Meertens number and its variations

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Abstract

In 1998, Bird introduced Meertens numbers as numbers that are invariant under a map similar to the Gödel encoding. In base 10, the only known Meertens number is 81312000. We look at some properties of Meertens numbers and consider variations of this concept. In particular, we consider variations where there is a finite time algorithm to decide whether such numbers exist.

1 Introduction

Kurt Gödel in his celebrated work on mathematical logic [2] uses an injective map from the set of finite sequences of symbols to the set of natural numbers in order to describe statements in logic as natural numbers and relating properties of mathematical proofs with properties of natural numbers. This approach is subsequently used by Alan Turing to define the notion of computable numbers [5], which are numbers that can be computed by his abstract computing model. This seminal work ushered in the field of theoretical computer science. The basic Gödel encoding is as follows: each symbol in an alphabet is mapped to a distinct positive integer. Thus a finite sequence of symbols $s_1, \cdots, s_n$ is mapped to a sequence of positive numbers $m_1, \cdots, m_n$. This sequence is then mapped to a natural number $G = \prod_{i=1}^{n} p_i^{m_i}$, where p_i is the i-th prime.

In Ref. [1], Richard Bird dedicated the number 81312000 to his friend Lambert Meertens on the occasion of his 25 years at the CWI institute and called it a *Meertens number*. He constructed this number using a mapping similar to the Gödel encoding.

Definition 1. *Given a decimal representation $d_1, \cdots, d_n$ of the number $m = \sum_{i=1}^{n} d_i 10^{n-i}$, if $m = \prod_{i=1}^{n} p_i^{d_i}$ then m is called a* Meertens number.

The only Meertens number known to date is $81312000 = 2^8 3^1 5^3 7^1 11^2 13^0 17^0 19^0$ [1]. David Applegate has conducted the search up to 10^{29} (see `https://oeis.org/A246532`) without finding any other Meertens number.

Definition 2. *Let $b \geq 2$ and $0 \leq d_i < b$ with $d_1 > 0$ be integers such that $m = \sum_{i=1}^{n} d_i b^{n-i}$, then $M_b(m)$ is defined as $M_b(m) = \prod_{i=1}^{n} p_i^{d_i}$.*

Thus a Meertens number is a fixed point of the function $M_{10}(\cdot)$. Note that the function M_{10} is similar to the Gödel encoding function. However, unlike the Gödel encoding, this function is not injective. In particular, $M_{10}(10^k) = 2$ for all $k \geq 0$. Since $d_i \leq 9$, the exponent of the prime 2 and 5 must be less than or equal to 9 and thus a Meertens number has at most 9 trailing zeros. In particular, the number of trailing zeros is the minimum of the first and third digit of m.

2 Meertens number in other bases

As noted in [1], the concept of a Meertens number can be defined in other number bases as well, i.e. m is a Meertens number in base b if m satisfies $m = \prod_{i=1}^{n} p_i^{d_i} = \sum_{i=1}^{n} d_i b^{n-i}$ for some nonnegative integers $d_i < b$ with $d_1 > 0$, i.e., $M_b(m) = m$. Since $d_1 \neq 0$, it is clear that a Meertens number must necessarily be even. Similarly, the number of trailing zeros in base 10 is the minimum of the first and third digit of m in base b. Table 1 lists some Meertens numbers found in various number bases.

The number 82944 is interesting as it is a Meertens number in base 8294 and shares the first 4 digits with the base 8294. The number 82944 in base 8294 is $A4$ (where we borrow from hexadecimal notation and use A to denote the digit 10) and $2^{10}3^4 = 82944$. Are there other numbers with this property?

Theorem 1. *If $1024 \cdot 3^c - c$ is divisible by 10 for some integer $c \geq 0$, then $1024 \cdot 3^c$ is a Meertens number in base $b = \frac{1024 \cdot 3^c - c}{10}$.*

Proof: First note that $b > 10$, $b > c$ and $1024 \cdot 3^c = 10b + c$ written in base b has digits 10 and c which maps to $1024 \cdot 3^c$ under the map M. □

There are two solutions with $c < 10$, i.e, $c = 4$ and $c = 6$, with $c = 4$ corresponding to the number 82944 above and $c = 6$ corresponding to a Meertens number 746496 in base 74649. Similarly, $2^{100}3^{96} - 96$ is a Meertens number in base $\frac{2^{100}3^{96}-96}{100}$ and the base in decimal is equal to the Meertens number in decimal minus the last 2 digits.

Since M_b is not injective, it is possible for a number to be a Meertens number in more than one number base. We note in Table 1 that 6, 10, 216 and 65536 are Meertens numbers in more than one number base. Are there any others? The answer is yes as a consequence of the following result.

Theorem 2. *If a, k and m are positive numbers such that $a + km = 2^a$ and $a < k$, then 2^{2^a} is a Meertens number in base 2^k. In particular for $a > 2$, 2^{2^a} is a Meertens number in base $2^{2^a - a}$.*

Proof: Since $2^{2^a} = 2^a 2^{km}$, this means that 2^{2^a} consists of a single digit of value $2^a < 2^k$ followed by m zeros. Thus $M_{2^k}\left(2^{2^a}\right) = 2^{2^a}$. For $a > 2$, $2^a - a > a$ and by setting $m = 1$, this shows that 2^{2^a} is a Meertens number in base $2^{2^a - a}$. □

Number base	Meertens number
2	2, 6, 10
3	10
4	200
5	6, 49000, 181500
6	54
7	100
8	216
9	4199040
10	81312000
14	47250
16	18
17	36
19	96
32	256
51	54
64	65536
71	216
158	162
160	324
323	1296
481	486
512	4294967296
1452	1458
1455	2916
1942	5832
4096	65536
4367	4374
7775	46656
8294	82944
13114	13122
13118	26244
26242	104976
39357	39366
52485	157464
74649	746496
118088	118098
209951	1679616
354283	354294
1062870	1062882
1119743	10077696

Table 1: Meertens numbers in various number bases.

In particular, we have the following Corollary:

Corollary 1. *If $k > a$ is a divisor of $2^a - a$, then 2^{2^a} is a Meertens number in base 2^k.*

For small values of a we list these divisors in Table 2.

a	k: divisors of $2^a - a$ larger than a
3	5
4	6, 12
5	9, 27
6	29, 58
7	11, 121
8	31, 62, 124, 248
9	503
10	13, 26, 39, 78, 169, 338, 507, 1014
11	21, 97, 291, 679, 2037
12	1021, 2042, 4084
13	8179
14	1637, 3274, 8185, 16370
15	4679, 32753

Table 2: Values of a and k such that 2^{2^a} is a Meertens number in base 2^k.

This shows that there are many numbers (for example $4294967296 = 2^{2^5}$) that are Meertens numbers in more than one base. For instance $2^{2^{16}}$ is a Meertens number in at least 105 different bases and $2^{2^{64}}$ is a Meertens number in at least 435 bases! In particular, for any integer $t > 2$, $2^{2^{t-k}} - 2^{t-k}$ is a divisor of $2^{2^t} - 2^t$ for $k = 0, \cdots, t$. Thus $2^{2^{2^t}}$ is a Meertens number in at least $t+1$ different bases, i.e. there are numbers which are Meertens numbers for an arbitrarily large number of bases. Even though there is only one known Meertens number in base 10, the above argument also implies that there are arbitrarily large bases for which Meertens numbers exist.

Theorem 3. *For integers $m \geq n \geq 0$,*

- *$2 \cdot 3^n$ is a Meertens number in base $2 \cdot 3^n - n$,*
- *$2^{2^n} 3^{2^m}$ is a Meertens number in base $2^{(2^n - n)} 3^{2^m} - 2^{m-n}$, and*
- *$2^{3^n} 3^{3^m}$ is a Meertens number in base $2^{3^n} 3^{(3^m - n)} - 3^{m-n}$.*

Proof: Since $2n < 2 \cdot 3^n$, $2 \cdot 3^n$ is written as $1n$ in base $b = 2 \cdot 3^n - n$, and $M_b(2 \cdot 3^n) = 2 \cdot 3^n$. Similarly, $2^{n+1} \leq 2^{m+1} < 2^{(2^n - n)} 3^{2^m}$ and the 2 digits in the base $2^{(2^n-n)} 3^{2^m} - 2^{m-n}$ representation of $2^{2^n} 3^{2^m}$ are 2^n and 2^m which is mapped by M_b into $2^{2^n} 3^{2^m}$. Next, $3^{n+1} \leq 3^{m+1} < 2^{3^n} 3^{(3^m - n)}$ and the 2 digits in the base $2^{3^n} 3^{(3^m-n)} - 3^{m-n}$ representation of $2^{3^n} 3^{3^m}$ are 3^n and 3^m which is mapped by M_b into $2^{3^n} 3^{3^m}$. □

3 Injective Gödel-like encodings

As mentioned earlier, the encoding defined by $M_b(m)$ is not a proper Gödel encoding as it is not one-to-one. Next we look at some injective Gödel-like encodings.

3.1 α-Meertens number

Definition 3. *Let $b \geq 2$ and $0 \leq d_i < b$ with $d_1 > 0$ be integers such that $m = \sum_{i=1}^{n} d_i b^{n-i}$, then $N_b(m) = \prod_{i=1}^{n} p_i^{d_i+1}$*

Note that by the unique factorization theorem of the integers, N_b is one-to-one on the set of positive integers. We will call numbers such that $N_b(m) = m$ an *α-Meertens number (in base b).* Since the encoding is one-to-one, there cannot be a number n that is a fixed point of this encoding in more than one base. This is easily seen as a number will have different digits in different bases. Some examples of α-Meertens numbers in various bases are listed in Table 3.

The following result shows that there are an infinite number of α-Meertens numbers.

Theorem 4. *For $t \geq 0$, $3 \cdot 2^{2^t+1}$ is an α-Meertens number in base $b = 3 \cdot 2^{2^t-t+1}$.*

Proof: First note that $2^t < 3 \cdot 2^{2^t-t+1}$. Then $3 \cdot 2^{2^t+1}$ in base $3 \cdot 2^{2^t-t+1}$ is the digit 2^t followed by the digit 0 which maps to $3 \cdot 2^{2^t+1}$ under the mapping N_b. □

On the other hand, for a fixed b, there are only a finite number of α-Meertens numbers in base b.

Definition 4. *Let p_i denote the i-th prime number, Let $p_n\#$ denote the primorial defined as $p_n\# = \prod_{i=1}^{n} p_i$. Let $\vartheta(t)$ denote the first Chebyshev function defined as $\vartheta(n) = \sum_{p \leq n} \log(p)$ where p ranges over all prime numbers less than or equal to n.*

Theorem 5. *$p_n\# > n^{0.5972n}$. If $n \geq 947$, then $p_n\# > n^{0.980n}$.*

Proof: For $n = 1$, the statement is trivially true. For $n > 1$, note that $p_n\# = e^{\vartheta(p_n)}$. Rosser [3] showed that for $n \geq 1$, $p_n > n \log n$. In [4, Theorem 10], it was shown that for $n \geq 7481$, $\vartheta(n) > 0.980n$. For primes $2 < p_n < 7481$, a simple computation shows that $\vartheta(p_n) > 0.5972p_n$. This implies that $p_n\# > e^{0.5972p_n} > e^{0.5972n \log n} = n^{0.5972n}$ for $n > 1$. The second part follows from the fact that the 947^{th} prime is 7481. □

Lemma 1. *If m is an α-Meertens number in base b with k digits, then $b^k > 2p_k\#$.*

Proof: Since m expressed in base b has k digits, $m < b^k$. On the other hand, $m = N_b(m) \geq 2p_k\#$. □

Theorem 6. *If m is an α-Meertens number in base b, then $m < b^{b^{1.675}}$.*

Proof: Suppose that m expressed as a base b number has k digits. Then by Lemma 1 and Theorem 5, $b^k > 2p_k\# > k^{0.5972k}$, implying that $k < b^{1.675}$. Thus $m < b^k < b^{b^{1.675}}$. □

base	α-Meertens number
12	12, 24
16	48
24	96
35	36
64	384
106	108
107	216
115	576
192	1536
321	324
329	2304
431	1296
968	972
970	1944
1943	7776
2048	24576
2911	2916
8742	8748
8745	17496
11662	34992
24576	393216
26237	26244
46655	279936
78724	78732
78728	157464
157462	629856
236187	236196
314925	944784

Table 3: α-Meertens numbers in various bases.

Corollary 2. *For a fixed b, let k^* be the largest integer k such that $b^k > 2p_k\#$. Then $k^* \leq b^{1.675}$. If m is an α-Meertens number in base b, then $m < b^{k^*}$.*

Proof: This is a consequence of Lemma 1 and Theorem 6. □

Corollary 3. *For $b \leq 10000$, if m is an α-Meertens number in base b, then $m < b^{b-1}$. If in addition $608 \leq b$, then $m < b^{\frac{b}{2}}$.*

Proof: This requires a computer-assisted proof by computing the value of k^* in Corollary 2 for various b. □

This allows us to improve Theorem 6.

Theorem 7. *If m is an α-Meertens number in base b, then $m < b^{b^{1.02041}}$.*

Proof: Suppose m has k digits in base b. Then $m < b^k$. If $k \geq 947$, then the proof of Theorem 6 combined with the second part of Theorem 5 shows that $k < b^{1.02041}$. Suppose $k < 947$. If $b \geq 826$, then $b^{1.02041} \geq 947$ and thus again $k < b^{1.02041}$. For $b < 826$, Corollary 3 shows that $m < b^{b-1} < b^{b^{1.02041}}$. □

Theorem 8. *If m is an α-Meertens number in base b, then $m < b^{b^{1+\epsilon}}$ where $\epsilon \to 0$ as $b \to \infty$.*

Proof: It is well known that $\vartheta(x)$ behaves asympotically as x. In particular, Ref. [4, Theorem 4] shows that $\vartheta(x) > (1-\delta)x$ where $\delta \to 0$ as $x \to \infty$. The rest of the proof is similar to the proof of Theorem 7 to show that $k < b^{\frac{1}{1-\delta}}$. □

We conjecture that k^* grows slower than the upper bound $b^{1.02041}$ or the asymptotic upper bound $b^{1+\epsilon}$ and that Corollary 3 is true for all b, i.e, all α-Meertens numbers m in base b satisfies $m < b^{b-1}$ and satisfies $m < b^{\frac{b}{2}}$ for large enough b. In particular, the first few values of k^* as a function of b is shown in Table 4 and a plot of k^* versus b is shown in Fig. 1 where k^* appears to be less than b for all b and less than $\frac{b}{3}$ for large b.

b	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
k^*	0	0	3	4	5	6	7	8	9	10	11	12	13	14	14

Table 4: Values of k^* as defined in Corollary 2 for various b.

The following result shows that 12 is the smallest base for which there exists an α-Meertens number.

Theorem 9. *There are no α-Meertens numbers in base $b < 12$.*

Proof: This again requires a computer-assisted proof. If m is an α-Meertens number in base b, then Corollary 3 implies that $m < b^{b-1}$. Next an exhaustive search up to b^{b-1} for $b < 12$ shows that there are no α-Meertens numbers in base $b < 12$. □

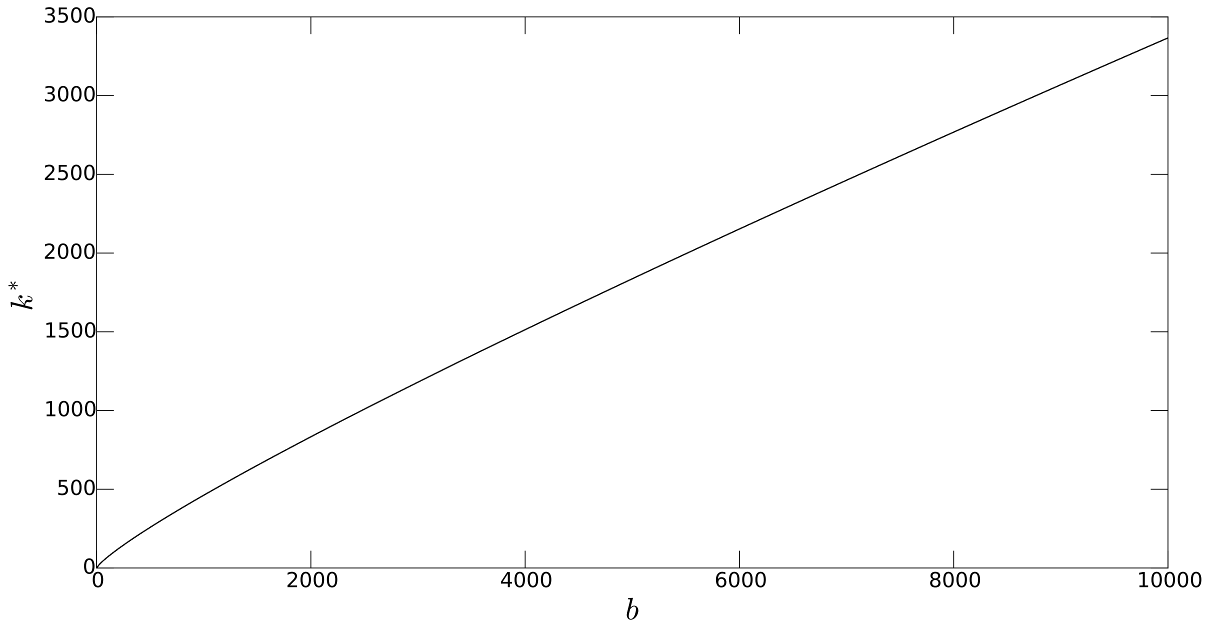

Figure 1: Plot of k^* as defined in Corollary 2 as a function of b.

3.2 Reverse Meertens number

Another way to define a one-to-one encoding is by reversing the digits and applying M_b, i.e. if the base-b representation of a number m is $d_n, \cdots, d_1$, then the encoding $M_b^r(m) = \prod_{i=1}^n p_i^{d_i}$ is one-to-one[1] and we define a *reverse Meertens number* in base b as a number m such that $M_b^r(m) = m$. As before, because this encoding is one-to-one, a number can be a reverse Meertens number in at most one number base. In base 10, $12 = 3^1 2^2$ is a reverse Meertens number. Reverse Meertens numbers in different bases are listed in Table 5.

Note that 17496 is both a reverse Meertens number and an α-Meertens number (in different bases). Clearly, Meertens number such as 6, 100, 36 and 1296 which are palindromes in their respective bases (5, 7, 17 and 323) are also reverse Meertens numbers.

Theorem 10. *For integers* $m \geq n \geq 0$*,*

- $3 \cdot 2^n$ *is a reverse Meertens number in base* $3 \cdot 2^n - n$*,*
- $2^{2^m} 3^{2^n}$ *is a reverse Meertens number in base* $2^{(2^m - n)} 3^{2^n} - 2^{m-n}$ *and*
- $2^{3^m} 3^{3^n}$ *is a reverse Meertens number in base* $2^{3^m} 3^{(3^n - n)} - 3^{m-n}$*.*

Proof: Since $2n < 3 \cdot 2^n$, $3 \cdot 2^n$ is written as $1n$ in base $b = 3 \cdot 2^n - n$, and $M_b^r(3 \cdot 2^n) = 3 \cdot 2^n$. Similarly, $2^{n+1} \leq 2^{m+1} < 2^{(2^m - n)} 3^{2^n}$ and the 2 digits in the base $2^{(2^m - n)} 3^{2^n} - 2^{m-n}$

[1] Note that in contrast to the definition of M_b, there is not a requirement here that $d_n > 0$, i.e. leading zeros in the base-b representation of m do not affect the value of $M_b^r(m)$.

base	reverse Meertens number
3	3, 10, 273
5	6, 175
7	100
9	27
10	12
17	36
21	24
25	3125
44	48
49	823543
70	144
71	216
91	96
97	486
186	192
194	972
285	576
323	1296
377	384
574	1728
760	768
1148	2304
1527	1536
2187	19683
2499	17496
3062	3072
4603	9216
4605	13824
5182	20736
6133	6144
7775	46656
9997	69984
12276	12288
12440	62208
18426	36864
24563	24576
36860	110592
49138	49152
73721	147456
98289	98304
209951	1679616
1119743	10077696

Table 5: Reverse Meertens numbers in various bases.

representation of $2^{2^m}3^{2^n}$ are 2^n and 2^m which is mapped by M_b^r into $2^{2^m}3^{2^n}$. Next, $3^{n+1} \leq 3^{m+1} < 3^{(3^n-n)}2^{3^m}$ and the 2 digits in the base $2^{3^m}3^{(3^n-n)} - 3^{m-n}$ representation of $2^{3^m}3^{3^n}$ are 3^n and 3^m which is mapped by M_b^r into $2^{3^m}3^{3^n}$. □

Theorem 11. *$p_{r+1}^{p_{r+1}}$ is a reverse Meertens number in base $b = p_{r+1}^{\frac{p_{r+1}-1}{r}}$ if r divides $p_{r+1} - 1$.*

Proof: Since $k < k^i$ for $k, i > 1$, consider a base k^i representation consisting of the digit k followed by r zeros, where $r = \frac{k-1}{i}$. This represents the number $m = k(k^i)^r = k^{ir+1} = k^k$. Under the mapping M_b^r, $M_b^r(m) = p_{r+1}^k$. Then the result follows if $k = p_{r+1}$. □

In particular, the first few primes p_{r+1} satisfying the condition in Theorem 11 are: 3, 5, 7, 31, 97, 101, 331, 1009, 1093, 1117, 1123, 1129, 3067, 64621, 480853, etc.

4 Zeroless Meertens numbers

Next we study Meertens numbers in base b without a zero digit when written in base b representation. We will call these numbers *zeroless Meertens numbers.* Examples include 6, 18, 36, 96, 54, 216, 1296 with corresponding bases 5, 16, 17, 19, 51, 71, 323. Similarly, examples of zeroless reverse Meertens numbers are: 6, 12, 36, 24, 48, 144, 1296 with corresponding bases 5, 10, 17, 21, 44, 70, 323. In fact, Theorems 3 and 10 show that there are an infinite number of bases with zeroless Meertens numbers or with zeroless reverse Meertens numbers. On the other hand, for a fixed b, the number of zeroless Meertens numbers and zeroless reverse Meertens numbers is finite.

Theorem 12. *If b is squarefree, then a zeroless Meertens number or a zeroless reverse Meertens number m in base b satisfies $m < b^{u-1}$, where p_u is the largest prime dividing b.*

Proof: Suppose m is a zeroless Meertens number in base b. Let S be the set of indices of primes which divide b, i.e. $b = \prod_{i \in S} p_i$. If m has u or more digits, then $d_i > 0$ for each $i \in S$, i.e. b divides $m = \sum_i p_i^{d_i}$, and m has a trailing zero digit in base b leading to a contradiction. The case of a zeroless reverse Meertens number is similar. □

The analysis in Section 3.1 can also be used to bound the number of zeroless (reverse) Meertens numbers.

Lemma 2. *For a fixed b, let l^* be the largest integer l such that $b^l > p_k\#$. Then $k^* \leq l^* \leq b^{1.675}$.*

Proof: The proof is similar to the proof of Corollary 2. □

Theorem 13. *If m is a zeroless Meertens number or a zeroless reverse Meertens number in base b, then $m < b^{l^*} \leq b^{b^{1.675}}$.*

Proof: The proof is similar to the proof of Theorem 6. □

Using Theorems 12 and 13 and an exhaustive computer search we show that:

Theorem 14.
- *The number 6 (associated with base 5) is the only zeroless Meertens number for bases < 12.*

b	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
l^*	0	2	4	5	6	7	8	9	10	11	12	12	13	14	15

Table 6: Values of l^* for various b.

- *The numbers* 6 *(associated with base* 5*) and* 12 *(associated with base* 10*) are the only zeroless reverse Meertens number for bases* < 12*.*
- *There are no zeroless Meertens numbers or zeroless reverse Meertens numbers in bases* 13*,* 14*, or* 15*.*
- *The number* 36 *is the only zeroless Meertens number and zeroless reverse Meertens number in base* 17*.*

We can also estimate the number of zero digits in a Meertens or reverse Meertens number:

Theorem 15. *If m is a Meertens or a reverse Meertens number in base b with u digits, then the number of zero digits in m is larger than*

$$u - e^{W(1.675u\log(b))} \tag{1}$$

where W is the Lambert W function.

Proof: Let z be the number of zero digits in m. Then

$$\begin{aligned} b^u &> m \geq p_{u-z}\# > (u-z)^{0.5972(u-z)} \\ u\log b &> 0.5972(u-z)\log(u-z) \\ 1.675u\log b &> (u-z)\log(u-z) \\ u-z &< e^{W(1.675u\log b)} \end{aligned}$$

□

5 Generalized Meertens numbers and generalized reverse Meertens numbers

Definition 5. *Given a pair of maps $f = \{f_1, f_2\}$ where $f_1 : \mathbb{N} \to \mathbb{N}$, $f_2 : \mathbb{N} \to \mathbb{N}$, define the map*

$$M^f(d_1, \cdots, d_n) = \prod_{i=1} f_1(i)^{f_2(d_i)}.$$

A generalized Meertens number (GMN) in base b is a number m such that $M^f(d_1, \cdots, d_n) = m$ where $(d_1, \cdots, d_n)$ are the digits of m in base b. A generalized reverse Meertens number (GRMN) in base b is a number m such that $M^f(d_n, \cdots, d_1) = m$.

In the cases we discussed in the sections above, $f_1(i)$ is the i-th prime and $f_2(i) = i$ or $f_2(i) = i + 1$. For these cases, since $p^d > d$ for all primes p and integers d, all GMN and GRMN in base b must be larger or equal to b. The tables above show that it it possible for a GMN or GRMN in base b to be equal to b. In particular, 2 is a Meertens number in base 2, 12 is an α-Meertens number in base 12 and 3 is a reverse Meertens number in base 3. In fact, since b written in base b is 10, applying the digits $(1, 0)$ (resp. the digits $(0, 1)$) to M^f will return a number b which is a GMN (resp. GRMN) in base b. This is summarized in the following result.

Theorem 16. *Suppose $f_1(i) > i$ for all i.*

- *If c is a GMN or a GRMN in base b, then $c \geq b$.*
- *If $f_1(1)^{f_2(1)} f_1(2)^{f_2(0)} > 1$, then b is a GMN in base b where $b = f_1(1)^{f_2(1)} f_1(2)^{f_2(0)}$.*
- *If $f_1(2)^{f_2(1)} f_1(1)^{f_2(0)} > 1$, then b is a GRMN in base b where $b = f_1(2)^{f_2(1)} f_1(1)^{f_2(0)}$.*

Consider the case where f_1 and f_2 are both the identity map, i.e. $f_1(i) = f_2(i) = i$. Clearly 1 is a GMN and a GRMN in this case. In base 10, $324 = 1^3 2^2 3^4$ is a GMN and $64 = 2^6 1^4$ is a GRMN. Table 7 lists some GMN and GRMN numbers under these f_i's.

base	generalized Meertens number	base	generalized reverse Meertens number
2	1350	2	2,6,12
4	108	3	120, 360
5	8	4	54
6	16	5	48
7	72	6	32
10	324	7	768
12	1458	8	216, 1728
23	1728	10	64
29	64	11	192, 729, 1536

Table 7: Generalized Meertens numbers and generalized Meertens numbers in various bases for the case when f_1 and f_2 are identity maps. The number 1 is omitted from this table.

6 Conclusions

We study Meertens numbers and their variations which are defined as fixed points of maps on the natural numbers. Depending on the map, the set of such numbers can be sparse or abundant. We showed that for α-Meertens numbers and zeroless (reverse) Meertens numbers these numbers are finite for a fixed base b. It would be interesting to investigate whether this is true for the other variations as well and under what conditions. Another open question is the asymptotic behavior of k^* and l^* as a function of b.

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