

# A catalog of matchstick graphs

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## Abstract

Classification of planar unit-distance graphs with up to 9 edges, by homeomorphism and isomorphism classes. With exactly nine edges, there are 633 nonisomorphic connected matchstick graphs, of which 197 are topologically distinct from each other. Increasing edges' number, their quantities rise more than exponentially, in a still unclear way.

## 1. Introduction

Two remarkable classes of graphs are planar graphs and unit distance graphs. Their intersection, that is the class of the planar graphs drawable using edges of unitary length, has been defined as the “matchstick graphs” class[1].

It is an NP-hard problem to test if a given graph can be represented as a matchstick graph (Eades, Wormland 1990 [2]; Cabello et al. 2007 [3]), and no formulas are known to determine how many distinct matchstick graphs exist for a given number of edges.

The problem of finding the number of topologically distinct matchstick graph with  $n$  edges is known as the “match problem” (Gardner 1969 [4]). R. Read (1968 [5]) solved the problem for  $n=8$ , reporting also the total number of matchstick graphs –up to isomorphism- with 7 and 8 edges.

Here are listed all the distinct matchstick graphs with less of 10 edges, grouped by topological equivalence.

## 2. Overview

Table 1 summarizes the number of non-homeomorphic and non-isomorphic connected planar unit distance graphs. For a quicker referring, henceforth will be assumed the following definitions:

**Definition 2.1.** Given a nonnegative integer  $n$ ,  $q(n)$  indicates the number of distinct connected matchstick graph, up to topological homeomorphism, with  $n$  edges.

**Definition 2.2.** Given a nonnegative integer  $n$ ,  $p(n)$  indicates the number of distinct connected matchstick graph, up to isomorphism, with  $n$  edges.

$n$	$q(n)$	$p(n)$
1	1	1
2	1	1
3	3	3
4	5	5
5	10	12
6	19	28
7	39	74
8	84	207
9	197	633

Table 1.

So far, the first 8 terms of both sequences have been known [6]. Here are first showed the values of  $q(9)$  and  $p(9)$ . The evolution of both sequences shall be discussed in section 4.

In table 2, topologically different matchstick graphs are reported by number of edges and number of faces (including the outer region).

In the next section, every one of the 964 distinct matchstick graph with  $|E| < 10$  will be showed. They are listed and progressively numbered, in order of priority, by:

1. Number of edges  $|E|$ ;
2. Number of faces  $F$ ;
3. Maximum degree  $\Delta$ .

		$F$					
		1	2	3	4	5	6
$ E $	1	1	0	0	0	0	0
	2	1	0	0	0	0	0
	3	2	1	0	0	0	0
	4	3	2	0	0	0	0
	5	5	4	1	0	0	0
	6	7	8	4	0	0	0
	7	11	15	12	1	0	0
	8	16	29	31	8	0	0
	9	26	56	75	38	3	0

Table 2.

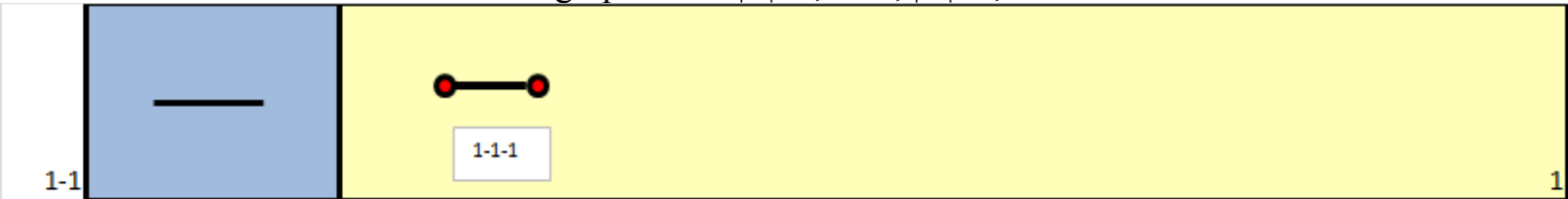
### 3. Classification

Drawings in the left cells of tables are aimed to illustrate the topological homeomorphism between graphs in the same box. The number on the bottom right of right cells indicates the number of graphs contained therein. Graphs 8-81-1, 9-84-1, 9-177-1, 9-180-1, 9-180-2 and 9-191-1 are represented magnified by a factor of 1.5 respect to others, and 9-164-1 by a factor of 2.5, to avoid apparent overlaps.

#### 3.1. Matchstick graphs with $|E|=1$

##### 3.1.1. Matchstick graphs with $|E|=1, F=1, |V|=2$

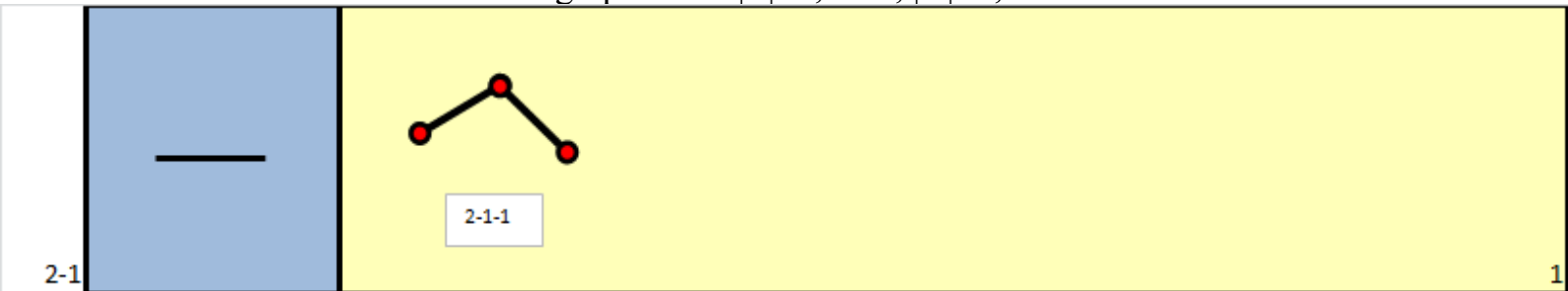
##### 3.1.1.1. Matchstick graphs with $|E|=1, F=1, |V|=2, \Delta=1$



#### 3.2. Matchstick graphs with $|E|=2$

##### 3.2.1. Matchstick graphs with $|E|=2, F=1, |V|=3$

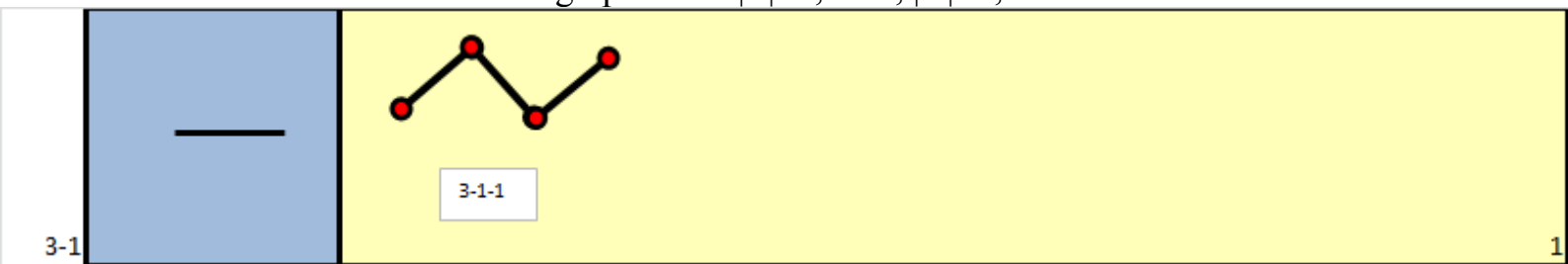
##### 3.2.1.1. Matchstick graphs with $|E|=2, F=1, |V|=3, \Delta=2$



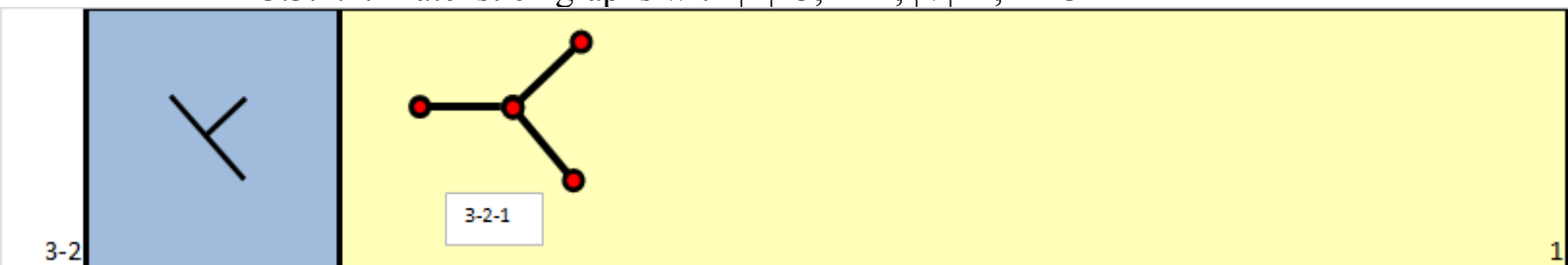
#### 3.3. Matchstick graphs with $|E|=3$

##### 3.3.1. Matchstick graphs with $|E|=3, F=1, |V|=4$

##### 3.3.1.1. Matchstick graphs with $|E|=3, F=1, |V|=4, \Delta=2$



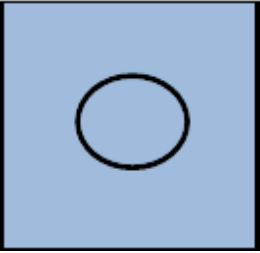
##### 3.3.1.2. Matchstick graphs with $|E|=3, F=1, |V|=4, \Delta=3$



##### 3.3.2. Matchstick graphs with $|E|=3, F=2, |V|=3$

##### 3.3.2.1. Matchstick graphs with $|E|=3, F=2, |V|=3, \Delta=2$

3-3



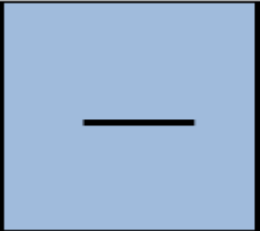
1

### 3.4. Matchstick graphs with $|E|=4$

#### 3.4.1. Matchstick graphs with $|E|=4, F=1, |V|=5$

##### 3.4.1.1. Matchstick graphs with $|E|=4, F=1, |V|=5, \Delta=2$

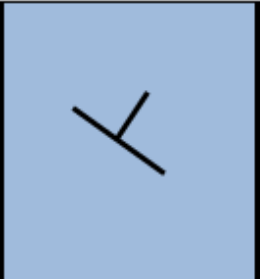
4-1



1

##### 3.4.1.2. Matchstick graphs with $|E|=4, F=1, |V|=5, \Delta=3$

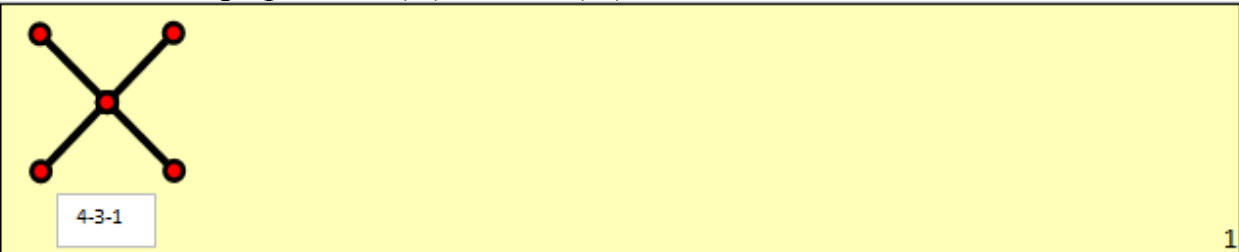
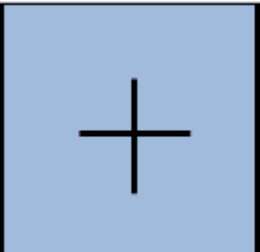
4-2



1

##### 3.4.1.3. Matchstick graphs with $|E|=4, F=1, |V|=5, \Delta=4$

4-3

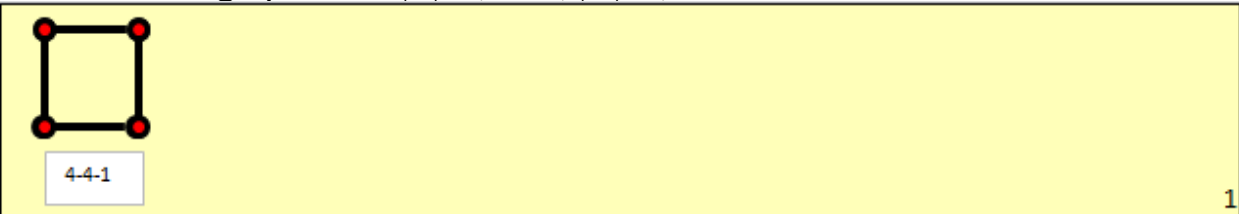
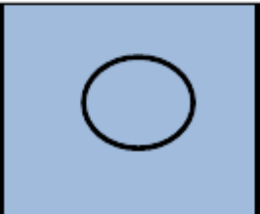


1

#### 3.4.2. Matchstick graphs with $|E|=4, F=2, |V|=4$

##### 3.4.2.1. Matchstick graphs with $|E|=4, F=2, |V|=4, \Delta=2$

4-4



1

##### 3.4.2.2. Matchstick graphs with $|E|=4, F=2, |V|=4, \Delta=3$

4-5



4-5-1

1

### 3.5. Matchstick graphs with $|E|=5$

#### 3.5.1. Matchstick graphs with $|E|=5, F=1, |V|=6$

##### 3.5.1.1. Matchstick graphs with $|E|=5, F=1, |V|=6, \Delta=2$

5-1



5-1-1

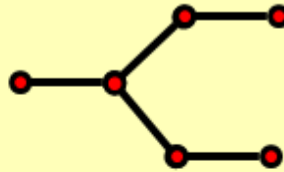
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##### 3.5.1.2. Matchstick graphs with $|E|=5, F=1, |V|=6, \Delta=3$

5-2



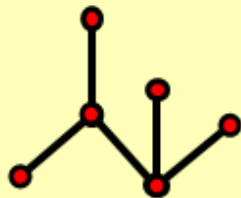
5-2-1



5-2-2

2

5-3

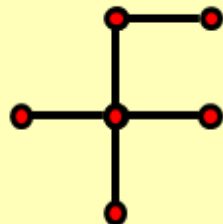


5-3-1

1

##### 3.5.1.3. Matchstick graphs with $|E|=5, F=1, |V|=6, \Delta=4$

5-4

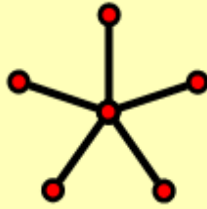


5-4-1

1

##### 3.5.1.3. Matchstick graphs with $|E|=5, F=1, |V|=6, \Delta=5$

5-5



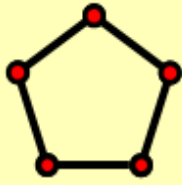
5-5-1

1

### 3.5.2. Matchstick graphs with $|E|=5, F=2, |V|=5$

#### 3.5.2.1. Matchstick graphs with $|E|=5, F=2, |V|=5, \Delta=2$

5-6

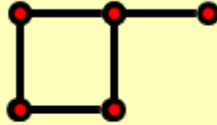


5-6-1

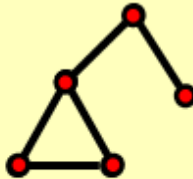
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#### 3.5.2.2. Matchstick graphs with $|E|=5, F=2, |V|=5, \Delta=3$

5-7



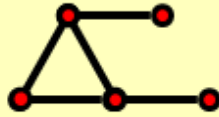
5-7-1



5-7-2

2

5-8

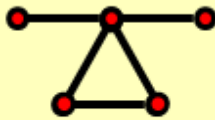


5-8-1

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#### 3.5.2.3. Matchstick graphs with $|E|=5, F=2, |V|=5, \Delta=4$

5-9



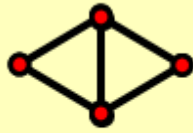
5-9-1

1

### 3.5.3. Matchstick graphs with $|E|=5, F=3, |V|=4$

#### 3.5.2.1. Matchstick graphs with $|E|=5, F=3, |V|=4, \Delta=3$

5-10



5-10-1

1

### 3.6. Matchstick graphs with $|E|=6$

#### 3.6.1. Matchstick graphs with $|E|=6, F=1, |V|=7$

##### 3.6.1.1. Matchstick graphs with $|E|=6, F=1, |V|=7, \Delta=2$

6-1



6-1-1

1

##### 3.6.1.2. Matchstick graphs with $|E|=6, F=1, |V|=7, \Delta=3$

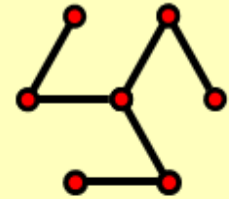
6-2



6-2-1



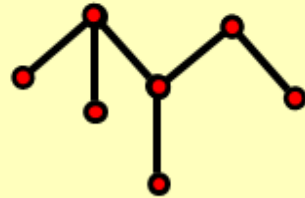
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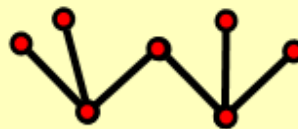
6-2-3

3

6-3



6-3-1

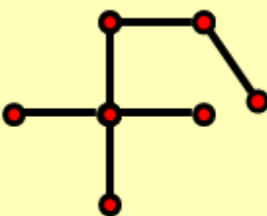


6-3-2

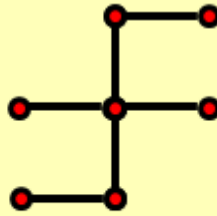
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##### 3.6.1.3. Matchstick graphs with $|E|=6, F=1, |V|=7, \Delta=4$

6-4



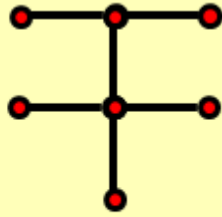
6-4-1



6-4-2

2

6-5

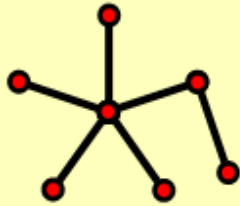


6-5-1

1

3.6.1.4. Matchstick graphs with  $|E|=6$ ,  $F=1$ ,  $|V|=7$ ,  $\Delta=5$

6-6

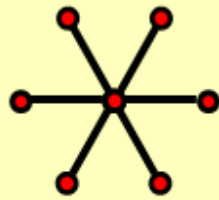


6-6-1

1

3.6.1.5. Matchstick graphs with  $|E|=6$ ,  $F=1$ ,  $|V|=7$ ,  $\Delta=6$

6-7



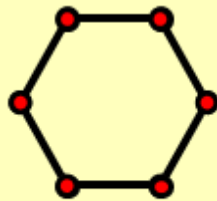
6-7-1

1

3.6.2. Matchstick graphs with  $|E|=6$ ,  $F=2$ ,  $|V|=6$

3.6.2.1. Matchstick graphs with  $|E|=6$ ,  $F=2$ ,  $|V|=6$ ,  $\Delta=2$


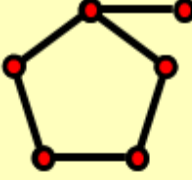
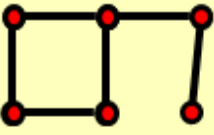
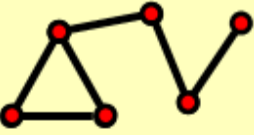

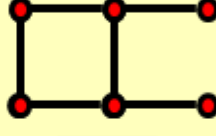
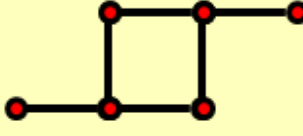
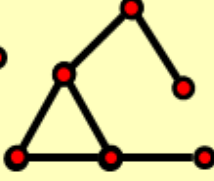
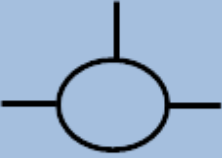
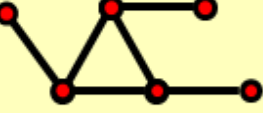


6-8




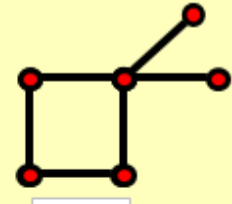
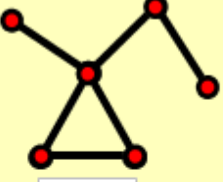
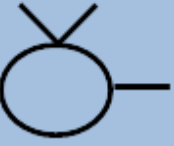
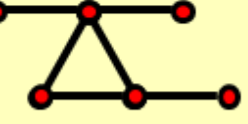
6-8-1

1

3.6.2.2. Matchstick graphs with  $|E|=6$ ,  $F=2$ ,  $|V|=6$ ,  $\Delta=3$

	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  6-9-1         </div> <div style="text-align: center;">  6-9-2         </div> <div style="text-align: center;">  6-9-3         </div> </div>	6-9	3
	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  6-10-1         </div> <div style="text-align: center;">  6-10-2         </div> <div style="text-align: center;">  6-10-3         </div> </div>	6-10	3
	<div style="display: flex; justify-content: center; align-items: center;"> <div style="text-align: center;">  6-11-1         </div> </div>	6-11	1
	<div style="display: flex; justify-content: center; align-items: center;"> <div style="text-align: center;">  6-12-1         </div> </div>	6-12	1

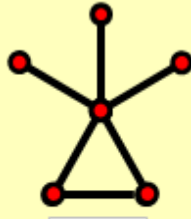
### 3.6.2.3. Matchstick graphs with $|E|=6$ , $F=2$ , $|V|=6$ , $\Delta=4$

	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  6-13-1         </div> <div style="text-align: center;">  6-13-2         </div> </div>	6-13	2
	<div style="display: flex; justify-content: center; align-items: center;"> <div style="text-align: center;">  6-14-1         </div> </div>	6-14	1

### 3.6.2.4. Matchstick graphs with $|E|=6$ , $F=2$ , $|V|=6$ , $\Delta=5$



6-15



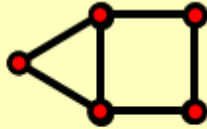
6-15-1

1

### 3.6.3. Matchstick graphs with $|E|=6$ , $F=3$ , $|V|=5$

#### 3.6.3.1. Matchstick graphs with $|E|=6$ , $F=3$ , $|V|=5$ , $\Delta=3$

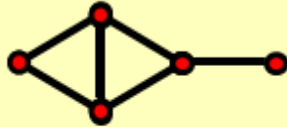
6-16



6-16-1

1

6-17

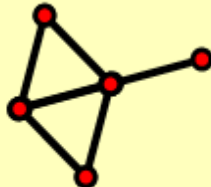


6-17-1

1

#### 3.6.3.2. Matchstick graphs with $|E|=6$ , $F=3$ , $|V|=5$ , $\Delta=4$

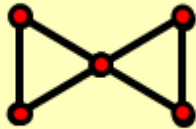
6-18



6-18-1

1

6-19



6-19-1

1

### 3.7. Matchstick graphs with $|E|=7$

#### 3.7.1. Matchstick graphs with $|E|=7$ , $F=1$ , $|V|=8$

##### 3.7.1.1. Matchstick graphs with $|E|=7$ , $F=1$ , $|V|=8$ , $\Delta=2$





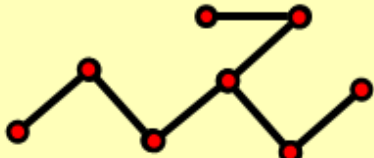

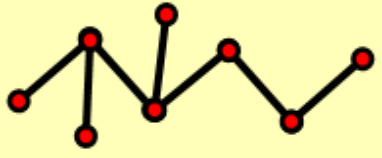
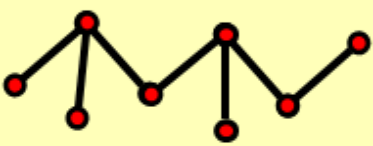

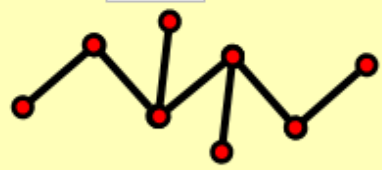
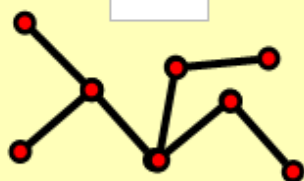

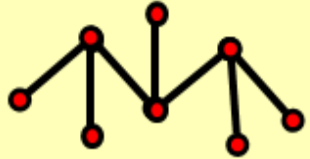
7-1




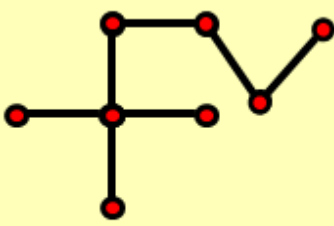
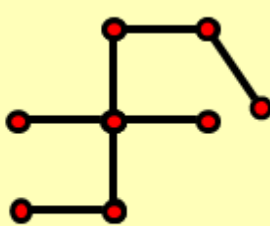
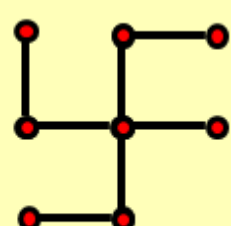
7-1-1


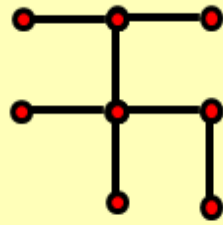
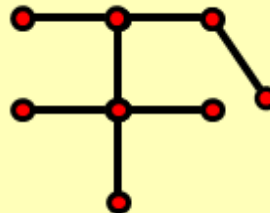
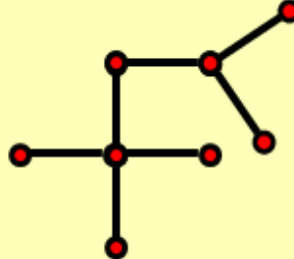
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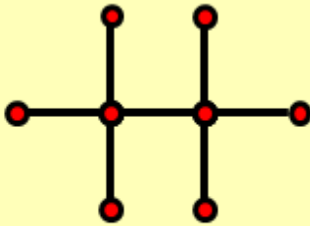
3.7.1.2. Matchstick graphs with  $|E|=7$ ,  $F=1$ ,  $|V|=8$ ,  $\Delta=3$

<p>7-2</p> 	 <p>7-2-1</p>  <p>7-2-2</p>  <p>7-2-3</p>  <p>7-4-1</p>	4
<p>7-3</p> 	 <p>7-3-1</p>  <p>7-3-2</p>  <p>7-3-3</p>  <p>7-3-4</p>  <p>7-3-5</p>	5
<p>7-4</p> 	 <p>7-4-1</p>	1


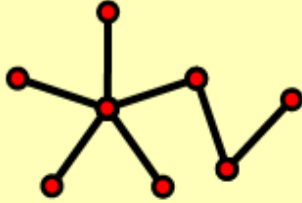
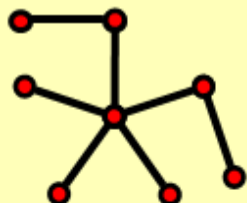
3.7.1.3. Matchstick graphs with  $|E|=7$ ,  $F=1$ ,  $|V|=8$ ,  $\Delta=4$


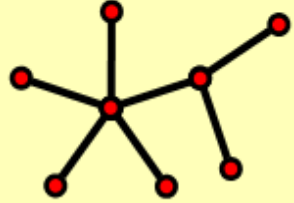
<p>7-5</p> 	 <p>7-5-1</p>  <p>7-5-2</p>  <p>7-5-3</p>	3
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	 <p>7-6-1</p>	 <p>7-6-2</p>	 <p>7-6-3</p>	3
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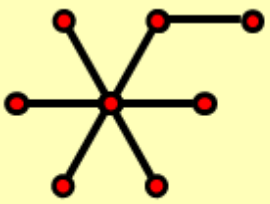
	 <p>7-7-1</p>	1
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3.7.1.4. Matchstick graphs with  $|E|=7, F=1, |V|=8, \Delta=5$

	 <p>7-8-1</p>	 <p>7-8-2</p>	2
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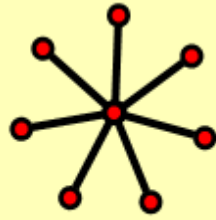
	 <p>7-9-1</p>	1
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3.7.1.5. Matchstick graphs with  $|E|=7, F=1, |V|=8, \Delta=6$

	 <p>7-10-1</p>	1
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3.7.1.6. Matchstick graphs with  $|E|=7, F=1, |V|=8, \Delta=7$

7-11



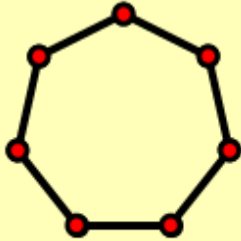
7-11-1

1

### 3.7.2. Matchstick graphs with $|E|=7$ , $F=2$ , $|V|=7$

#### 3.7.2.1. Matchstick graphs with $|E|=7$ , $F=2$ , $|V|=7$ , $\Delta=2$

7-12

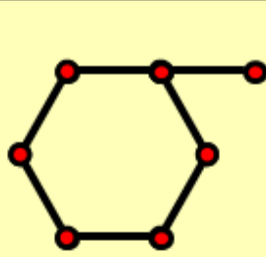


7-12-1

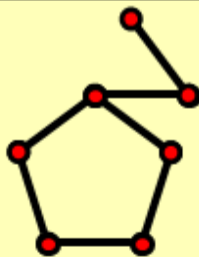
1

#### 3.7.2.2. Matchstick graphs with $|E|=7$ , $F=2$ , $|V|=7$ , $\Delta=3$

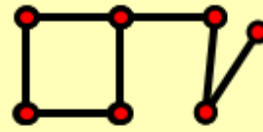
7-13



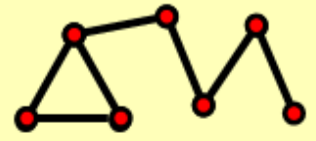
7-13-1



7-13-2



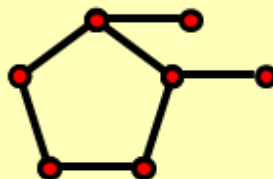
7-13-3



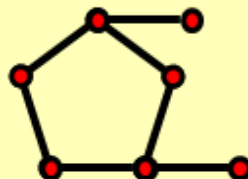
7-13-4

4

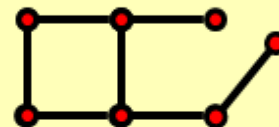
7-14



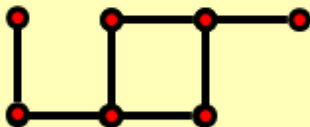
7-14-1



7-14-2



7-14-3



7-14-4



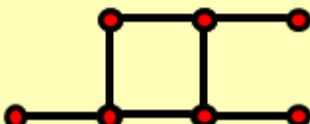
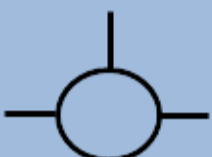
7-14-5



7-14-6

6

7-15


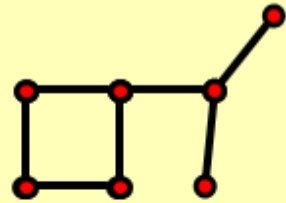
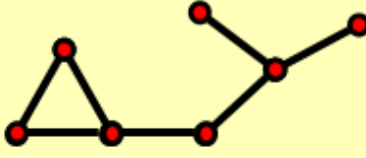

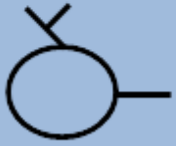
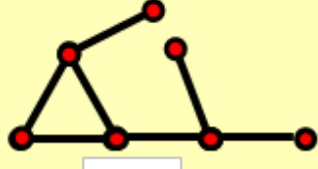


7-15-1


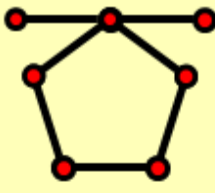
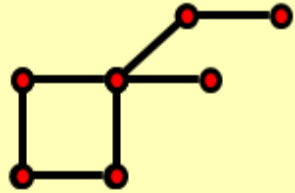
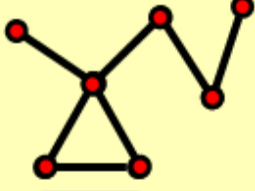
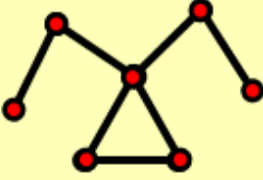
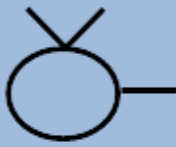
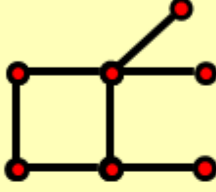
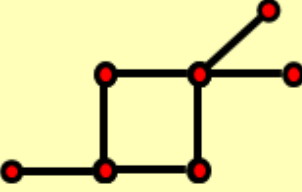
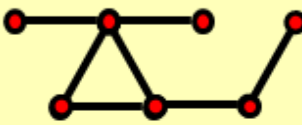
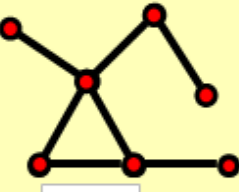

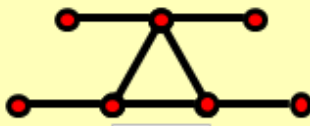

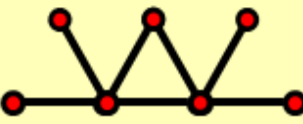



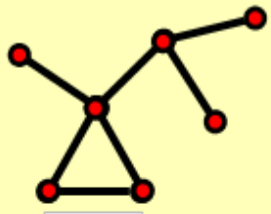
7-15-2


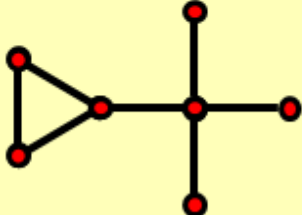
2

7-16		<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>7-16-1</p> </div> <div style="text-align: center;">  <p>7-16-2</p> </div> <div style="text-align: center;">  <p>7-16-3</p> </div> </div>	3
7-17		<div style="text-align: center;">  <p>7-17-1</p> </div>	1


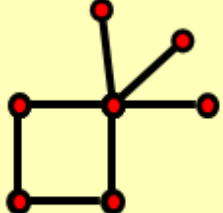
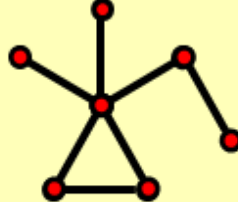
3.7.2.3. Matchstick graphs with  $|E|=7$ ,  $F=2$ ,  $|V|=7$ ,  $\Delta=4$


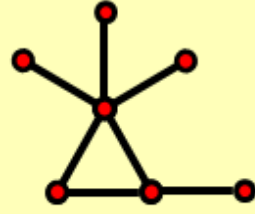
7-18		<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>7-18-1</p> </div> <div style="text-align: center;">  <p>7-18-2</p> </div> <div style="text-align: center;">  <p>7-18-3</p> </div> <div style="text-align: center;">  <p>7-18-4</p> </div> </div>	4
7-19		<div style="display: flex; flex-direction: column; align-items: center;"> <div style="display: flex; justify-content: space-around; width: 100%;"> <div style="text-align: center;">  <p>7-19-1</p> </div> <div style="text-align: center;">  <p>7-19-2</p> </div> <div style="text-align: center;">  <p>7-19-3</p> </div> </div> <div style="margin-top: 20px; text-align: center;">  <p>7-19-4</p> </div> </div>	4
7-20		<div style="text-align: center;">  <p>7-20-1</p> </div>	1
7-21		<div style="text-align: center;">  <p>7-21-1</p> </div>	1

7-22		 <p data-bbox="454 392 550 436">7-22-1</p>	1
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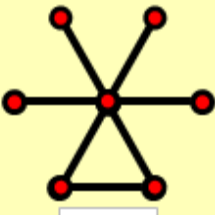
7-23		 <p data-bbox="454 683 550 728">7-23-1</p>	1
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3.7.2.4. Matchstick graphs with  $|E|=7$ ,  $F=2$ ,  $|V|=7$ ,  $\Delta=5$

7-24		 <p data-bbox="422 1131 518 1176">7-24-1</p>  <p data-bbox="678 1131 774 1176">7-24-2</p>	2
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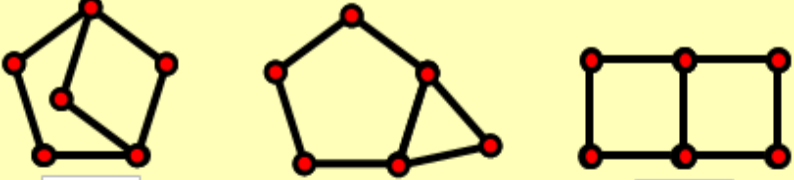

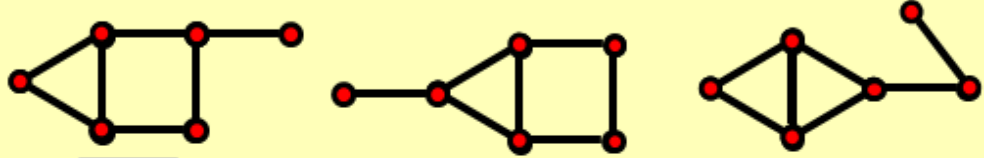


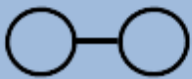
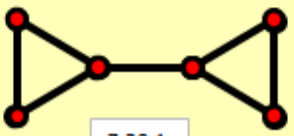
7-25		 <p data-bbox="438 1444 534 1489">7-25-1</p>	1
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3.7.2.5. Matchstick graphs with  $|E|=7$ ,  $F=2$ ,  $|V|=7$ ,  $\Delta=6$


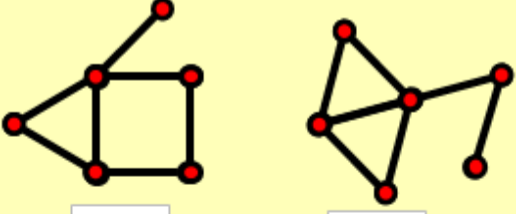
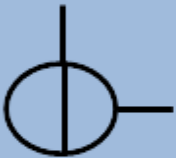
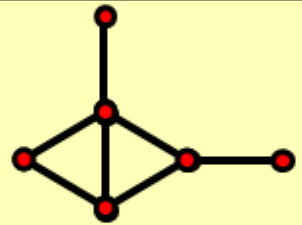
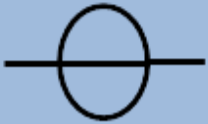
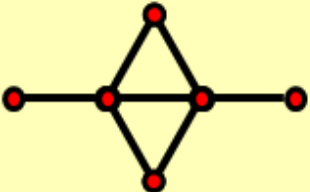
7-26		 <p data-bbox="454 1803 550 1848">7-26-1</p>	1
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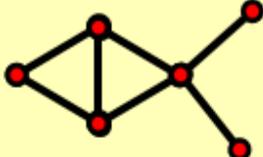

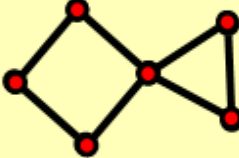


3.7.3. Matchstick graphs with  $|E|=7$ ,  $F=3$ ,  $|V|=6$

3.7.3.1. Matchstick graphs with  $|E|=7$ ,  $F=3$ ,  $|V|=6$ ,  $\Delta=3$


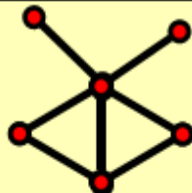

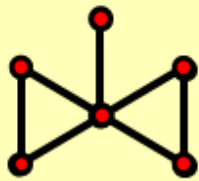
7-27			3
7-28			3
7-29			1
7-30			1

3.7.3.2. Matchstick graphs with  $|E|=7, F=3, |V|=6, \Delta=4$

7-31			2
7-32			1
7-33			1


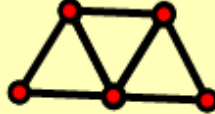
7-34		 <p data-bbox="454 324 550 369">7-34-1</p>	1
7-35		 <p data-bbox="454 582 534 627">7-35-1</p>	1
7-36		 <p data-bbox="454 806 534 851">7-36-1</p>	1

3.7.3.3. Matchstick graphs with  $|E|=7, F=3, |V|=6, \Delta=5$

7-37		 <p data-bbox="454 1153 550 1198">7-37-1</p>	1
7-38		 <p data-bbox="454 1422 534 1467">7-38-1</p>	1

3.7.4. Matchstick graphs with  $|E|=7, F=4, |V|=5$

3.7.4.1. Matchstick graphs with  $|E|=7, F=4, |V|=5, \Delta=4$

7-39		 <p data-bbox="454 1769 534 1814">7-39-1</p>	1
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3.8. Matchstick graphs with  $|E|=8$

3.8.1. Matchstick graphs with  $|E|=8, F=1, |V|=9$

3.8.1.1. Matchstick graphs with  $|E|=8, F=1, |V|=9, \Delta=2$



8-1

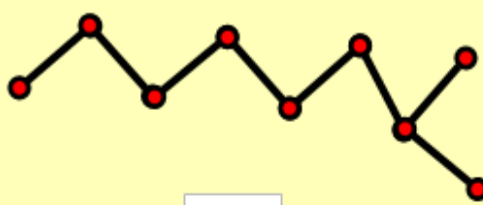


8-1-1

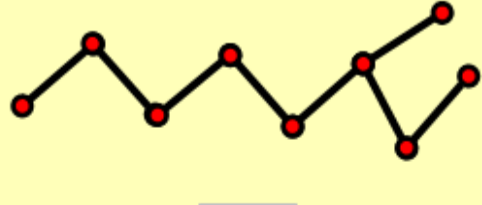
1

3.8.1.2. Matchstick graphs with  $|E|=8$ ,  $F=1$ ,  $|V|=9$ ,  $\Delta=3$

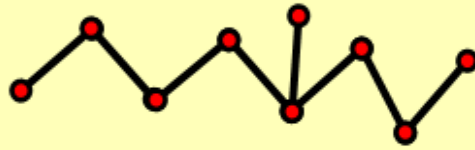
8-2



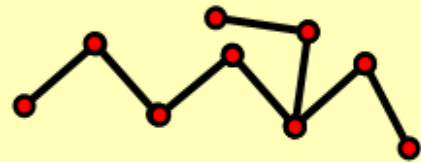
8-2-1



8-2-2



8-2-3

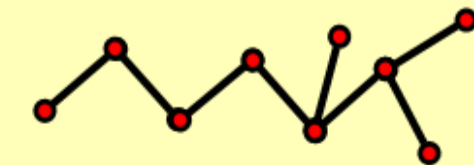


8-2-4



8-2-5

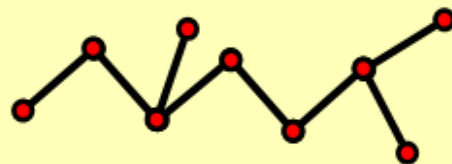
5



8-3-1



8-3-2



8-3-3



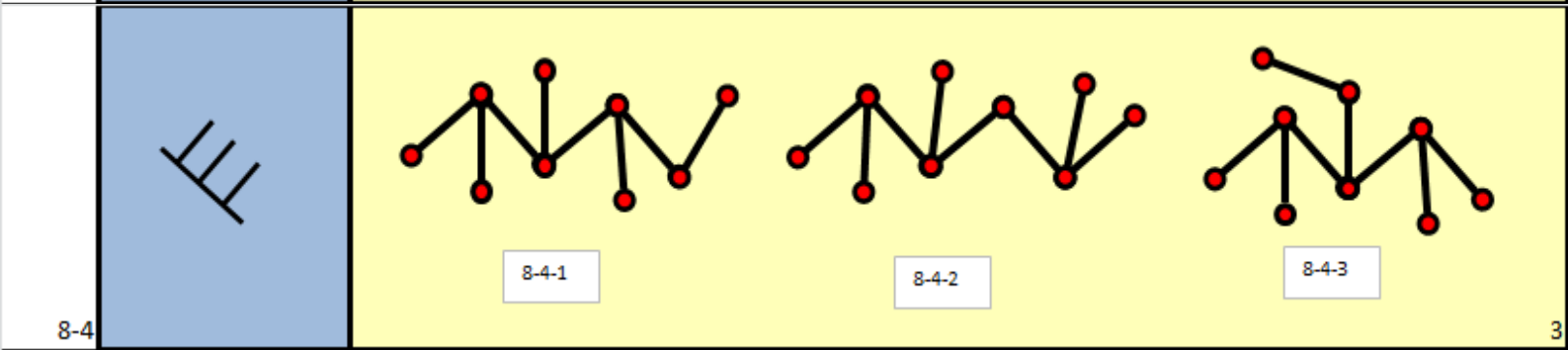
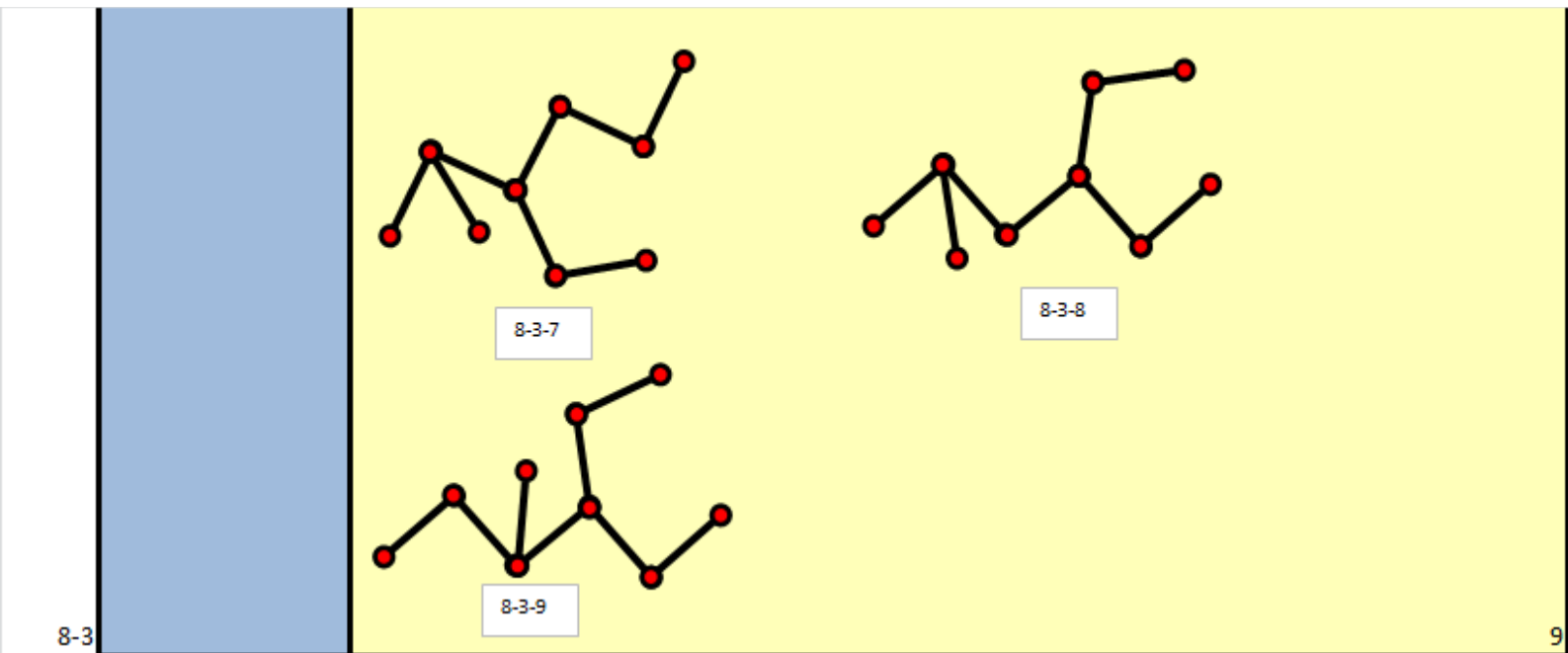
8-3-4



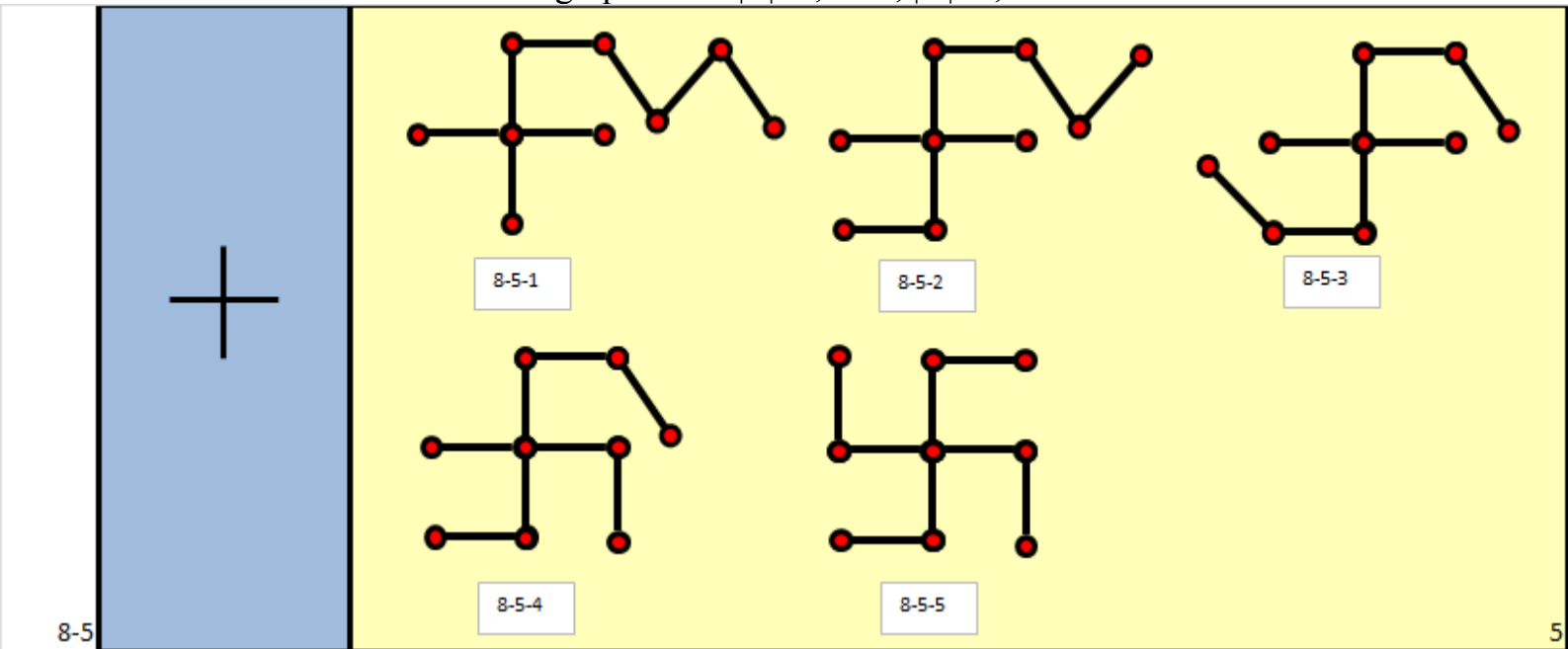
8-3-5


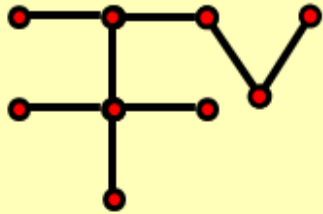
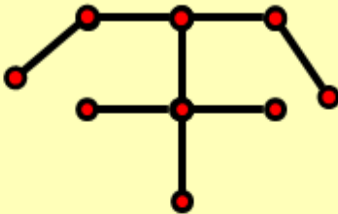
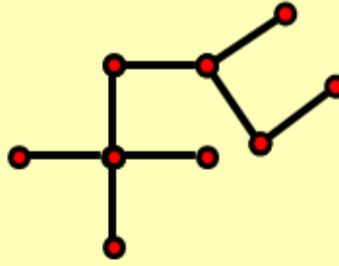
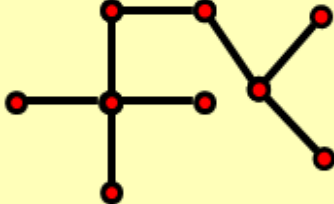
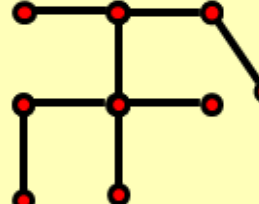
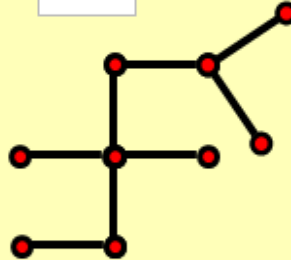
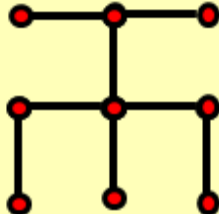
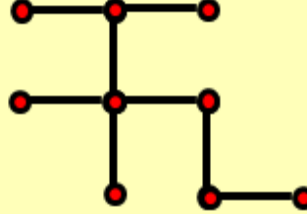

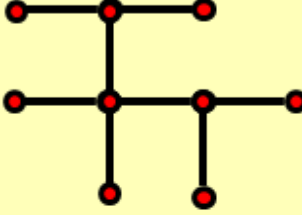

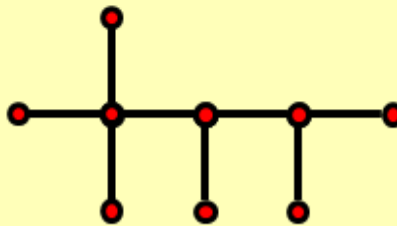



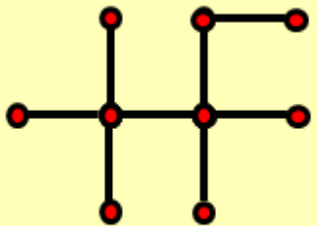
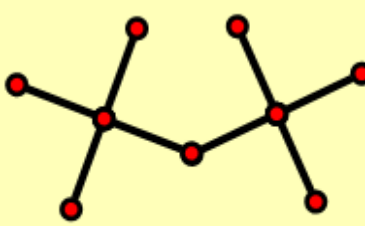
8-3-6




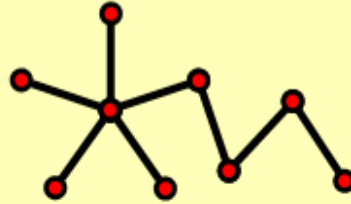
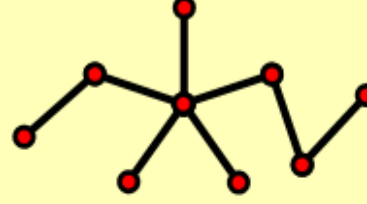
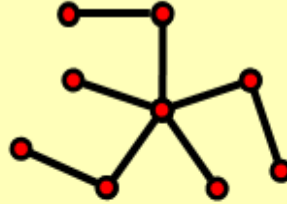
3.8.1.3. Matchstick graphs with  $|E|=8$ ,  $F=1$ ,  $|V|=9$ ,  $\Delta=4$


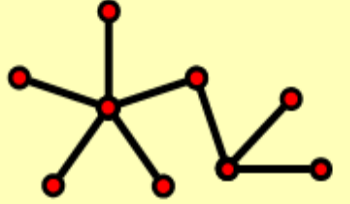
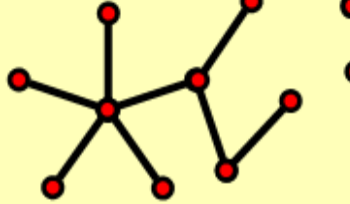
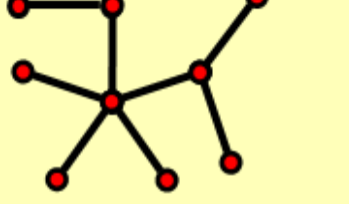



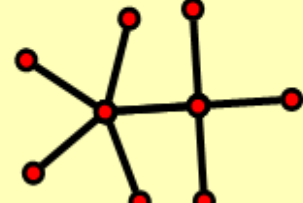
<p>8-6</p> 	 <p>8-6-1</p>  <p>8-6-2</p>  <p>8-6-3</p>  <p>8-6-4</p>  <p>8-6-5</p>  <p>8-6-6</p>  <p>8-6-7</p>  <p>8-6-8</p>
<p>8-7</p> 	 <p>8-7-1</p>
<p>8-8</p> 	 <p>8-8-1</p>

8-9		 <p>8-9-1</p>	 <p>8-9-2</p>	2
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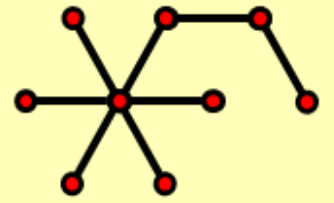
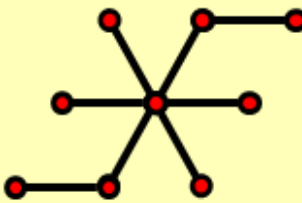
3.8.1.4. Matchstick graphs with  $|E|=8, F=1, |V|=9, \Delta=5$


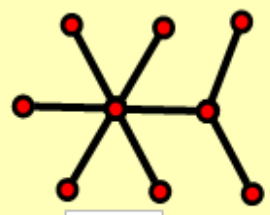
8-10		 <p>8-10-1</p>	 <p>8-10-2</p>	 <p>8-10-3</p>	3
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8-11		 <p>8-11-1</p>	 <p>8-11-2</p>	 <p>8-11-3</p>	3
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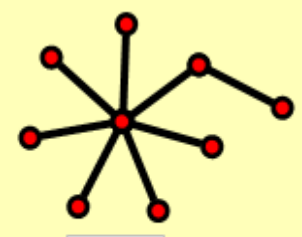
8-12		 <p>8-12-1</p>	1
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3.8.1.5. Matchstick graphs with  $|E|=8, F=1, |V|=9, \Delta=6$


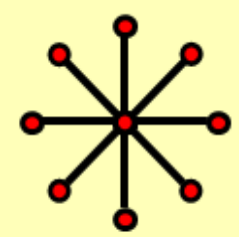
8-13		 <p>8-13-1</p>	 <p>8-13-2</p>	2
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8-14		 <p data-bbox="446 414 542 459">8-14-1</p>	1
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3.8.1.6. Matchstick graphs with  $|E|=8$ ,  $F=1$ ,  $|V|=9$ ,  $\Delta=7$


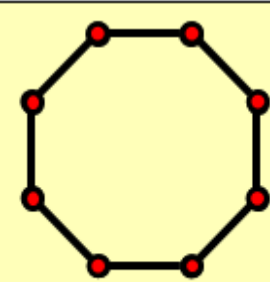
8-15		 <p data-bbox="462 795 558 840">8-15-1</p>	1
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3.8.1.7. Matchstick graphs with  $|E|=8$ ,  $F=1$ ,  $|V|=9$ ,  $\Delta=8$

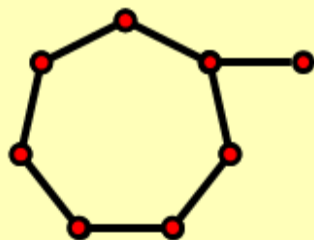
8-16		 <p data-bbox="462 1198 558 1243">8-16-1</p>	1
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3.8.2. Matchstick graphs with  $|E|=8$ ,  $F=2$ ,  $|V|=8$

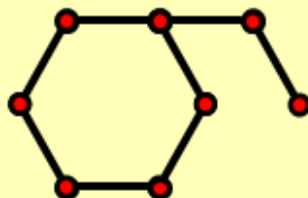
3.8.2.1. Matchstick graphs with  $|E|=8$ ,  $F=2$ ,  $|V|=8$ ,  $\Delta=2$

8-17		 <p data-bbox="494 1691 590 1736">8-17-1</p>	1
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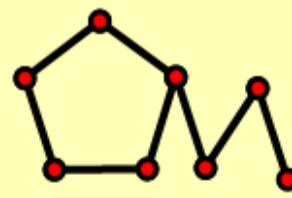
3.8.2.2. Matchstick graphs with  $|E|=8$ ,  $F=2$ ,  $|V|=8$ ,  $\Delta=3$



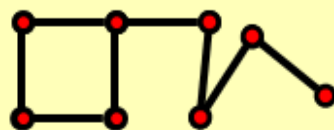
8-18-1



8-18-2



8-18-3



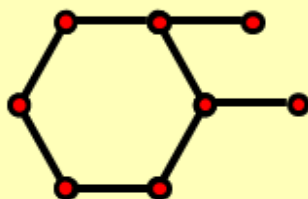
8-18-4



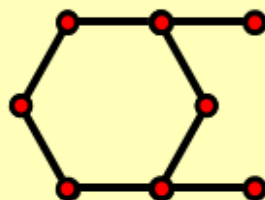
8-18-5

8-18

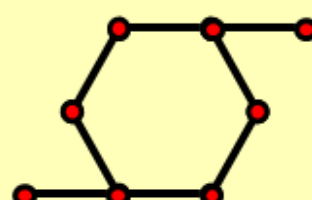
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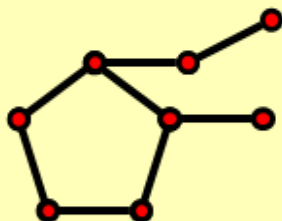
8-19-1



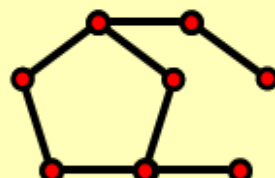
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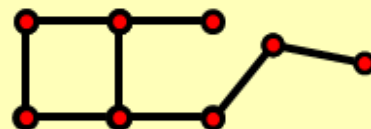
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8-19-4



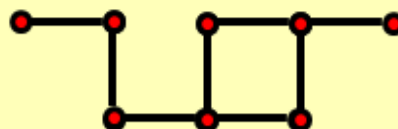
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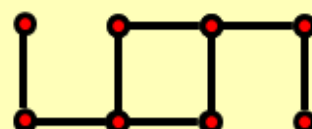
8-19-6



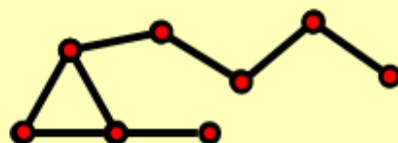
8-19-7



8-19-8



8-19-9



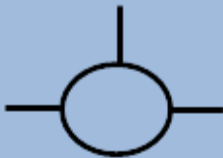
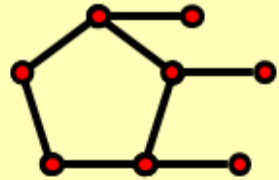
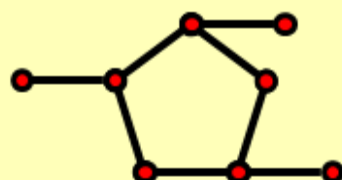
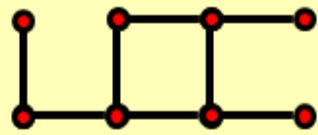
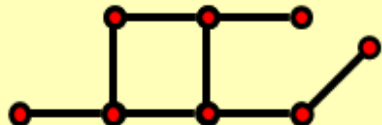

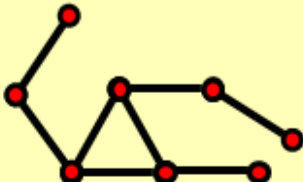
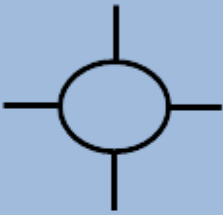
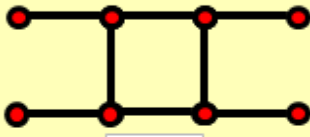

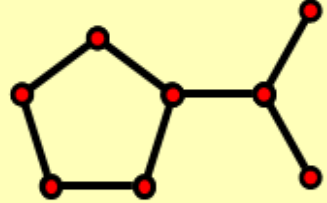
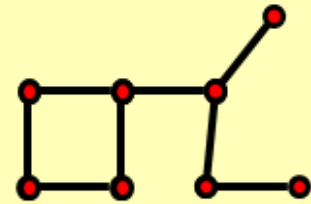
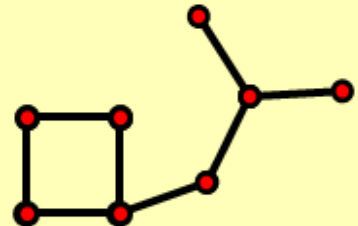
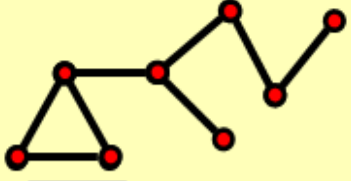
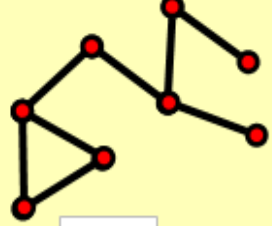

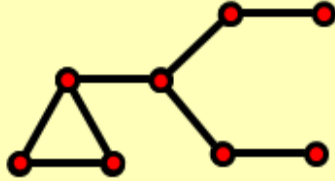
8-19-10



8-19-11

8-19

11

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8-21		<div style="text-align: center;">  <p>8-21-1</p> </div>	1
8-22		<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p>8-22-1</p> </div> <div style="text-align: center;">  <p>8-22-2</p> </div> <div style="text-align: center;">  <p>8-22-3</p> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;">  <p>8-22-4</p> </div> <div style="text-align: center;">  <p>8-22-5</p> </div> <div style="text-align: center;">  <p>8-22-6</p> </div> </div> <div style="text-align: center; margin-top: 20px;">  <p>8-22-7</p> </div>	7

8-23

8-23-1 8-23-2 8-23-3  
8-23-4 8-23-5

8-24

8-24-1

8-25

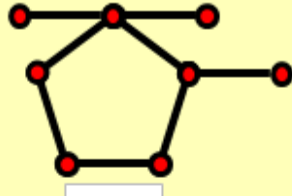
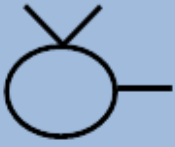
8-25-1

3.8.2.3. Matchstick graphs with  $|E|=8$ ,  $F=2$ ,  $|V|=8$ ,  $\Delta=4$

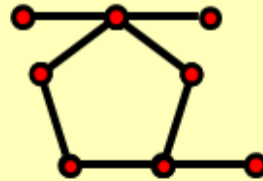
8-26

8-26-1 8-26-2 8-26-3  
8-26-4 8-26-5 8-26-6

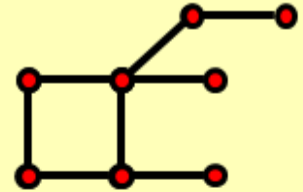




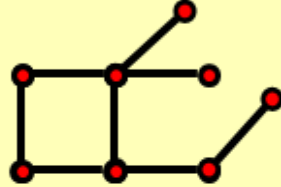
8-27-1



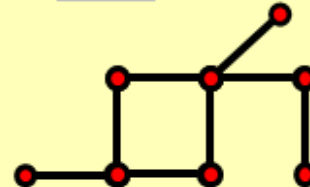
8-27-2



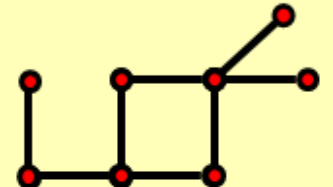
8-27-3



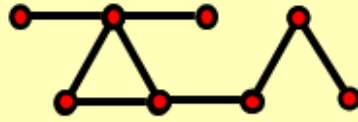
8-27-4



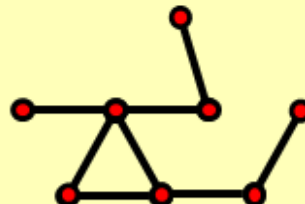
8-27-5



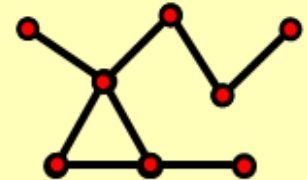
8-27-6



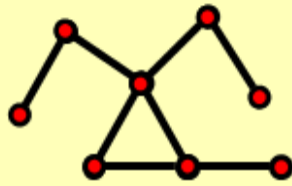
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8-27-8



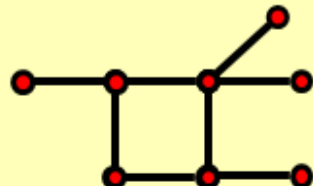
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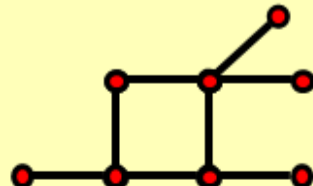
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8-27

10



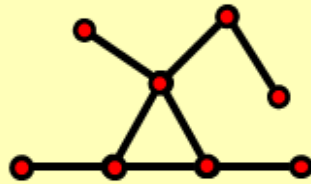
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8-28-2



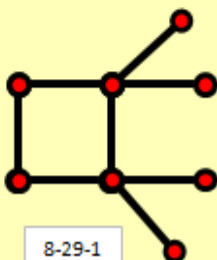
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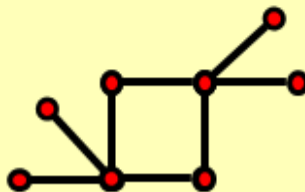
8-28-4

8-28

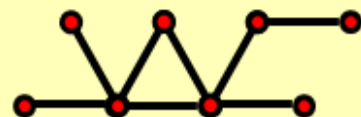
4



8-29-1




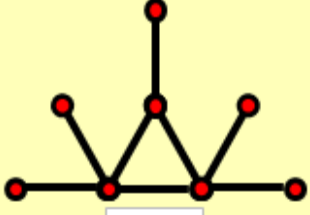

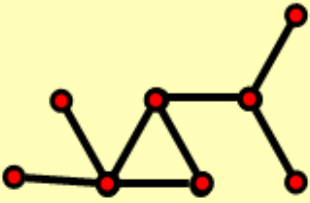

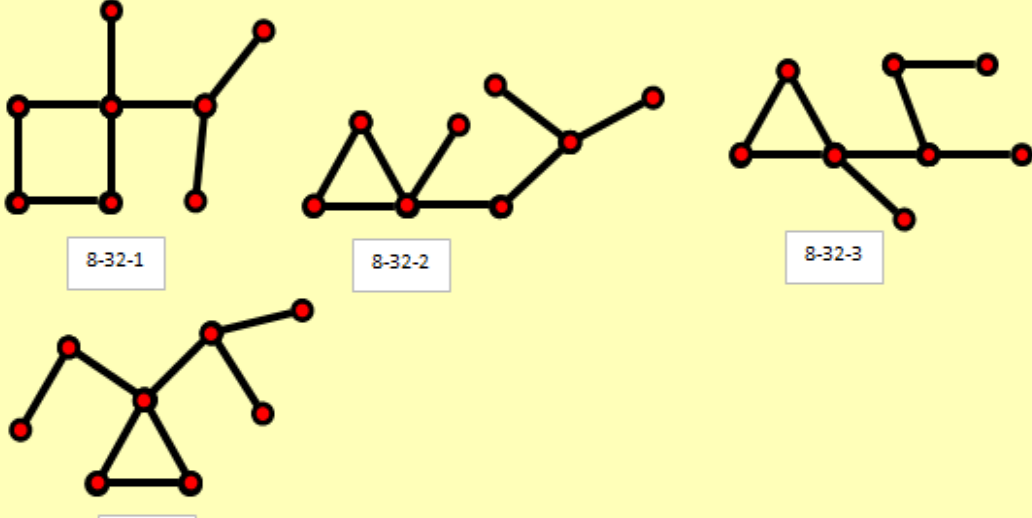
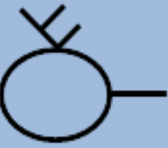
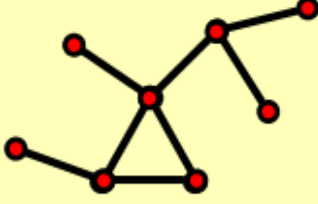

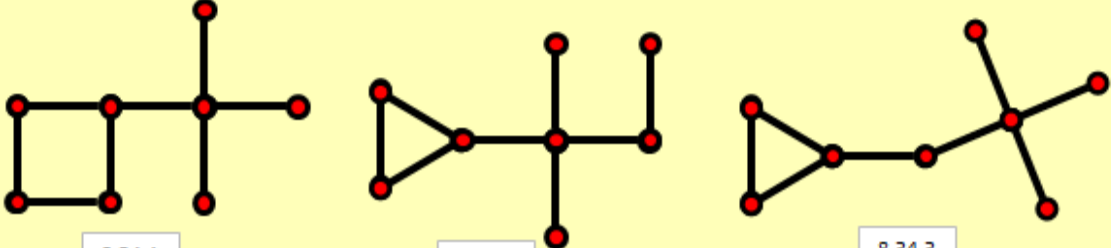
8-29-2


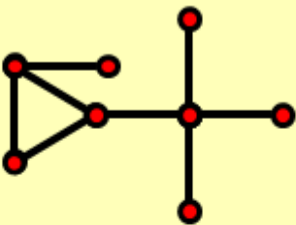



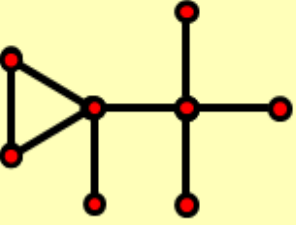
8-29-3

8-29


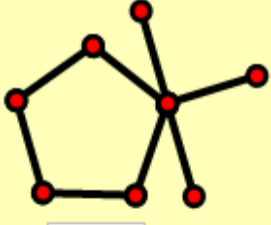
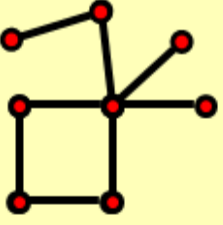
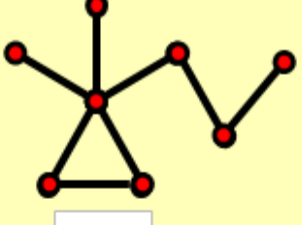
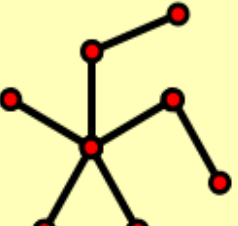
3

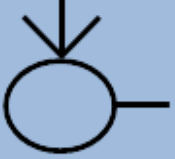
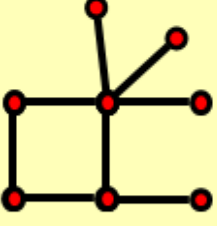
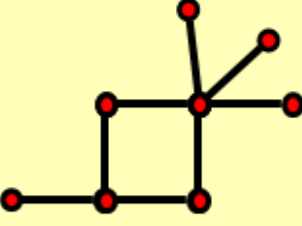
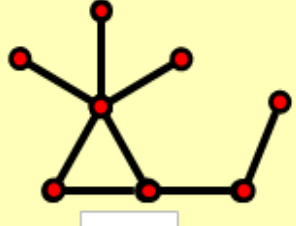
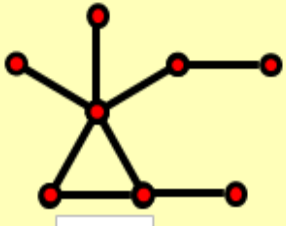
8-30		 <p>8-30-1</p>	1
8-31		 <p>8-31-1</p>	1
8-32		 <p>8-32-1      8-32-2      8-32-3</p> <p>8-32-4</p>	4
8-33		 <p>8-33-1</p>	1
8-34		 <p>8-34-1      8-34-2      8-34-3</p>	3


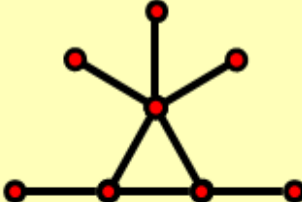

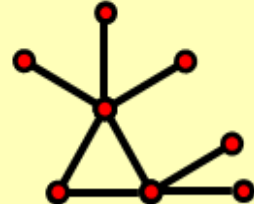

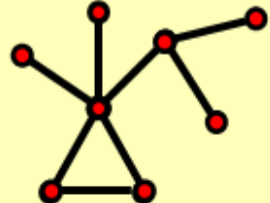

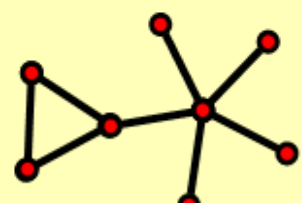
8-35		 <p data-bbox="450 385 545 430">8-35-1</p>	1
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8-36		 <p data-bbox="450 721 545 766">8-36-1</p>	1
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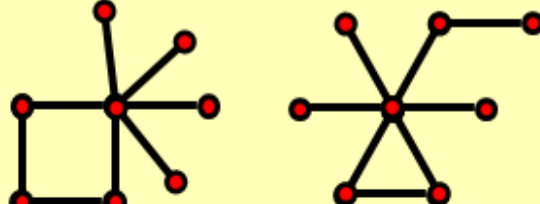

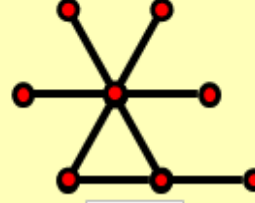
3.8.2.4. Matchstick graphs with  $|E|=8$ ,  $F=2$ ,  $|V|=8$ ,  $\Delta=5$

8-37		 <p data-bbox="450 1124 545 1169">8-37-1</p>  <p data-bbox="785 1124 880 1169">8-37-2</p>  <p data-bbox="1104 1124 1200 1169">8-37-3</p>  <p data-bbox="450 1438 545 1482">8-37-4</p>	4
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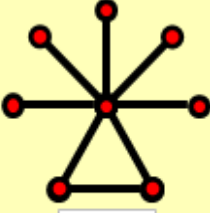
8-38		 <p data-bbox="450 1751 545 1796">8-38-1</p>  <p data-bbox="769 1751 865 1796">8-38-2</p>  <p data-bbox="1104 1751 1200 1796">8-38-3</p>  <p data-bbox="450 2042 545 2087">8-38-4</p>	4
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8-39		 <p>8-39-1</p>	1
8-40		 <p>8-40-1</p>	1
8-41		 <p>8-41-1</p>	1
8-42		 <p>8-42-1</p>	1

3.8.2.5. Matchstick graphs with  $|E|=8$ ,  $F=2$ ,  $|V|=8$ ,  $\Delta=6$


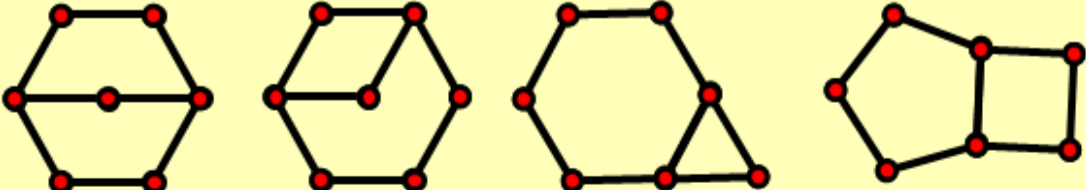
8-43		 <p>8-43-1      8-43-2</p>	2
8-44		 <p>8-44-1</p>	1


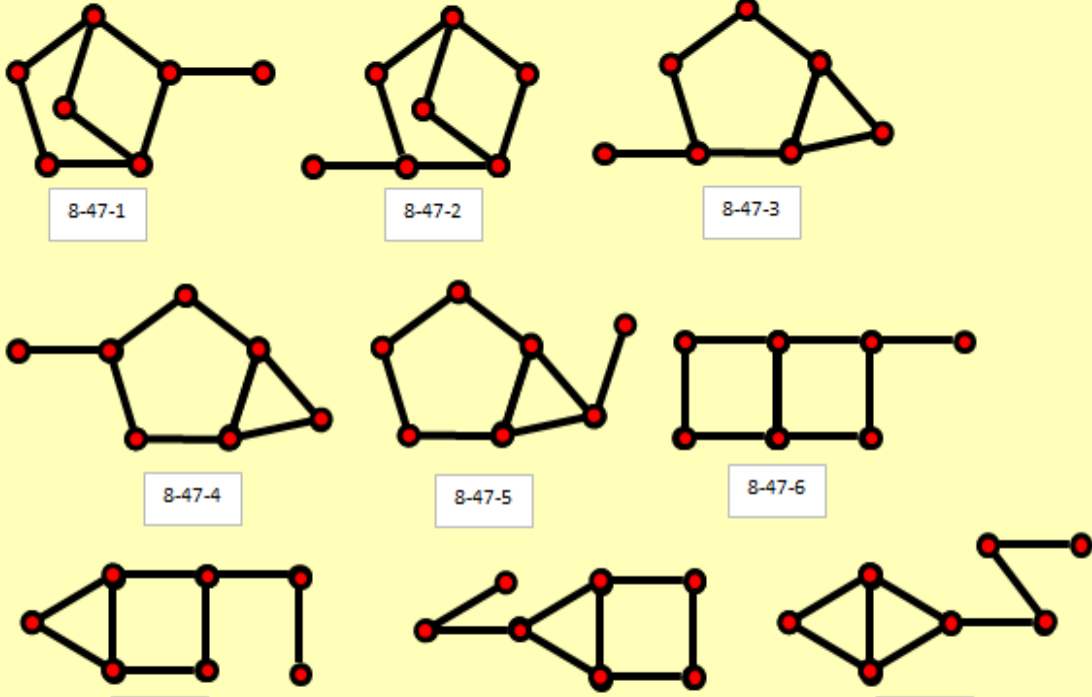
3.8.2.6. Matchstick graphs with  $|E|=8, F=2, |V|=8, \Delta=7$


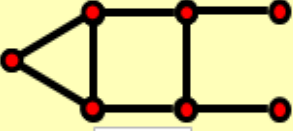
	 <p>8-45-1</p>
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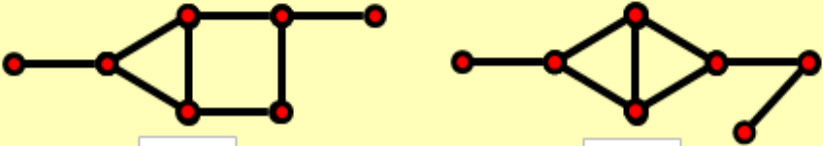
3.8.3. Matchstick graphs with  $|E|=8, F=3, |V|=7$


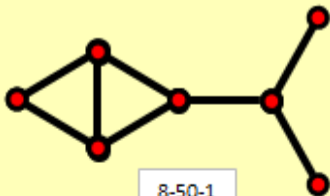
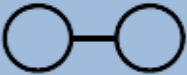
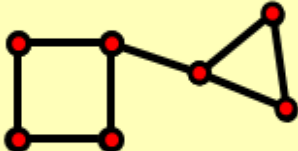

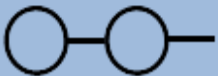
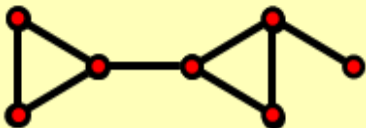
3.8.3.1. Matchstick graphs with  $|E|=8, F=3, |V|=7, \Delta=3$

	 <p>8-46-1      8-46-2      8-46-3      8-46-4</p>
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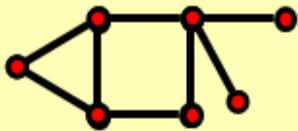
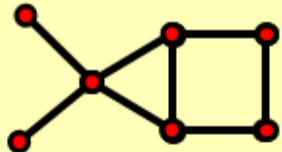
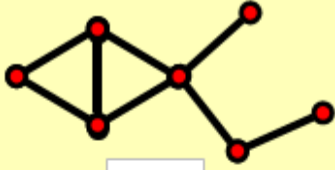

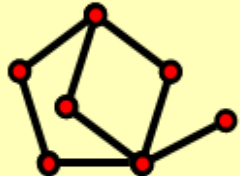
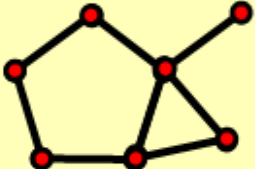
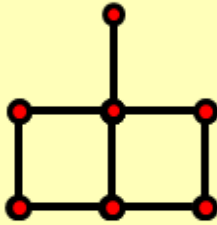
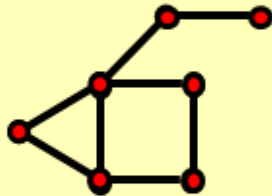
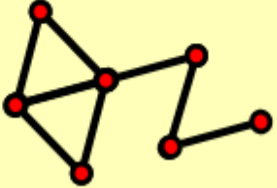
	 <p>8-47-1      8-47-2      8-47-3</p> <p>8-47-4      8-47-5      8-47-6</p> <p>8-47-7      8-47-8      8-47-9</p>
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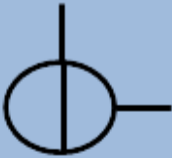
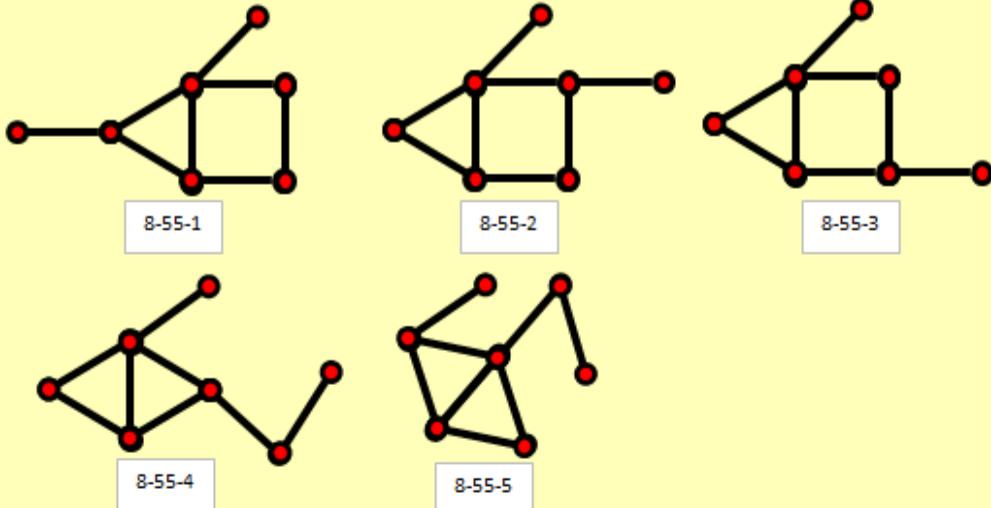
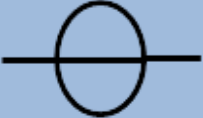
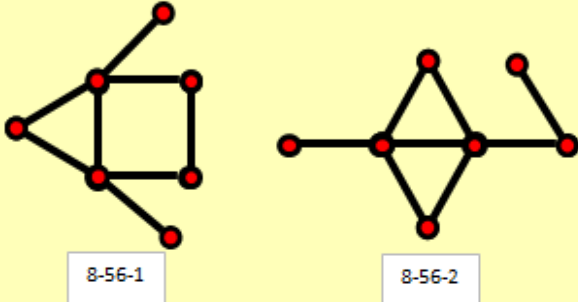
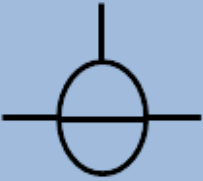
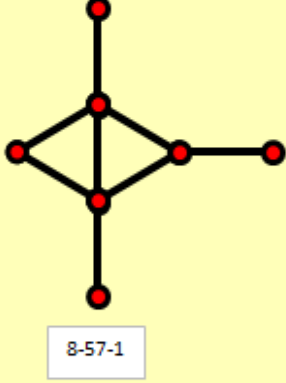

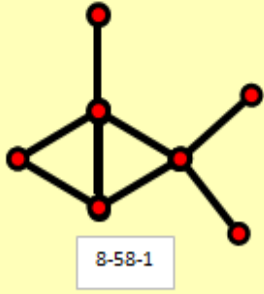

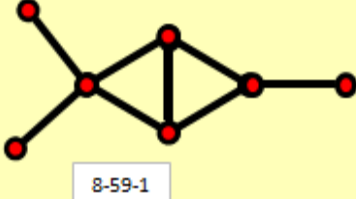
	 <p>8-48-1</p>
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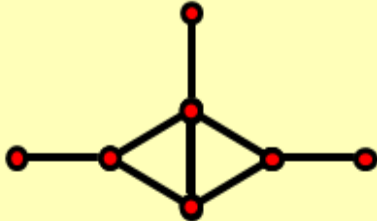

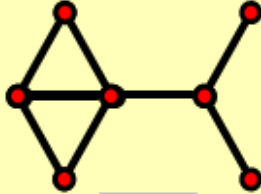

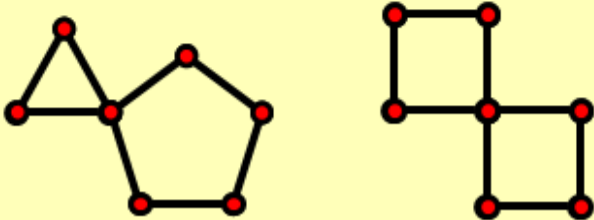

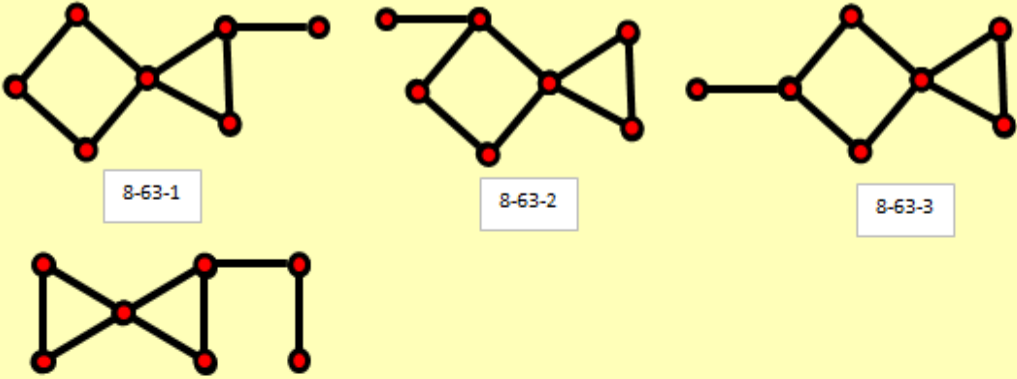

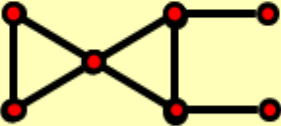


	 <p>8-49-1      8-49-2</p>
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8-50		 <p>8-50-1</p>	1
8-51		 <p>8-51-1</p>  <p>8-51-2</p>	2
8-52		 <p>8-52-1</p>	1


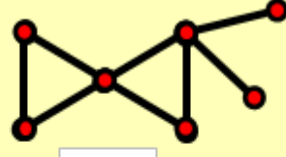

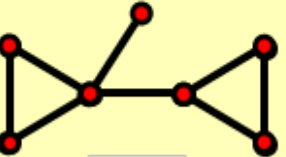
3.8.3.2. Matchstick graphs with  $|E|=8$ ,  $F=3$ ,  $|V|=7$ ,  $\Delta=4$

8-53		 <p>8-53-1</p>  <p>8-53-2</p>  <p>8-53-3</p>	3
8-54		 <p>8-54-1</p>  <p>8-54-2</p>  <p>8-54-3</p>  <p>8-54-4</p>  <p>8-54-5</p>	5


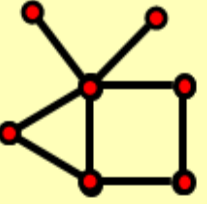
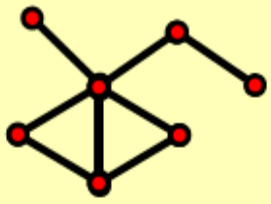

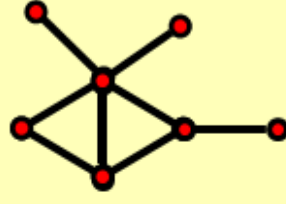

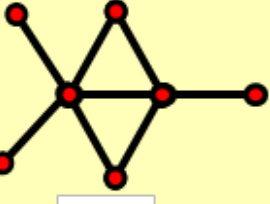



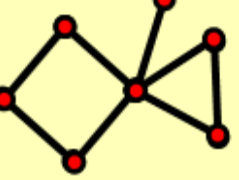

8-55			5
8-56			2
8-57			1
8-58			1
8-59			1


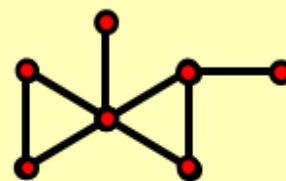
8-60		 <p>8-60-1</p>	1
8-61		 <p>8-61-1</p>	1
8-62		 <p>8-62-1</p> <p>8-62-2</p>	2
8-63		 <p>8-63-1</p> <p>8-63-2</p> <p>8-63-3</p> <p>8-63-4</p>	4
8-64		 <p>8-64-1</p>	1
8-65		 <p>8-65-1</p>	1




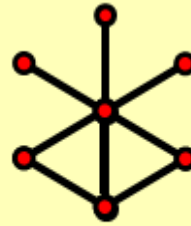
8-66		 <div data-bbox="454 313 550 369">8-66-1</div>	1
8-67		 <div data-bbox="470 548 566 604">8-67-1</div>	1


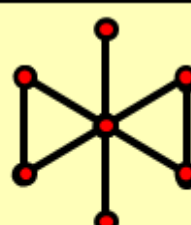
3.8.3.3. Matchstick graphs with  $|E|=8$ ,  $F=3$ ,  $|V|=7$ ,  $\Delta=5$

8-68		 <div data-bbox="470 907 566 963">8-68-1</div>  <div data-bbox="742 907 837 963">8-68-2</div>	2
8-69		 <div data-bbox="454 1198 550 1254">8-69-1</div>	1
8-70		 <div data-bbox="454 1512 550 1568">8-70-1</div>	1
8-71		 <div data-bbox="470 1758 566 1814">8-71-1</div>	1
8-72		 <div data-bbox="470 2049 566 2105">8-72-1</div>  <div data-bbox="742 2049 837 2105">8-72-2</div>	2

8-73		 <p data-bbox="446 336 542 392">8-73-1</p>	1
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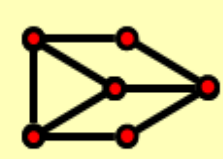
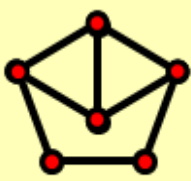
3.8.3.4. Matchstick graphs with  $|E|=8$ ,  $F=3$ ,  $|V|=7$ ,  $\Delta=6$

8-74		 <p data-bbox="446 716 542 772">8-74-1</p>	1
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8-75		 <p data-bbox="446 1008 542 1064">8-75-1</p>	1
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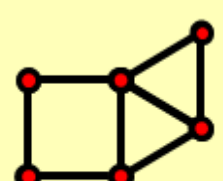
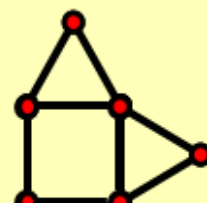
3.8.4. Matchstick graphs with  $|E|=8$ ,  $F=4$ ,  $|V|=6$

3.8.4.1. Matchstick graphs with  $|E|=8$ ,  $F=4$ ,  $|V|=6$ ,  $\Delta=3$

8-76		 <p data-bbox="446 1411 542 1467">8-76-1</p>	 <p data-bbox="734 1411 829 1467">8-76-2</p>	2
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8-77		 <p data-bbox="446 1635 542 1691">8-77-1</p>	1
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3.8.4.2. Matchstick graphs with  $|E|=8$ ,  $F=4$ ,  $|V|=6$ ,  $\Delta=4$

8-78		 <p data-bbox="446 1993 542 2049">8-78-1</p>	 <p data-bbox="734 1993 829 2049">8-78-2</p>	2
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8-79		<p>8-79-1</p>	1
8-80		<p>8-80-1</p>	1
8-81		<p>8-81-1</p>	1
8-82		<p>8-82-1</p>	1

3.8.4.3. Matchstick graphs with  $|E|=8, F=4, |V|=6, \Delta=5$

8-83		<p>8-83-1</p>	1
8-84		<p>8-84-1</p>	1

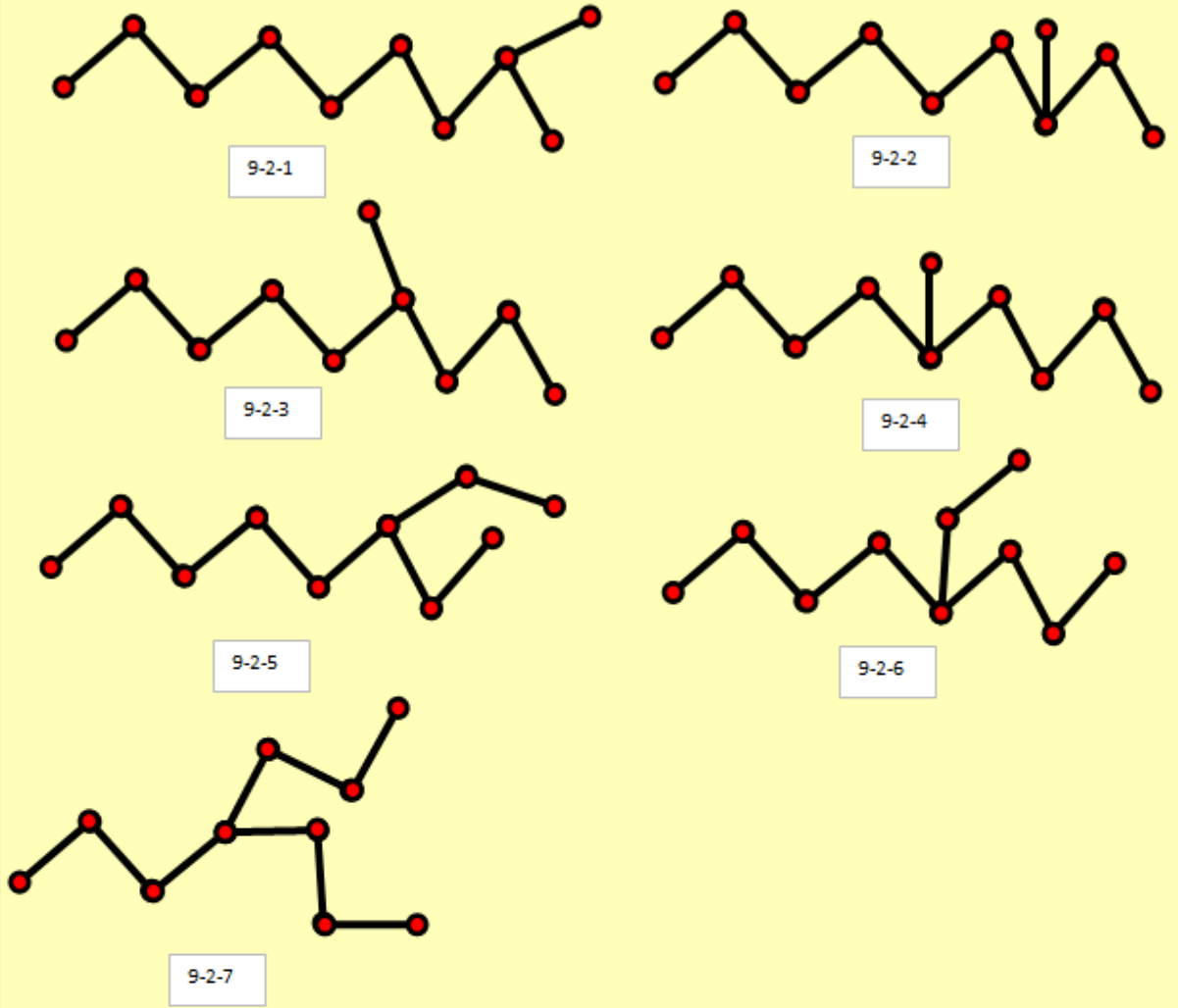
3.9. Matchstick graphs with  $|E|=9$

3.9.1. Matchstick graphs with  $|E|=9, F=1, |V|=10$

3.9.1.1. Matchstick graphs with  $|E|=9, F=1, |V|=10, \Delta=2$

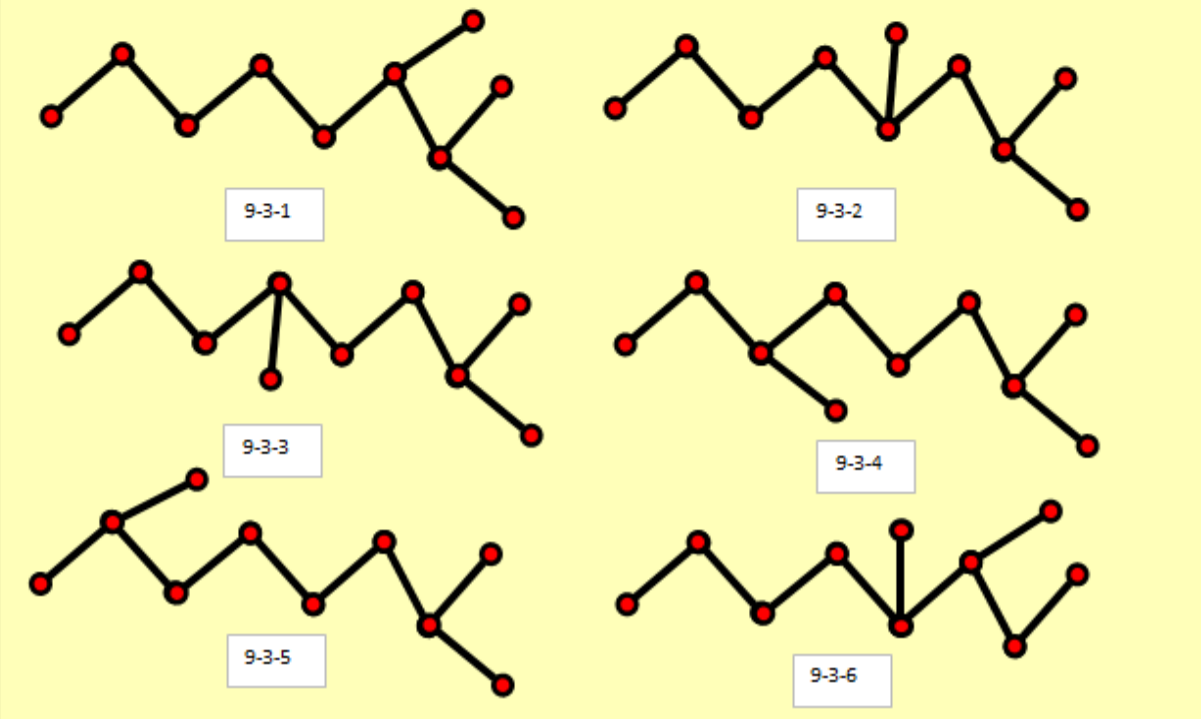
9-1		<p>9-1-1</p>	1
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3.9.1.2. Matchstick graphs with  $|E|=9$ ,  $F=1$ ,  $|V|=10$ ,  $\Delta=3$



9-2

7

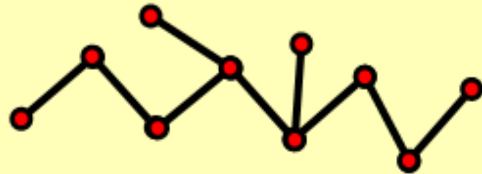




9-3-7



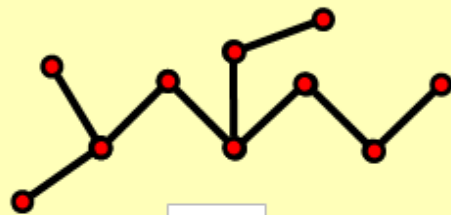
9-3-8



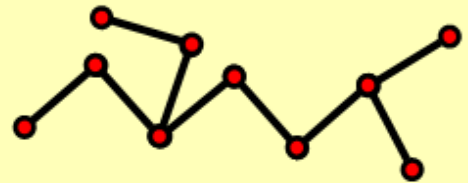
9-3-9



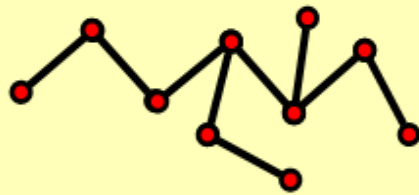
9-3-10



9-3-11



9-3-12



9-3-13



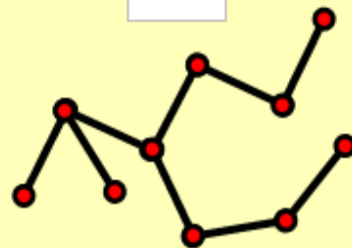
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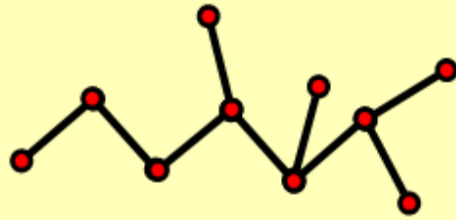
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9-3-16



9-3-17



9-4-1



9-4-2



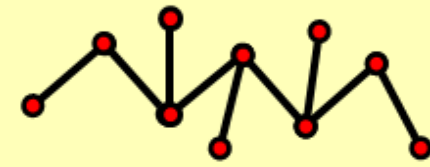
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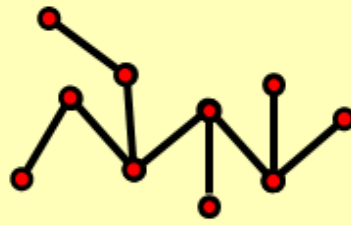
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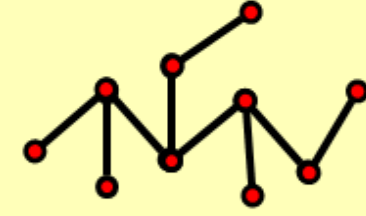
9-4-5



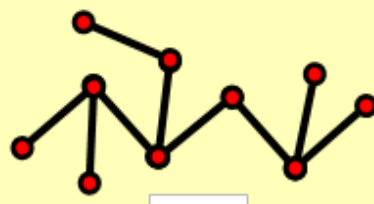
9-4-6



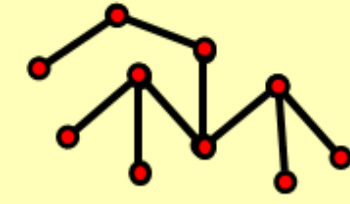
9-4-7



9-4-8



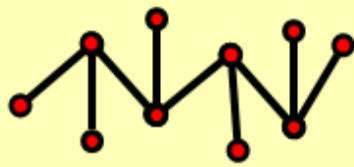
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9-4-10

9-4

10

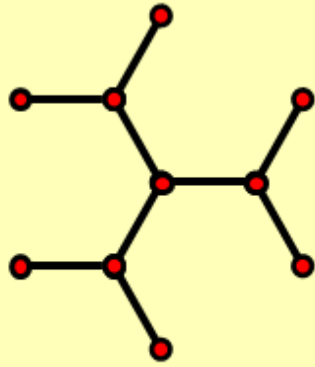


9-5-1

9-5

1

9-6

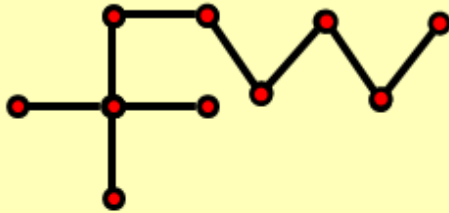


9-6-1

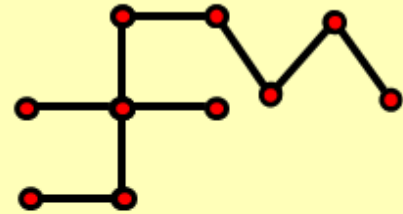
1

3.9.1.3. Matchstick graphs with  $|E|=9, F=1, |V|=10, \Delta=4$

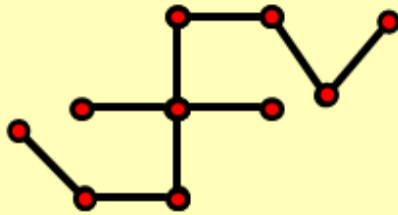
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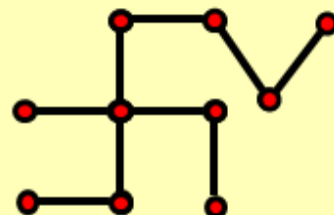
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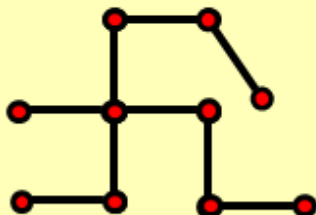
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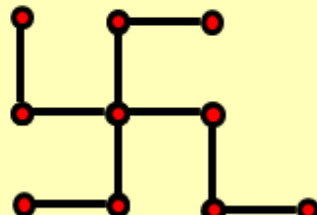
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9-7-4

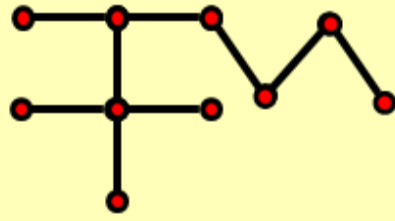


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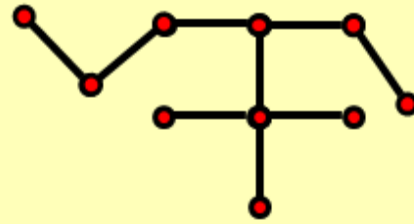


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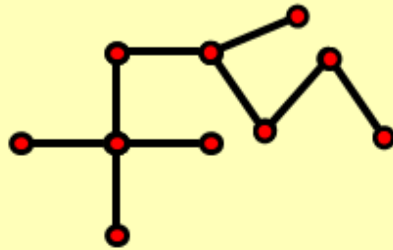
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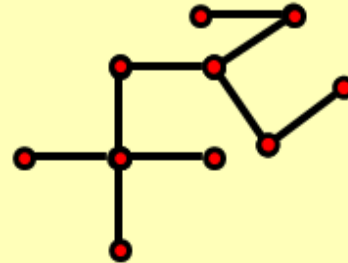
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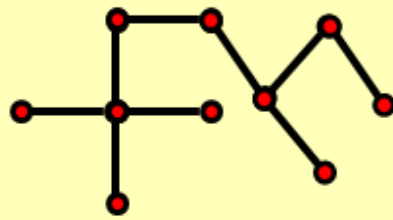
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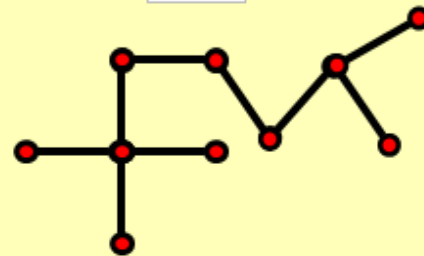
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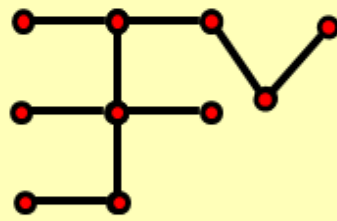
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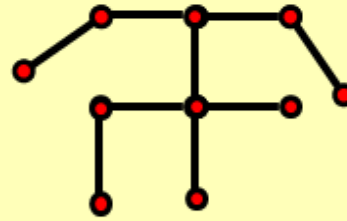
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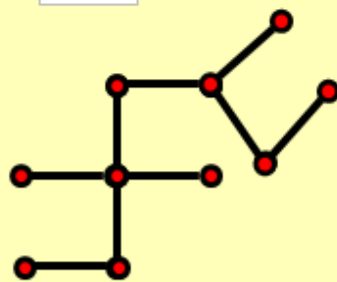
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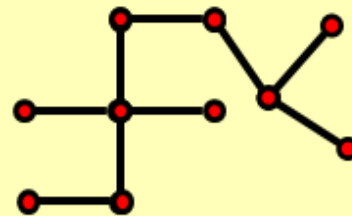
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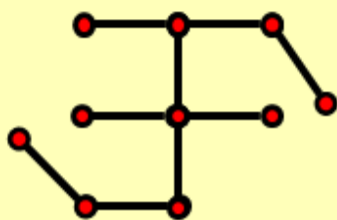
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9-8-9



9-8-10



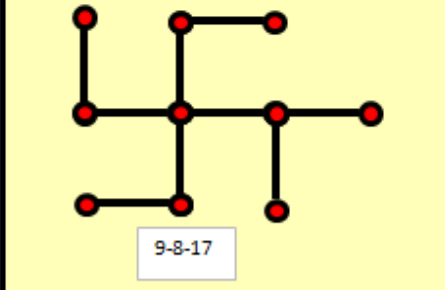
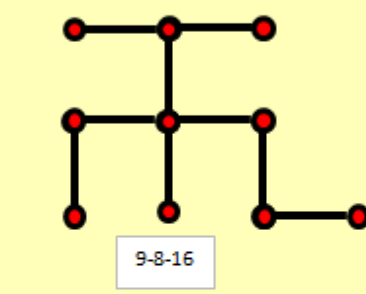
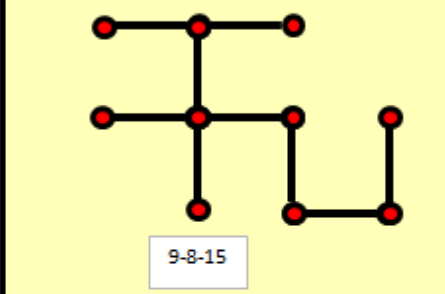
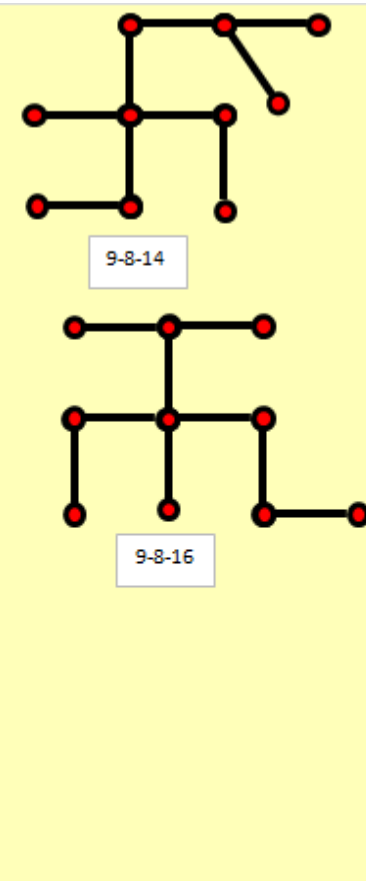
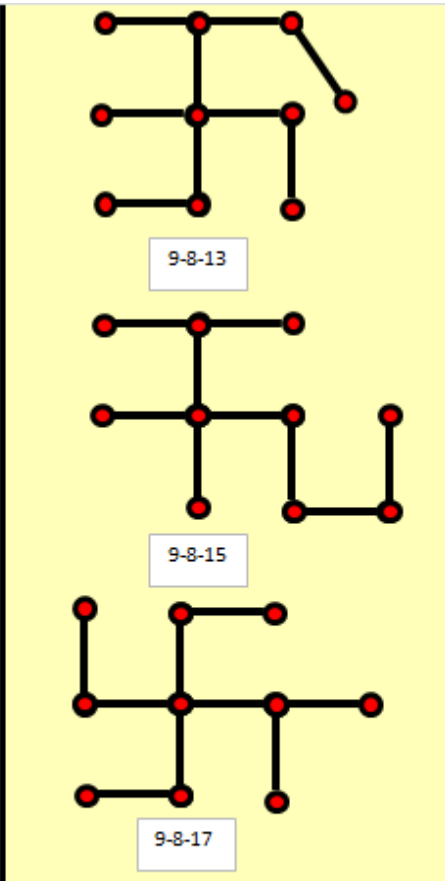
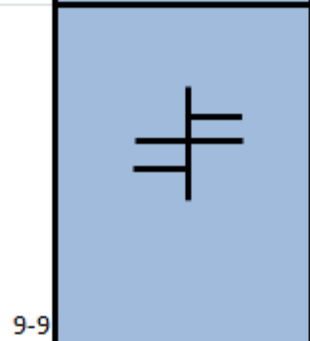
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9-8-12

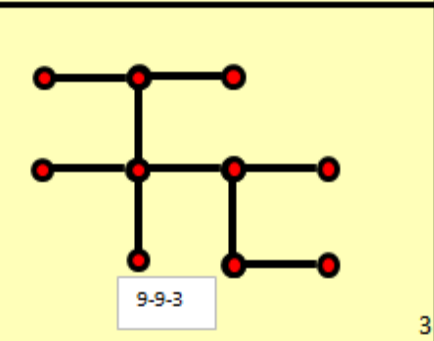
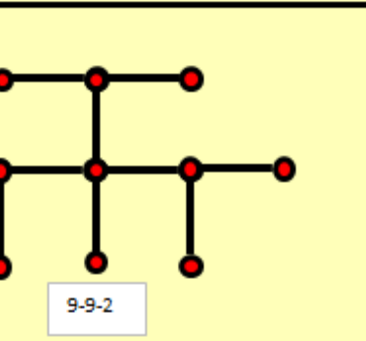
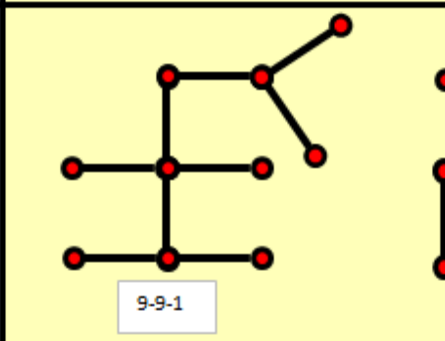
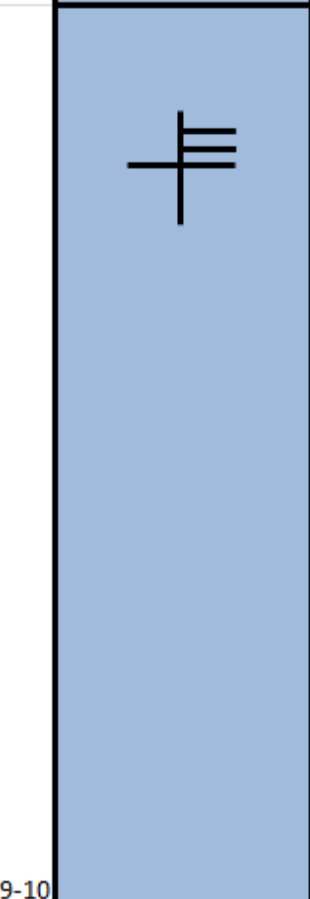


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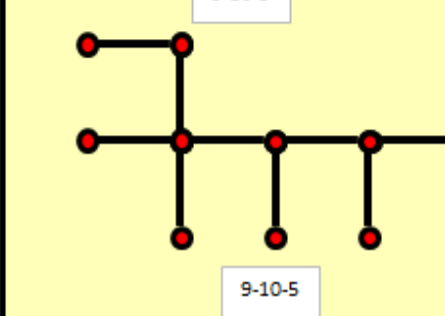
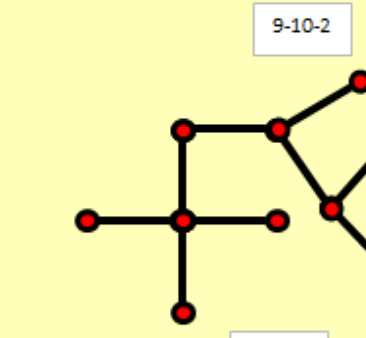
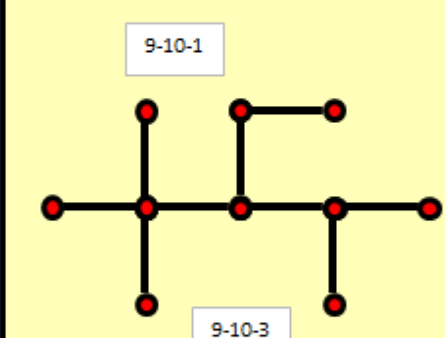
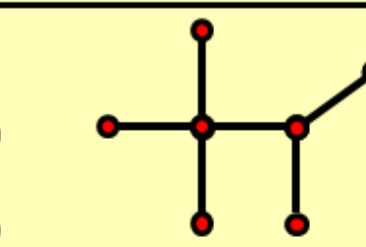
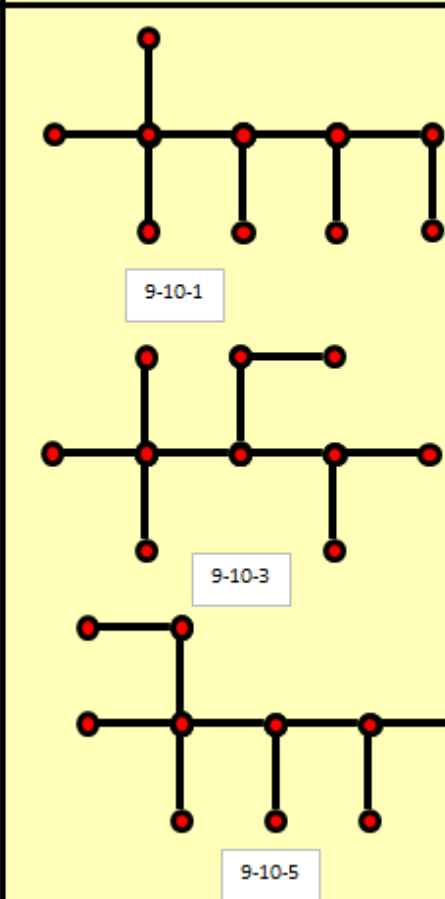
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9-9


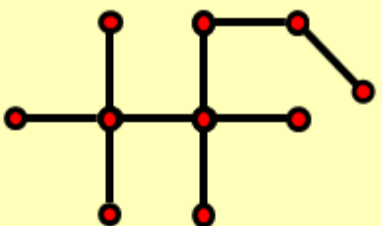
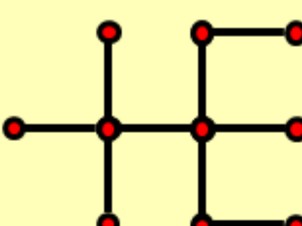
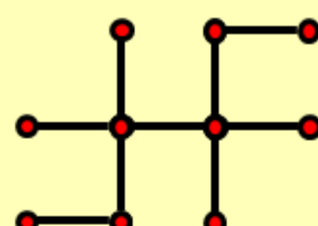
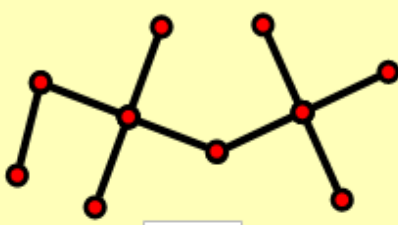




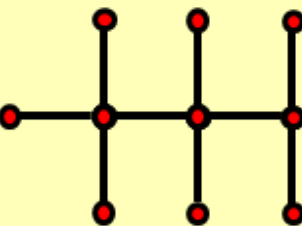
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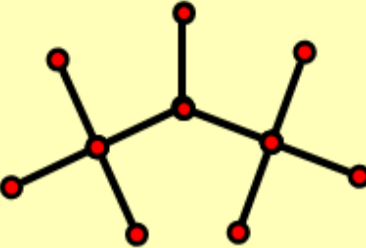
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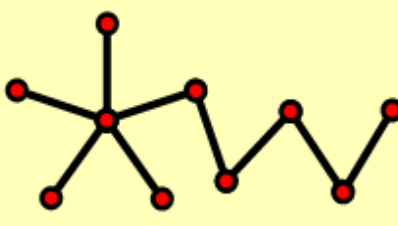
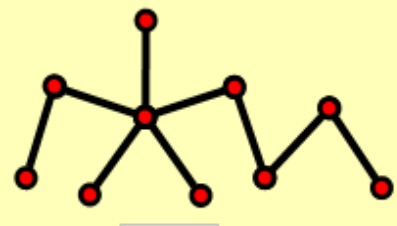
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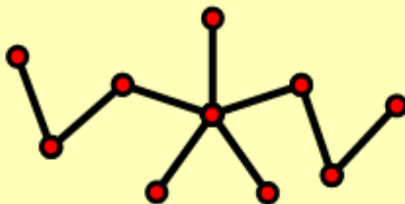

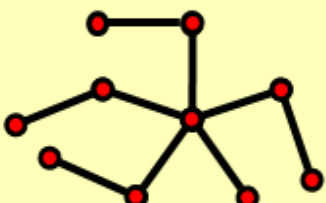

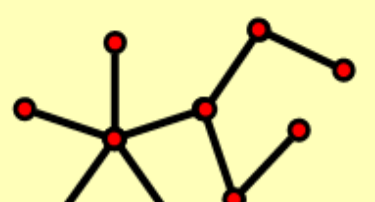
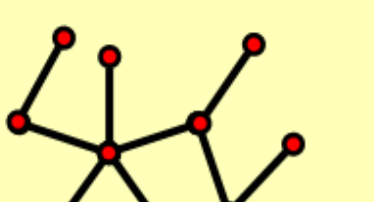
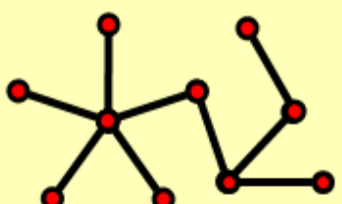
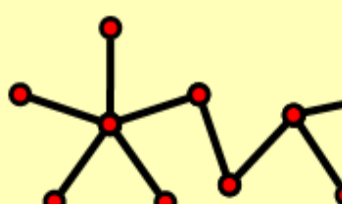
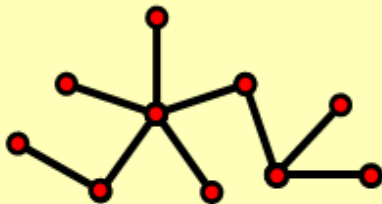
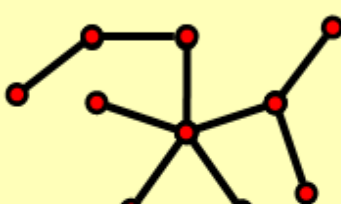
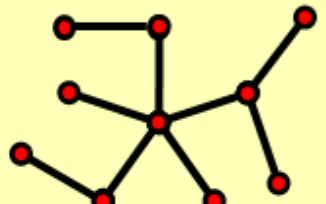
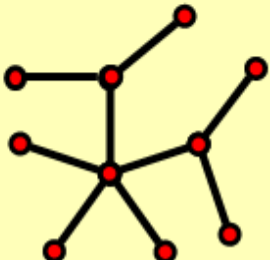
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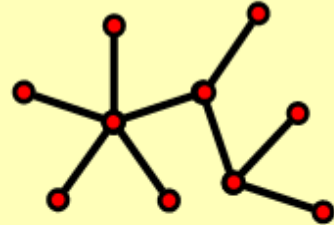

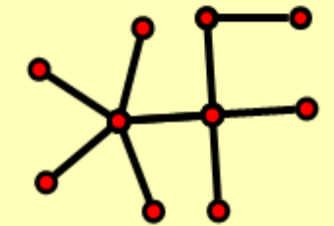
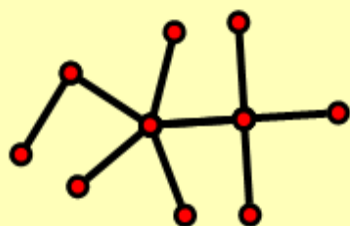
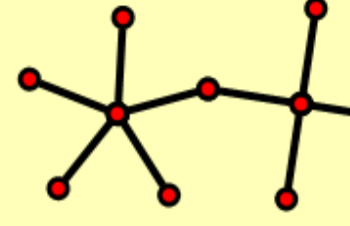

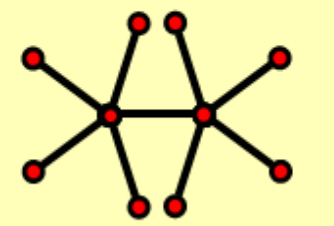
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<p>9-13</p> 	 <p>9-13-1</p>
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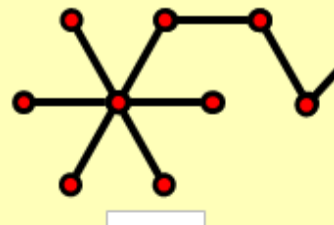
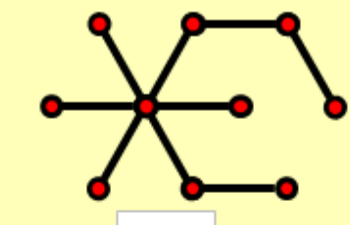
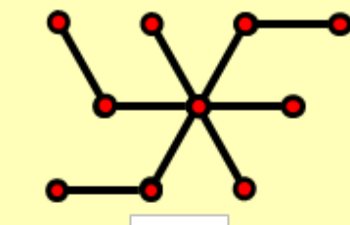

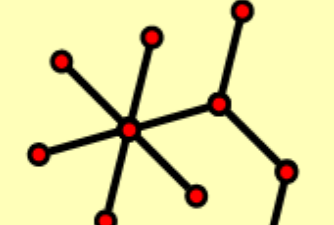
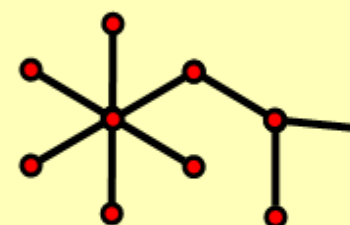
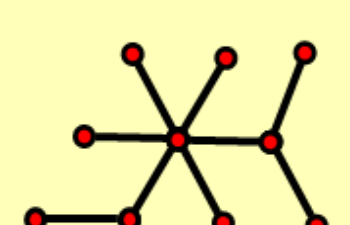

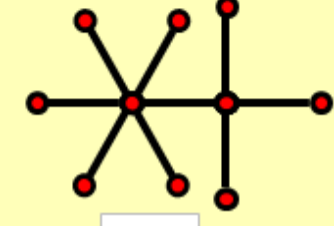
3.9.1.4. Matchstick graphs with  $|E|=9$ ,  $F=1$ ,  $|V|=10$ ,  $\Delta=5$

	 <p>9-14-1</p>  <p>9-14-2</p>
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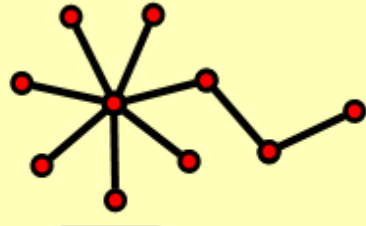
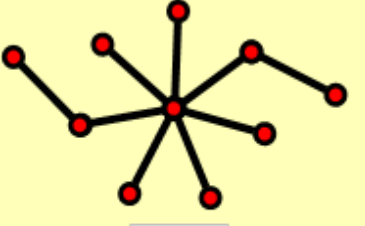
9-14	 <p>9-14-3</p>  <p>9-14-4</p>  <p>9-14-5</p>	5
9-15	 <p>9-15-1</p>  <p>9-15-2</p>  <p>9-15-5</p>  <p>9-15-3</p>  <p>9-15-4</p>  <p>9-15-6</p>  <p>9-15-7</p>  <p>9-15-8</p>	8
9-16	 <p>9-16-1</p>	1


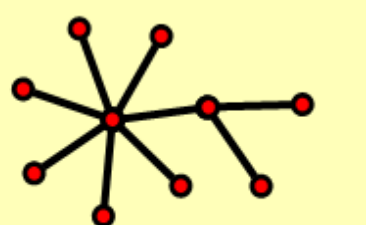
9-17		 <p style="text-align: center;">9-17-1</p>	1
9-18		 <p style="text-align: center;">9-18-1</p>  <p style="text-align: center;">9-18-2</p>  <p style="text-align: center;">9-18-3</p>	3
9-19		 <p style="text-align: center;">9-19-1</p>	1

3.9.1.5. Matchstick graphs with  $|E|=9$ ,  $F=1$ ,  $|V|=10$ ,  $\Delta=6$


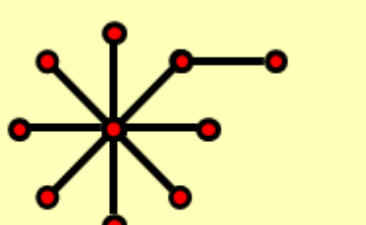
9-20		 <p style="text-align: center;">9-20-1</p>  <p style="text-align: center;">9-20-2</p>  <p style="text-align: center;">9-20-3</p>	3
9-21		 <p style="text-align: center;">9-21-1</p>  <p style="text-align: center;">9-21-2</p>  <p style="text-align: center;">9-21-3</p>	3
9-22		 <p style="text-align: center;">9-22-1</p>	1

3.9.1.6. Matchstick graphs with  $|E|=9$ ,  $F=1$ ,  $|V|=10$ ,  $\Delta=7$


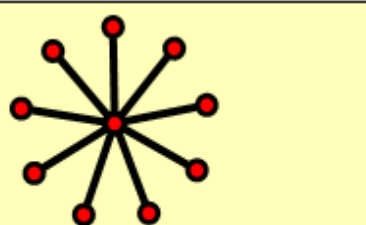
<p>9-23</p> 	 <p>9-23-1</p>  <p>9-23-2</p>
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<p>9-24</p> 	 <p>9-24-1</p>
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3.9.1.7. Matchstick graphs with  $|E|=9$ ,  $F=1$ ,  $|V|=10$ ,  $\Delta=8$


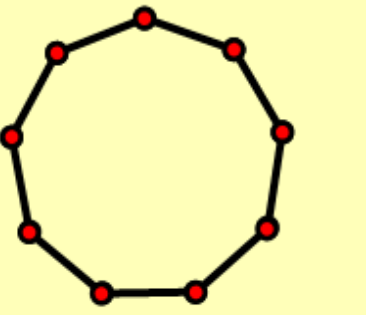
<p>9-25</p> 	 <p>9-25-1</p>
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3.9.1.8. Matchstick graphs with  $|E|=9$ ,  $F=1$ ,  $|V|=10$ ,  $\Delta=9$

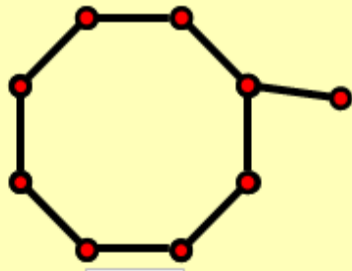
<p>9-26</p> 	 <p>9-26-1</p>
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3.9.2. Matchstick graphs with  $|E|=9$ ,  $F=2$ ,  $|V|=9$

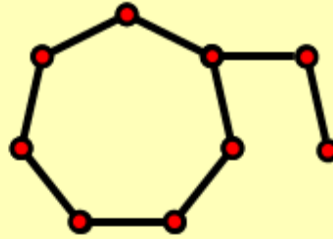
3.9.2.1. Matchstick graphs with  $|E|=9$ ,  $F=2$ ,  $|V|=9$ ,  $\Delta=2$

<p>9-27</p> 	 <p>9-27-1</p>
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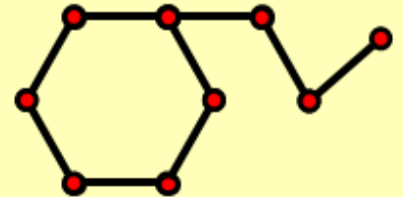
3.9.2.2. Matchstick graphs with  $|E|=9$ ,  $F=2$ ,  $|V|=9$ ,  $\Delta=3$



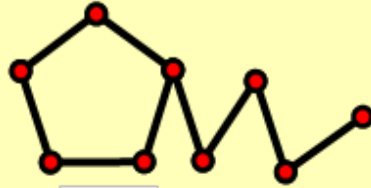
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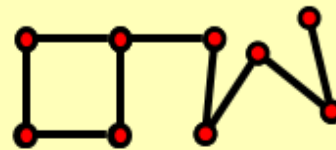
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9-28-3



9-28-4



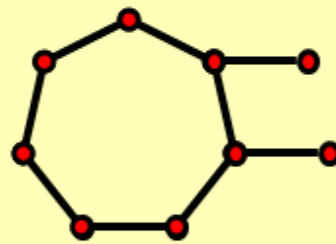
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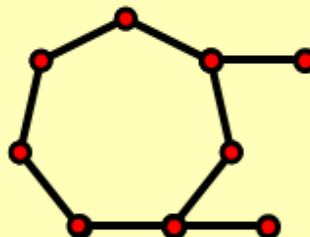
9-28-6

9-28

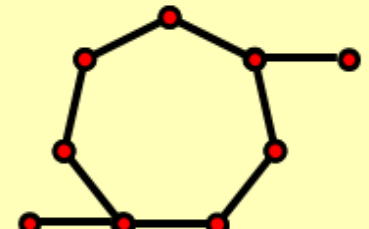
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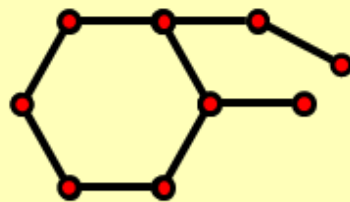
9-29-1



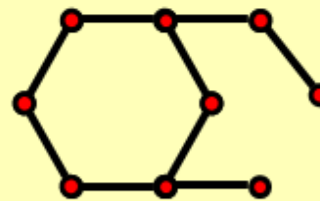
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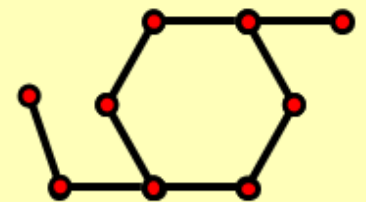
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9-29-4



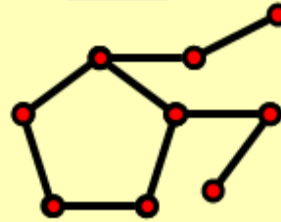
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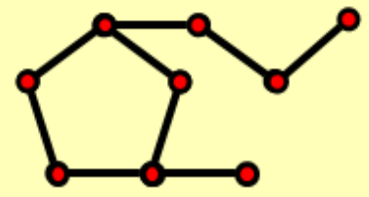
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9-29-7

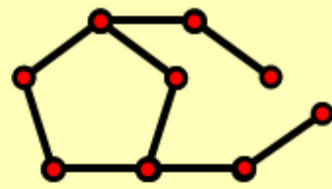


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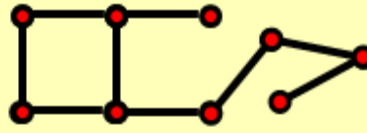


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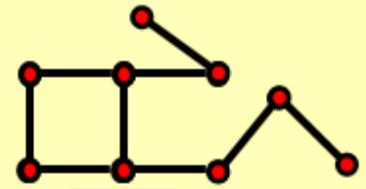
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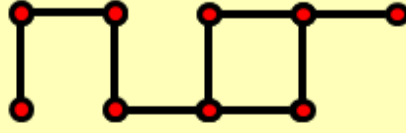
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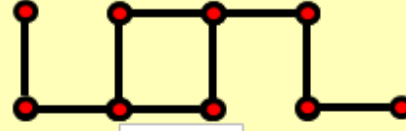
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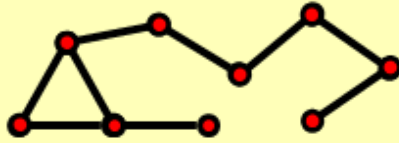
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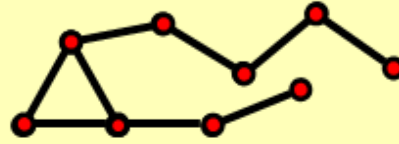
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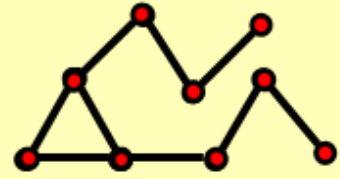
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9-29-15

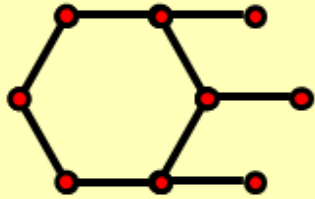
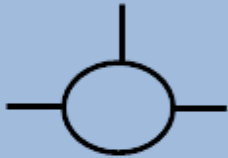


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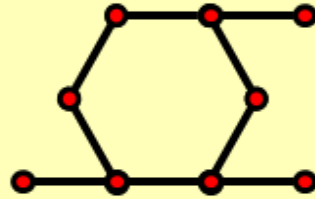


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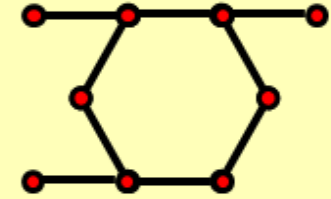
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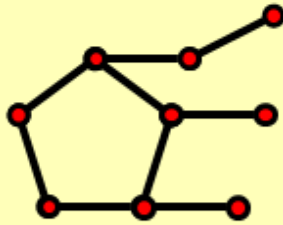
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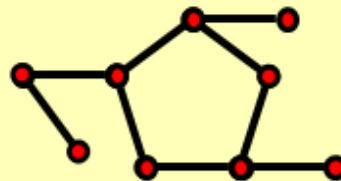
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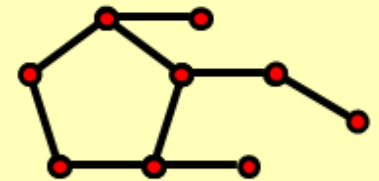
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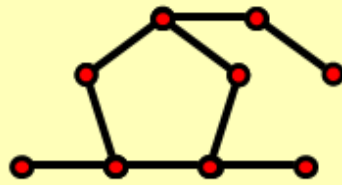
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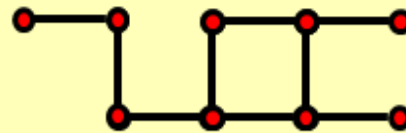
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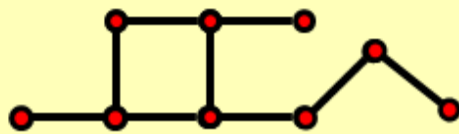
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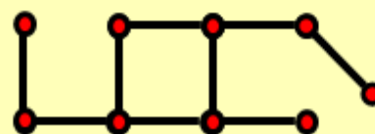
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9-30-8

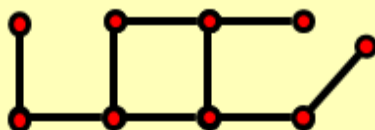


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9-30-10

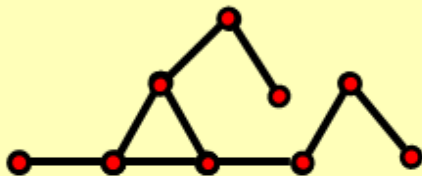
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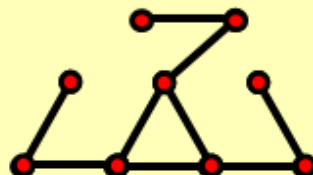
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9-30-12



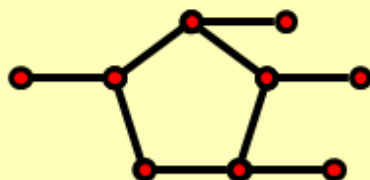
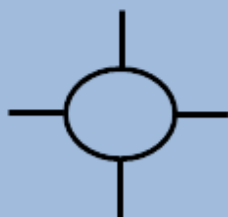
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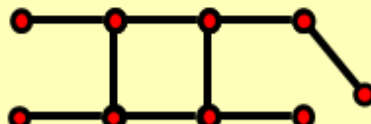
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14

9-31

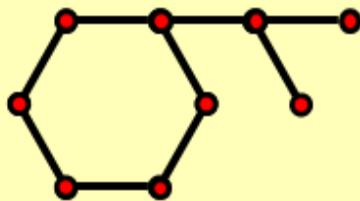


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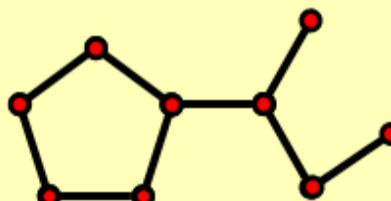


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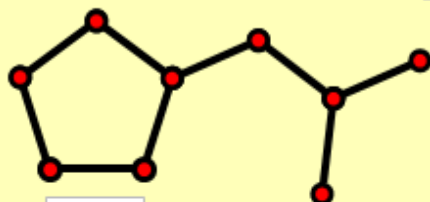
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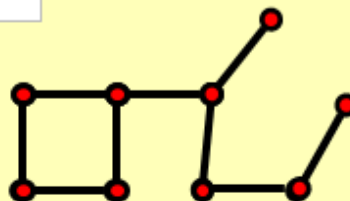
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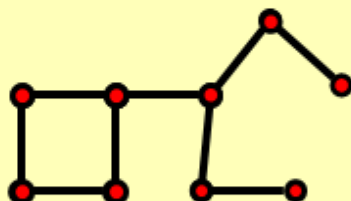
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9-32-3



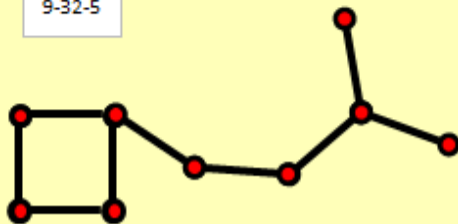
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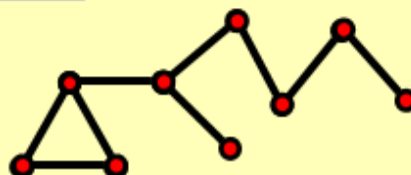
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9-32-6



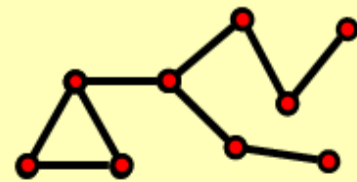
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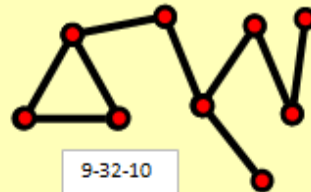
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9-32



9-32-9



9-32-10



9-32-11

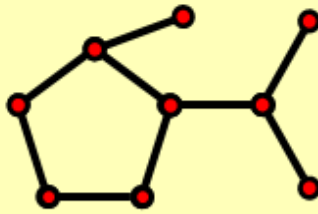
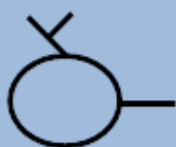


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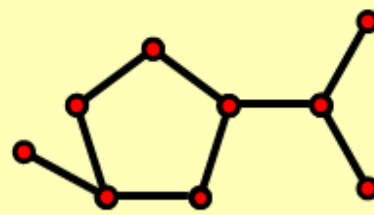


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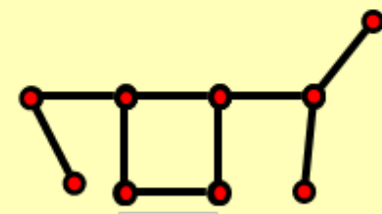
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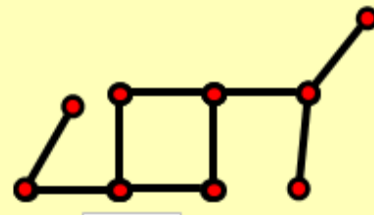
9-33-1



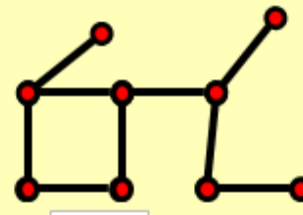
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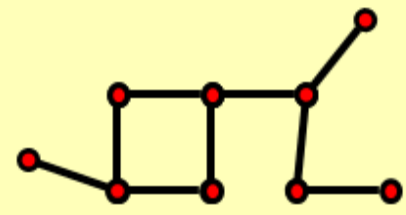
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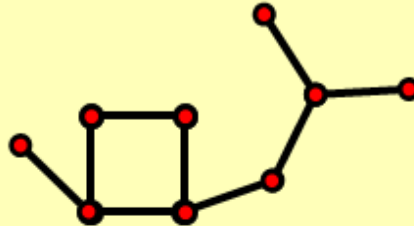
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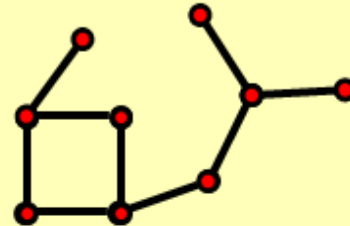
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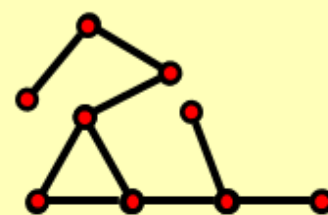
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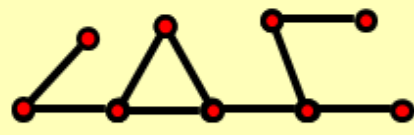
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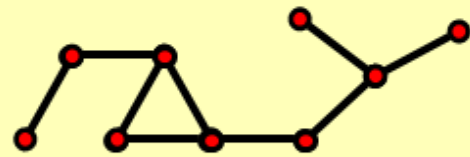
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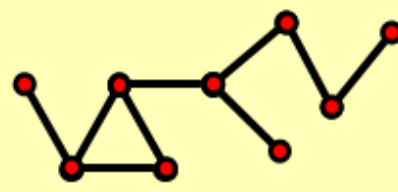
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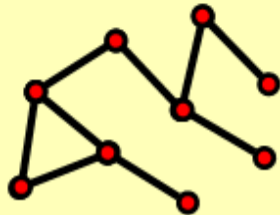
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9-33-11

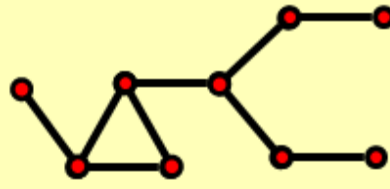


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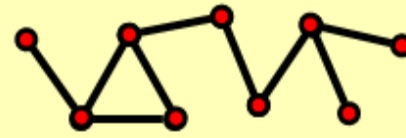


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9-33



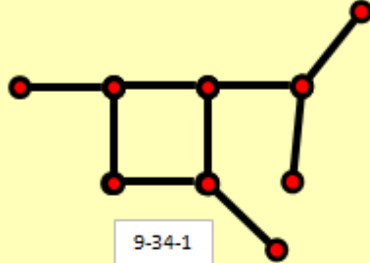
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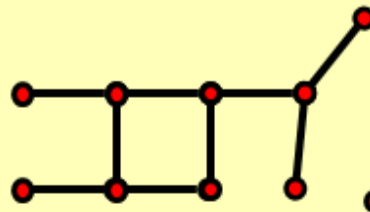
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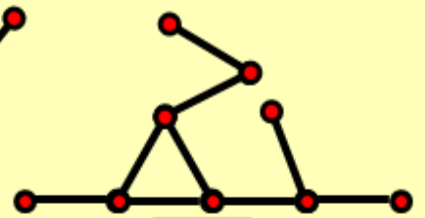
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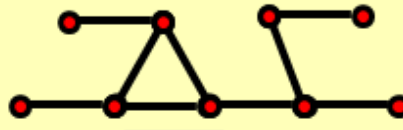
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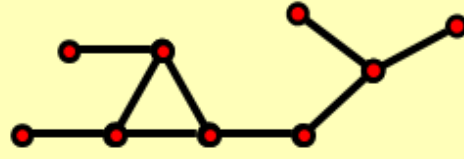
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9-34-3



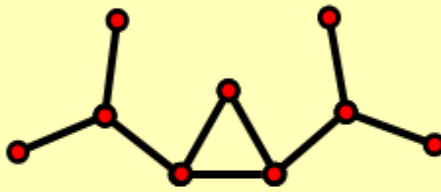
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9-34-5

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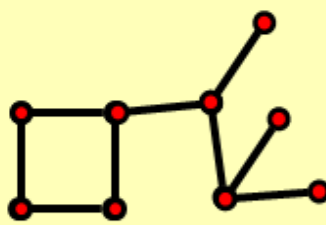
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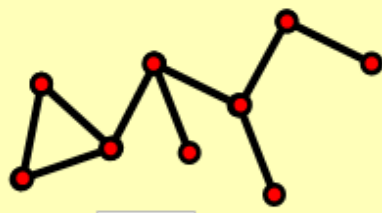
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9-36



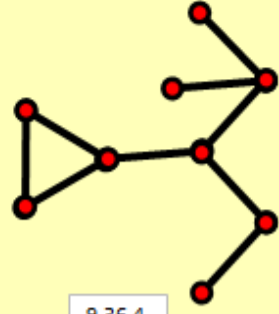
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9-36-2



9-36-3



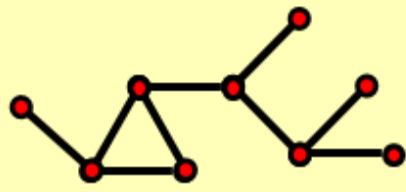
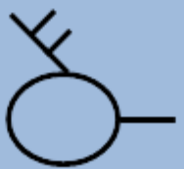
9-36-4



9-36-5

5

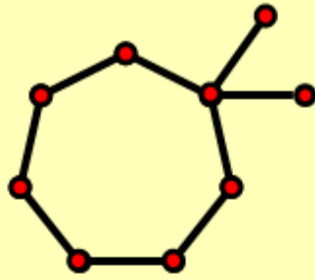
9-37



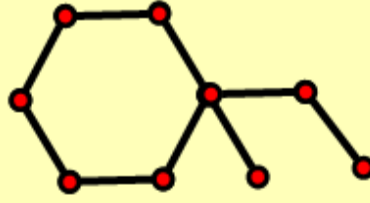
9-37-1

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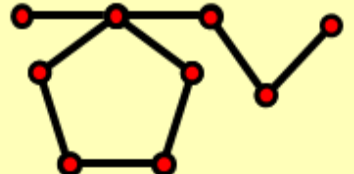
3.9.2.3. Matchstick graphs with  $|E|=9$ ,  $F=2$ ,  $|V|=9$ ,  $\Delta=4$



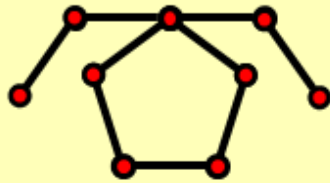
9-38-1



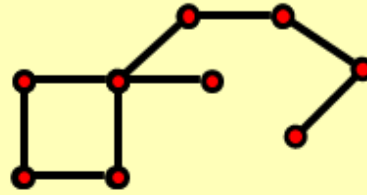
9-38-2



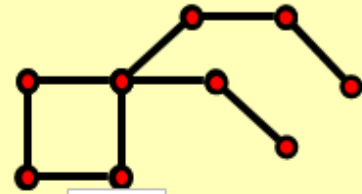
9-38-3



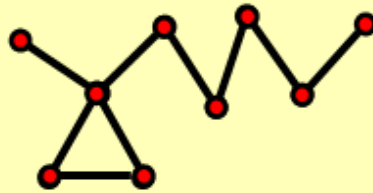
9-38-4



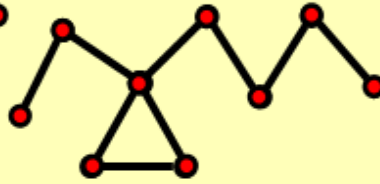
9-38-5



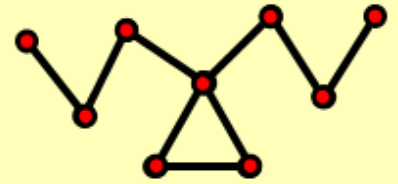
9-38-6



9-38-7



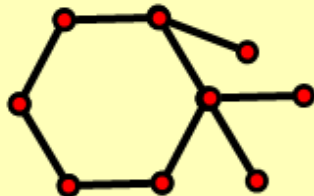
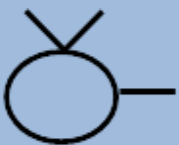
9-38-8



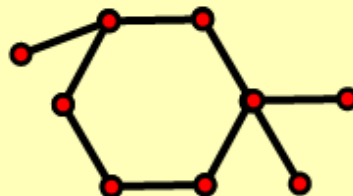
9-38-9

9-38

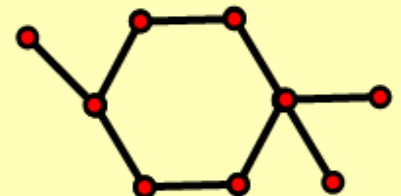
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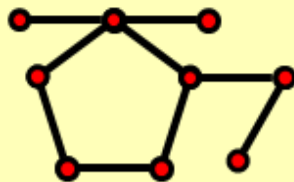
9-39-1



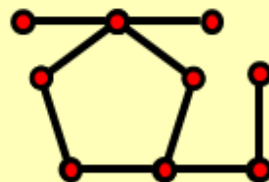
9-39-2



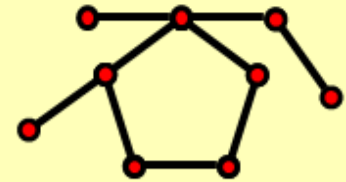
9-39-3



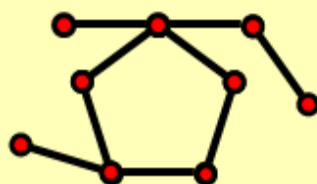
9-39-4



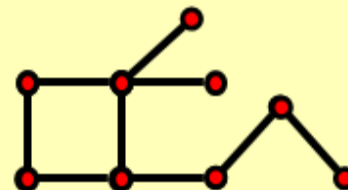
9-39-5



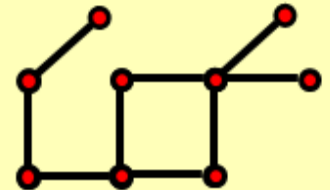
9-39-6



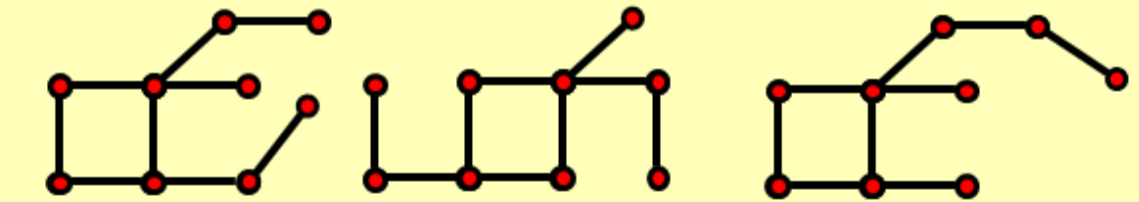
9-39-7



9-39-8



9-39-9



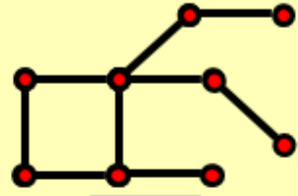
9-39-10

9-39-11

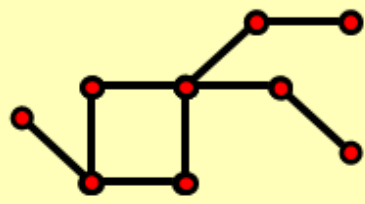
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9-39-13



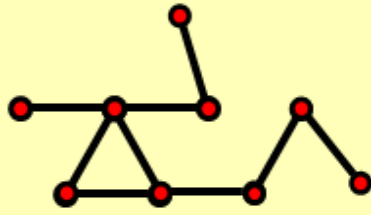
9-39-14



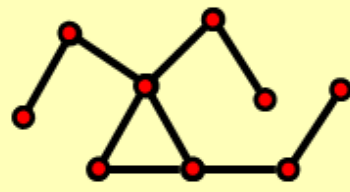
9-39-15



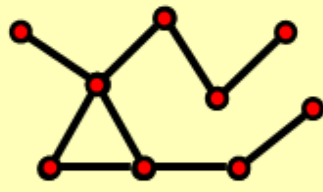
9-39-16



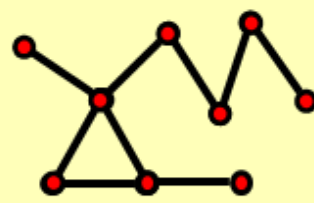
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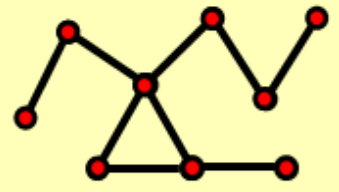
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9-39-19



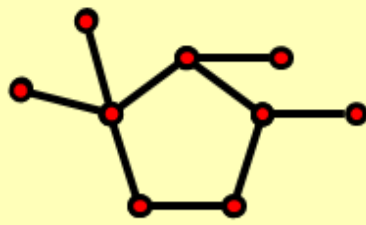
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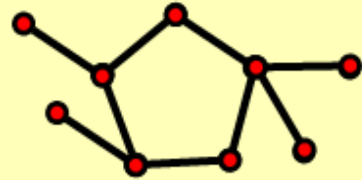
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9-39

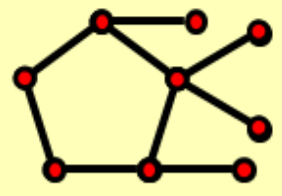
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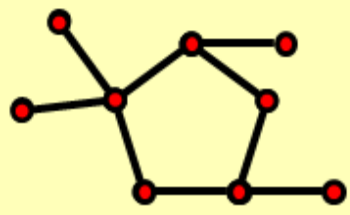
9-40-1



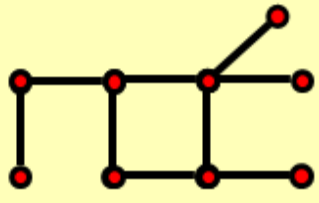
9-40-2



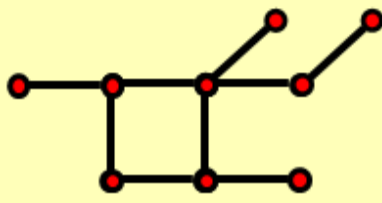
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9-40-4

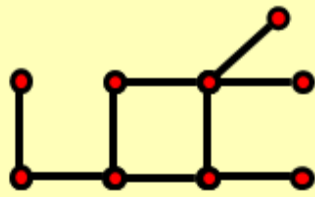


9-40-5

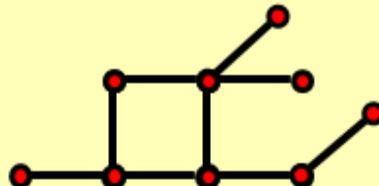


9-40-6

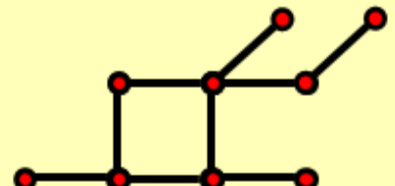
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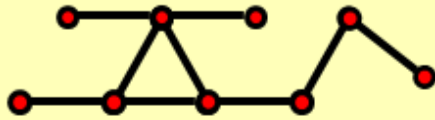
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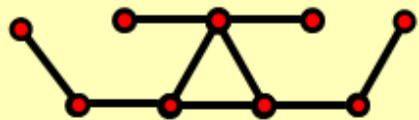
9-40-8



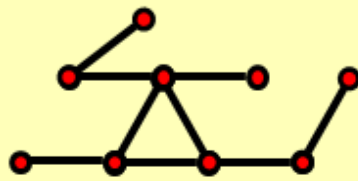
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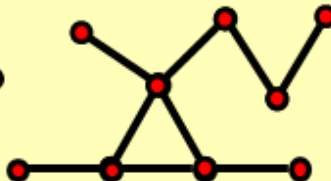
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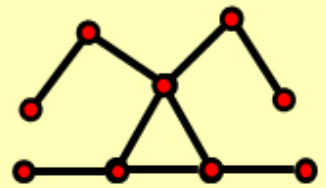
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9-40-12



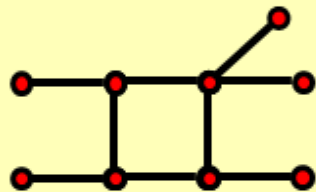
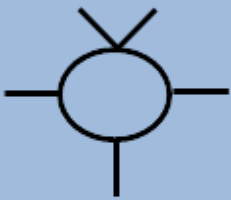
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9-40-14

14

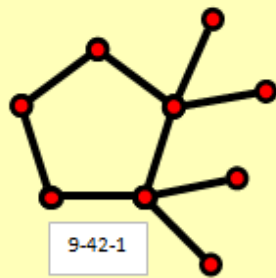
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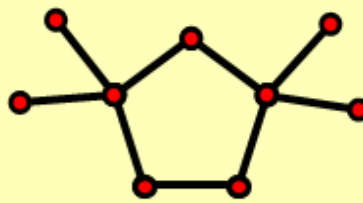
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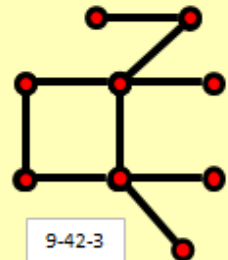
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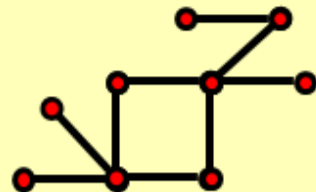
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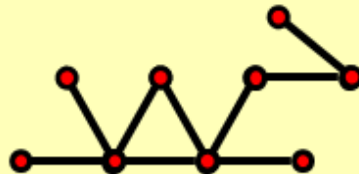
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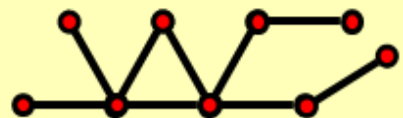
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9-42-4



9-42-5


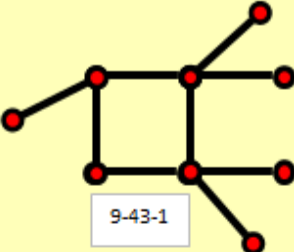
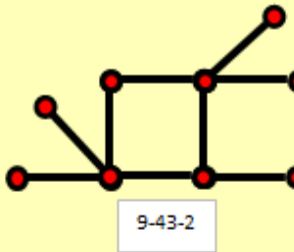
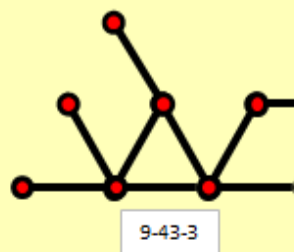
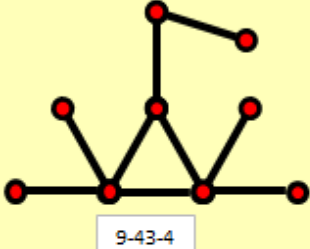

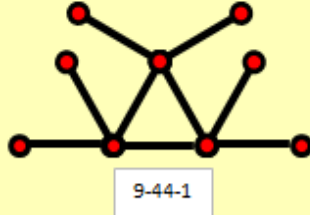

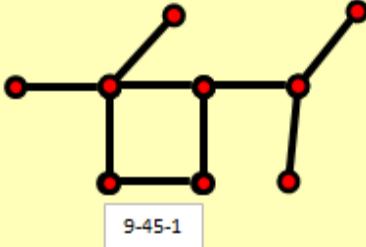
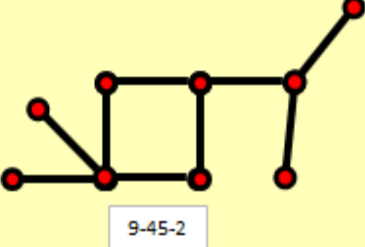
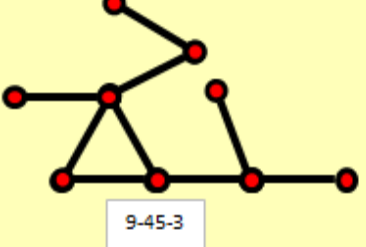
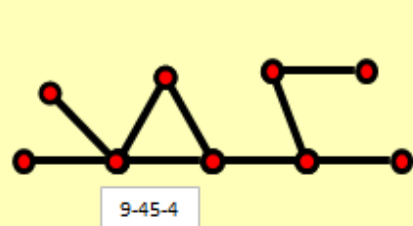
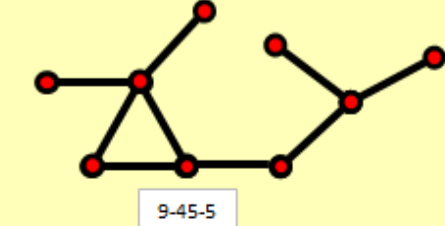
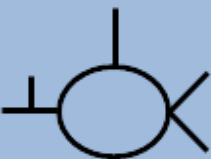
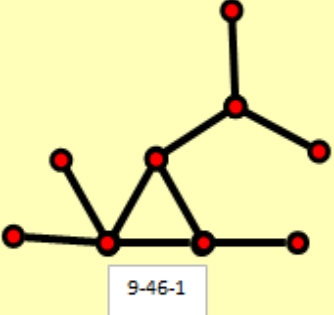

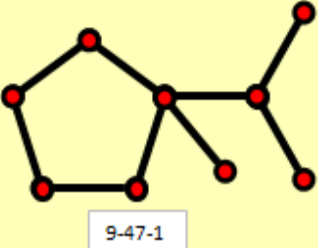
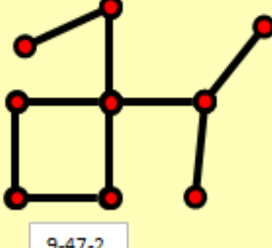
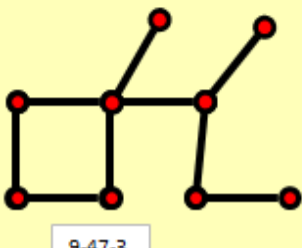


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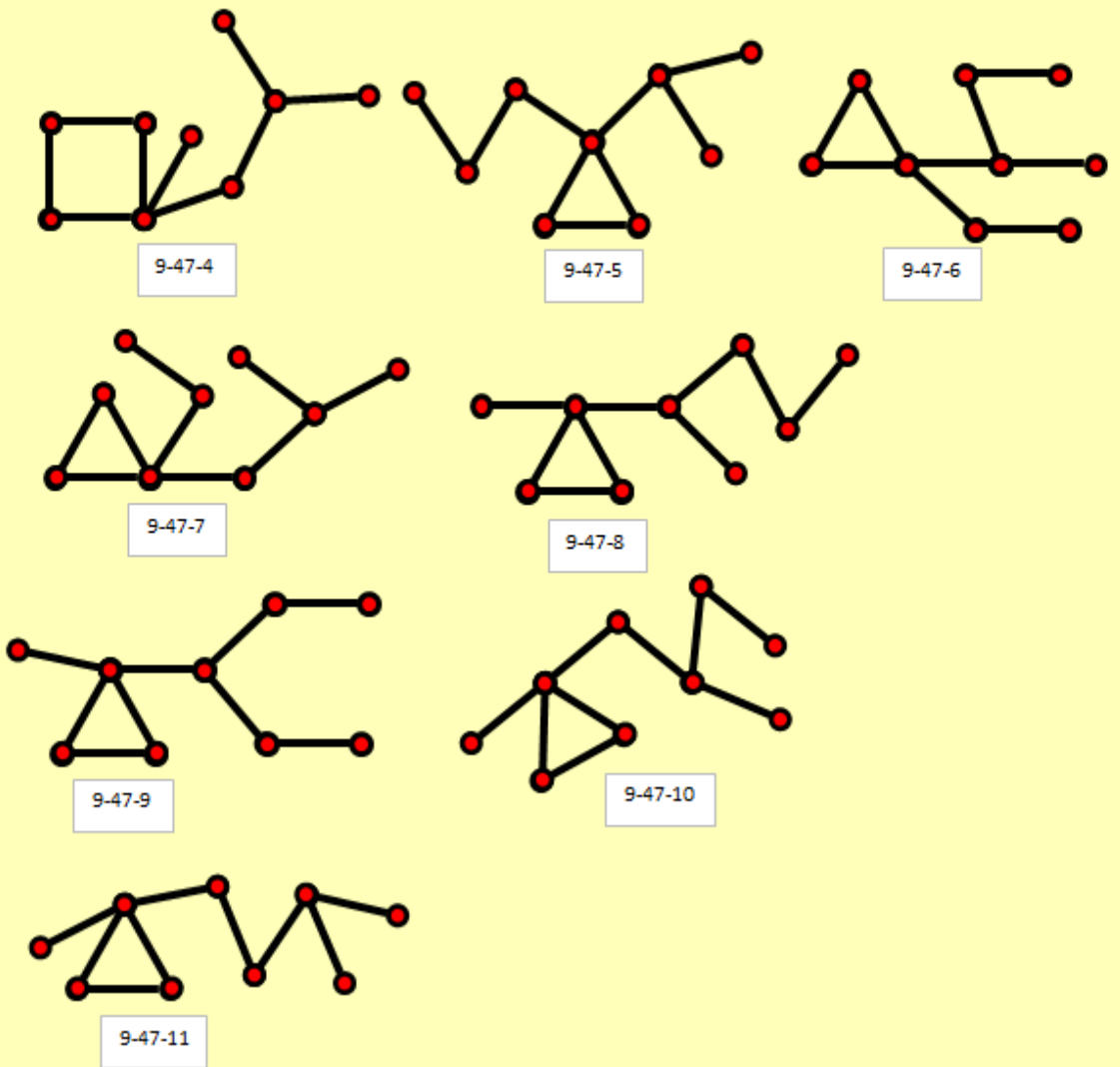


9-42-7

7

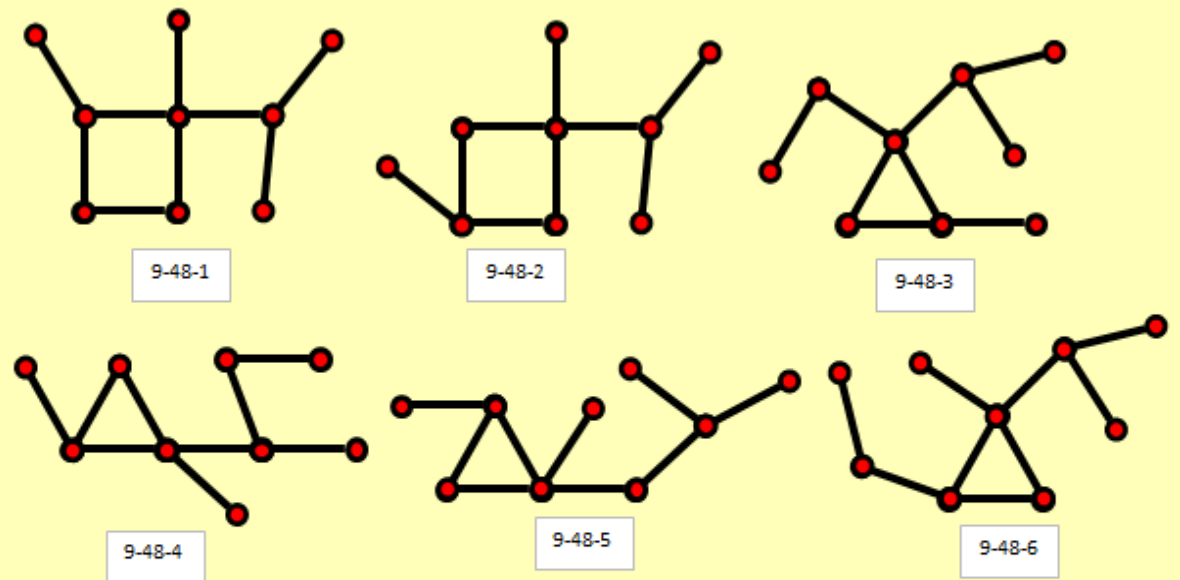
<p>9-43</p> 	 <p>9-43-1</p>  <p>9-43-2</p>  <p>9-43-3</p>  <p>9-43-4</p>	4
<p>9-44</p> 	 <p>9-44-1</p>	1
<p>9-45</p> 	 <p>9-45-1</p>  <p>9-45-2</p>  <p>9-45-3</p>  <p>9-45-4</p>  <p>9-45-5</p>	5
<p>9-46</p> 	 <p>9-46-1</p>	1
	 <p>9-47-1</p>  <p>9-47-2</p>  <p>9-47-3</p>	

9-47



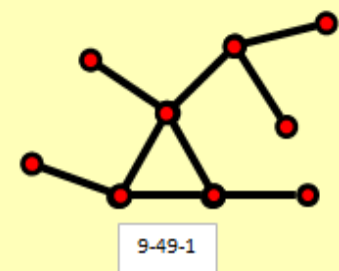
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9-48



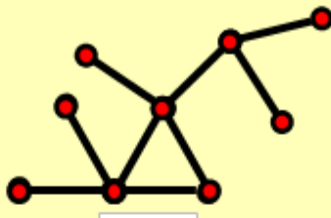
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9-49



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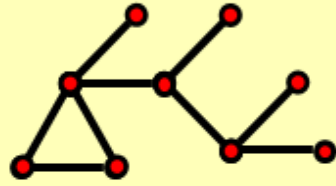
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9-50-1

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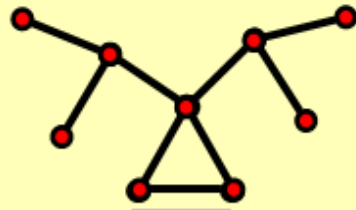
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9-51-1

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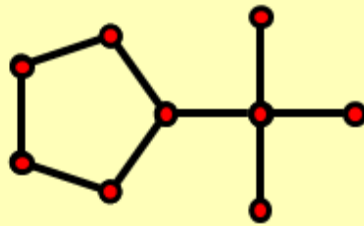
9-52



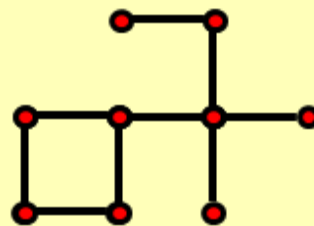
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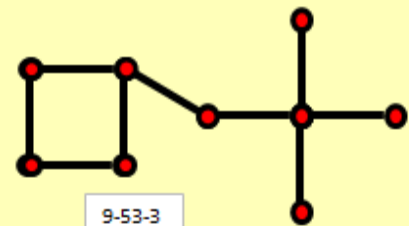
9-53



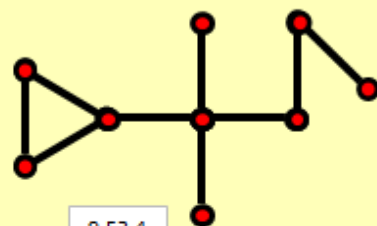
9-53-1



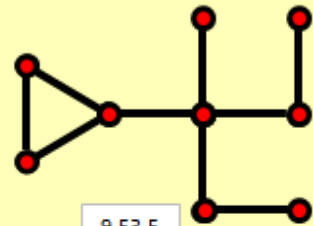
9-53-2



9-53-3



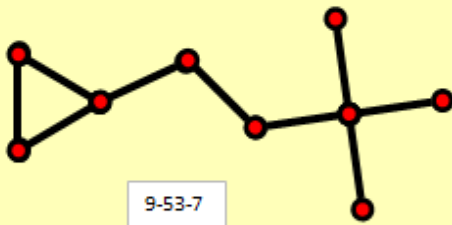
9-53-4



9-53-5



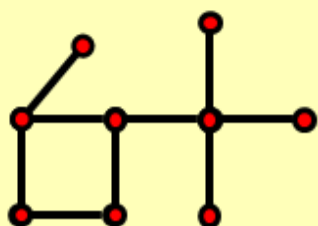
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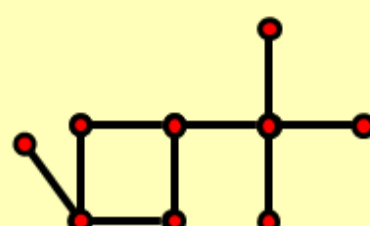
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7

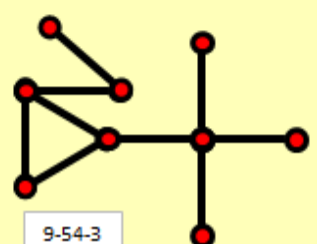
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9-54-1



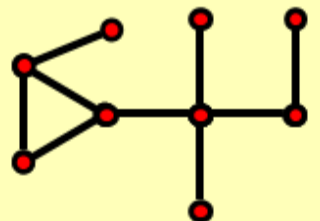
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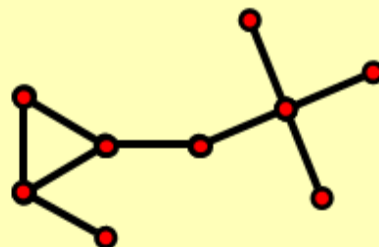
9-54-3



9-54



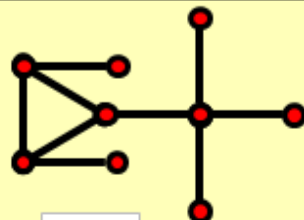
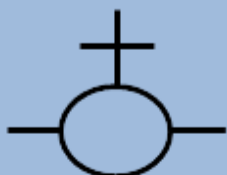
9-54-4



9-54-5

5

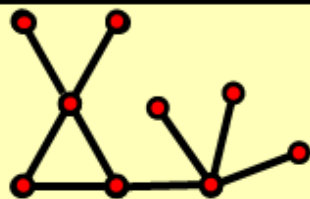
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9-55-1

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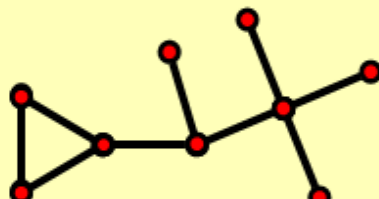
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9-56-1

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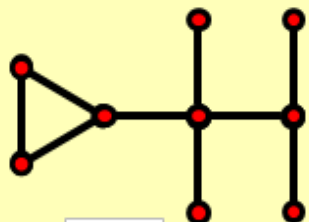
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9-57-1

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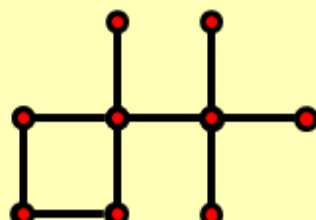
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9-58-1

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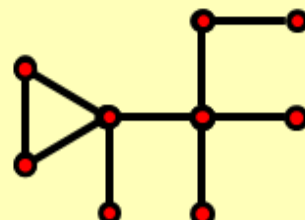
9-59



9-59-1



9-59-2



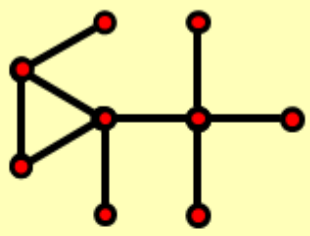
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9-59-4

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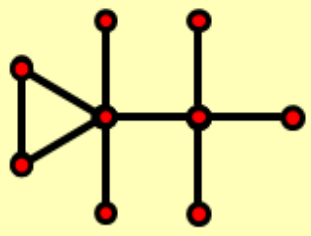
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9-60-1

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9-61

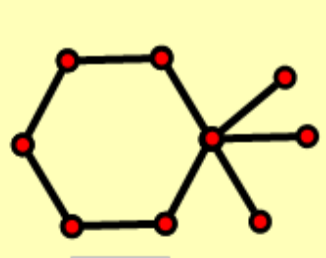


9-61-1

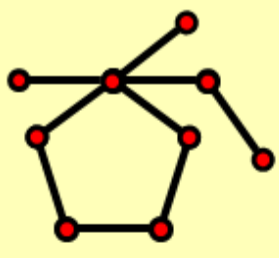
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3.9.2.4. Matchstick graphs with  $|E|=9$ ,  $F=2$ ,  $|V|=9$ ,  $\Delta=5$

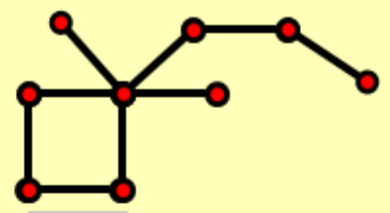
9-62



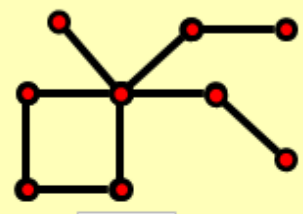
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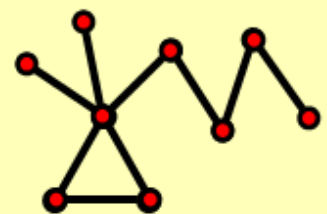
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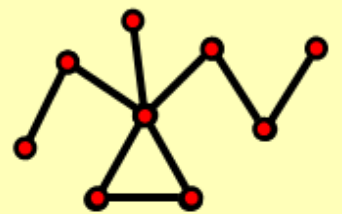
9-62-3



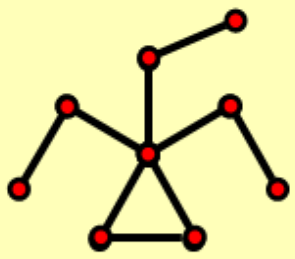
9-62-4



9-62-5



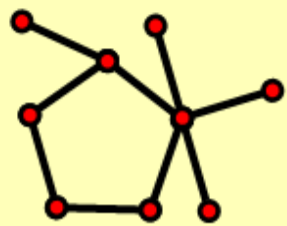
9-62-6



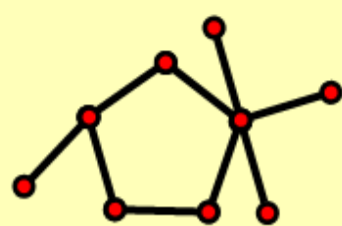
9-62-7

7

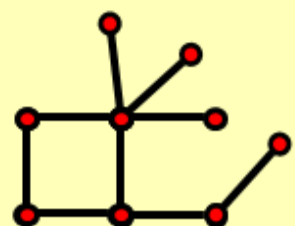
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9-63-1

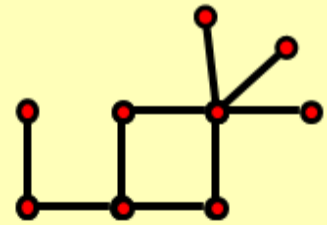


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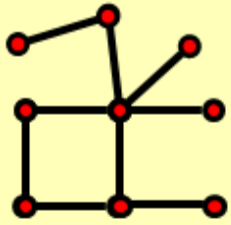


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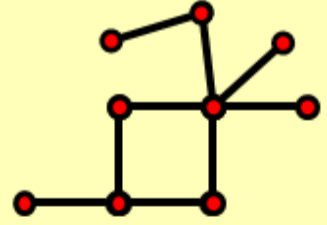
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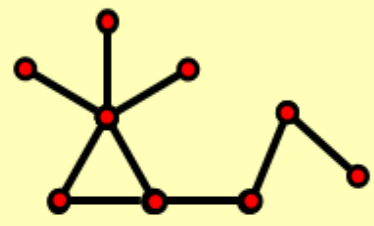
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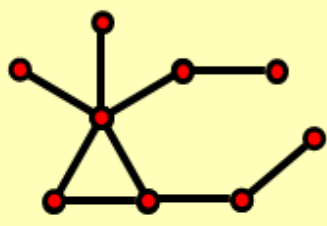
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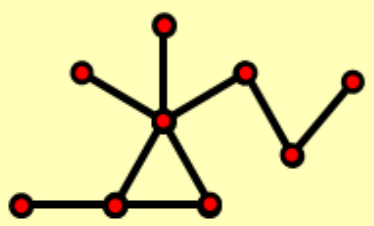
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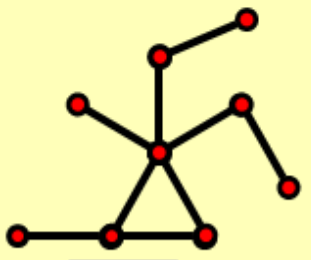
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9-63-8



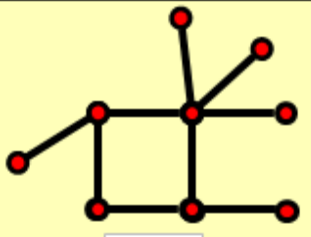
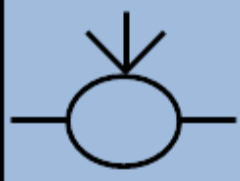
9-63-9



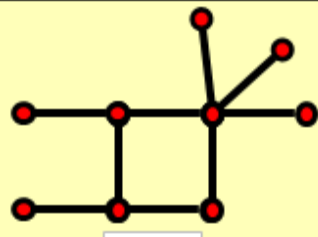
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10

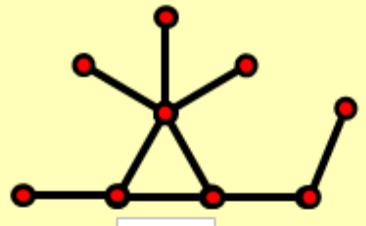
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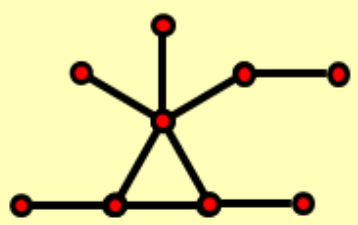
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9-64-2



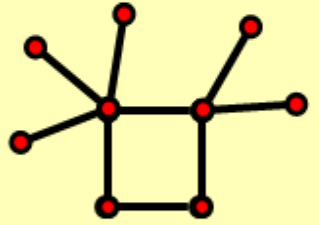
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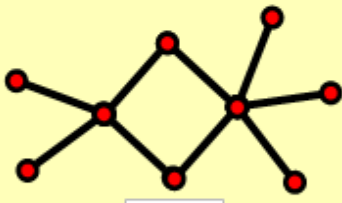
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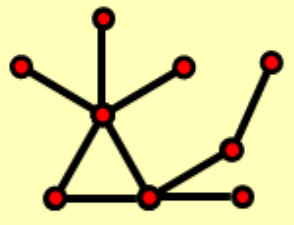
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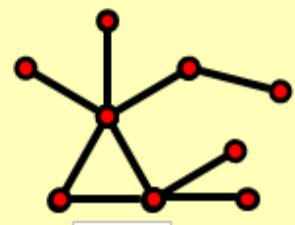
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9-65-2


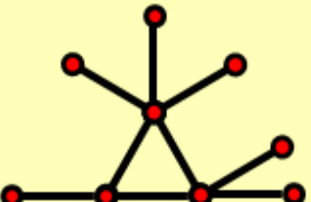

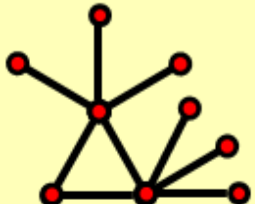

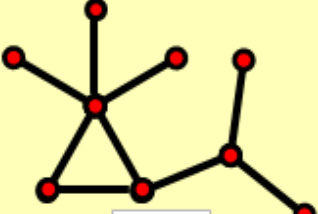

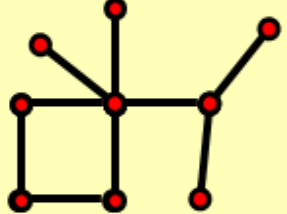
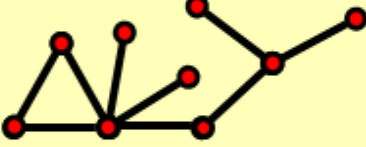
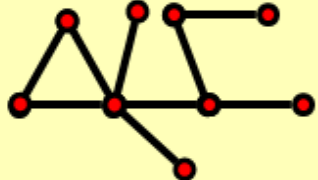
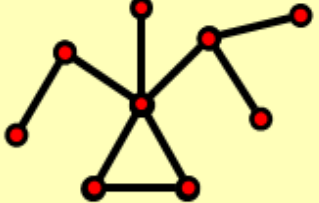

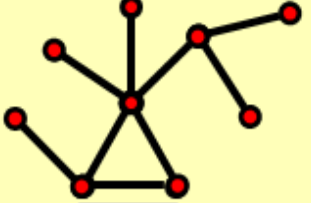

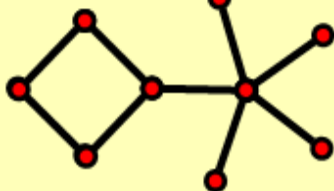




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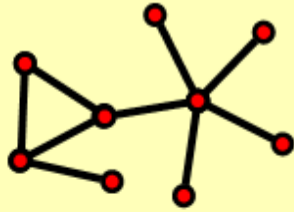


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4

<p>9-66</p> 	 <p>9-66-1</p>	1
<p>9-67</p> 	 <p>9-67-1</p>	1
<p>9-68</p> 	 <p>9-68-1</p>	1
<p>9-69</p> 	 <p>9-69-1</p>  <p>9-69-2</p>  <p>9-69-3</p>  <p>9-69-4</p>	4
<p>9-70</p> 	 <p>9-70-1</p>	1
<p>9-71</p> 	 <p>9-71-1</p>  <p>9-71-2</p>  <p>9-71-3</p>	3

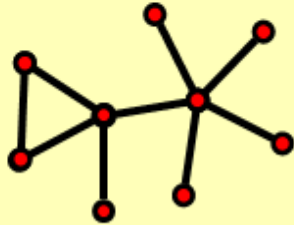
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9-73

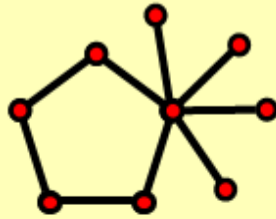


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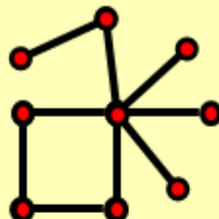
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3.9.2.5. Matchstick graphs with  $|E|=9, F=2, |V|=9, \Delta=6$

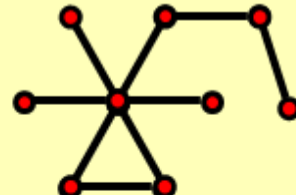
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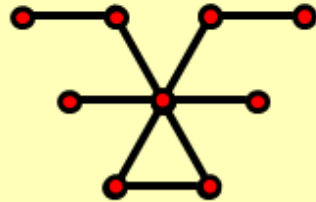
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9-74-2



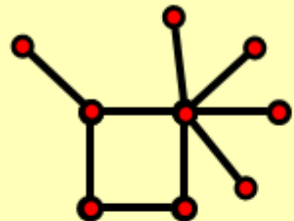
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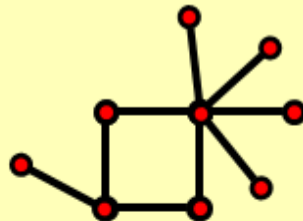
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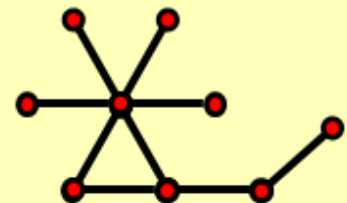
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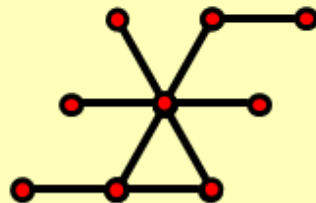
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9-75-2

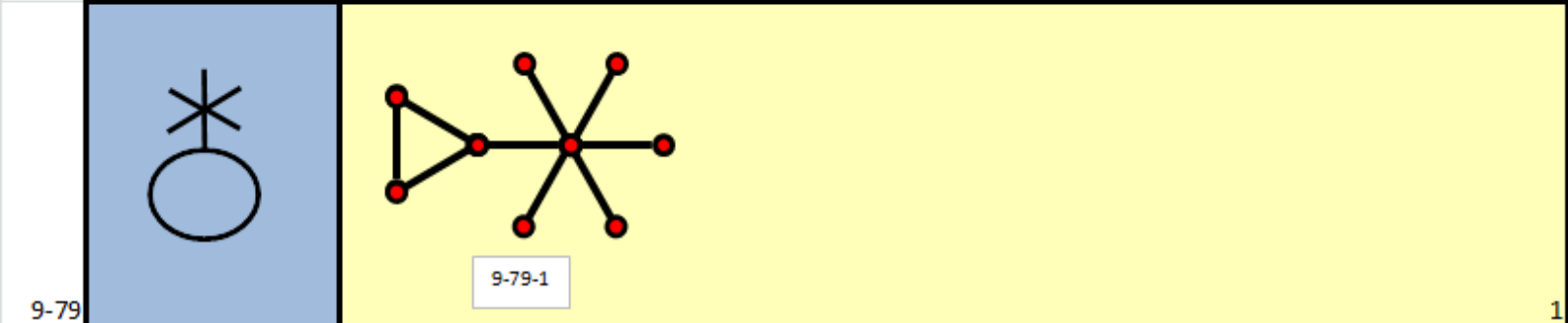
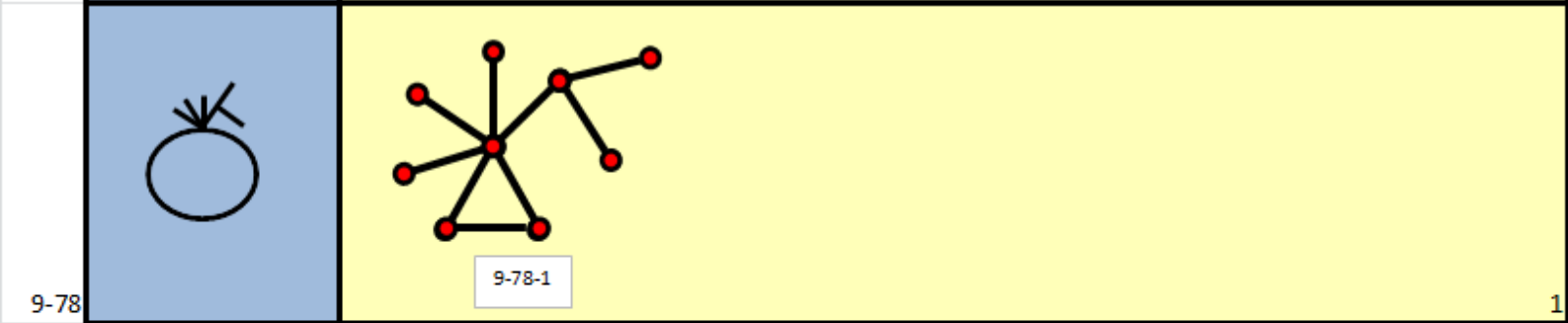
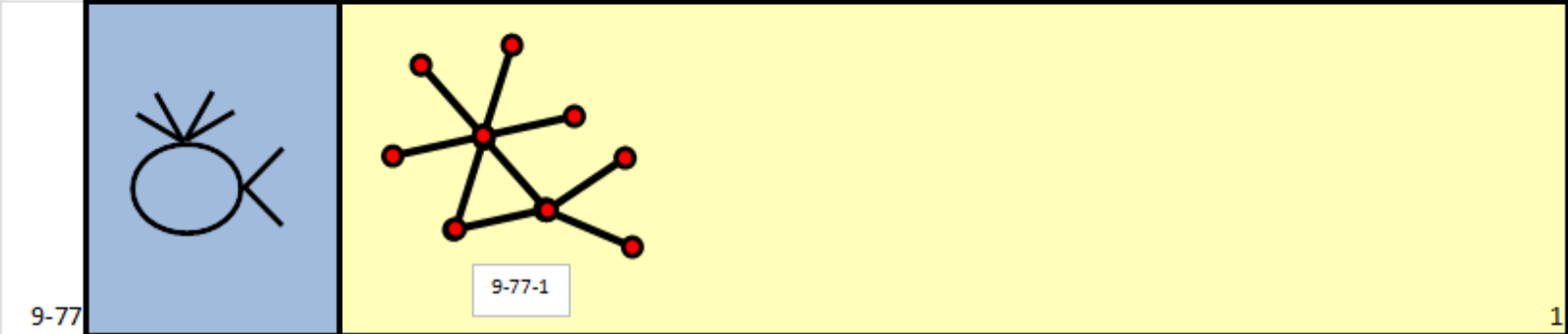
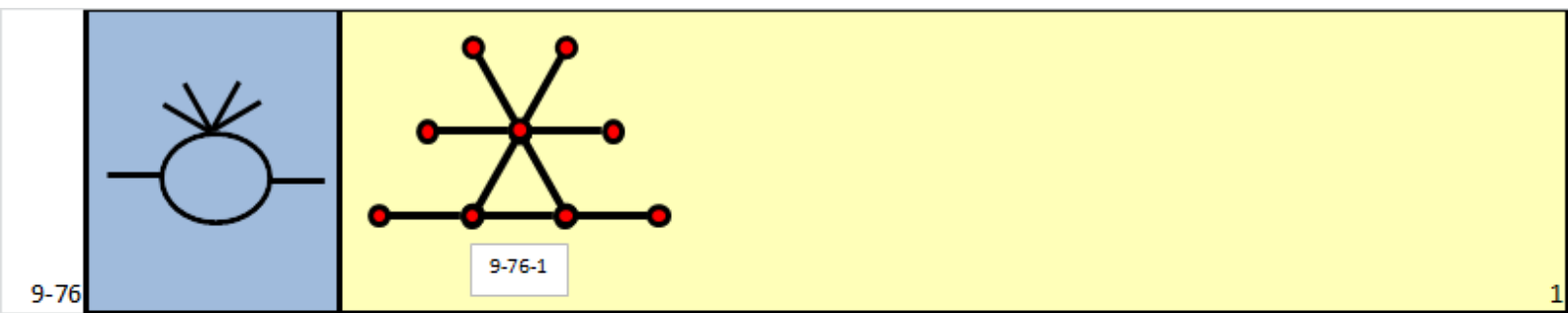


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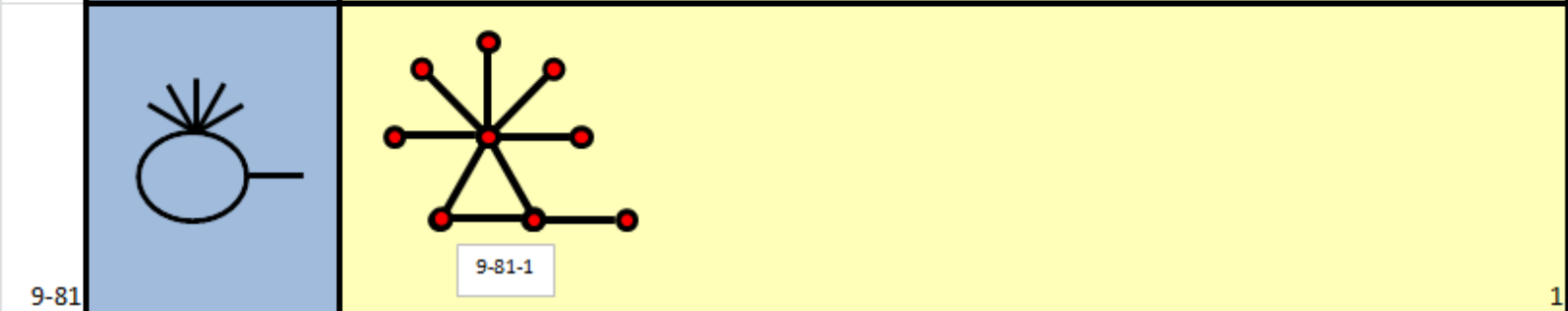
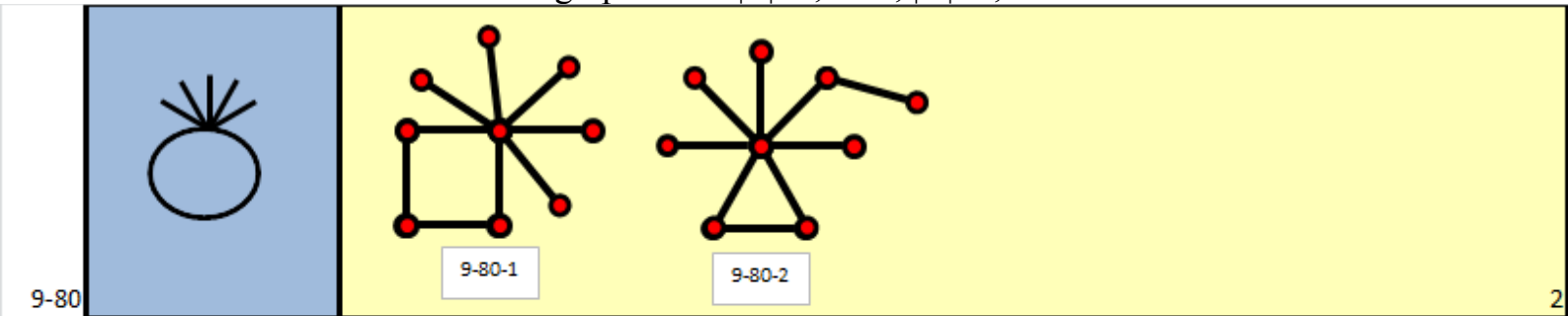


9-75-4

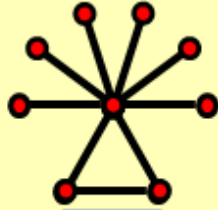
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3.9.2.6. Matchstick graphs with  $|E|=9, F=2, |V|=9, \Delta=7$



3.9.2.7. Matchstick graphs with  $|E|=9, F=2, |V|=9, \Delta=8$



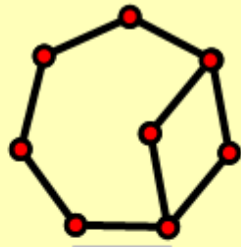
9-82-1

9-82

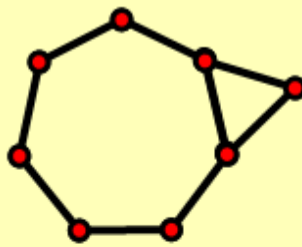
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3.9.3. Matchstick graphs with  $|E|=9, F=3, |V|=8$

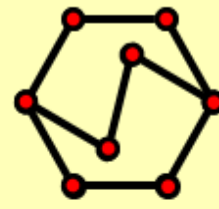
3.9.3.1. Matchstick graphs with  $|E|=9, F=3, |V|=8, \Delta=3$



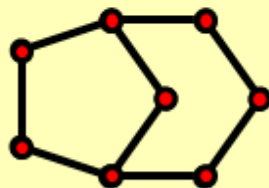
9-83-1



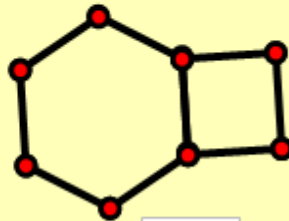
9-83-2



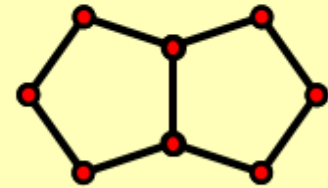
9-83-3



9-83-4



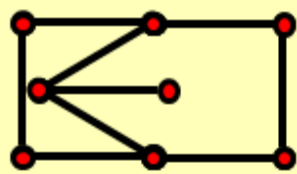
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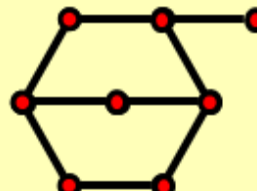
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9-83

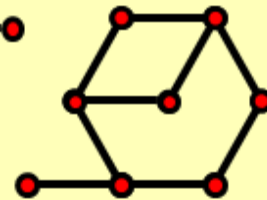
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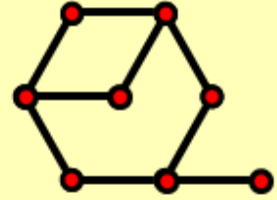
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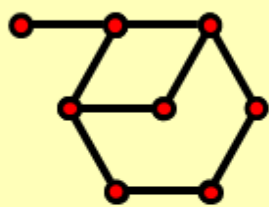
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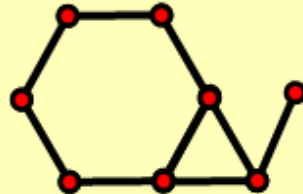
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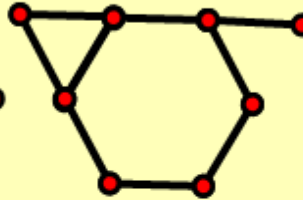
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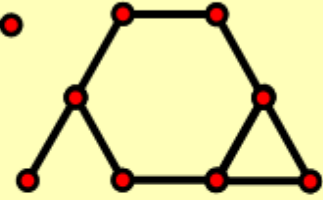
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9-84-6



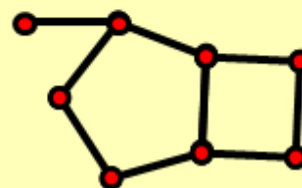
9-84-7



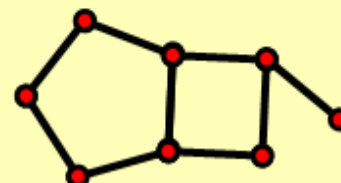
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9-84-9

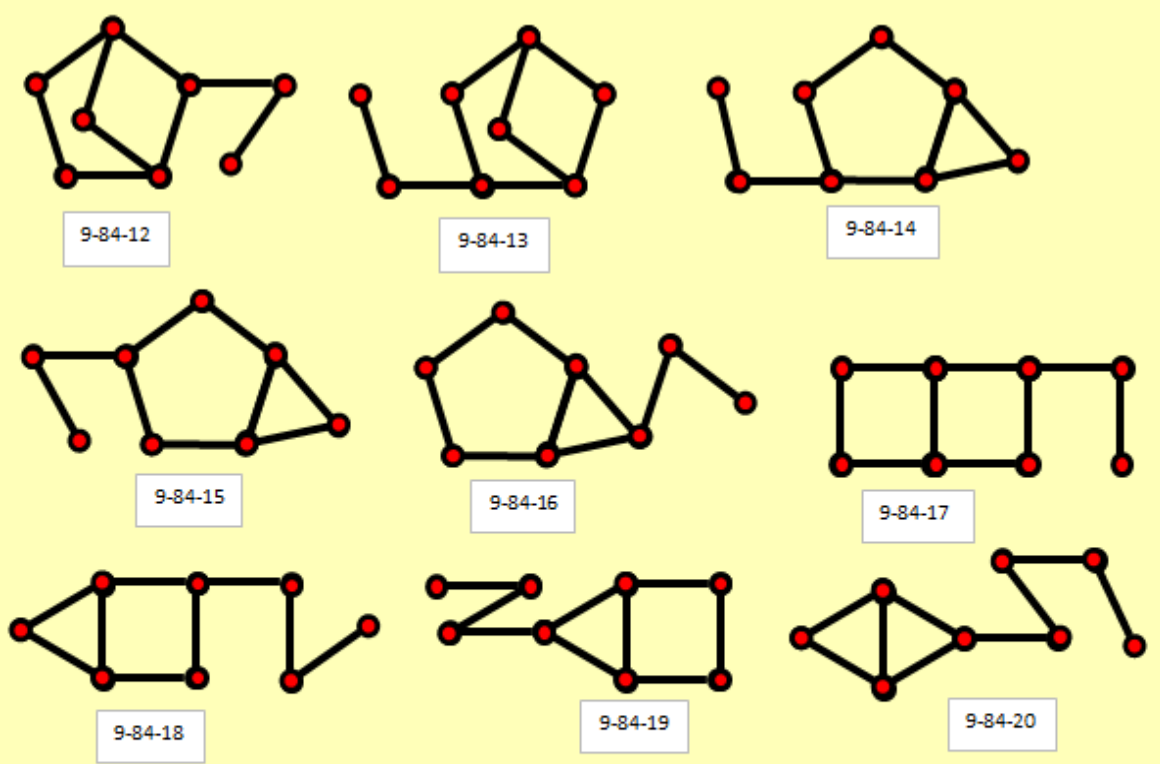


9-84-10



9-84-11

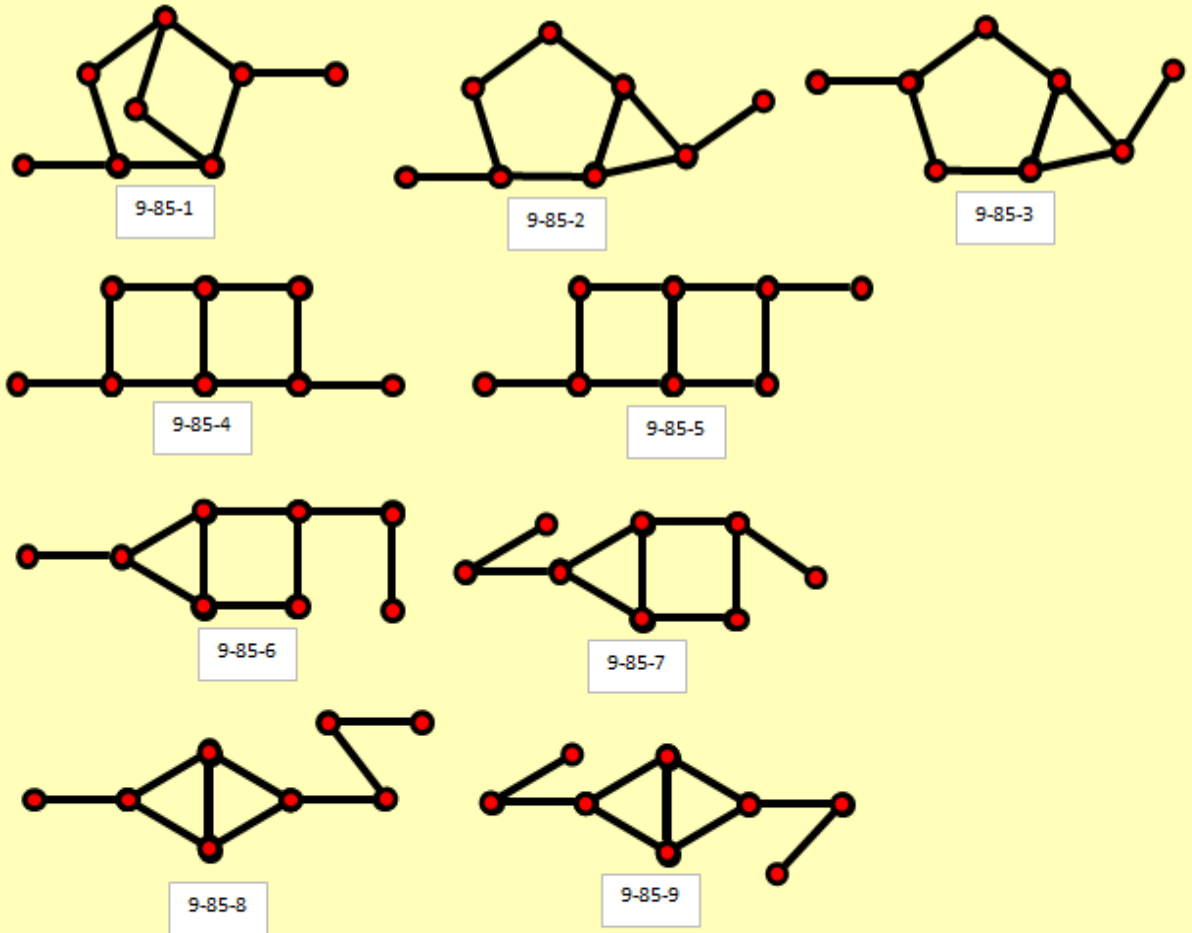
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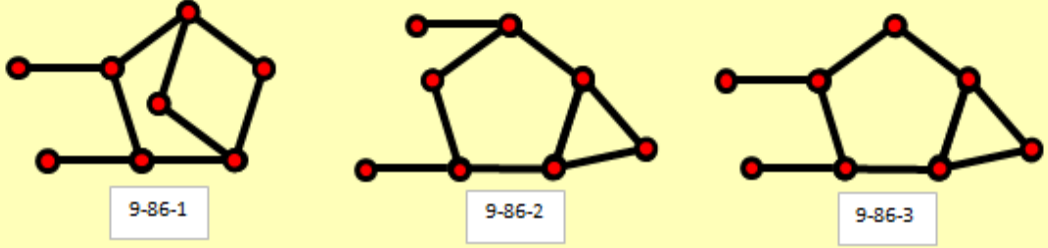
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9-85

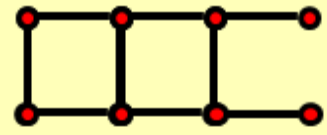


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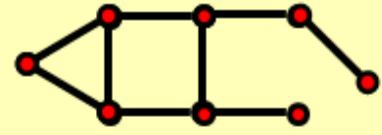




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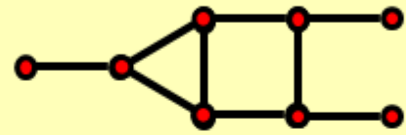
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9-86-5

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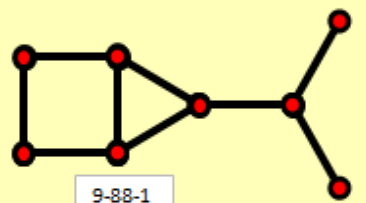
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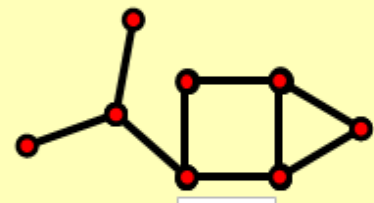
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9-88



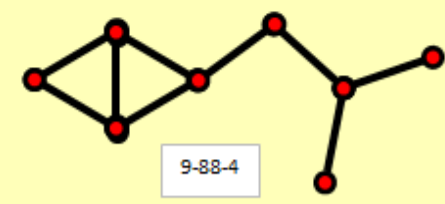
9-88-1



9-88-2



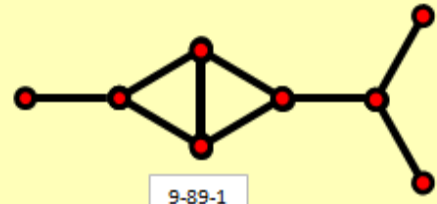
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9-88-4

4

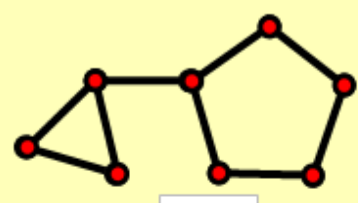
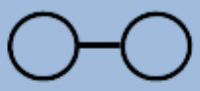
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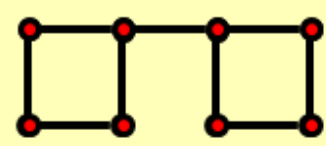
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9-90



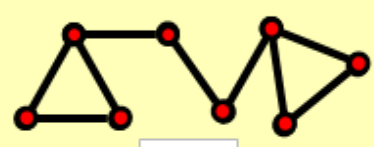
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9-90-2



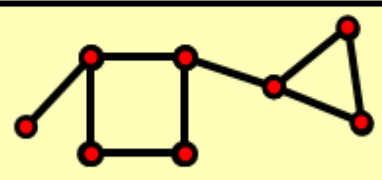
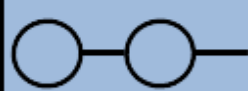
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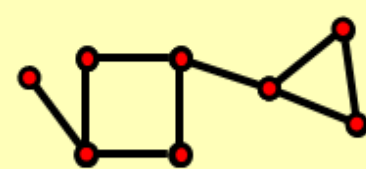
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9-91



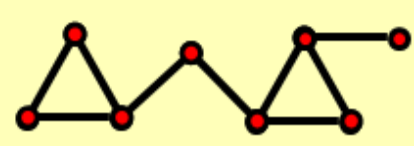
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9-91-2



9-91-3

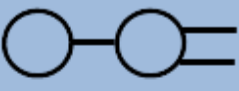
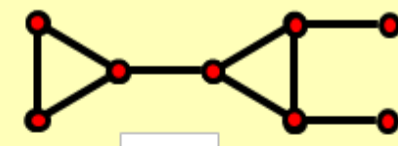
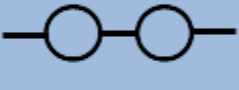
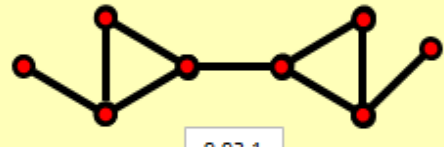
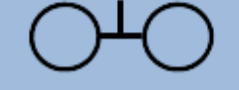



9-91-4


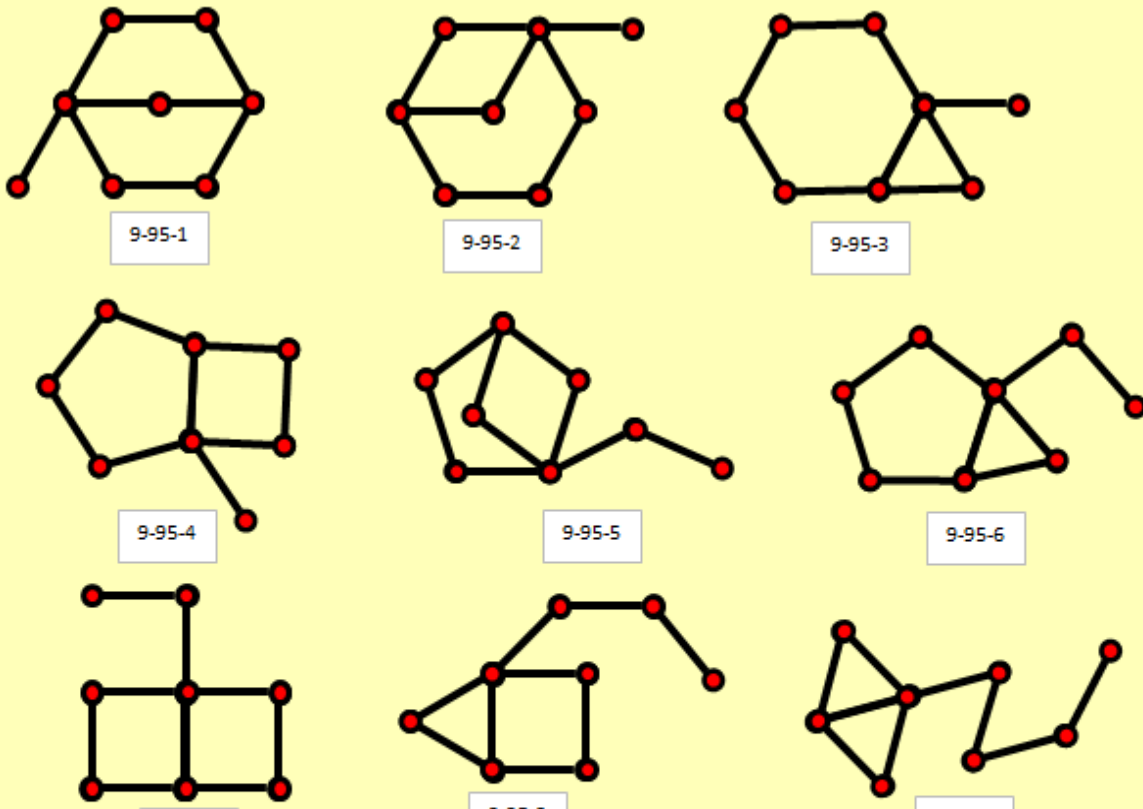
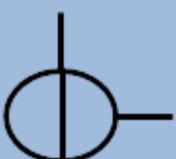
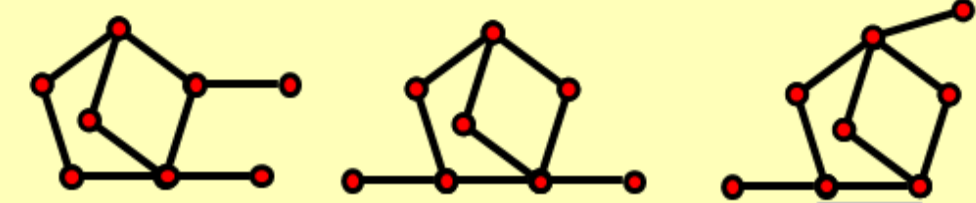


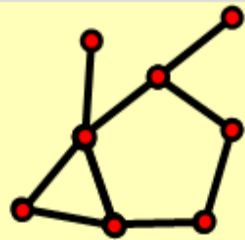
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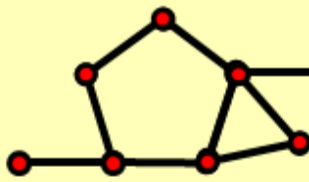
9-92		 <p style="text-align: center;">9-92-1</p>	1
9-93		 <p style="text-align: center;">9-93-1</p>	1
9-94		 <p style="text-align: center;">9-94-1</p>	1

3.9.3.2. Matchstick graphs with  $|E|=9$ ,  $F=3$ ,  $|V|=8$ ,  $\Delta=4$

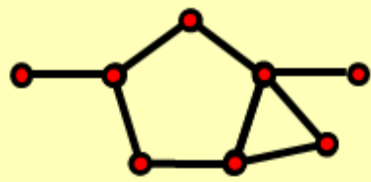
9-95		 <p style="text-align: center;">9-95-1      9-95-2      9-95-3</p> <p style="text-align: center;">9-95-4      9-95-5      9-95-6</p> <p style="text-align: center;">9-95-7      9-95-8      9-95-9</p>	9
		 <p style="text-align: center;">9-96-1      9-96-2      9-96-3</p>	



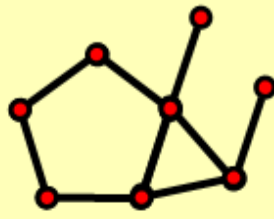
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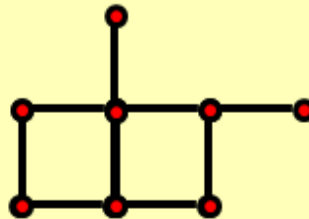
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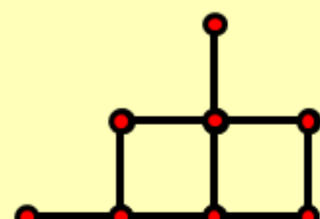
9-96-6



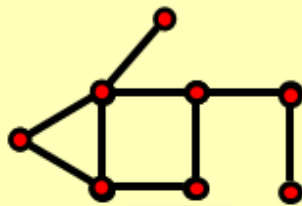
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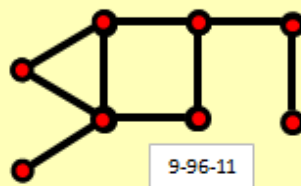
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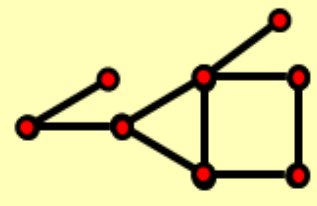
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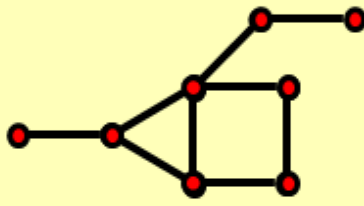
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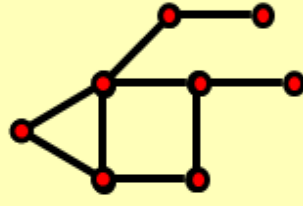
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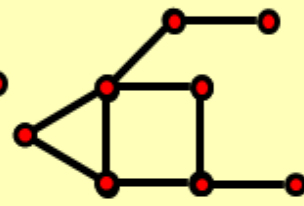
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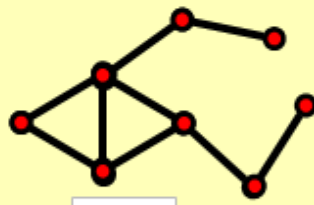
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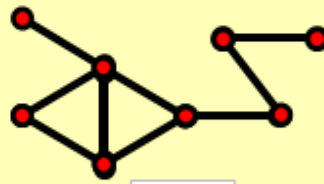
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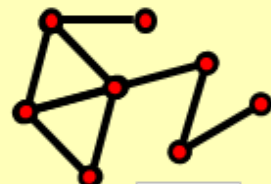
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9-96-16



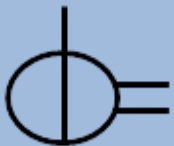
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9-96-18

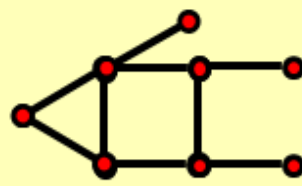
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18



9-97

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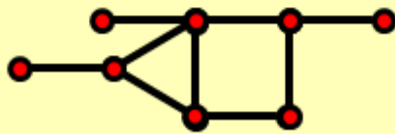


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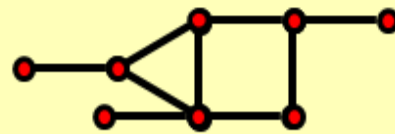


9-98

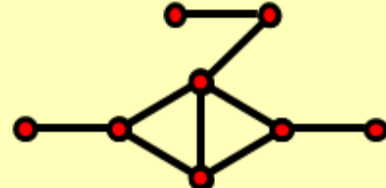
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9-98-1



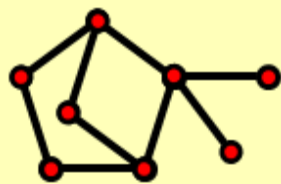
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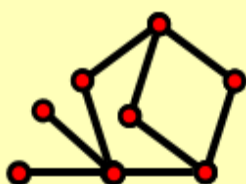
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9-98-4



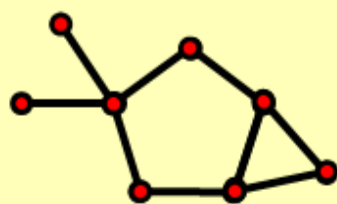
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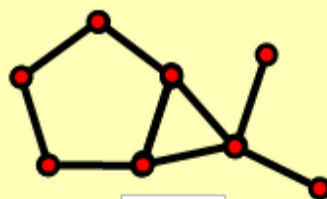
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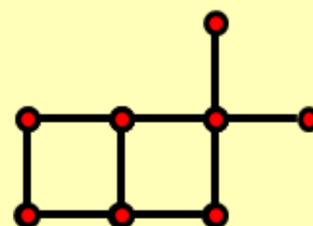
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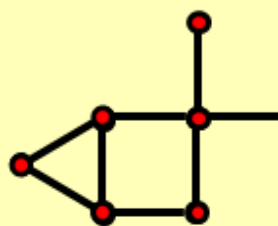
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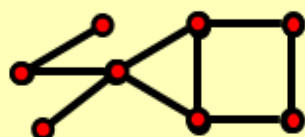
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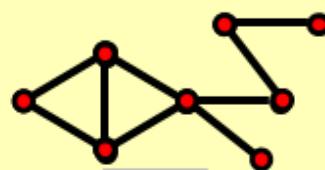
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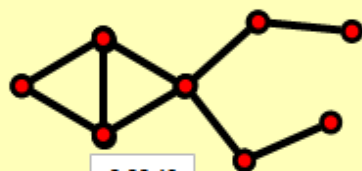
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9-99-8



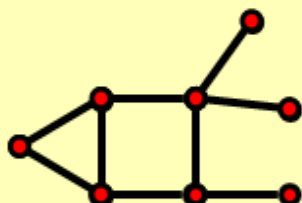
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9-99-10

9-99

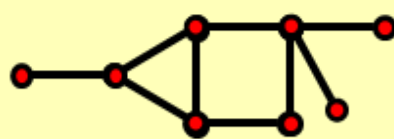
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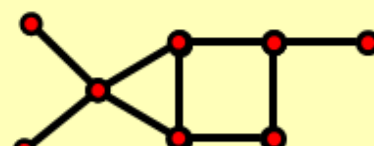
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9-100

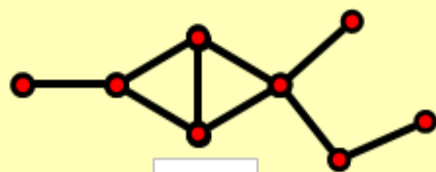
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9-101-1



9-101-2



9-101-3



9-101-4

9-101


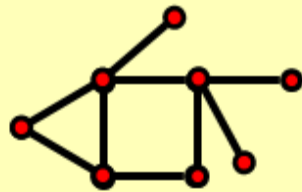
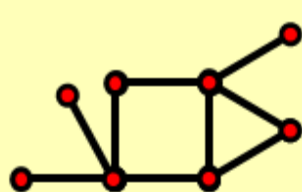
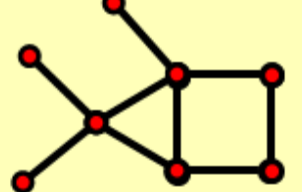
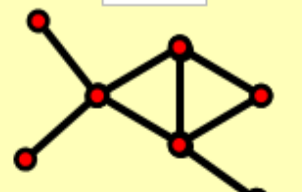
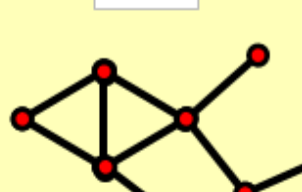

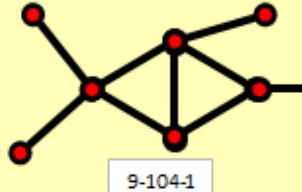
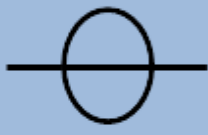
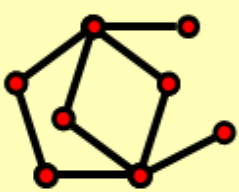
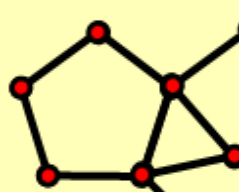
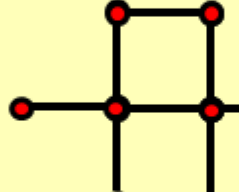
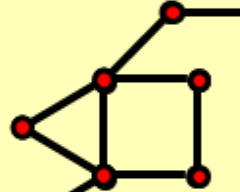
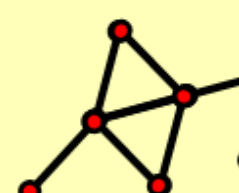
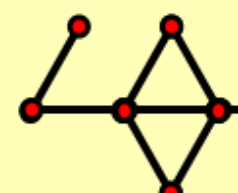
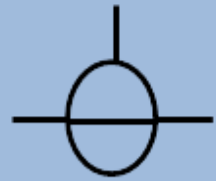
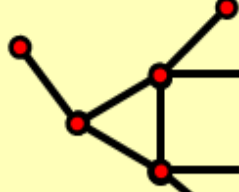
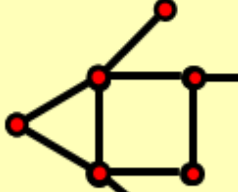
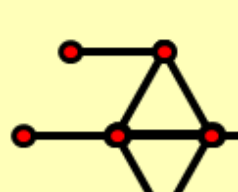
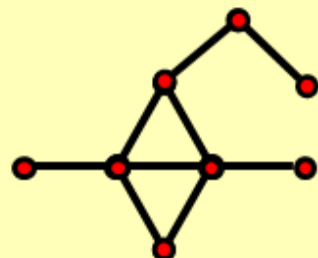
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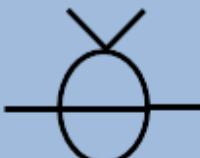
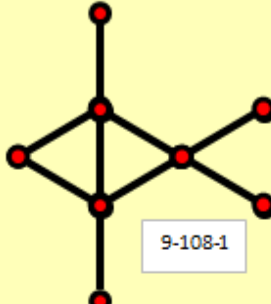

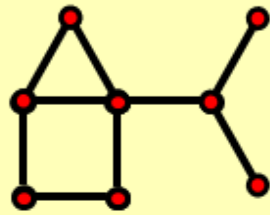
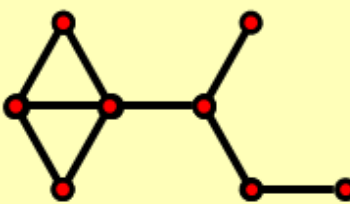
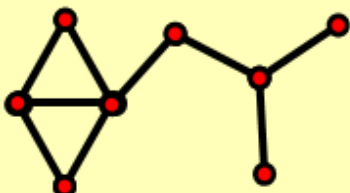

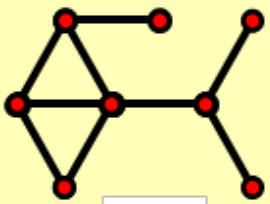

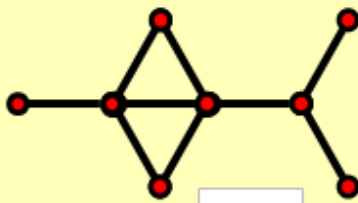

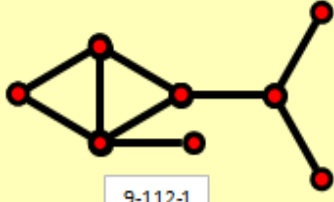

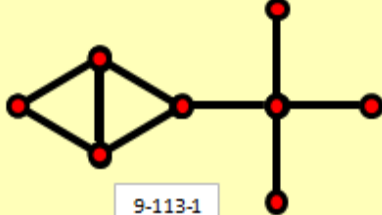



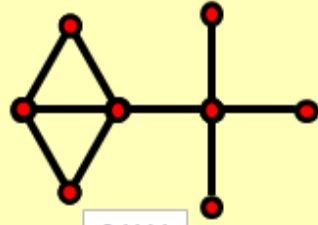

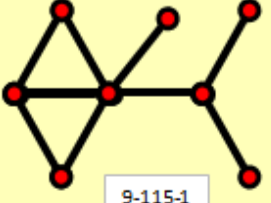

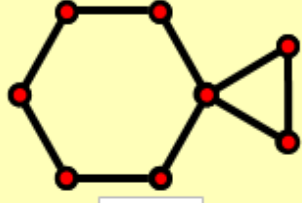
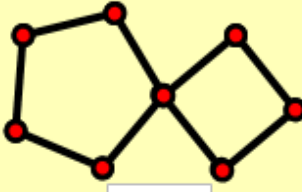

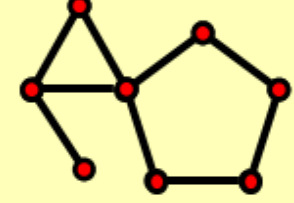
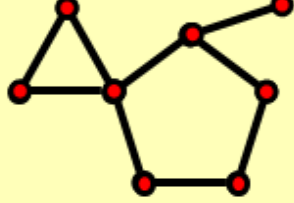
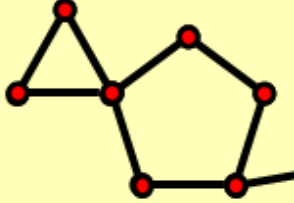
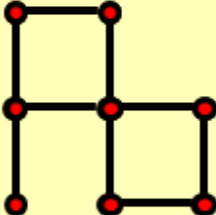
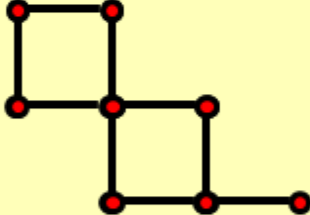
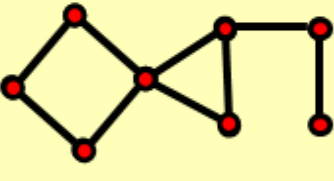
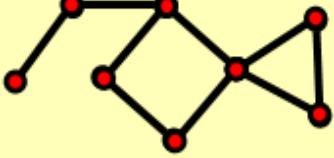
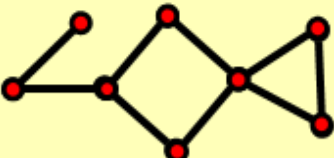
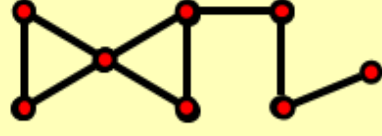

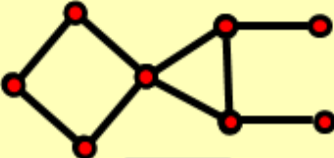
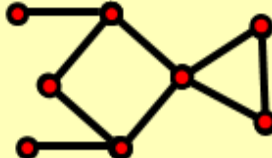
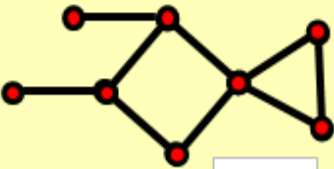
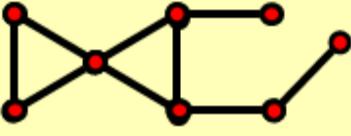
9-102-1

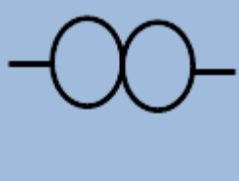
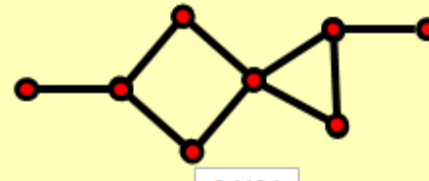
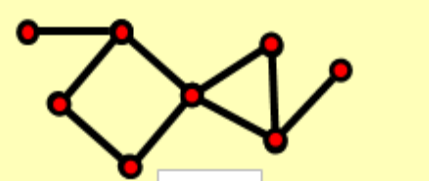
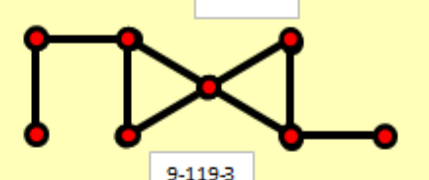

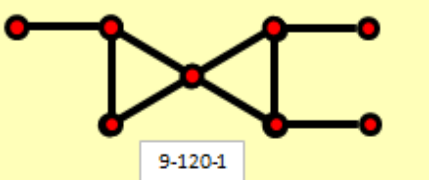

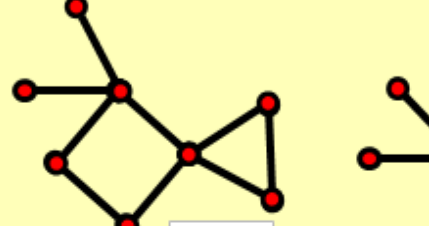
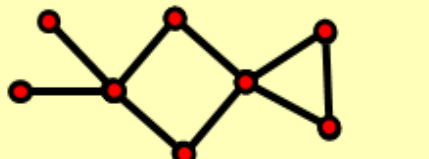
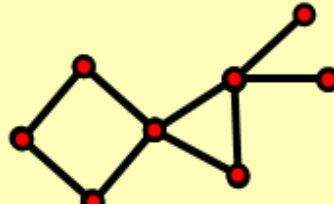
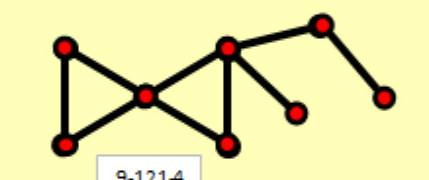

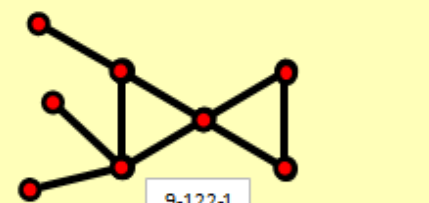

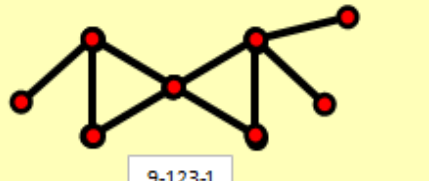

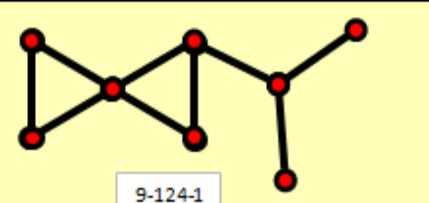
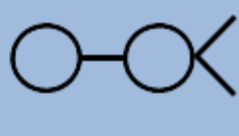
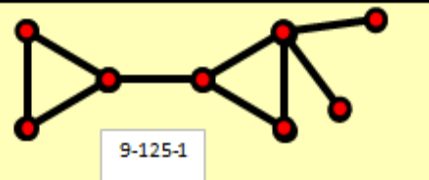
9-102

1



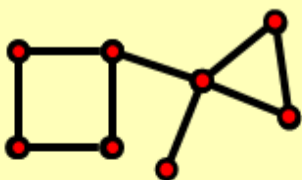
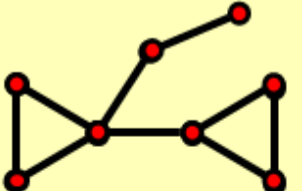
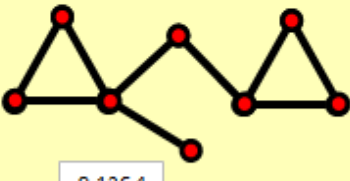

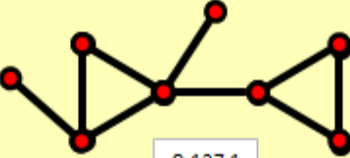

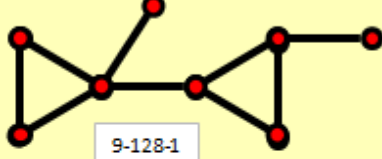

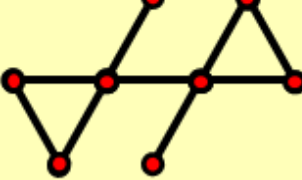
<p>9-103</p> 	 <p>9-103-1</p>  <p>9-103-2</p>  <p>9-103-3</p>  <p>9-103-4</p>  <p>9-103-5</p>	5
<p>9-104</p> 	 <p>9-104-1</p>	1
<p>9-105</p> 	 <p>9-105-1</p>  <p>9-105-2</p>  <p>9-105-3</p>  <p>9-105-4</p>  <p>9-105-5</p>  <p>9-105-6</p>	6
<p>9-106</p> 	 <p>9-106-1</p>  <p>9-106-2</p>  <p>9-106-3</p>  <p>9-106-4</p>	4

9-107		 <p>9-107-1</p>	1
9-108		 <p>9-108-1</p>	1
9-109		 <p>9-109-1</p>  <p>9-109-2</p>  <p>9-109-3</p>	3
9-110		 <p>9-110-1</p>	1
9-111		 <p>9-111-1</p>	1
9-112		 <p>9-112-1</p>	1
9-113		 <p>9-113-1</p>	1


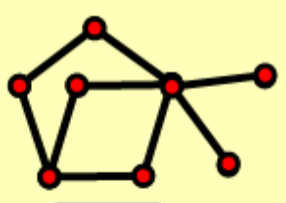
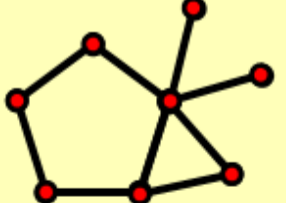
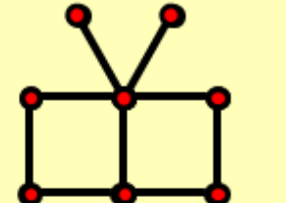
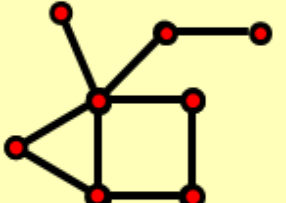

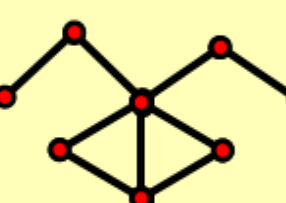


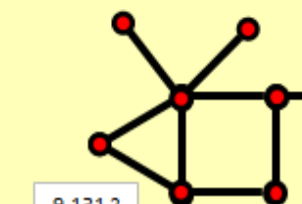

9-114	  <p>9-114-1</p>	1
9-115	  <p>9-115-1</p>	1
9-116	  <p>9-116-1</p>  <p>9-116-2</p>	2
9-117	  <p>9-117-1</p>  <p>9-117-2</p>  <p>9-117-3</p>  <p>9-117-4</p>  <p>9-117-5</p>  <p>9-117-6</p>  <p>9-117-7</p>  <p>9-117-8</p>  <p>9-117-9</p>	9
9-118	  <p>9-118-1</p>  <p>9-118-2</p>  <p>9-118-3</p>  <p>9-118-4</p>	4

<p>9-119</p> 	 <p>9-119-1</p>  <p>9-119-2</p>  <p>9-119-3</p>	<p>3</p>
<p>9-120</p> 	 <p>9-120-1</p>	<p>1</p>
<p>9-121</p> 	 <p>9-121-1</p>  <p>9-121-2</p>  <p>9-121-3</p>  <p>9-121-4</p>	<p>4</p>
<p>9-122</p> 	 <p>9-122-1</p>	<p>1</p>
<p>9-123</p> 	 <p>9-123-1</p>	<p>1</p>
<p>9-124</p> 	 <p>9-124-1</p>	<p>1</p>
<p>9-125</p> 	 <p>9-125-1</p>	<p>1</p>


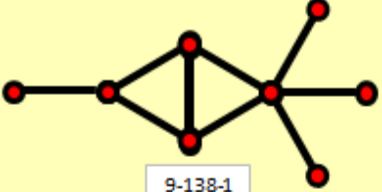

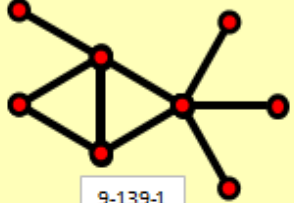

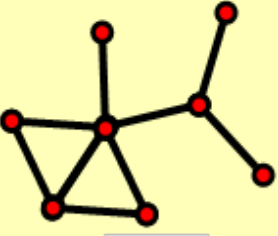



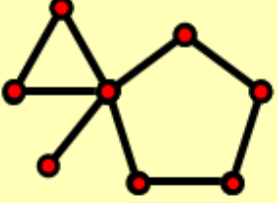
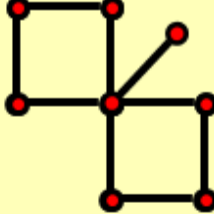
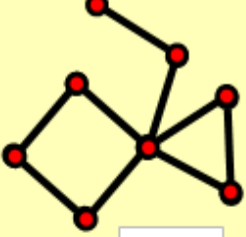
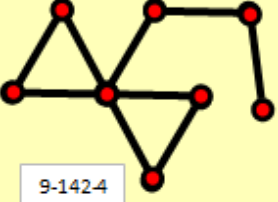

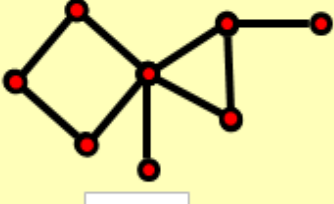
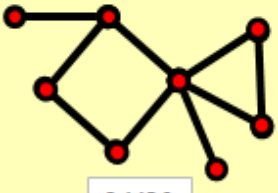
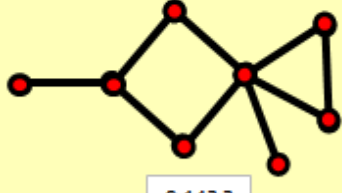




<p>9-126</p> 	 <p>9-126-1</p>  <p>9-126-2</p>  <p>9-126-3</p>  <p>9-126-4</p>	4
<p>9-127</p> 	 <p>9-127-1</p>	1
<p>9-128</p> 	 <p>9-128-1</p>	1
<p>9-129</p> 	 <p>9-129-1</p>	1

3.9.3.3. Matchstick graphs with  $|E|=9$ ,  $F=3$ ,  $|V|=8$ ,  $\Delta=5$

<p>9-130</p> 	 <p>9-130-1</p>  <p>9-130-2</p>  <p>9-130-3</p>  <p>9-130-4</p>  <p>9-130-5</p>  <p>9-130-6</p>	6
	 <p>9-131-1</p>  <p>9-131-2</p>  <p>9-131-3</p>	

9-131		<p>9-131-4</p>	<p>9-131-5</p>	5
9-132		<p>9-132-1</p>		1
9-133		<p>9-133-1</p>		1
9-134		<p>9-134-1</p>		1
9-135		<p>9-135-1</p>	<p>9-135-2</p>	<p>9-135-3</p>
9-136		<p>9-136-1</p>		1
9-137		<p>9-137-1</p>	<p>9-137-2</p>	<p>9-137-3</p>

9-138	  <p>9-138-1</p>	1
9-139	  <p>9-139-1</p>	1
9-140	  <p>9-140-1</p>	1
9-141	  <p>9-141-1</p>	1
9-142	  <p>9-142-1</p>  <p>9-142-2</p>  <p>9-142-3</p>  <p>9-142-4</p>	4
9-143	  <p>9-143-1</p>  <p>9-143-2</p>  <p>9-143-3</p>  <p>9-143-4</p>  <p>9-143-5</p>	5

9-144			1
9-145			1
9-146			1
9-147			1
9-148			1

3.9.3.4. Matchstick graphs with  $|E|=9$ ,  $F=3$ ,  $|V|=8$ ,  $\Delta=6$

9-149				2
9-150			1	

9-151			1
9-152			1
9-153		 	2
9-154			1

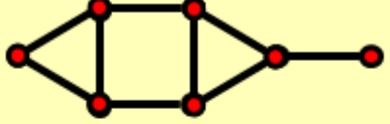
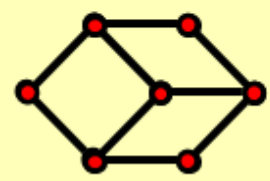
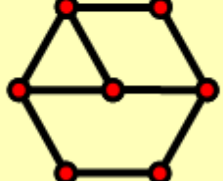
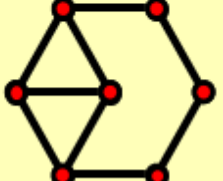
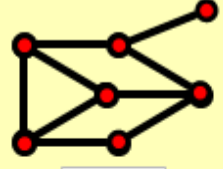
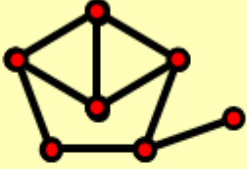
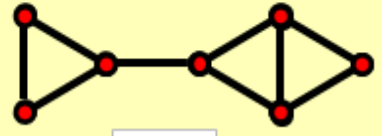
3.9.3.5. Matchstick graphs with  $|E|=9$ ,  $F=3$ ,  $|V|=8$ ,  $\Delta=7$

9-155			1
9-156			1

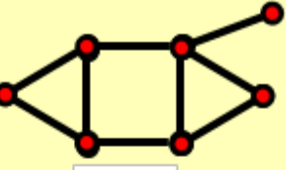

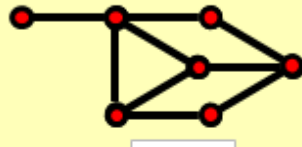
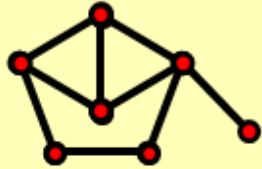
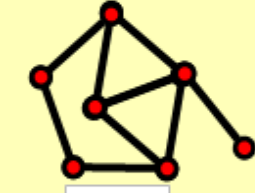
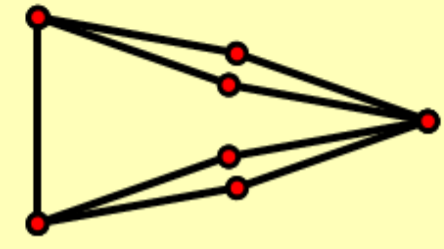
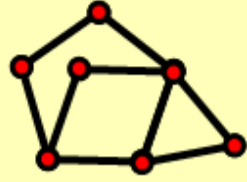
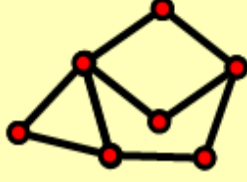
3.9.4. Matchstick graphs with  $|E|=9$ ,  $F=4$ ,  $|V|=7$

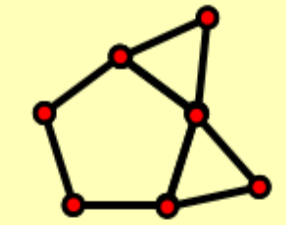
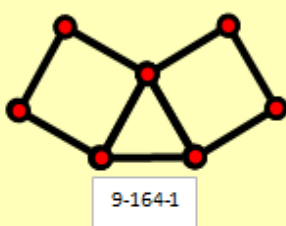
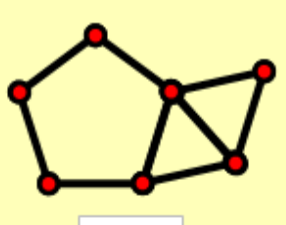
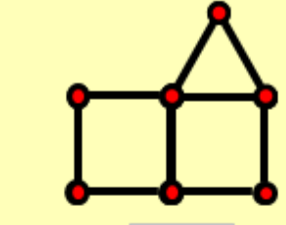
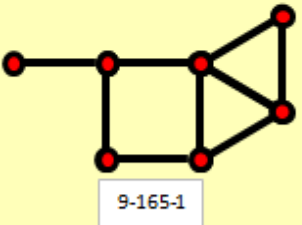
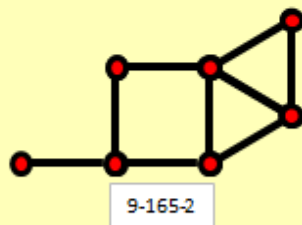
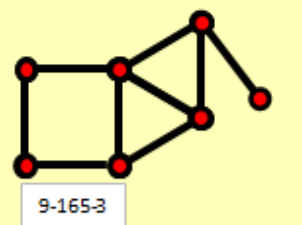
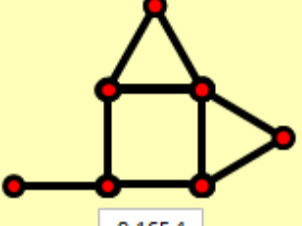
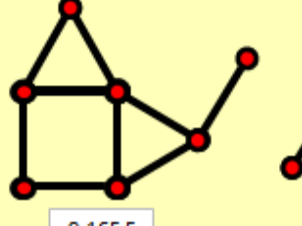
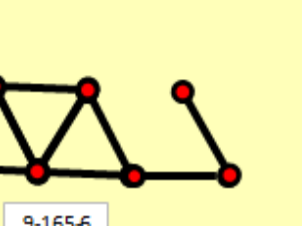
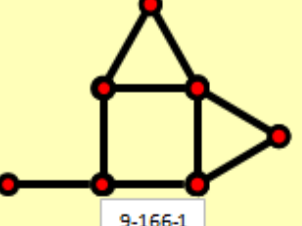
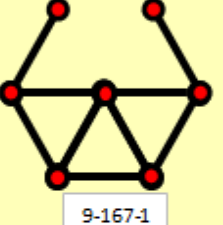
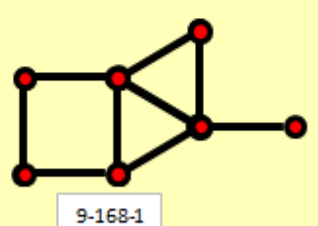
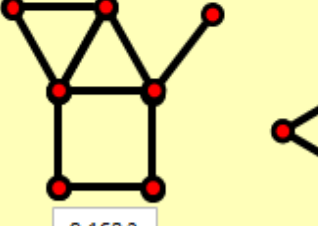
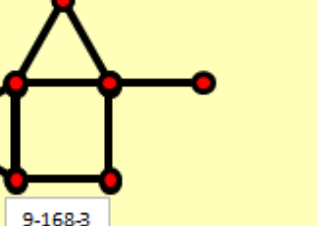
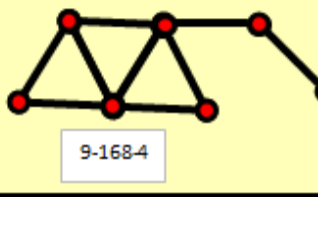
3.9.4.1. Matchstick graphs with  $|E|=9$ ,  $F=4$ ,  $|V|=7$ ,  $\Delta=3$




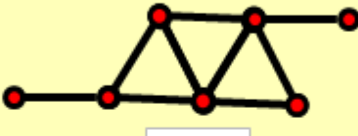



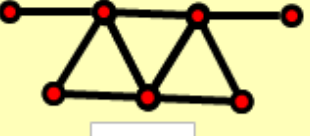

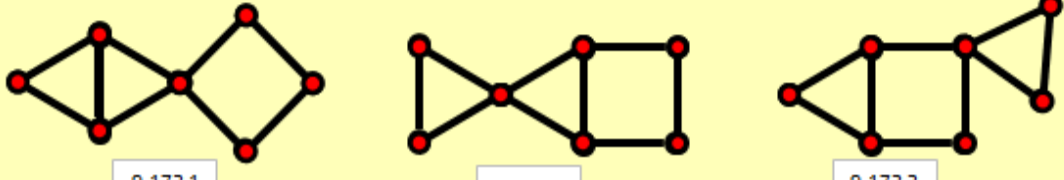





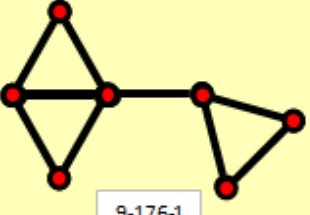
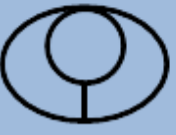
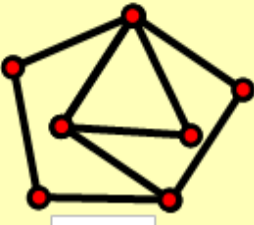
9-157		  	3
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9-158	 <p>9-158-1</p>	1
9-159	 <p>9-159-1</p>  <p>9-159-2</p>  <p>9-159-3</p>	3
9-160	 <p>9-160-1</p>  <p>9-160-1</p>	2
9-161	 <p>9-161-1</p>	1


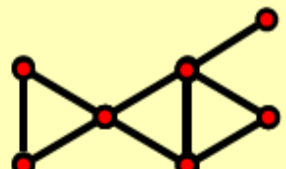

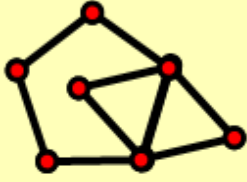
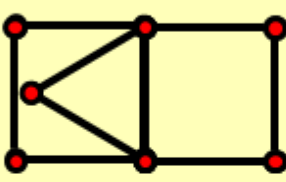

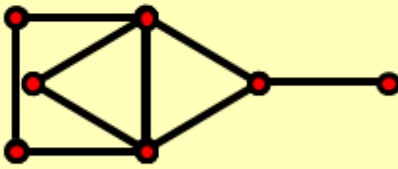
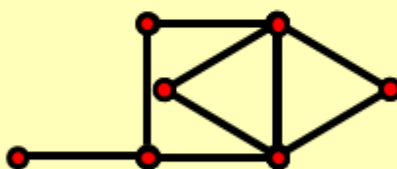

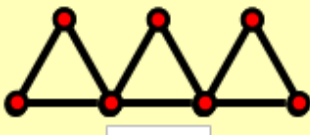
3.9.4.2. Matchstick graphs with  $|E|=9$ ,  $F=4$ ,  $|V|=7$ ,  $\Delta=4$

9-162	 <p>9-162-1</p>	1
9-163	 <p>9-163-1</p>  <p>9-163-2</p>  <p>9-163-3</p>  <p>9-163-4</p>	4
9-164	 <p>9-164-1</p>  <p>9-164-2</p>  <p>9-164-3</p>	


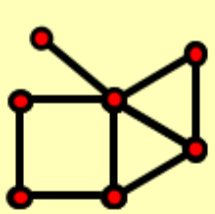
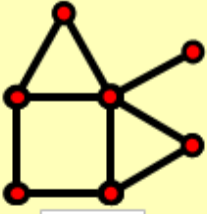

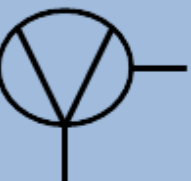
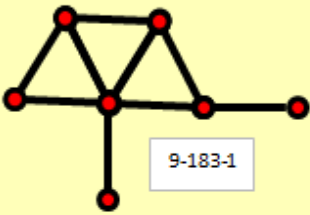

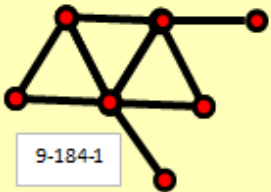

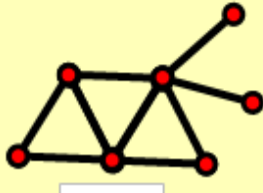
9-164	 <p>9-164-1</p>  <p>9-164-4</p>  <p>9-164-5</p>  <p>9-164-6</p>	7
9-165	 <p>9-165-1</p>  <p>9-165-2</p>  <p>9-165-3</p>  <p>9-165-4</p>  <p>9-165-5</p>  <p>9-165-6</p>	6
9-166	 <p>9-166-1</p>	1
9-167	 <p>9-167-1</p>	1
9-168	 <p>9-168-1</p>  <p>9-168-2</p>  <p>9-168-3</p>  <p>9-168-4</p>	4




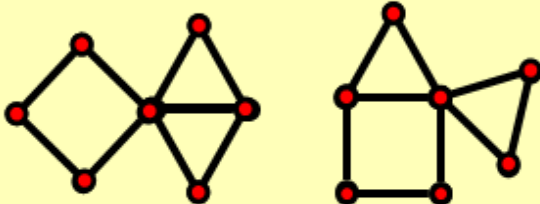

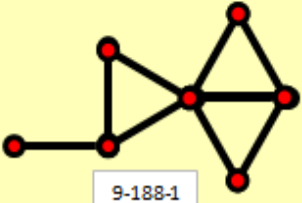

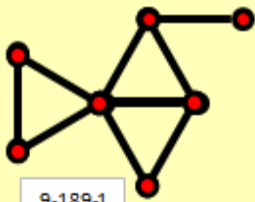

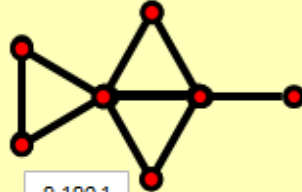

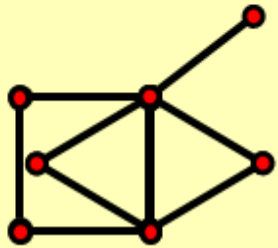
9-169		 <p>9-169-1</p>	1
9-170		 <p>9-170-1</p>	1
9-171		 <p>9-171-1</p>	1
9-172		 <p>9-172-1</p>	1
9-173		 <p>9-173-1      9-173-2      9-173-3</p>	3
9-174		 <p>9-174-1</p>	1
9-175		 <p>9-175-1</p>	1
9-176		 <p>9-176-1</p>	1
9-177		 <p>9-177-1</p>	1




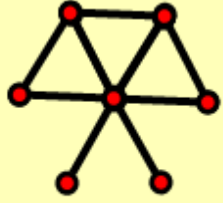
9-178		 9-178-1	1	
9-179		 9-179-1	 9-179-2	2
9-180		 9-180-1	 9-180-2	2
9-181		 9-181-1	1	


3.9.4.3. Matchstick graphs with  $|E|=9$ ,  $F=4$ ,  $|V|=7$ ,  $\Delta=5$

9-182		 9-182-1	 9-182-2	 9-182-3	3
9-183		 9-183-1			1
9-184		 9-184-1			1
9-185		 9-185-1			1

9-186		 <p data-bbox="438 224 534 268">9-186-1</p>	1
9-187		 <p data-bbox="438 515 534 560">9-187-1</p> <p data-bbox="710 515 805 560">9-187-2</p>	2
9-188		 <p data-bbox="454 772 550 817">9-188-1</p>	1
9-189		 <p data-bbox="391 1030 486 1075">9-189-1</p>	1
9-190		 <p data-bbox="391 1299 486 1344">9-190-1</p>	1
9-191		 <p data-bbox="454 1635 550 1680">9-191-1</p>	1

3.9.4.4. Matchstick graphs with  $|E|=9$ ,  $F=4$ ,  $|V|=7$ ,  $\Delta=6$


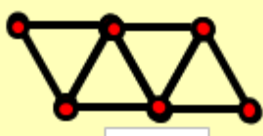
9-192		 <p data-bbox="470 1982 566 2027">9-192-1</p>	1
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9-193		 <p>9-193-1</p>	1
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9-194		 <p>9-194-1</p>	1
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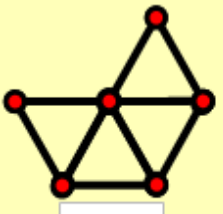
3.9.5. Matchstick graphs with  $|E|=9$ ,  $F=5$ ,  $|V|=6$

3.9.5.1. Matchstick graphs with  $|E|=9$ ,  $F=5$ ,  $|V|=6$ ,  $\Delta=4$

9-195		 <p>9-195-1</p>	1
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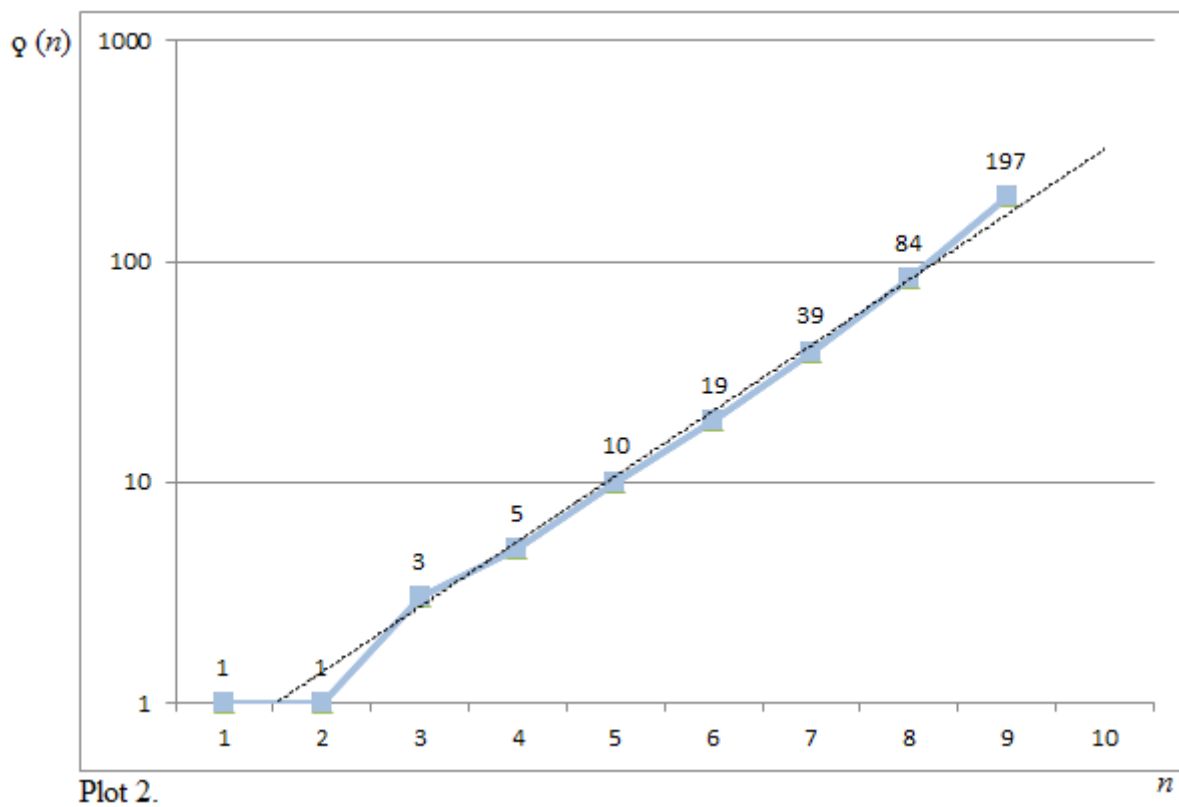
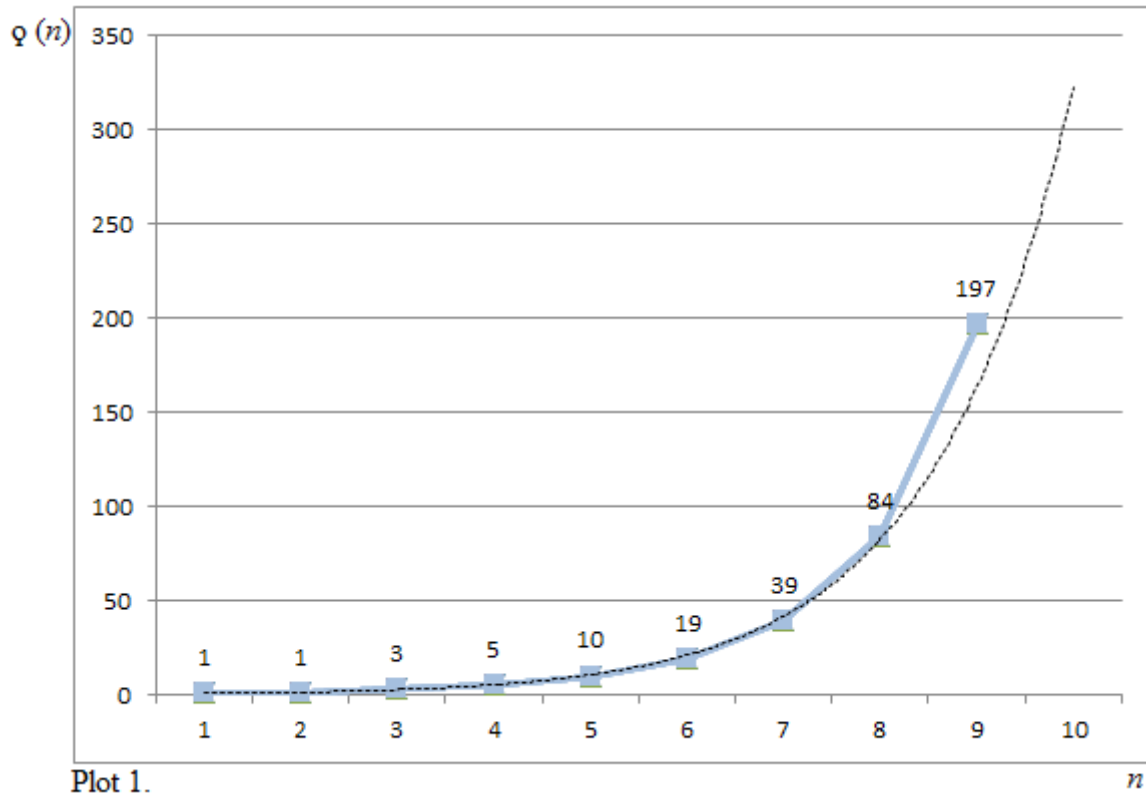
9-196		 <p>9-196-1</p>	1
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3.9.5.2. Matchstick graphs with  $|E|=9$ ,  $F=5$ ,  $|V|=6$ ,  $\Delta=5$

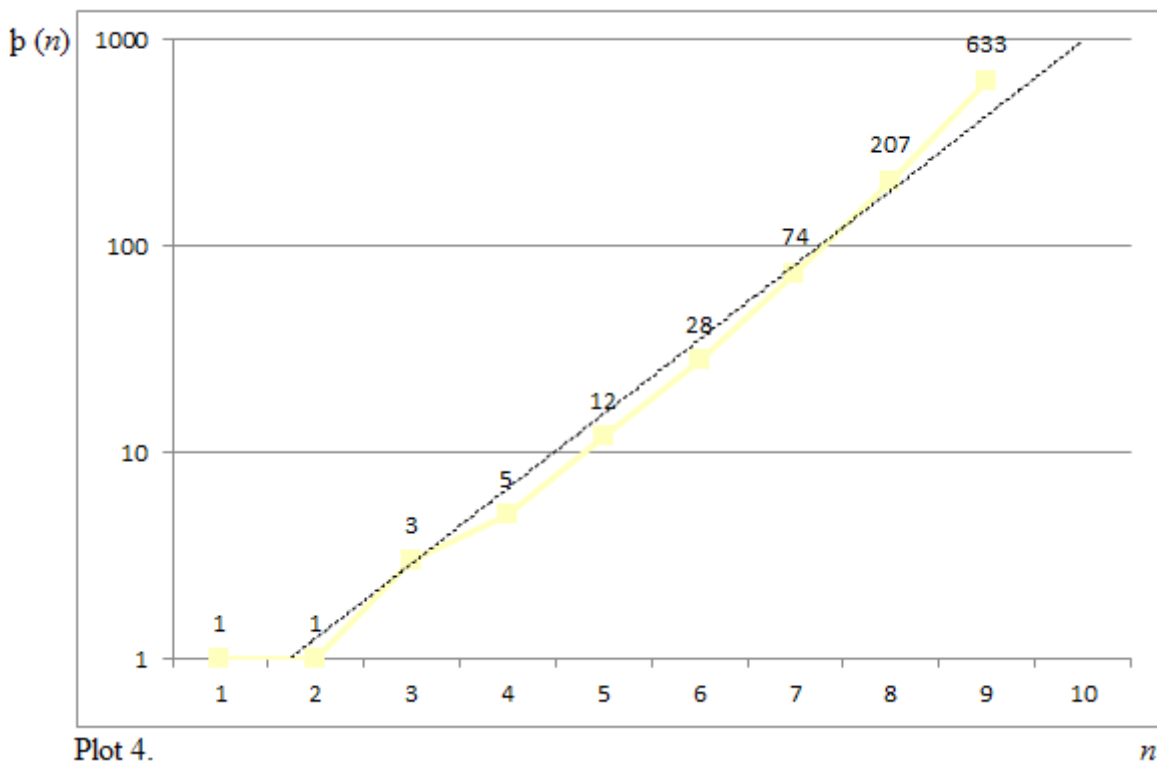
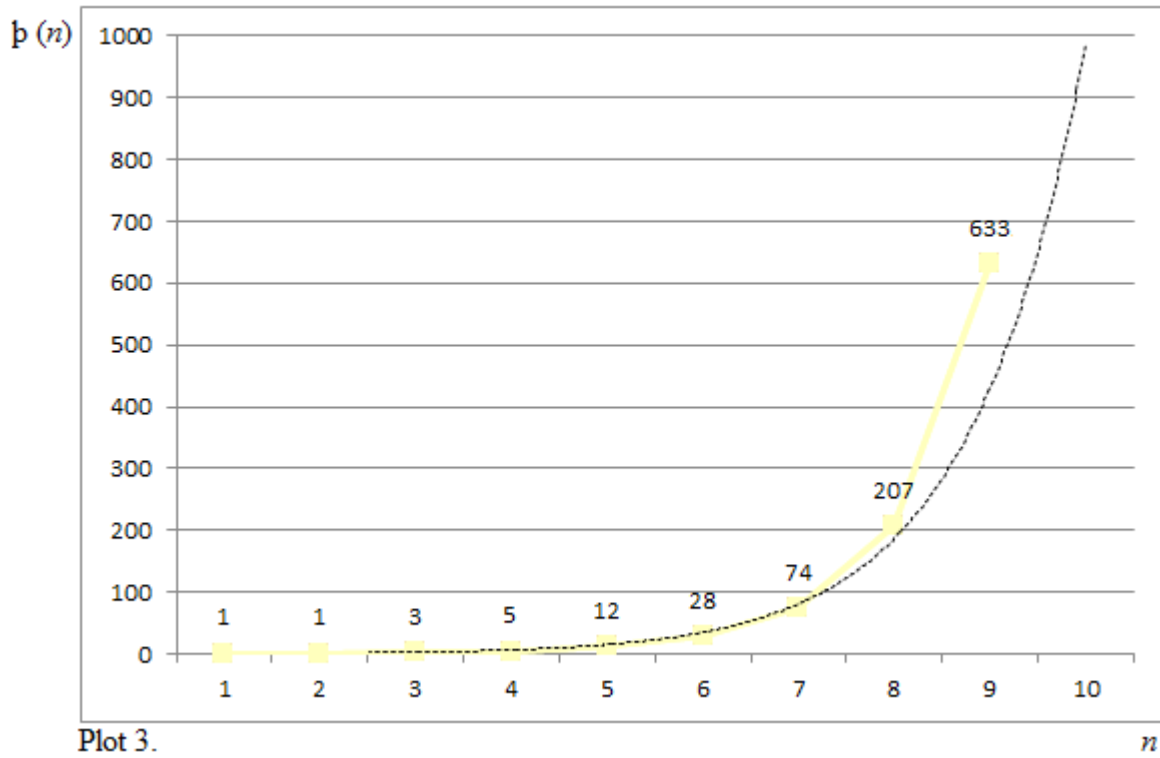
9-197		 <p>9-197-1</p>	1
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## 4. Plots

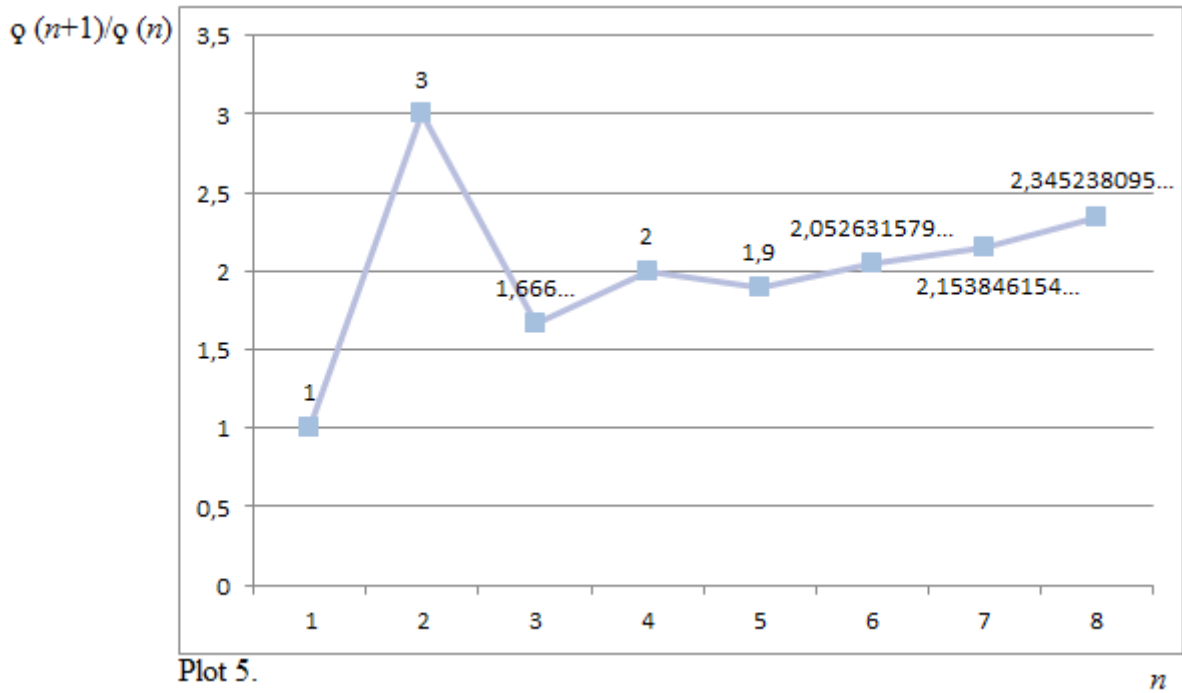
Plots 1 and 2 displays the value of  $q(n)$ , respectively in linear and logarithmic scale.



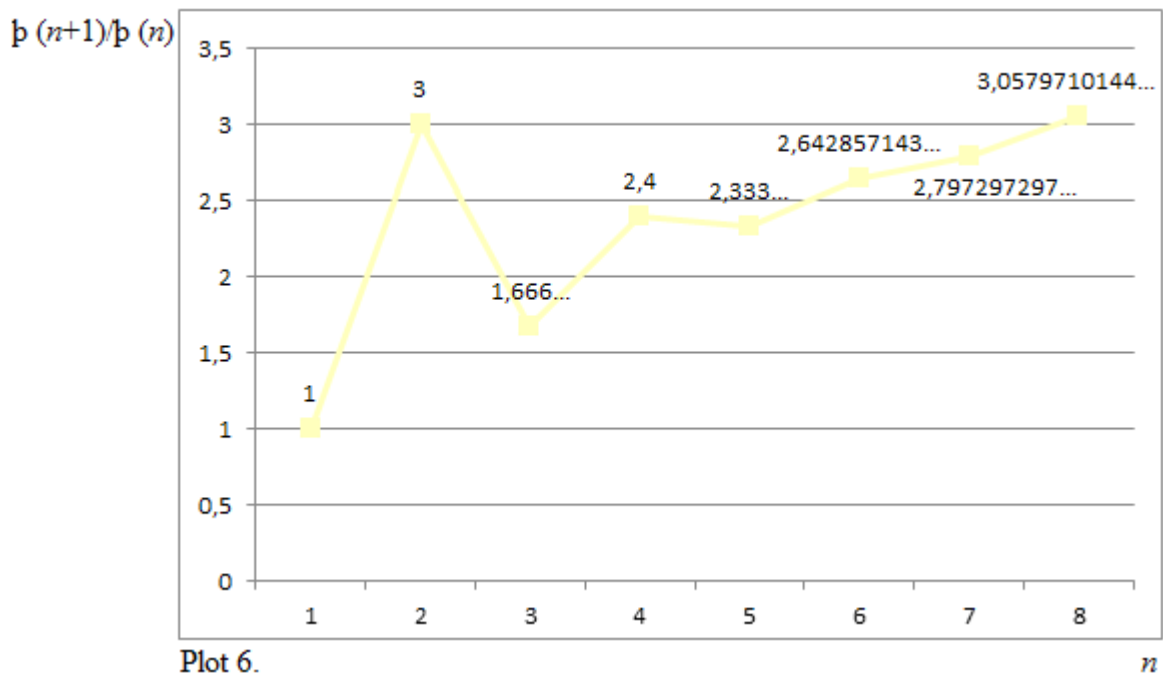
The same is done for  $p(n)$  in plots 3 and 4.



The incremental ratios between consecutive terms of  $q(n)$  and  $p(n)$  are shown in plots 5 and 6. After an initial oscillation, with  $n > 4$  both quotients grow monotonically as  $n$  increases.



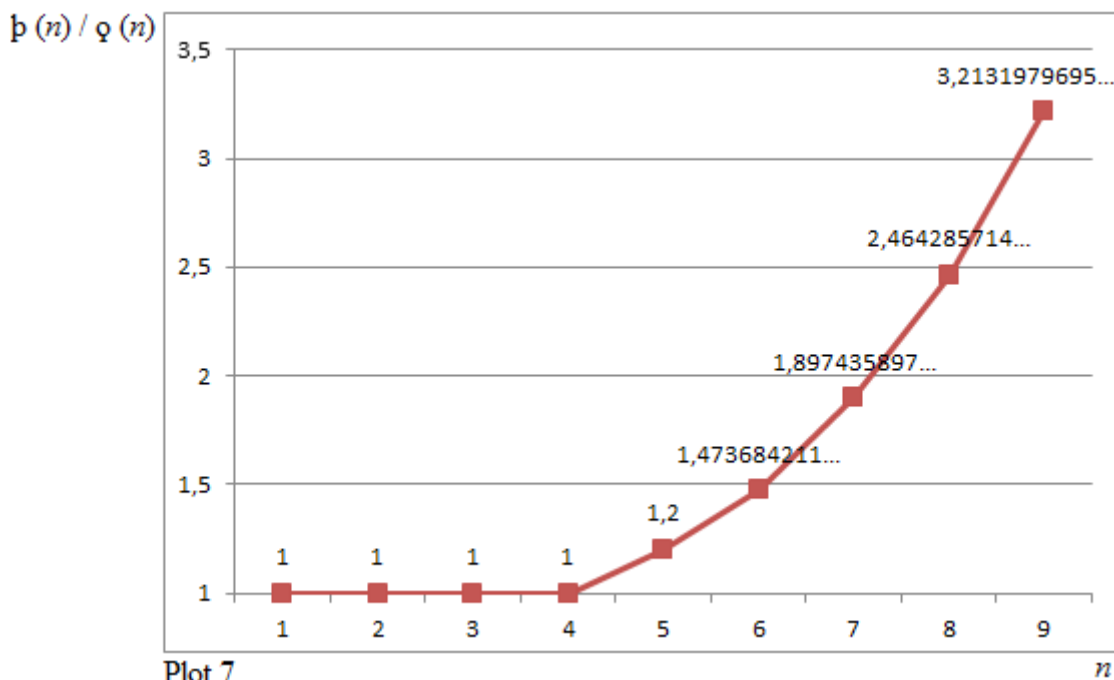
Plot 5.



Plot 6.

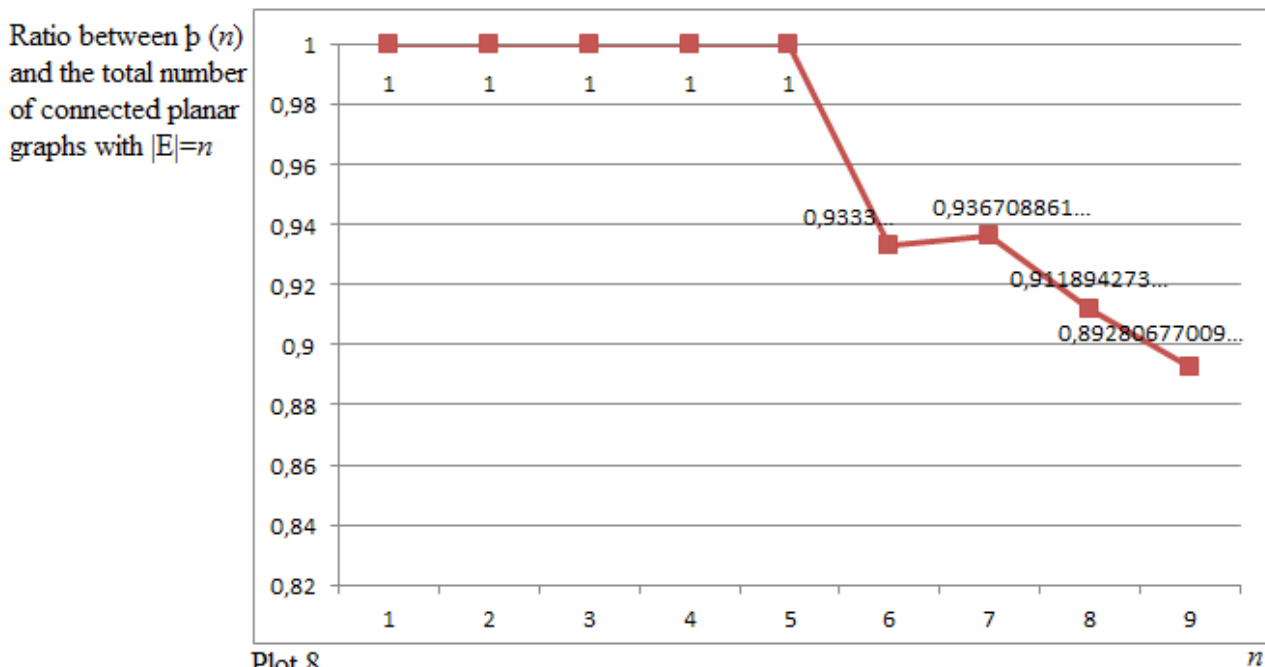
If we suppose that the trend continues also for  $n=10$ , this would place a lower limit of 463 for  $q(10)$  and of 1936 for  $p(10)$ . It seems reasonable to expect  $q(10)$  to be about from 500 to 600.

Plot 7 reports the average number of non-isomorphic matchstick graphs for homeomorphism class. Also this sequence shows a monotonically growing behavior with  $n > 4$ .



Plot 7.

Plot 8 shows what fraction of all the connected planar graphs consists in matchstick graphs. Up to 5 edges, every planar graph is a matchstick graph as well. Then the quotient starts decreasing quite irregularly. Since we know there are 2318 connected planar graphs with 10 edges (Steinbach 1990 [7]), to made a coarse intepolation, would be plausible to expect a value of about 2000-2100 for  $p(n)$ .



Plot 8.

## 5. References

- [1] Harborth, Heiko (1994). "Match sticks in the plane", in Guy, R. K.; Woodrow, R. E., *The Lighter Side of Mathematics: Proceedings of the Eugène Strens Memorial Conference of Recreational Mathematics and its History*, Calgary, Canada, 1986, Washington, D.C.: Mathematical Association of America, pp. 281–288.
- [2] Eades, Peter; Wormald, Nicholas C. (1990). "Fixed edge-length graph drawing is NP-hard". *Discrete Applied Mathematics* **28** (2): 111–134

- [3] Cabello, Sergio; Demaine, Erik D.; Rote, Günter (2007). “*Planar embeddings of graphs with specified edge lengths*”. *Journal of Graph Algorithms and Applications* **11** (1): 259–276.
- [4] Gardner, M. (1969). “*The Problem of the Six Matches*”, in *The Unexpected Hanging and Other Mathematical Diversions*. Simon and Schuster, NY, p. 80.
- [5] Read, R. C. (July 1968). “*From Forests to Matches*”. *Journal of Recreational Mathematics*, **1** (3): 60-172.
- [6] Cfr. OEIS sequences A003055 (<http://oeis.org/history/view?seq=A003055&v=21>) and A066951 (<http://oeis.org/history/view?seq=A066951&v=13>).
- [7] Steinbach, P. (1990). “*Field Guide to Simple Graphs*”. Design lab, Albuquerque, NM

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