th, 2002
L. Marmet	(Riyadh)	46.0×10^{15} to 46.3×10^{15}	October 15th, 2002
N. Robertson	(Maya)	45.7×10^{15} to 46.0×10^{15}	November 3rd, 2002
N. Robertson	(Laika.Droica)	45.4×10^{15} to 45.7×10^{15}	October 30th, 2002
J.-P. Bernier	(Pentium 600MHz)	45.1×10^{15} to 45.4×10^{15}	September 12th, 2002
D. Bernier	(Pentium 500MHz)	44.8×10^{15} to 45.1×10^{15}	September 15th, 2002
J.-P. Bernier	(Pentium 600MHz)	44.5×10^{15} to 44.8×10^{15}	August 13th, 2002
D. Bernier	(Pentium 500MHz)	44.2×10^{15} to 44.5×10^{15}	August 12th, 2002
L. Marmet	(Riyadh)	43.9×10^{15} to 44.2×10^{15}	August 7th, 2002

User Name	(Computer Name)	*Range*	*DateCompleted*
N. Robertson	(Maya)	43.6×10^{15} to 43.9×10^{15}	July 31st, 2002
N. Robertson	(Laika.Droica)	43.3×10^{15} to 43.6×10^{15}	July 27th, 2002
J.-P. Bernier	(Pentium 600MHz)	43.0×10^{15} to 43.3×10^{15}	July 23rd, 2002
D. Bernier	(Pentium 500MHz)	42.7×10^{15} to 43.0×10^{15}	July 11th, 2002
L. Marmet	(Riyadh)	42.4×10^{15} to 42.7×10^{15}	July 11th, 2002
J.-P. Bernier	(Pentium 600MHz)	42.1×10^{15} to 42.4×10^{15}	June 24th, 2002
N. Robertson	(Laika.Droica)	41.8×10^{15} to 42.1×10^{15}	June 17th, 2002
L. Marmet	(Riyadh)	41.5×10^{15} to 41.8×10^{15}	June 14th, 2002
N. Robertson	(Maya)	41.2×10^{15} to 41.5×10^{15}	June 9th, 2002
N. Robertson	(Laika.Droica)	40.9×10^{15} to 41.2×10^{15}	May 28th, 2002
J.-P. Bernier	(Pentium 600MHz)	40.6×10^{15} to 40.9×10^{15}	June 5th, 2002
D. Bernier	(Pentium 500MHz)	40.3×10^{15} to 40.6×10^{15}	May 26th, 2002
N. Robertson	(Maya)	40.0×10^{15} to 40.3×10^{15}	March 6th, 2003
J.-P. Bernier	(Pentium 600MHz)	39.7×10^{15} to 40.0×10^{15}	May 2nd, 2002
D. Bernier	(Pentium 500MHz)	39.4×10^{15} to 39.7×10^{15}	May 1st, 2002
N. Robertson	(Maya)	39.1×10^{15} to 39.4×10^{15}	April 23rd, 2002
N. Robertson	(Laika.Droica)	38.8×10^{15} to 39.1×10^{15}	April 18th, 2002
L. Marmet	(Riyadh)	38.5×10^{15} to 38.8×10^{15}	April 26th, 2002
N. Robertson	(Maya)	38.2×10^{15} to 38.5×10^{15}	March 25th, 2002
N. Robertson	(Laika.Droica)	37.9×10^{15} to 38.2×10^{15}	March 20th, 2002
L. Marmet	(Riyadh)	37.6×10^{15} to 37.9×10^{15}	March 27th, 2002
J.-P. Bernier	(Pentium 600MHz)	37.3×10^{15} to 37.6×10^{15}	March 6th, 2002
D. Bernier	(Pentium 500MHz)	37.0×10^{15} to 37.3×10^{15}	March 12th, 2002
N. Robertson	(Maya)	36.7×10^{15} to 37.0×10^{15}	March 4th, 2002
N. Robertson	(Laika.Droica)	36.4×10^{15} to 36.7×10^{15}	February 28th, 2002
D. Bernier	(Pentium 500MHz)	36.2×10^{15} to 36.4×10^{15}	June 17th, 2002
N. Robertson	(Arthurus)	36.0×10^{15} to 36.2×10^{15}	October 2nd, 2002
L. Marmet	(Riyadh)	35.7×10^{15} to 36.0×10^{15}	February 18th, 2002
C.R. Ward	(Cosmos)	35.4×10^{15} to 35.7×10^{15}	March 10th, 2002
J.-P. Bernier	(Pentium 600MHz)	35.1×10^{15} to 35.4×10^{15}	February 5th, 2002
D. Bernier	(Pentium 500MHz)	34.8×10^{15} to 35.1×10^{15}	February 2nd, 2002
L. Marmet	(Riyadh)	34.5×10^{15} to 34.8×10^{15}	January 6th, 2002
N. Robertson	(Maya)	34.2×10^{15} to 34.5×10^{15}	December 30th, 2001
N. Robertson	(Laika.Droica)	33.9×10^{15} to 34.2×10^{15}	December 25th, 2001
J.-P. Bernier	(Pentium 600MHz)	33.6×10^{15} to 33.9×10^{15}	January 10th, 2002
C.R. Ward	(Cosmos)	33.3×10^{15} to 33.6×10^{15}	January 16th, 2002
L. Marmet	(Riyadh)	33.0×10^{15} to 33.3×10^{15}	December 8th, 2001
N. Robertson	(Maya)	32.7×10^{15} to 33.0×10^{15}	December 4th, 2001
N. Robertson	(Laika.Droica)	32.4×10^{15} to 32.7×10^{15}	November 27th, 2001
D. Bernier	(Pentium 500MHz)	32.1×10^{15} to 32.4×10^{15}	December 26th, 2001

User Name	(Computer Name)	*Range*	*DateCompleted*
J.-P. Bernier	(Pentium 600MHz)	31.8×10^{15} to 32.1×10^{15}	December 6th, 2001
J.-P. Bernier	(Pentium 600MHz)	31.5×10^{15} to 31.8×10^{15}	November 1st, 2001
C.R. Ward	(Cosmos)	31.2×10^{15} to 31.5×10^{15}	November 25th, 2001
L. Marmet	(Riyadh)	30.9×10^{15} to 31.2×10^{15}	November 2nd, 2001
N. Robertson	(Maya)	30.6×10^{15} to 30.9×10^{15}	October 26th, 2001
N. Robertson	(Laika.Droica)	30.3×10^{15} to 30.6×10^{15}	October 20th, 2001
J.-P. Bernier	(Pentium 600MHz)	30.0×10^{15} to 30.3×10^{15}	September 29th, 2001
N. Robertson	(Maya)	29.7×10^{15} to 30.0×10^{15}	September 18th, 2001
N. Robertson	(Laika.Droica)	29.4×10^{15} to 29.7×10^{15}	September 16th, 2001
L. Marmet	(Riyadh)	29.1×10^{15} to 29.4×10^{15}	September 19th, 2001
N. Robertson	(Rosette.Droica)	28.9×10^{15} to 29.1×10^{15}	December 12th, 2001
N. Robertson	(Laika.Droica)	28.6×10^{15} to 28.9×10^{15}	August 28th, 2001
J.-P. Bernier	(Pentium 600MHz)	28.3×10^{15} to 28.6×10^{15}	September 2nd, 2001
N. Robertson	(Maya)	28.0×10^{15} to 28.3×10^{15}	August 28th, 2001
C.R. Ward	(Cosmos)	27.7×10^{15} to 28.0×10^{15}	September 22nd, 2001
L. Marmet	(Riyadh)	27.4×10^{15} to 27.7×10^{15}	August 22nd, 2001
N. Robertson	(Maya)	27.1×10^{15} to 27.4×10^{15}	August 7th, 2001
J.-P. Bernier	(Pentium 600MHz)	26.8×10^{15} to 27.1×10^{15}	August 8th, 2001
D. Bernier	(Pentium 500MHz)	26.5×10^{15} to 26.8×10^{15}	April 6th, 2002
C.R. Ward	(Cosmos)	26.2×10^{15} to 26.5×10^{15}	April 29th, 2002
D. Bernier	(Pentium 500MHz)	25.9×10^{15} to 26.2×10^{15}	November 2nd, 2001
N. Robertson	(Maya)	25.6×10^{15} to 25.9×10^{15}	July 16th, 2001
J.-P. Bernier	(Pentium 600MHz)	25.3×10^{15} to 25.6×10^{15}	July 15th, 2001
L. Marmet	(Riyadh)	25.0×10^{15} to 25.3×10^{15}	July 23rd, 2001
C.R. Ward	(Cosmos)	24.7×10^{15} to 25.0×10^{15}	August 2nd, 2001
J.-P. Bernier	(Pentium 600MHz)	24.4×10^{15} to 24.7×10^{15}	April 8th, 2002
Z. McGregor-Dorsey	(Abzug)	24.1×10^{15} to 24.4×10^{15}	July 7th, 2001
N. Robertson	(Arthurus)	23.9×10^{15} to 24.1×10^{15}	November 30th, 2001
L. Marmet	(Riyadh)	23.6×10^{15} to 23.9×10^{15}	June 14th, 2001
N. Robertson	(Maya)	23.3×10^{15} to 23.6×10^{15}	June 18th, 2001
Z. McGregor-Dorsey	(Hayduke)	23.0×10^{15} to 23.3×10^{15}	July 7th, 2001
J.-P. Bernier	(Pentium 600MHz)	22.7×10^{15} to 23.0×10^{15}	June 16th, 2001
J.-P. Bernier	(Pentium 600MHz)	22.4×10^{15} to 22.7×10^{15}	May 26th, 2001
D. Bernier	(Pentium 500MHz)	22.1×10^{15} to 22.4×10^{15}	June 19th, 2001
Z. McGregor-Dorsey	(Castalia)	21.8×10^{15} to 22.1×10^{15}	June 5th, 2001
C.R. Ward	(Cosmos)	21.5×10^{15} to 21.8×10^{15}	June 7th, 2001
J.-P. Bernier	(Pentium 600MHz)	21.2×10^{15} to 21.5×10^{15}	May 8th, 2001
L. Marmet	(Riyadh)	20.9×10^{15} to 21.2×10^{15}	May 16th, 2001
D. Bernier	(Pentium 500MHz)	20.6×10^{15} to 20.9×10^{15}	May 3rd, 2001
L. Marmet	(Fontaine)	20.3×10^{15} to 20.6×10^{15}	May 8th, 2001

User Name	(Computer Name)	*Range*	*DateCompleted*
J.-P. Bernier	(Pentium 600MHz)	20.0×10^{15} to 20.3×10^{15}	April 20th, 2001
C.R. Ward	(Cosmos)	19.8×10^{15} to 20.0×10^{15}	April 20th, 2001
L. Marmet	(Riyadh)	19.6×10^{15} to 19.8×10^{15}	April 18th, 2001
Z. McGregor-Dorsey	(Abzug)	19.4×10^{15} to 19.6×10^{15}	May 27th, 2001
Z. McGregor-Dorsey	(Hayduke)	19.2×10^{15} to 19.4×10^{15}	May 10th, 2001
D. Bernier	(Pentium 500MHz)	19.0×10^{15} to 19.2×10^{15}	April 11th, 2001
L. Marmet	(Fontaine)	18.8×10^{15} to 19.0×10^{15}	April 7th, 2001
D. Bernier	(Pentium 500MHz)	18.6×10^{15} to 18.8×10^{15}	April 4th, 2001
J.-P. Bernier	(Pentium 600MHz)	18.4×10^{15} to 18.6×10^{15}	March 27th, 2001
L. Marmet	(Riyadh)	18.2×10^{15} to 18.4×10^{15}	April 1st, 2001
D. Bernier	(Pentium 500MHz)	18.0×10^{15} to 18.2×10^{15}	March 18th, 2001
L. Marmet	(Fontaine)	17.8×10^{15} to 18.0×10^{15}	March 21st, 2001
L. Marmet	(Riyadh)	17.6×10^{15} to 17.8×10^{15}	March 15th, 2001
J.-P. Bernier	(Pentium 600MHz)	17.4×10^{15} to 17.6×10^{15}	March 14th, 2001
D. Bernier	(Pentium 500MHz)	17.2×10^{15} to 17.4×10^{15}	March 5th, 2001
J.-P. Bernier	(Pentium 600MHz)	17.0×10^{15} to 17.2×10^{15}	March 2nd, 2001
L. Marmet	(Fontaine)	16.8×10^{15} to 17.0×10^{15}	March 3rd, 2001
L. Marmet	(Riyadh)	16.6×10^{15} to 16.8×10^{15}	February 26th, 2001
D. Bernier	(Pentium 500MHz)	16.4×10^{15} to 16.6×10^{15}	February 19th, 2001
J.-P. Bernier	(Pentium 600MHz)	16.2×10^{15} to 16.4×10^{15}	February 17th, 2001
N. Robertson	(Arthurus)	16.0×10^{15} to 16.2×10^{15}	May 27th, 2001
L. Marmet	(Fontaine)	15.8×10^{15} to 16.0×10^{15}	February 11th, 2001
Z. McGregor-Dorsey	(Castalia)	15.6×10^{15} to 15.8×10^{15}	April 25th, 2001
Z. McGregor-Dorsey	(Abzug)	15.4×10^{15} to 15.6×10^{15}	April 17th, 2001
L. Marmet	(Riyadh)	15.2×10^{15} to 15.4×10^{15}	February 8th, 2001
Z. McGregor-Dorsey	(Hayduke)	15.0×10^{15} to 15.2×10^{15}	April 5th, 2001
J.-P. Bernier	(Pentium 600MHz)	14.8×10^{15} to 15.0×10^{15}	February 4th, 2001
L. Marmet	(Riyadh)	14.6×10^{15} to 14.8×10^{15}	January 22nd, 2001
Z. McGregor-Dorsey	(Castalia)	14.4×10^{15} to 14.6×10^{15}	April 3rd, 2001
Z. McGregor-Dorsey	(Abzug)	14.2×10^{15} to 14.4×10^{15}	March 19th, 2001
Z. McGregor-Dorsey	(Hayduke)	14.0×10^{15} to 14.2×10^{15}	March 19th, 2001
L. Marmet	(Strontium)	13.8×10^{15} to 14.0×10^{15}	December 17th, 2000
L. Marmet	(Riyadh)	13.6×10^{15} to 13.8×10^{15}	January 5th, 2001
L. Marmet	(Strontium)	13.4×10^{15} to 13.6×10^{15}	December 11th, 2000
L. Marmet	(Fontaine)	13.2×10^{15} to 13.4×10^{15}	December 13th, 2000
L. Marmet	(Strontium)	13.0×10^{15} to 13.2×10^{15}	December 4th, 2000
L. Marmet	(Riyadh)	12.8×10^{15} to 13.0×10^{15}	December 9th, 2000
L. Marmet	(Strontium)	12.6×10^{15} to 12.8×10^{15}	November 26th, 2000
L. Marmet	(Fontaine)	12.4×10^{15} to 12.6×10^{15}	November 24th, 2000
Z. McGregor-Dorsey	(Abzug)	12.2×10^{15} to 12.4×10^{15}	January 21st, 2001

User Name	(Computer Name)	*Range*	*DateCompleted*
L. Marmet	(Riyadh)	12.0×10^{15} to 12.2×10^{15}	November 22nd, 2000
L. Marmet	(Strontium)	11.8×10^{15} to 12.0×10^{15}	November 17th, 2000
Z. McGregor-Dorsey	(Castalia)	11.6×10^{15} to 11.8×10^{15}	January 22nd, 2001
L. Marmet	(Riyadh)	11.4×10^{15} to 11.6×10^{15}	November 7th, 2000
Z. McGregor-Dorsey	(Hayduke)	11.2×10^{15} to 11.4×10^{15}	January 21st, 2001
L. Marmet	(Riyadh)	11.0×10^{15} to 11.2×10^{15}	November 1st, 2000
Z. McGregor-Dorsey	(Hayduke)	10.8×10^{15} to 11.0×10^{15}	December 10th, 2000
Z. McGregor-Dorsey	(Castalia)	10.6×10^{15} to 10.8×10^{15}	December 10th, 2000
L. Marmet	(Riyadh)	10.4×10^{15} to 10.6×10^{15}	October 13th, 2000
N. Robertson	(Arthurus)	10.2×10^{15} to 10.4×10^{15}	January 27th, 2001
N. Marmet	(Computer)	10.0×10^{15} to 10.2×10^{15}	October 21st, 2000
L. Marmet	(Riyadh)	9.8×10^{15} to 10.0×10^{15}	September 27th, 2000
Z. McGregor-Dorsey	(Castalia)	9.6×10^{15} to 9.8×10^{15}	November 16th, 2000
Z. McGregor-Dorsey	(Hayduke)	9.4×10^{15} to 9.6×10^{15}	January 26th, 2001
L. Marmet	(Fontaine)	9.2×10^{15} to 9.4×10^{15}	January 26th, 2001
Z. McGregor-Dorsey	(Castalia)	9.0×10^{15} to 9.2×10^{15}	October 24th, 2000
Z. McGregor-Dorsey	(Castalia)	8.8×10^{15} to 9.0×10^{15}	September 27th, 2000
Z. McGregor-Dorsey	(Hayduke)	8.6×10^{15} to 8.8×10^{15}	September 11th, 2000
L. Marmet	(Riyadh)	8.4×10^{15} to 8.6×10^{15}	September 11th, 2000
Z. McGregor-Dorsey	(Castalia)	8.2×10^{15} to 8.4×10^{15}	September 3rd, 2000
Z. McGregor-Dorsey	(Abzug)	8.0×10^{15} to 8.2×10^{15}	October 16th, 2000
Z. McGregor-Dorsey	(Hayduke)	7.8×10^{15} to 8.0×10^{15}	October 16th, 2000
Z. McGregor-Dorsey	(Abzug)	7.6×10^{15} to 7.8×10^{15}	September 27th, 2000
Z. McGregor-Dorsey	(Hayduke)	7.4×10^{15} to 7.6×10^{15}	August 26th, 2000
N. Marmet	(Computer)	7.2×10^{15} to 7.4×10^{15}	September 17th, 2000
L. Marmet	(Riyadh)	7.0×10^{15} to 7.2×10^{15}	August 25th, 2000
G. Engebreth	(Computer)	6.8×10^{15} to 7.0×10^{15}	August 28th, 2000
Z. McGregor-Dorsey	(Hayduke)	6.6×10^{15} to 6.8×10^{15}	August 17th, 2000
Z. McGregor-Dorsey	(Abzug)	6.4×10^{15} to 6.6×10^{15}	September 2nd, 2000
Z. McGregor-Dorsey	(Castalia)	6.2×10^{15} to 6.4×10^{15}	August 26th, 2000
Z. McGregor-Dorsey	(Abzug)	6.0×10^{15} to 6.2×10^{15}	August 7th, 2000
G. Engebreth	(Computer)	5.8×10^{15} to 6.0×10^{15}	August 10th, 2000
L. Marmet	(Riyadh)	5.6×10^{15} to 5.8×10^{15}	August 6th, 2000
Z. McGregor-Dorsey	(Abzug)	5.4×10^{15} to 5.6×10^{15}	August 26th, 2000
Z. McGregor-Dorsey	(Hayduke)	5.2×10^{15} to 5.4×10^{15}	July 28th, 2000
Z. McGregor-Dorsey	(Castalia)	5.0×10^{15} to 5.2×10^{15}	August 16th, 2000
Z. McGregor-Dorsey	(Castalia)	4.8×10^{15} to 5.0×10^{15}	July 28th, 2000
N. Marmet	(Computer)	4.6×10^{15} to 4.8×10^{15}	August 15th, 2000
E. Wong	(Computer)	4.4×10^{15} to 4.6×10^{15}	September 27th, 2000
Z. McGregor-Dorsey	(Hayduke)	4.2×10^{15} to 4.4×10^{15}	July 14th, 2000

User Name	(Computer Name)	*Range*	*DateCompleted*
L. Marmet	(Riyadh)	4.0×10^{15} to 4.2×10^{15}	July 20^{th}, 2000
Z. McGregor-Dorsey	(Abzug)	3.8×10^{15} to 4.0×10^{15}	July 18^{th}, 2000
Z. McGregor-Dorsey	(Castalia)	3.6×10^{15} to 3.8×10^{15}	July 10^{th}, 2000
Z. McGregor-Dorsey	(Abzug)	3.4×10^{15} to 3.6×10^{15}	July 10^{th}, 2000
Z. McGregor-Dorsey	(Hayduke)	3.2×10^{15} to 3.4×10^{15}	July 1^{st}, 2000
L. Marmet	(Riyadh)	3.0×10^{15} to 3.2×10^{15}	June 29^{th}, 2000
Z. McGregor-Dorsey	(Castalia)	2.8×10^{15} to 3.0×10^{15}	June 18^{th}, 2000
G. Engebreth	(Computer)	2.6×10^{15} to 2.8×10^{15}	July 22^{nd}, 2000
Z. McGregor-Dorsey	(Castalia)	2.4×10^{15} to 2.6×10^{15}	June 8^{th}, 2000
N. Marmet	(Computer)	2.2×10^{15} to 2.4×10^{15}	July 5^{th}, 2000
L. Marmet	(Riyadh)	2.0×10^{15} to 2.2×10^{15}	June 9^{th}, 2000
Z. McGregor-Dorsey	(Abzug)	1.8×10^{15} to 2.0×10^{15}	May 27^{th}, 2000
A. Simpson	(Computer)	1.6×10^{15} to 1.8×10^{15}	June 30^{th}, 2000
Z. McGregor-Dorsey	(Hayduke)	1.4×10^{15} to 1.6×10^{15}	May 13^{th}, 2000
L. Marmet	(Riyadh)	1.2×10^{15} to 1.4×10^{15}	May 16^{th}, 2000
G. Engebreth	(Computer)	1.1×10^{15} to 1.2×10^{15}	June 1^{st}, 2000
E. Wong	(Computer)	1.0×10^{15} to 1.1×10^{15}	May 31^{st}, 2000
L. Marmet	(Lion)	9×10^{14} to 10×10^{14}	June 13^{th}, 2000
L. Marmet	(Riyadh)	8×10^{14} to 9×10^{14}	April 5^{th}, 2000
Z. McGregor-Dorsey	(Castalia)	7×10^{14} to 8×10^{14}	May 1^{st}, 2000
L. Marmet	(Fontaine)	6×10^{14} to 7×10^{14}	April 22^{nd}, 2000
E. Wong	(Computer)	5×10^{14} to 6×10^{14}	April 8^{th}, 2000
L. Marmet	(Fontaine)	4×10^{14} to 5×10^{14}	March 5^{th}, 2000
D. Bernier	(Pentium 500MHz)	3×10^{14} to 4×10^{14}	March 24^{th}, 2000
L. Marmet	(Riyadh)	2×10^{14} to 3×10^{14}	February 4^{th}, 2000
D. Bernier	(Pentium 500MHz)	1.5×10^{14} to 2.0×10^{14}	January 24^{th}, 2000
L. Marmet	(Lion)	4×10^{0} to 1500×10^{11}	December 20^{th}, 1999