# A Conjectured Explicit Determinant Evaluation Whose Proof Would Make Me Happy (and the OEIS richer)

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**Abstract**: I conjecture a certain explicit determinant evaluation, whose proof would imply the solution of certain enumeration problem that I have been working on, and that I find interesting. I am pledging $500 to the OEIS Foundation (in honor of the prover!) for a proof, and $50 (in honor of the disprover or his or her computer) for a disproof, as well as (in the affirmative case only) a co-authorship in a good enumeration paper, that would immediately bequest a Zeilberger-number 1, an Erdös number $\leq 3$, an Einstein number $\leq 4$ and numerous other prestigious numbers.

In order to complete the proof of a certain enumeration problem that I have been working on for the last few weeks, I need a proof of the following conjecture.

Let $d$ be a positive integer, and Let $M = M(d)$ be the following $2d \times 2d$ matrix with entries in $\{-1, 0, 1\}$. For $1 \leq a \leq 2d$ and $1 \leq b \leq d$,

$$M_{a,2b-1} = \begin{cases} 1 & \text{if } a = 2b \ ; \\ -1 & \text{if } a = 3b+1; \\ 0. & \text{otherwise} \end{cases}$$

$$M_{a,2b} = \begin{cases} 1 & \text{if } a = 2b-1 \ ; \\ -1 & \text{if } a = b-1; \\ 0. & \text{otherwise} \end{cases}$$

**Conjecture**: For every positive integer $d$, the following is true:

$$\det M(d) = (-1)^d \quad .$$

**Comments**:

**1.** This conjecture came up in my current work in enumerative combinatorics. Shalosh B. EKhad kindly verified it for $d \leq 200$. I have no idea how hard it is, and it is possibly not that hard, but right now I am busy with other problems. I believe that the powerful and versatile techniques of Karattenthaler[K1][K2] may be applicable, and possibly the computer-assisted approach described in [Z] and already nicely exploited in [KKZ] and [KT].

**2.** The Short Maple code in: `http://www.math.rutgers.edu/~zeilberg/tokhniot/DetConj` defines the matrix $M(d)$ (procedure `M(d)`) and procedure `C(N)` verifies it empirically for all $d \leq n$. So far `C(200);` returned `true`.

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`http://www.math.rutgers.edu/~zeilberg/` . Jan. 7, 2014. downloadable from Zeilberger's website. Supported in part by the NSF.

**3.** I am offering to donate $500 to the OEIS Foundation for a proof and $50 for a disproof, with an explicit statement that the donation is in honor of the prover (or disprover).

**References**

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