On the Number of Cycles in a Graph

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Abstract

There is a sizable literature on investigating the minimum and maximum numbers of cycles in a class of graphs. However, the answer is known only for special classes. This paper presents a result on the smallest number of cycles in hamiltonian 3-connected cubic graphs. Further, it describes a proof technique that could improve an upper bound of the largest number of cycles in a hamiltonian graph.

1 Introduction

One of the oldest problems in graph theory that in fact goes back to the end of the 19th century, see [1], is the question: "What are the smallest and the largest number of cycles in a class of graphs?"

It turns out that it is most convenient to study the largest number of cycles in a graph $G = (V, E)$ with respect to its cyclomatic number $r(G) = |E| - |V| + 1$. Let $M(r)$ be the largest number of cycles among all graphs with the cyclomatic number r. In 1897 Ahrens showed that $M(r) \leq 2^r - 1$ [1]. A big step forward is due to Entringer and Slater [6]. They showed that when studying the value of $M(r)$ one can confine himself/herself to cubic graphs. Namely, they proved that there is a cubic graph with the cyclomatic number r and $M(r)$ cycles. Also they conjectured that $M(r)$ is asymptotically equal to 2^{r-1}. There are upper bounds on $M(r)$ where the error-term is not exponential, see for example [12] and the references given there. Aldred and Thomassen [3] proved that $M(r) \leq \frac{15}{16}2^r + o(2^r)$. This is still the only upper bound that improves on the coefficient of the leading term. As to the lower bounds, the best one so far was published recently in [8], where the error term is exponential.

As for the smallest number of cycles, it has been shown in [4] that a 2-connected cubic graph of n vertices contains at least $(n^2 + 14n)/8$ cycles, and that the bound is best possible. In the same paper, it was conjectured that the difference between the 2-connected and 3-connected cubic graphs, in a sense, is dramatic. More precisely, it was conjectured that $f(n)$, the minimum number of cycles in a 3-connected cubic graph, is superpolynomial. The conjecture was proved in [2]. It is shown there that, for n sufficiently large, $2^{n^{0.17}} < f(n) < 2^{n^{0.95}}$. In the same paper, it is suggested that replacing the condition 3-connected by the condition cyclically 4-edge-connected may increase the growth of cycles' number to be exponential in terms of n. R. Aldred has conjectured (unpublished) that restricting the graphs to be cubic hamiltonian 3-connected might lead to the same property. The first main result of this paper supports Aldred's conjecture. We conjecture that a graph H_{2n}, defined in this paper, has the smallest number of cycles among all cubic hamiltonian 3-connected graphs. Then we show that the number of cycles in H_{2n} grows faster than Fibonacci sequence. The second result provides a proof technique whose refinement could lead to an improvement of the upper bound on the largest number of cycles in a graph.

Our research has been motivated by a computer science application. Cyclomatic complexity is a software metric used to quantify the structure of a computer program. A program source code can be modeled by a Flow Control Graph (FCG) in which nodes and edges represent code blocks and the possible execution paths among them [10]. The cyclomatic complexity is based on the cyclomatic number of a program FCG. A software of high cyclomatic complexity indicates a large number of possible execution paths. Such a software would be difficult to test and expensive to maintain. As pointed out in [11] and [13], one should estimate cyclomatic complexity in advance during the software design to avoid a complex code structure. Therefore, investigating the number of cycles in graphs could help developers and automated code generators in avoiding structures that may lead to high cyclomatic complexity. It also could help in defining software design templates that lead to building low cyclomatic complexity software.

2 Preliminaries

In what follows it is assumed that $G = (V, E)$ is a cubic hamiltonian 3-connected graph, where $V = \{v_0, ..., v_{n-1}\}, H = v_0 v_1 v_2 ... v_{n-1} v_0$ is a hamiltonian cycle of G. The set of edges in G but not in H will be denoted by S, and called spokes. We will say that a set of spokes $F \subseteq S$ forms a cycle if there is a cycle C in G so that all spokes of F belong to C and no other spoke is in C. Such cycle C will be called an F-cycle. $v_i - v_j$ denotes the path $v_i v_{i+1} ... v_j$, which is a part of H, indices taken modulo n. $E(T)$ stand for the edge set of a graph T. The fact that a vertex v_k is an internal vertex of the path $v_i - v_j$ will be denoted by $v_i \prec v_k \prec v_j$. The expression $v_i \prec v_k \prec v_j \prec v_m$ is a shorthand for $v_i \prec v_k \prec v_j$ and $v_k \prec v_j \prec v_m$. The next lemma constitutes a simple but useful observation.

Lemma 1. *Let F be a non-empty set of spokes, $|F| = k$, and let the spokes of F be incident with vertices $v_{i_1}, ..., v_{i_{2k}}$, $i_1 < ... < i_{2k}$. If F forms a cycle C in G, then $E(C) = E_1$ or $E(C) = E_2$, where*

$$E_1 = F \cup \bigcup_{j=1}^{k} E(v_{i_{2j-1}} - v_{i_{2j}}), \text{ and}$$

$$E_2 = F \cup \bigcup_{j=1}^{k-1} E(v_{i_{2j}} - v_{i_{2j+1}}) \cup E(v_{i_{2k}} - v_{i_1}).$$

In particular, F forms at most two cycles in G.

Proof. Let F be a non-empty set of spokes. It is worth noting that no vertex is incident with both an edge in $E_1 - F$ and an edge in $E_2 - F$. Clearly, both E_1 and E_2 induce 2-regular graphs. The set F forms a cycle in G if E_1 or E_2 induces a cycle.

It will be shown that there are at most two ways how to choose the set H' of edges from the hamiltonian cycle H so that $F \cup H'$ forms a cycle of G. Suppose that v_i, v_j, and v_m, are vertices of G, $v_i \prec v_j \prec v_m$ so that each of the three vertices is incident to a spoke of F, and for all $k, i < k < j$, and $j < k < m$, the vertex v_k is not incident to a spoke of F. Then either all the edges of the path $v_i - v_j$ belong to H' and no edge of the path $v_j - v_m$ is in H' or no edge of the path $v_i - v_j$ is in H' and the path $v_j - v_k$ is in H'. In general, if $v_{i_1}, ..., v_{i_{2k}}, i_1 < ... < i_{2k}$, were vertices of G incident to a spoke of F, then either all the edges of paths $v_{i_1} - v_{i_2}, v_{i_3} - v_{i_4}, ..., v_{i_{2k-1}} - v_{i_{2k}}$, would be in H', or all the edges of the paths $v_{i_2} - v_{i_3}, v_{i_4} - v_{i_5}, ..., v_{i_{2k}} - v_{i_1}$, would be in H'. Thus, if F forms a cycle in G, then the edge set of C is either E_1 or E_2, and consequently, F forms at most two cycles in G. □

3 Smallest Number of Cycles

Let $H_{2n} = (V, E)$ be a cubic hamiltonian 3-connected graph where $n \geq 2$, $V = \{v_0, v_1, ..., v_{2n-1}\}$, and E comprises a hamiltonian cycle $C = v_0, v_1, ..., v_{2n-1}, v_0$ and a set of spokes S_n, $|S_n| = n$ such that:

$$e_0 = v_1 v_{2n-1} \in S_n$$

$$e_i = \begin{cases} v_{i-1} v_{2n-i-2} \in S_n, & \text{for } 1 \leq i < n-1, \ i \text{ is odd} \\ v_{i+1} v_{2n-i} \in S_n, & \text{for } 2 \leq i < n-1, \ i \text{ is even} \end{cases}$$

$$e_{n-1} = \begin{cases} v_{n-1} v_{n+1} \in S_n, & n \text{ is odd} \\ v_{n-2} v_n \in S_n, & n \text{ is even} \end{cases}$$

See Figure 1 and Figure 2 for H_{16} and H_{18}.

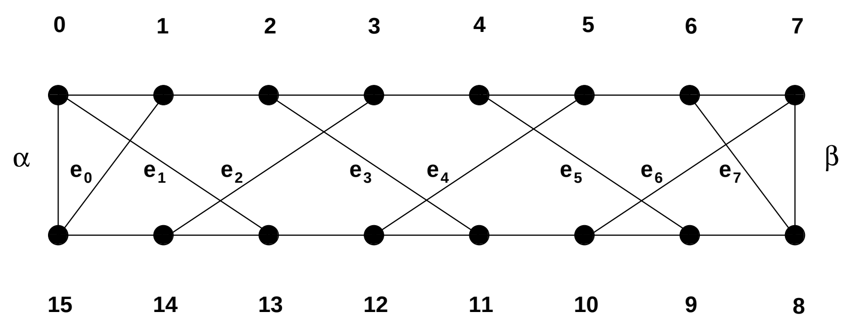

Figure 1: H_{16}

We believe that:

Conjecture 2. *The graph H_{2n} has the smallest number of cycles among all hamiltonian cubic 3-connected graphs on n vertices.*

Let $I(n) = (S_n, E')$ denote the graph where the vertex set is the set of spokes of H_{2n} and two vertices are adjacent if the corresponding spokes

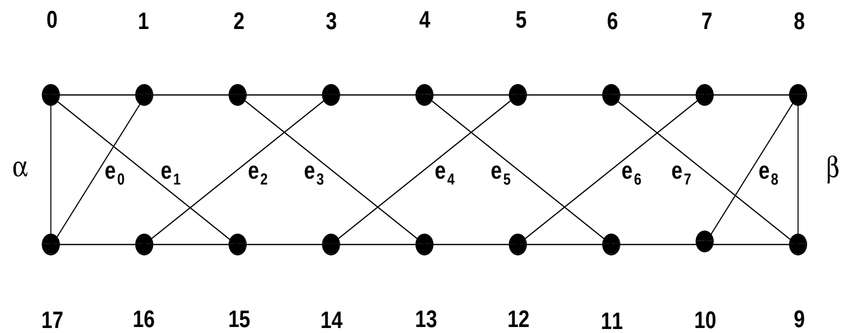

Figure 2: H_{18}

intersect. Since H_{2n} is 3-connected, $I(n)$ is a connected graph. $I(n)$ being a path is one reason to believe that Conjecture 2 is true. Note that $I(n)$ is a path with the first vertex e_0 and the last e_{n-1}.

Let α be the edge (v_0, v_{2n-1}), and β be the edge (v_{n-1}, v_n). The following claim states a very important property of the graph H_{2n}.

Claim 3. *Let H be a graph obtained by removing an edge $e \in \{\alpha, \beta, e_0, e_{n-1}\}$ from H_{2n} and suppressing two vertices of degree 2. Then H is isomorphic to H_{2n-2}.*

Proof. The situation after removing the edge α or e_0 is clear from Figure 3 and Figure 4, respectively. The situation after deleting the edge β or e_{n-1} is analogous. □

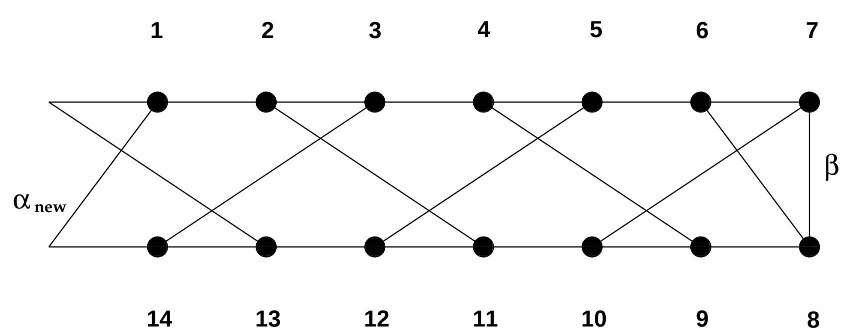

Figure 3: $H_{16} - \alpha \simeq H_{14}$

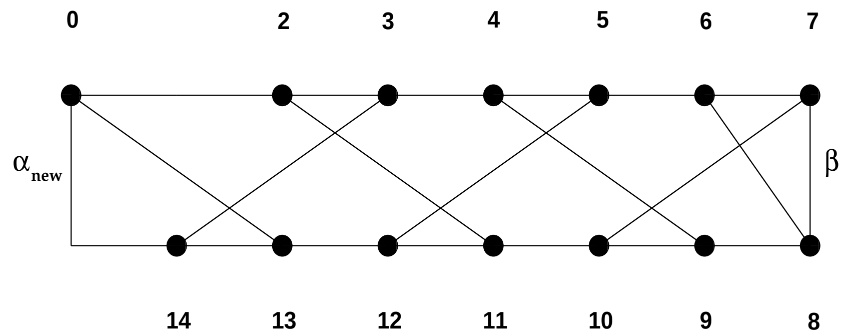

Figure 4: $H_{16} - e_0 \simeq H_{14}$

Let $F \subseteq S_n$ be a set of spokes. The Basic Interval Representation of F, the BIR of F, is a partition of F into minimal number of parts $F_1, ..., F_k$ such that (i) For each $i, i = 1, 2, ..., k$, F_i is a set of spokes so that the indices of spokes in F_i are consecutive numbers, that is, they form an interval; (ii) If $e_s \in F_i, e_t \in F_j$, and $i < j$, then $s < t$.

Clearly, for each set F its BIR is determined in a unique way. The part F_k will be called the last part of BIR of F.

If a set of spokes comprises spokes so that their indices form an interval, then for short F will be called a set of consecutive spokes.

Lemma 4. *Let F be a set of consecutive spokes. Then there are two F-cycles in H_{2n}. Further, if $|F|$ is even, then one of the two cycles contains both edges α and β, while the other cycle contains neither α nor β. For $|F|$ odd, one of the two cycles contains α and not β, the other contains β but not α.*

Proof. The statement follows directly from Lemma 1. It is easy to check in this case that both E_1 and E_2 induce a single cycle. □

As a direct consequence:

Corollary 5. *Let $F_1, ..., F_k, k \geq 2$, be BIR of a set F of spokes of H_{2n}. Then there is at most one F-cycle in H_{2n}. An F-cycle C exists iff $|F_i|$ is even for all $i = 2, ..., k-1$. Further, C contains the edge α (the edge β) if and only if $|F_1|$ is even ($|F_k|$ is even).*

Proof. It follows directly from the above lemma. □

Let c_n be the total number of cycles in H_{2n}, and let α_n stand for the number of cycles in H_{2n} containing the edge α. By inspection, using Corollary 5, we get, $c_2 = 7, c_3 = 14, c_4 = 26, c_5 = 46$, and $\alpha_2 = 4$, $\alpha_3 = 7$, $\alpha_4 = 12$, and $\alpha_5 = 20$. Further, let E_n and O_n be the number of cycles C in H_{2n} containing the edge α so that if F_C is the set of all spokes of C, then the last part in the BIR of F_C is of even, or odd parity, respectively. For short, these cycles will be called $\alpha-$*even* and $\alpha-$*odd* cycles, respectively. By definition,

$$\alpha_n = E_n + O_n$$

Clearly, if F is a set of spokes of H_{2n} not containing the last spoke e_{n-1}, then there is an F-cycle in H_{2n} iff there is an F'-cycle in H_{2n-2}.

Lemma 6. *For* $n \geq 4$, *(i)* $O_n = \alpha_{n-1}$; *(ii)* $E_n = \alpha_{n-1} - O_{n-2}$.

Proof. Let C be an $\alpha-$*odd* cycle in H_{2n}. If F_C, the set of spokes of C, contains the last spoke e_{n-1}, then the $F_C - e_{n-1}$ cycle is an $\alpha-$*even* cycle of H_{2n-2}, otherwise F_C-cycle forms an $\alpha-$*odd* cycle of H_{2n-2}. Thus, $O_n = E_{n-1} + O_{n-1} = \alpha_{n-1}$ and (i) follows. For C being an $\alpha-$*even* cycle, if F_C does not contain the last spoke e_{n-1}, then F_C-cycle forms an $\alpha-$*even* cycle of H_{2n-2}, otherwise $F_C - e_{n-1}$ cycle forms an $\alpha-$*odd* cycle of H_{2n-2} that contains e_{n-1} the last spoke of H_{2n-2}. Hence, $E_n = E_{n-1} + E_{n-2} = (\alpha_{n-1} - O_{n-1}) + (\alpha_{n-2} - O_{n-2}) = \alpha_{n-1} - O_{n-2}$, since $O_{n-1} = \alpha_{n-2}$. □

Corollary 7. *For* $n \geq 4$, $\alpha_n = \alpha_{n-1} + \alpha_{n-2} + 1$.

Proof. By Lemma 6, $\alpha_n = E_n + O_n = 2\alpha_{n-1} - O_{n-2} = 2\alpha_{n-1} - \alpha_{n-3}$. We show by induction that, for $n \geq 4$, $\alpha_n = \alpha_{n-1} + \alpha_{n-2} + 1$. It is $\alpha_4 = 12 = 7 + 4 + 1$. By the induction hypothesis, for $n > 4$, $\alpha_{n-1} - \alpha_{n-3} = \alpha_{n-2} + 1$. Therefore, $\alpha_n = 2\alpha_{n-1} - \alpha_{n-3} = \alpha_{n-1} + (\alpha_{n-1} - \alpha_{n-3}) = \alpha_{n-1} + \alpha_{n-2} + 1$. □

Now it is possible to proceed with counting the number of cycles in H_{2n}.

Theorem 8. *The number of cycles of* H_{2n} *is:*

$$c_n = \left(\frac{2}{\sqrt{5}} + 1\right)\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} + \left(1 - \frac{2}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^{n+2} - (n+4).$$

Proof. Since the number of cycles of H_{2n} not containing the edge α equals the number of cycles of H_{2n-2} , then:

$$c_n = c_{n-1} + \alpha_n$$

To determine c_n we first prove:

$$c_n = \alpha_{n+2} - (n+3) \text{ for } n \geq 2.$$

Using the initial values given above, the formula can be verified to be true for $n = 2$. For $n > 2$, by the induction hypothesis and Corollary 7, it is $c_n = c_{n-1} + \alpha_n = \alpha_{n+1} - (n+2) + \alpha_n = \alpha_{n+2} - (n+3)$.

Solving the recurrence relation $\alpha_n = \alpha_{n-1} + \alpha_{n-2} + 1$, with the initial conditions $\alpha_2 = 4, \alpha_3 = 7$, one gets:

$$\alpha_n = \left(\frac{2}{\sqrt{5}} + 1\right)\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(1 - \frac{2}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n - 1$$

which in turn implies:

$$c_n = \left(\frac{2}{\sqrt{5}} + 1\right)\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} + \left(1 - \frac{2}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^{n+2} - (n-4).$$

□

Remark. The "pseudo" Fibonacci sequence $\{c_n\}$ is identical to sequence A001924 electronically published at The On-Line Encyclopedia of Integer Sequences (OEIS) - https://oeis.org/A001924. The same sequence appears also in page 58 of [5], and in [9].

4 Largest Number of Cycles

It is widely believed, but not proved, that the largest number of cycles among all cubic graphs is attained at a hamiltonian graph. In this section it is shown that the largest number of cycles among all hamiltonian cubic graphs on n vertices, denoted by $T(n)$, can be upper bounded by $T(n) \leq 2^{\frac{n}{2}+1} - f(n)$, where f is an exponential function. We note that the cyclomatic number r of a cubic graph on n vertices equals $\frac{n}{2} + 1$. This result is superseded by

Aldred and Thomasen in [3] who proved that $M(r) \leq \frac{15}{16}2^r + o(2^r)$. However, we believe that a refinement of the method used here could lead to further improvement of the above mentioned result.

Theorem 9. $T(n) < 2^{\frac{n}{2}+1} - 2^{\frac{n}{2}-2\sqrt{n}-3}$.

Before proving the statement, some more notions are introduced. Two spokes $e = v_iv_j, f = v_kv_m$ are intersecting if the path $v_i - v_j$ contains one of the two vertices v_k, v_m, and the path $v_j - v_i$ contains the other of the two vertices, otherwise it is said that they are parallel. Further, let F be a set of spokes, $e, f \in F$. Then, e and f are consecutive spokes in F, if each $g \in F$, $e \neq g \neq f$, either intersects both e and f, or g is parallel to both e and f. It is easy to see that if e, f are two spokes of F incident with vertices v_i, v_j, v_k, v_m, where, say $v_i \prec v_j \prec v_k \prec v_m$, then e and f are consecutive iff either no internal vertex of the paths $v_i - v_j$ and $v_k - v_m$ or no internal vertex of the paths $v_j - v_k$ and $v_m - v_i$ is incident with a spoke in F.

Proof. We start with a series of claims.

Claim 9.1. Let F be a set of spokes so that e, f be consecutive spokes in F, and the number of spokes of F intersecting both e and f be even. Then F forms at most one cycle in G.

Proof of Claim 9.1. The statement is immediate for $|F| = 2$. Assume now $|F| \geq 3$. Let spokes e, f be incident to vertices v_i, v_j, v_k, v_m, where, $v_i \prec v_j \prec v_k \prec v_m$. In addition, we suppose WLOG that no internal vertex of the paths $v_i - v_j$ and $v_k - v_m$ is incident with a spoke in F. Since the number of spokes in F intersecting e and f is even, hence, we have that if a cycle C formed by the spokes of F contains all edges of the path $v_i - v_j$, then C would have to contain also all the edges of the path $v_k - v_m$. However, then the two path together with e and f form a cycle. Therefore, at least one of E_1, E_2 defined in Lemma 1 does not induce a single cycle. Thus, F forms at most one cycle in G.

Claim 9.2. Let $F, |F| > 1$, be a set of spokes, $e \in F$ be so that no spoke in F intersects e. Then F forms at most one cycle in G.

Proof of Claim 9.2. Let $e = v_iv_j$. Suppose first that both $v_i - v_j$ path and $v_j - v_i$ path contain an internal vertex incident to a spoke in F. Then clearly F does not form any cycle in G. Otherwise, let no internal vertex of the path $P = v_i - v_j$ be incident to a spoke in F. Then e and the path P form a cycle, hence at least one of E_1, E_2 does not induce a single cycle.

Claim 9.3. There is in G a set F of spokes, $|F| \geq \frac{n}{2} - 2\lfloor\sqrt{n}\rfloor$ so that either F contains a pair of consecutive spokes, or F contains a spoke that is intersected by no spoke in F.

Proof of Claim 9.3. First, let P be a partition of the set $0, 1, ..., n-1$ into k parts $P_1, ..., P_k$, so that P_i contains a set of consecutive integers, and $k \leq |P_i| \leq k+2$ for $i = 1, ..., k$. To see that such partition is possible, set $k = \lfloor\sqrt{n}\rfloor$. Choose t so that $(t-1)^2 \leq n < t^2$. Then $k = t-1$, and the existence of P follows from $k(k+2) = (t-1)(t+1) = t^2 - 1 \geq n$.

Suppose first that there is a spoke $e = v_i v_j, \in G, i < j$ so that both $i, j \in P_m$, for some $1 \leq m \leq k$. To get a set F with the required properties it suffices to remove from S, the set of all spokes, those spokes that are incident to internal vertices of the path $v_i - v_j$. As at most k spokes can be removed, and e is not intersected by any spoke in F, the proof follows.

In the other case, let T be the set of spokes incident to vertices in P_1. Since no spoke in T is incident to two vertices in P_1, by the pigeon hole principle, there is an m, $1 < m \leq k$, so that there are at least two spokes in T, say $e = v_i v_a$ and $f = v_j v_b$, $i, j \in P_1$ so that a, b belong to P_m. Let $i < j, a < b$. To get a requested set F of spokes, it suffices to remove from S all spokes incident to an internal vertex of $v_i - v_j$, and all spokes incident to an internal vertex of $v_a - v_b$. As $|P_t| \leq k+2$ for all $t = 1, ..., k$, at most $2k$ spokes can be removed. Thus, $|F| \geq \frac{n}{2} - 2k$.

Now we are ready to prove the theorem. Let F be a set of spokes guaranteed by Claim 9.3. Suppose that F contains two consecutive spokes e and f, and that there are t spokes in F intersecting both e and f. Then there are $\frac{1}{2}2^t 2^{|F|-t-2} = 2^{|F|-3}$ subsets of F satisfying the assumption of Claim 9.1, and the statement follows. In the case that F contains a spoke not intersected by any spoke in F, then there are $2^{|F|-1}$ subsets of F fulfilling the assumptions of Claim 9.2. The proof is complete. □

5 Computational Results

To provide supporting evidence for Conjecture 2 on the smallest number of cycles, we used an extensive computer search to verify it for all H_{2n}, $2n \leq 16$. It turns out that there are several extremal graphs for small values of $2n$ vertices as shown in Table 1.

2n	Number of Graphs
6	1
8	2
10	2
12	5
14	7
16	14

Table 1: Number of Extremal Graphs Exhibiting The Smallest Number of Cycles Including H_{2n}

Figures 5 to 8 show the extremal graphs for $2n = 8$ to $2n = 14$ excluding H_{2n}.

6 Concluding Remarks

So far there is no viable conjecture as to the largest number of cycles in cubic graphs. Guichard [7] found $T(n)$ by an extensive computer search for all $n \leq 18$. Unfortunately, it is not clear from these results what is the structure of the extremal graph.

Unlike the case of the largest number of cycles, this paper conjectures a structure H_{2n} for the smallest number of cycles in 3-connected hamiltonian graphs. The number of cycles in H_{2n} is derived, and the conjecture is verified using extensive computer searches for up to $2n = 16$. The paper also presents a proof technique that could be refined to improve the known upper bound on the largest number of cycles in a hamiltonian graph.

Extensive computer searches shall be carried on in future to verify the conjecture for $2n > 16$. The searches will also find all the graphs exhibiting the largest number of cycles. Hopefully, this would result in identifying common extremal graph structures across different graph sizes. Investigating these structure, if any, could lead to new venues on how to determine the largest number of cycles for this class of graphs.

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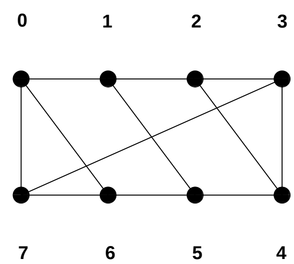

Figure 5: The extremal graph other than H_8 with the smallest number of cycles for $2n = 8$

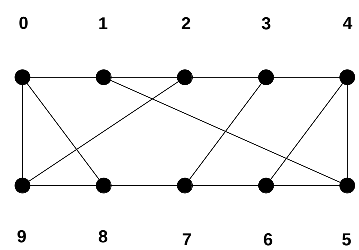

Figure 6: The extremal graph other than H_{10} with the smallest number of cycles for $2n = 10$

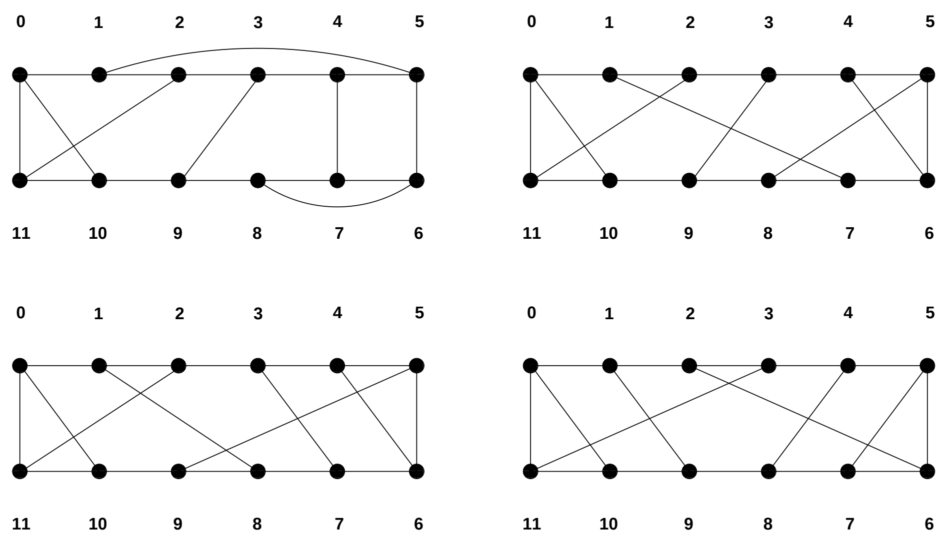

Figure 7: The extremal graphs for $2n = 12$ with the smallest number of cycles excluding H_{12}

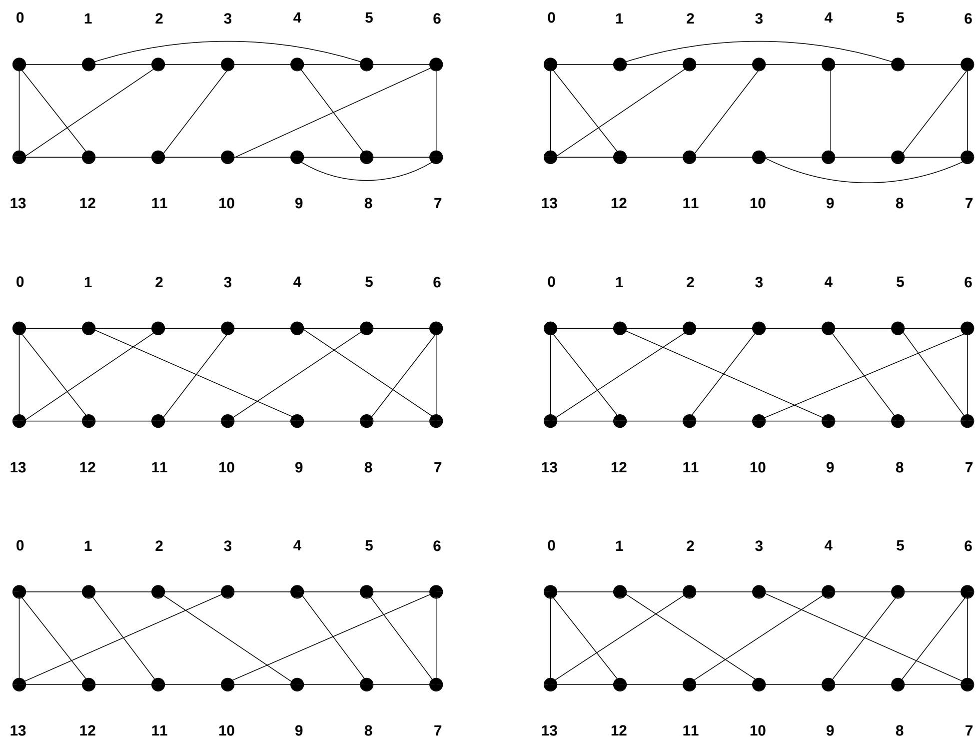

Figure 8: The extremal graphs for $2n = 14$ with the smallest number of cycles excluding H_{14}