Quantum return probability of a system of N non-interacting lattice fermions

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Abstract. We consider N non-interacting fermions performing continuous-time quantum walks on a one-dimensional lattice. The system is launched from a most compact configuration where the fermions occupy neighboring sites. We calculate exactly the quantum return probability (sometimes referred to as the Loschmidt echo) of observing the very same compact state at a later time t. Remarkably, this probability depends on the parity of the fermion number – it decays as a power of time for even N, while for odd N it exhibits periodic oscillations modulated by a decaying power law. The exponent also slightly depends on the parity of N, and is roughly twice smaller than what it would be in the continuum limit. We also consider the same problem, and obtain similar results, in the presence of an impenetrable wall at the origin constraining the particles to remain on the positive half-line. We derive closed-form expressions for the amplitudes of the power-law decay of the return probability in all cases. The key point in the derivation is the use of Mehta integrals, which are limiting cases of the Selberg integral.

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1. Introduction

The advent of techniques to manipulate cold atoms has led to the experimental realization of low-dimensional quantum gases [1] that, for many decades, were mainly thought of as being toy models for theoreticians [2, 3]. The continuous tuning of the pair interaction between atoms thanks to Feshbach resonances allows one to create many-body systems ranging from free to strongly interacting. This revolution triggered a profusion of studies of the non-equilibrium dynamics of quantum systems [4, 5, 6], including quenches, entanglement and thermalization, that resulted in many new ways of exploring the quantum world [7].

Non-interacting tight-binding fermions on a lattice form the simplest of all quantum gases. They can be viewed as continuous-time quantum walkers. Quantum walks were initially introduced in the context of quantum information theory, as a general framework to state quantum algorithms [8, 9, 10]. Many fundamental problems that have been studied for classical random walks have natural counterparts in the realm of quantum walks, often leading to results that look unexpected and surprising from a classical point of view. First of all, quantum walkers behave ballistically rather than diffusively [11, 12]. The survival in the presence of traps [13, 14] and the ballistic spreading of bound states [15, 16] provide yet other examples of qualitatively different behavior in the classical and quantum cases.

A foremost question that has been investigated since the earlier days of quantum mechanics concerns the decay of individual quantum states [17, 18, 19, 20, 21, 22, 23]. In many circumstances the return probability of a quantum system to its very initial state, also referred to as the Loschmidt echo [24, 25, 26, 27], falls off exponentially in time. Quantum revival therefore corresponds to a rare event, somehow analogously to those described by the laws of large deviations in classical statistical mechanics [28, 29]. The return probability has been recently studied for various systems of bosons or fermions confined to one dimension. There, a wide variety of temporal decays can be found, ranging from a power law to a superexponential decay. These recent works include two-particle systems [30, 31, 32], many-particle systems [33, 34, 35, 36], and systems with infinitely many particles [37, 38, 39]. For instance, for free lattice fermions with domain-wall initial condition, the return probability was found to obey a pure Gaussian decay in time [37, 38]. More recently, the XXZ quantum spin chain with generic anisotropy Δ with domain-wall initial condition has been investigated in the massless phase ($\Delta = \cos\gamma$) [39]. There, the decay of the return probability is found to be either Gaussian or exponential, depending on whether the parameter γ is commensurate to π or not. This highly discontinuous asymptotic behaviour is attributed to integrability.

The aim of the present work is to study the return probability for a system of N non-interacting fermions hopping on a one-dimensional lattice. The fermions are launched from the most compact state where they occupy N successive lattice sites. We consider two different settings: an infinite chain (section 2) and a semi-infinite chain bounded by an impenetrable wall (section 3). In the latter situation, the compact initial state lies near the wall. We first derive general expressions for the return probability at arbitrary finite times (sections 2.1 and 3.1). The Andréief identity provides an explicit way of checking the equivalence of the first-quantized and second-quantized approaches. We then perform an asymptotic analysis of the regime of late times (sections 2.3 and 3.2). In both settings the return probability manifests a dependence on the parity of the fermion number, oscillating forever for odd N and decaying monotonically for even N. In all cases it falls off as a power of time, whose exponent also slightly depends on the parity of N, and is roughly twice smaller than what it would be in the continuum limit. We also derive closed-form expressions for the amplitudes of the power-law decay of the return probability in all these situations. This derivation relies on the usage of Mehta integrals [40, Ch. 17]. Our results suggest a non-trivial crossover behavior in the scaling regime where time t and the fermion number N are both large and proportional to each other (sections 2.4 and 3.3). The determination of the corresponding scaling functions F and $F^{(\mathrm{w})}$ remains a challenging open problem. Section 4 contains a brief summary and a discussion of our findings. Two appendices are respectively devoted to the Andréief identity (Appendix A) and to Barnes' G-function (Appendix B).

2. Free fermions on the infinite chain

We consider N non-interacting tight-binding fermions hopping on an infinite chain. At the initial time ($t = 0$), the fermions are launched from N consecutive sites, labeled $n = 1, \ldots, N$ (see figure 1).

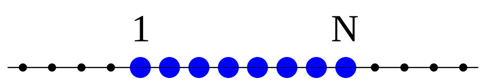

Figure 1. The initial configuration of the N fermions on the chain.

We are interested in the return probability $R_N(t)$, i.e., the probability that the fermions occupy the same sites at a later time t, and especially in the asymptotic decay of this quantity in the regime of long times.

2.1. General expressions at finite times

General expressions for the return probability $R_N(t)$ at arbitrary finite times can be derived as follows. Within the formalism of the second quantization, the Hamiltonian of the system reads

$$\mathcal{H} = \sum_n \left(a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n \right), \tag{2.1}$$

in dimensionless units, with the standard fermionic anti-commutation rules

$$\{a_m, a_n\} = \{a_m^\dagger, a_n^\dagger\} = 0, \qquad \{a_m, a_n^\dagger\} = \delta_{m,n}. \tag{2.2}$$

The many-body wavefunction $|\Psi(t)\rangle$ obeys the time-dependent Schrödinger equation

$$\mathrm{i}\,\frac{\mathrm{d}|\Psi\rangle}{\mathrm{d}t} = \mathcal{H}|\Psi\rangle, \tag{2.3}$$

and so

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\mathcal{H}t}|\Psi(0)\rangle. \tag{2.4}$$

The return probability therefore reads

$$R_N(t) = |A_N(t)|^2, \tag{2.5}$$

with

$$A_N(t) = \langle\Psi(0)|\Psi(t)\rangle = \langle\Psi(0)|\mathrm{e}^{-\mathrm{i}\mathcal{H}t}|\Psi(0)\rangle. \tag{2.6}$$

The above expression can be made explicit by bringing the Hamiltonian $\mathcal{H}$ to the diagonal form

$$\mathcal{H} = \int_0^{2\pi} \frac{\mathrm{d}q}{2\pi}\, \varepsilon_q\, \widehat{a}_q^\dagger \widehat{a}_q, \tag{2.7}$$

with

$$\varepsilon_q = 2\cos q \qquad (0 \le q \le 2\pi). \tag{2.8}$$

The operators

$$\widehat{a}_q = \sum_n \mathrm{e}^{\mathrm{i}nq} a_n, \qquad \widehat{a}_q^\dagger = \sum_n \mathrm{e}^{-\mathrm{i}nq} a_n^\dagger \tag{2.9}$$

obey the anti-commutation relation

$$\{\widehat{a}_q, \widehat{a}_{q'}^\dagger\} = 2\pi\, \delta(q - q'). \tag{2.10}$$

We find after some elementary algebra

$$A_N(t) = \int_0^{2\pi} \cdots \int_0^{2\pi} \prod_{n=1}^N \frac{\mathrm{d}q_n}{2\pi}\, \mathrm{e}^{-2\mathrm{i}t\cos q_n}\, \left|\langle \Psi(0)|\mathbf{q}\rangle\right|^2, \tag{2.11}$$

where $\mathbf{q} = (q_1, \ldots, q_N)$, and the q_n are the momenta of the N fermions. The many-body amplitude $\langle \Psi(0)|\mathbf{q}\rangle$ is given by a normalized Slater determinant. In the present situation, where the fermions are launched from the sites $m = 1, \ldots, N$, we have

$$\langle \Psi(0)|\mathbf{q}\rangle = \frac{1}{\sqrt{N!}} \det\left(\mathrm{e}^{\mathrm{i}mq_n}\right)_{1\le m,n\le N}, \tag{2.12}$$

and therefore

$$A_N(t) = \frac{1}{N!} \int_0^{2\pi} \cdots \int_0^{2\pi} \prod_{n=1}^N \frac{\mathrm{d}q_n}{2\pi}\, \mathrm{e}^{-2\mathrm{i}t\cos q_n} \times \left|\det\left(\mathrm{e}^{\mathrm{i}mq_n}\right)_{1\le m,n\le N}\right|^2. \tag{2.13}$$

The formula (2.12) can be recast as

$$\langle \Psi(0)|\mathbf{q}\rangle = \frac{1}{\sqrt{N!}} \prod_{n=1}^N \mathrm{e}^{\mathrm{i}q_n} \prod_{1\le m<n\le N} \left(\mathrm{e}^{\mathrm{i}q_n} - \mathrm{e}^{\mathrm{i}q_m}\right), \tag{2.14}$$

where the last product is a Vandermonde determinant. We are thus left with the following integral expression for the amplitude:

$$A_N(t) = \frac{1}{N!} \int_0^{2\pi} \cdots \int_0^{2\pi} \prod_{n=1}^N \frac{\mathrm{d}q_n}{2\pi}\, \mathrm{e}^{-2\mathrm{i}t\cos q_n} \times \prod_{1\le m<n\le N} \left|\mathrm{e}^{\mathrm{i}q_n} - \mathrm{e}^{\mathrm{i}q_m}\right|^2. \tag{2.15}$$

Applying the Andréief identity (A.1) to (2.13), we get the alternative expression

$$A_N(t) = \det\bigl(\psi_{m,n}(t)\bigr)_{1\le m,n\le N} = \det\bigl(J_{m-n}(2t)\bigr)_{1\le m,n\le N}, \tag{2.16}$$

where

$$\psi_{m,n}(t) = \int_0^{2\pi} \frac{\mathrm{d}q}{2\pi}\, \mathrm{e}^{-2\mathrm{i}t\cos q + \mathrm{i}(n-m)q} = \mathrm{i}^{m-n}\, J_{n-m}(2t) \tag{2.17}$$

is the one-body wavefunction at site n at time t for a particle launched from site m at time 0, and the J_n are Bessel functions.

The expressions (2.15) and (2.16) are complementary. The first one is more amenable to some analytical investigations, including the asymptotic analysis of the long-time regime performed in section 2.3. The second one could have been obtained by more elementary means within the formalism of the first quantization, along the lines of [14, 15, 16]. It demonstrates explicitly that the amplitude is properly normalized, i.e., it obeys $A_N(0) = 1$. It is also more suited for numerical evaluations and power-series expansions in t.

2.2. The first few values of N

It is interesting to first look at the first few values of the fermion number N.

• $N = 1$. For a single particle, we have $A_1(t) = J_0(2t)$, and so

$$R_1(t) = J_0^2(2t) \approx \frac{\cos^2(2t - \pi/4)}{\pi t}. \tag{2.18}$$

The return probability therefore oscillates forever, becoming exactly zero at infinitely many times $t_k \approx (k - 1/4)\pi/2$. These oscillations can be averaged out by replacing the squared cosine by a factor 1/2. We thus obtain the power-law decay

$$\overline{R}_1(t) \approx \frac{1}{2\pi t} \tag{2.19}$$

for the mean return probability.

• $N = 2$. The situation of two fermions is the simplest one where quantum statistics plays a role. We have $A_2(t) = J_0^2(2t) + J_1^2(2t)$, and so

$$R_2(t) = (J_0^2(2t) + J_1^2(2t))^2 \approx \frac{1}{\pi^2 t^2}. \tag{2.20}$$

In this case the return probability falls off monotonically to zero.

• $N = 3$. We have

$$\begin{aligned} A_3(t) &= (J_0(2t) + J_2(2t))(J_0^2(2t) + 2J_1^2(2t) - J_0(2t)J_2(2t)) \\ &= \frac{2J_1(2t)}{t}(J_0^2(2t) + J_1^2(2t)) - \frac{J_0(2t)J_1^2(2t)}{t^2}, \end{aligned} \tag{2.21}$$

where the first line gives the raw determinantal expression (2.16). The second one, obtained by means of the recursion

$$J_{n+1}(2t) + J_{n-1}(2t) = \frac{n}{t} J_n(2t), \tag{2.22}$$

is more suitable to study the late-time behavior of the return probability. Keeping only the first group of terms in the second line of (2.21), we obtain

$$R_3(t) \approx \frac{4\cos^2(2t - 3\pi/4)}{\pi^3 t^5}. \tag{2.23}$$

The return probability again oscillates forever, becoming exactly zero at infinitely many times $t_k \approx (k + 1/4)\pi/2$. The situation is therefore qualitatively similar to that of a single particle. Averaging again over the oscillations, we obtain

$$\overline{R}_3(t) \approx \frac{2}{\pi^3 t^5}. \tag{2.24}$$

2.3. Asymptotic analysis in the long-time regime

In this section we determine the asymptotic decay of the return probability $R_N(t)$ in the long-time regime, for arbitrary values of the fermion number N. To do so, we evaluate the multiple integral entering (2.15) by means of the saddle-point approximation. Saddle points are defined by the condition that every momentum q_n is either 0 or π. The vicinity of the most general saddle point can thus be parametrized by choosing M momenta near 0, of the form $q_n = x_n$, and the remaining $N - M$ momenta around π, of the form $q_n = \pi + y_n$. For a fixed M, there are $\binom{N}{M}$ ways of choosing which momenta are near 0 and near π. Expanding in (2.15) the arguments of the exponentials and the products to quadratic order in the variables x_n and y_n, we obtain the estimate

$$A_N(t) \approx \sum_{M=0}^{N} \frac{I_{N,M}(t)}{M!(N-M)!}, \tag{2.25}$$

with

$$\begin{aligned} I_{N,M}(t) &= 2^{2M(N-M)}\,\mathrm{e}^{2\mathrm{i}(N-2M)t} \\ &\times \int_{-\infty}^{\infty}\!\!\cdots\int_{-\infty}^{\infty} \prod_{n=1}^{M} \frac{\mathrm{d}x_n}{2\pi}\,\mathrm{e}^{\mathrm{i}tx_n^2} \prod_{1\le m<n\le M} (x_n - x_m)^2 \\ &\times \int_{-\infty}^{\infty}\!\!\cdots\int_{-\infty}^{\infty} \prod_{n=1}^{N-M} \frac{\mathrm{d}y_n}{2\pi}\,\mathrm{e}^{-\mathrm{i}ty_n^2} \prod_{1\le m<n\le N-M} (y_n - y_m)^2. \end{aligned} \tag{2.26}$$

The above integrals can be performed exactly. They are indeed given be the analytical continuation to imaginary values of a of the Mehta integral [40, Eq. (17.6.7), $\gamma = 1$]

$$\begin{aligned} \mathcal{I}_N(a) &= \int_{-\infty}^{\infty}\!\!\cdots\int_{-\infty}^{\infty} \prod_{n=1}^{N} \mathrm{d}x_n\,\mathrm{e}^{-ax_n^2} \prod_{1\le m<n\le N} (x_n - x_m)^2 \\ &= (2\pi)^{N/2}(2a)^{-N^2/2} \prod_{j=1}^{N} j! \\ &= (2\pi)^{N/2}(2a)^{-N^2/2} G(N+2), \end{aligned} \tag{2.27}$$

where G is Barnes' G-function (see Appendix B). We thus obtain

$$\begin{aligned} I_{N,M}(t) &= 2^{2M(N-M)}(2\pi)^{-N/2}\,\mathrm{e}^{\mathrm{i}(N-2M)(2t-N\pi/4)} \\ &\times G(M+2)G(N-M+2)(2t)^{M(N-M)-N^2/2}. \end{aligned} \tag{2.28}$$

All the quantities $I_{N,M}(t)$ which enter the estimate (2.25) thus fall off as power laws in time, albeit with an M-dependent exponent. The behavior of the return probability at long times is governed by the term $I_{N,M}(t)$ with the slowest decay. Even and odd values of the fermion number N yield different behaviors, and have to be dealt with separately. The emerging picture fully corroborates the observations made in section 2.2 for the first few values of N.

• If $N = 2m$ is even, the slowest decay is reached for $M = m$, i.e., for equal numbers of momenta near 0 and π. We thus obtain the estimate

$$A_{2m}(t) \approx \frac{2^{m(m-1)} G(m+1)^2}{\pi^m\, t^{m^2}}, \tag{2.29}$$

showing that the amplitude is positive and decays monotonically, at least for large times. The echo therefore falls off monotonically as a power law

$$R_{2m}(t) \approx \frac{2^{2m(m-1)} G(m+1)^4}{\pi^{2m}\, t^{2m^2}}, \tag{2.30}$$

with exponent $2m^2 = \frac{1}{2}N^2$.

• If $N = 2m+1$ is odd, the slowest decay is reached both for $M = m$ and $M = m+1$. We thus obtain the estimate

$$A_{2m+1}(t) \approx \frac{2^{m^2} G(m+1) G(m+2)}{\pi^{m+1/2}\, t^{m^2+m+1/2}} \cos(2t - (2m+1)\pi/4), \tag{2.31}$$

showing that the amplitude behaves for large times as an oscillatory function modulated by a decaying power law. Averaging over the oscillations, we obtain the following power-law decay for the mean echo:

$$\overline{R}_{2m+1}(t) \approx \frac{2^{2m^2-1} G(m+1)^2 G(m+2)^2}{\pi^{2m+1}\, t^{2m^2+2m+1}}, \tag{2.32}$$

with exponent $2m^2 + 2m + 1 = \frac{1}{2}(N^2 + 1)$.

Figure 2 illustrates the above results, showing log-log plots of the return probability $R_N(t)$ against time t for $N = 1$, 2, 3 and 4. Data have been obtained using the expression (2.16). For odd N (left), the echo oscillates forever. For even N (right), it falls off monotonically and exhibits mild damped oscillations. The blue straight lines – slightly translated for a better readability – have the predicted slopes 1, 2, 5 and 8.

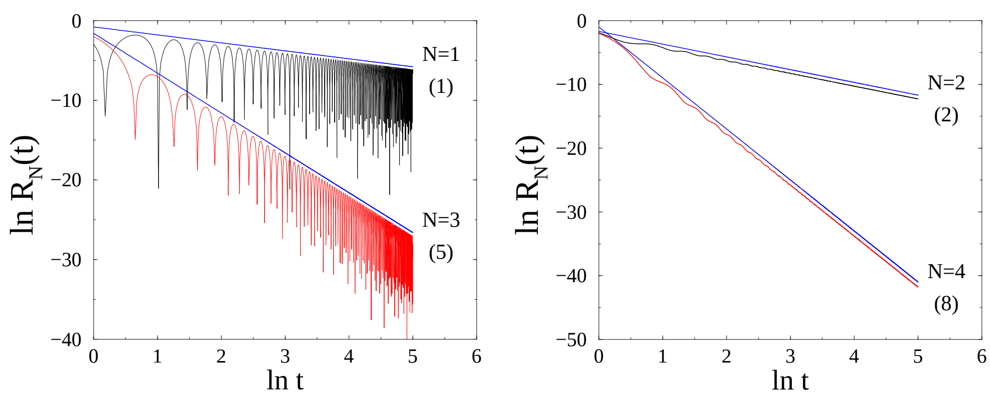

Figure 2. Log-log plots of the return probability $R_N(t)$ on the infinite chain against time t, for $N = 1$, 2, 3 and 4. Left: odd N. Right: even N. The blue straight lines have the predicted slopes 1, 2, 5 and 8.

It is worth comparing the above results for lattice fermions to the corresponding predictions in the continuum limit. Within the present framework, taking the continuum limit just amounts to approximating the dispersion relation (2.8) by a quadratic law of the form

$$\varepsilon_q \approx 2 - q^2, \tag{2.33}$$

and therefore to restricting the saddle-point approximation to the sector where all momenta are near zero (i.e., $M = N$). This yields

$$R_N^{(\mathrm{cont})}(t) \approx \frac{G(N+1)^2}{2^{N(N+1)}\pi^N t^{N^2}}. \tag{2.34}$$

We still obtain a power-law decay of the return probability, albeit with no parity effect and with a different – roughly twice larger – decay exponent.

The above decay of the return probability in the continuum can be alternatively derived by the following heuristic reasoning. Suppose the fermions have mass m and are launched from the interval $0 < x < \ell$. In reduced units ($\hbar = 1$), the typical momentum of each particle is $p \sim 1/\ell$, and so its wavefunction spreads ballistically over a region of size

$$L(t) \sim \frac{t}{m\ell}. \tag{2.35}$$

The return probability of one single particle therefore scales as $R_1(t) \sim \ell/L(t) \sim m\ell^2/t$. The exponent of this decay is in agreement with (2.32) and (2.34). In the same setting, the modulus of the wavefunction of N non-interacting fermions at late times can be estimated as

$$|\psi_N(x_1,\ldots,x_N;t)| \sim C_N(t) \prod_{1\le m<n\le N} |x_m - x_n|\,, \tag{2.36}$$

as long as all the coordinates x_n are less than $L(t)$, whereas it falls off very fast at larger distances. Dimensional analysis implies that the normalization of the wavefunction scales as $C_N(t) \sim L(t)^{-N^2/2}$, and that the return probability scales as

$$R_N(t) \sim \left(\frac{\ell}{L(t)}\right)^{N^2} \sim \left(\frac{m\ell^2}{t}\right)^{N^2}. \tag{2.37}$$

The exponent of this decay is in agreement with (2.34). Furthermore, comparing the prefactors of the estimates (2.34) and (2.37) at large N yields the identification $m\ell^2 \approx N/(2\mathrm{e}^{3/2})$. The scaling $\ell \sim \sqrt{N}$ of the length of the confining interval is an artifact of the continuum framework.

2.4. Large-N asymptotics

In the $N \to \infty$ limit, the amplitude $A_N(t)$ admits the following remarkably simple expression for all finite times:

$$A_\infty(t) = \lim_{N\to\infty} \det\left(J_{m-n}(2t)\right)_{1\le m,n\le N} = \mathrm{e}^{-t^2}. \tag{2.38}$$

This expression has been derived independently in two recent works, devoted to quantum quenches of fermionic chains [38], and to volumes of balls in unitary groups [41]. The return probability therefore reads

$$R_\infty(t) = \mathrm{e}^{-2t^2}. \tag{2.39}$$

It can be verified, by expanding the result (2.16) as a power series in t for the first values of N, that the expressions for $R_N(t)$ and $R_\infty(t)$ start differing at order t^{2N+2}. This phenomenon has already been noticed in [41].

On the other hand, for large but finite N, the results of section 2.3 can be given more explicit forms. Using the asymptotic expansion (B.5), the expressions (2.30) for N even and (2.32) for N odd respectively yield

$$\begin{aligned} N \text{ even:} \quad \ln R_N(t) = &-\frac{N^2}{2}\left(\ln\frac{t}{N}+\frac{3}{2}\right) \\ &-\frac{1}{3}\ln N+\frac{1}{3}\ln 2+4\zeta'(-1)+\cdots, \qquad (2.40) \\ N \text{ odd:} \quad \ln \overline{R}_N(t) = &-\frac{N^2+1}{2}\left(\ln\frac{t}{N}+\frac{3}{2}\right) \\ &-\frac{1}{3}\ln N-\frac{2}{3}\ln 2+\frac{3}{4}+4\zeta'(-1)+\cdots, \qquad (2.41) \end{aligned}$$

where the remainders go to zero for large N. To leading order, both expressions read

$$\ln R_N(t) \approx -\frac{N^2}{2}\left(\ln\frac{t}{N}+\frac{3}{2}\right). \qquad (2.42)$$

The results (2.39) and (2.42) suggest a scaling law of the form

$$\ln R_N(t) \approx -\frac{N^2}{2}\,F(x), \qquad x=\frac{t}{N}, \qquad (2.43)$$

all over the regime where t and N are large and comparable, with

$$F(x) \approx \begin{cases} 4x^2 & (x \ll 1), \\ \ln x + \dfrac{3}{2} & (x \gg 1). \end{cases} \qquad (2.44)$$

The form of the scaling variable x reflects the ballistic nature of the dynamics of a free tight-binding particle.

3. Free fermions near a wall

In this section we consider the same problem on a semi-infinite chain ending with an impenetrable wall. At time $t=0$, the fermions are launched from the N sites which are closest to the wall (see figure 3).

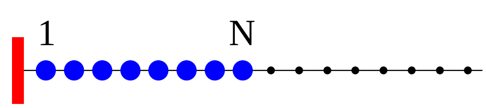

Figure 3. The initial configuration of the N fermions near a wall ending a semi-infinite chain.

We are interested in the temporal decay of the return probability $R_N^{(\mathrm{w})}(t)$, where the superscript (w) reminds of the presence of a wall.

3.1. General expressions at finite times

General expressions for the return probability $R_N^{(\mathrm{w})}(t)$ at arbitrary finite times can be derived along the very lines of section 2.1.

The presence of an impenetrable wall imposes Dirichlet boundary conditions at site $n = 0$. With these boundary conditions, the Hamiltonian $\mathcal{H}$ is brought to the diagonal form

$$\mathcal{H} = \int_0^\pi \frac{\mathrm{d}q}{\pi}\, \varepsilon_q\, \widehat{a}_q^\dagger \widehat{a}_q, \tag{3.1}$$

with

$$\varepsilon_q = 2\cos q \qquad (0 \le q \le \pi). \tag{3.2}$$

The operators

$$\widehat{a}_q = \sqrt{2}\sum_n \sin nq\, a_n, \qquad \widehat{a}_q^\dagger = \sqrt{2}\sum_n \sin nq\, a_n^\dagger \tag{3.3}$$

obey the anti-commutation relation

$$\{\widehat{a}_q, \widehat{a}_{q'}^\dagger\} = \pi\,\delta(q - q'). \tag{3.4}$$

The many-body amplitude $\langle\Psi(0)|\mathbf{q}\rangle$ is again given by a normalized Slater determinant. Since the fermions are launched from the sites $m = 1, \ldots, N$, the latter reads

$$\langle\Psi(0)|\mathbf{q}\rangle = \sqrt{\frac{2^N}{N!}}\,\det\left(\sin mq_n\right)_{1\le m,n\le N}. \tag{3.5}$$

The analogue of (2.11) therefore reads

$$A_N^{(\mathrm{w})}(t) = \frac{2^N}{N!}\int_0^\pi \!\cdots\! \int_0^\pi \prod_{n=1}^N \frac{\mathrm{d}q_n}{\pi}\, \mathrm{e}^{-2\mathrm{i}t\cos q_n} \times \Big(\det\left(\sin mq_n\right)_{1\le m,n\le N}\Big)^2. \tag{3.6}$$

The formula (3.5) can be brought to a form similar to (2.14). We recall that

$$\sin mq = \sin q\, U_{m-1}(\cos q), \tag{3.7}$$

where the U_m are the Tchebyshev polynomials of the second kind [42, Vol. II, Ch. X]. The mth polynomial has degree m and the leading term $U_m(z) = (2z)^m + \cdots$ We have therefore

$$\begin{aligned}\det\left(\sin mq_n\right)_{1\le m,n\le N} &= \prod_{n=1}^N \sin q_n\ \det\left(U_{m-1}(q_n)\right)_{1\le m,n\le N} \\ &= 2^{N(N-1)/2}\prod_{n=1}^N \sin q_n \\ &\times \prod_{1\le m<n\le N} (\cos q_n - \cos q_m),\end{aligned} \tag{3.8}$$

where the last product is again a Vandermonde determinant. We are thus left with the following integral expression for the amplitude:

$$A_N^{(\mathrm{w})}(t) = \frac{2^{N^2}}{N!}\int_0^\pi \!\cdots\! \int_0^\pi \prod_{n=1}^N \frac{\mathrm{d}q_n}{\pi}\ \sin^2 q_n\, \mathrm{e}^{-2\mathrm{i}t\cos q_n} \times \prod_{1\le m<n\le N} (\cos q_n - \cos q_m)^2. \tag{3.9}$$

Applying the Andréief identity (A.1) to (3.6), we get the alternative expression

$$A_N^{(\mathrm{w})}(t) = \det\bigl(\psi_{m,n}^{(\mathrm{w})}(t)\bigr)_{1\le m,n\le N}, \tag{3.10}$$

where

$$\begin{aligned}\psi_{m,n}^{(\mathrm{w})}(t) &= \psi_{m,n}(t) - \psi_{-m,n}(t)\\ &= \mathrm{i}^{-(n-m)}\,J_{n-m}(2t) - \mathrm{i}^{-(n+m)}\,J_{n+m}(2t)\end{aligned} \tag{3.11}$$

is the one-body wavefunction at site n at time t for a particle located on the semi-infinite chain and launched from site m at time 0. The expression (3.11) can be recovered by applying the reflection principle (i.e., the method of images) to (2.17).

The complementary expressions (3.9) and (3.10) are the exact analogues on the semi-infinite chain of their counterparts (2.15) and (2.16) on the infinite chain.

3.2. Asymptotic analysis in the long-time regime

In this section we determine the asymptotic decay of the return probability $R_N^{(\mathrm{w})}(t)$. We again apply the saddle-point approximation to the multiple integral in (3.9). Saddle points are still defined by the condition that every momentum q_n is either 0 or π. The vicinity of the most general saddle point will be parametrized by choosing M momenta near 0, of the form $q_n = x_n$, and the remaining $N - M$ momenta around π, of the form $q_n = \pi - y_n$. At variance with the previous situation (section 2.3), the variables x_n and y_n are now positive. We thus obtain the estimate

$$A_N^{(\mathrm{w})}(t) \approx 2^{N^2}\sum_{M=0}^{N}\frac{I_{N,M}^{(\mathrm{w})}(t)}{M!(N-M)!}, \tag{3.12}$$

with

$$\begin{aligned}I_{N,M}^{(\mathrm{w})}(t) &= 2^{N-(N-2M)^2}\,\mathrm{e}^{2\mathrm{i}(N-2M)t}\\ &\times\int_0^\infty\!\!\cdots\int_0^\infty\prod_{n=1}^{M}\frac{\mathrm{d}x_n}{\pi}\,x_n^2\,\mathrm{e}^{\mathrm{i}tx_n^2}\prod_{1\le m<n\le M}(x_n^2-x_m^2)^2\\ &\times\int_0^\infty\!\!\cdots\int_0^\infty\prod_{n=1}^{N-M}\frac{\mathrm{d}y_n}{\pi}\,y_n^2\,\mathrm{e}^{-\mathrm{i}ty_n^2}\prod_{1\le m<n\le N-M}(y_n^2-y_m^2)^2.\end{aligned} \tag{3.13}$$

The above integrals can still be performed exactly. They are indeed proportional to the analytical continuation to imaginary values of a of the Mehta integral [40, Eq. (17.6.6), $\alpha = 3/2$, $\gamma = 1$]

$$\begin{aligned}\mathcal{I}_N^{(\mathrm{w})}(a) &= \int_{-\infty}^{\infty}\!\!\cdots\int_{-\infty}^{\infty}\prod_{n=1}^{N}\mathrm{d}x_n\,x_n^2\,\mathrm{e}^{-ax_n^2}\prod_{1\le m<n\le N}(x_n^2-x_m^2)^2\\ &= a^{-N(2N+1)/2}\prod_{j=1}^{N}j!\,\Gamma(j+1/2)\\ &= \pi^{N/2}(2a)^{-N(2N+1)/2}\sqrt{N!\,G(2N+2)},\end{aligned} \tag{3.14}$$

where G is again Barnes' G-function (see Appendix B). We thus obtain

$$\begin{aligned}I_{N,M}^{(\mathrm{w})}(t) &= 2^{-(N-2M)^2}\pi^{-N/2}\,\mathrm{e}^{\mathrm{i}(N-2M)(2t-(N+1/2)\pi/4)}\\ &\times\sqrt{M!(N-M)!\,G(2M+2)G(2N-2M+2)}\\ &\times(2t)^{2M(N-M)-N(2N+1)/2}.\end{aligned} \tag{3.15}$$

The behavior of the return probability at long times is still governed by the term with the slowest decay. Even and odd values of the fermion number N again yield different behaviors.

• If $N = 2m$ is even, the slowest decay is reached for $M = m$, i.e., for equal numbers of momenta near 0 and π. We thus obtain the estimate

$$A_{2m}^{(\mathrm{w})}(t) \approx \frac{2^{m(2m-1)} G(2m+2)}{\pi^m\, m!\, t^{m(2m+1)}}, \tag{3.16}$$

showing that the amplitude is positive and decays monotonically, at least for large times. The echo therefore falls off monotonically as a power law

$$R_{2m}^{(\mathrm{w})}(t) \approx \frac{2^{2m(2m-1)} G(2m+2)^2}{\pi^{2m}\, m!^2\, t^{2m(2m+1)}}, \tag{3.17}$$

with exponent $2m(2m+1) = N(N+1)$.

• If $N = 2m+1$ is odd, the slowest decay is reached both for $M = m$ and $M = m+1$. We thus obtain the estimate

$$A_{2m+1}^{(\mathrm{w})}(t) \approx \frac{2^{m(2m+1)} G(2m+3)}{\pi^{m+1/2}\, m!\, t^{2m^2+3m+3/2}} \cos(2t - (2m+3/2)\pi/4), \tag{3.18}$$

showing that the amplitude behaves for large times as an oscillatory function modulated by a decaying power law. Averaging over the oscillations, we obtain the following power-law decay for the mean echo:

$$\overline{R}_{2m+1}^{(\mathrm{w})}(t) \approx \frac{2^{2m(2m+1)-1} G(2m+3)^2}{\pi^{2m+1}\, m!^2\, t^{4m^2+6m+3}}, \tag{3.19}$$

with exponent $4m^2 + 6m + 3 = N^2 + N + 1$.

Figure 4 illustrates the above results, showing log-log plots of the return probability $R_N^{(\mathrm{w})}(t)$ against time t for $N = 1$, 2, 3 and 4. Data have been obtained using the expression (3.10). For odd N (left), the echo oscillates forever. For even N (right), it falls off monotonically and exhibits mild damped oscillations. The blue straight lines – slightly translated for a better readability – have the predicted slopes 3, 6, 13 and 20.

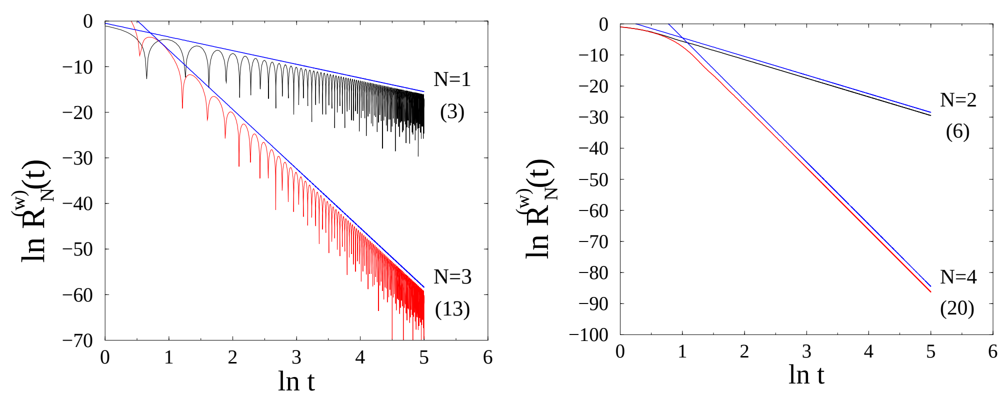

Figure 4. Log-log plots of the return probability $R_N^{(\mathrm{w})}(t)$ near a wall ending a semi-infinite chain against time t, for N up to 4. Left: odd N. Right: even N. The blue straight lines have the predicted slopes 3, 6, 13 and 20.

It is again worth comparing the above results for lattice fermions to the corresponding prediction in the continuum limit. Restricting the saddle-point approximation to the sector where all momenta are near zero (i.e., $M = N$) yields

$$R_N^{(\mathrm{w,cont})}(t) \approx \frac{G(2N+2)}{2^{N(2N+1)}\pi^N N!\, t^{N(2N+1)}}. \tag{3.20}$$

We again obtain a power-law decay of the return probability, albeit with no parity effect and with a different, larger decay exponent.

The above power-law decay can still be recovered by means of heuristic reasoning. Suppose the fermions are launched from the interval $0 < x < \ell$ near the wall. The modulus of the many-body wavefunction at time t now reads approximately

$$\left|\psi_N^{(\mathrm{w})}(x_1,\dots,x_N;t)\right| \sim C_N^{(\mathrm{w})}(t)\prod_{n=1}^{N} x_n \prod_{1\le m<n\le N}\left|x_m^2 - x_n^2\right|. \tag{3.21}$$

Dimensional analysis determines the scaling of the normalization of the wavefunction, $C_N^{(\mathrm{w})}(t) \sim L(t)^{-N(2N+1)}$, and of the return probability,

$$R_N^{(\mathrm{w})}(t) \sim \left(\frac{\ell}{L(t)}\right)^{N(2N+1)} \sim \left(\frac{m\ell^2}{t}\right)^{N(2N+1)}. \tag{3.22}$$

A very similar result can be found in [33]. The decay exponent of the above estimate agrees with (3.20). Furthermore, comparing the prefactors of the estimates (3.20) and (3.22) at large N yields $m\ell^2 \approx N/\mathrm{e}^{3/2}$. The latter estimate is twice larger than its counterpart on the infinite line, given below (2.37). An intuitive interpretation of this factor two will be given below (3.27).

3.3. Large-N asymptotics

In the $N \to \infty$ limit, the return probability is expected to be equal to the square root of its counterpart (2.39) on the infinite chain, namely

$$R_\infty^{(\mathrm{w})}(t) = \mathrm{e}^{-t^2}. \tag{3.23}$$

A heuristic way of showing this goes as follows. On the infinite chain, the compact fermionic state has two ends, and can therefore decay through either end, whereas it has only one right end if confined near a wall. Hence we can expect $R_\infty(t) = (R_\infty^{(\mathrm{w})}(t))^2$. The latter result can be checked by expanding the expression (3.10) as a power series in t for the first values of N. Doing so confirms our expectation and shows that $R_N^{(\mathrm{w})}(t)$ and $R_\infty^{(\mathrm{w})}(t)$ again start differing at order t^{2N+2}.

On the other hand, using the asymptotic formula (B.5), we can still derive more explicit forms of the above results for large N. The expressions (3.17) for N even and (3.19) for N odd respectively yield

$$N \text{ even:} \quad \ln R_N^{(\mathrm{w})}(t) = -N(N+1)\left(\ln\frac{t}{2N+1} + \frac{3}{2}\right) - \frac{1}{6}\ln N + \ln 2 - \frac{3}{8} + 2\zeta'(-1) + \cdots, \tag{3.24}$$

$$N \text{ odd:} \quad \ln \overline{R}_N^{(\mathrm{w})}(t) = -(N^2+N+1)\left(\ln\frac{t}{2N+1} + \frac{3}{2}\right) - \frac{1}{6}\ln N - 2\ln 2 + \frac{9}{8} + 2\zeta'(-1) + \cdots \tag{3.25}$$

The argument $t/(2N+1)$ of the logarithms has been chosen in order to minimize the order of magnitude of the correction terms given in the second lines of the above expressions. To leading order, both results read

$$\ln R_N^{(\mathrm{w})}(t) \approx -N^2 \left(\ln \frac{t}{2N} + \frac{3}{2} \right). \tag{3.26}$$

The results (3.23) and (3.26) again suggest a scaling law of the form

$$\ln R_N^{(\mathrm{w})}(t) \approx -N^2 \, F^{(\mathrm{w})}(x), \qquad x = \frac{t}{2N}. \tag{3.27}$$

The effective fermion number $2N$ entering the scaling variable x can be interpreted as the total distance to be traveled by an excitation entering the compact fermionic state from the right, bouncing at the wall, and exiting from the right. Finally, the scaling function $F^{(\mathrm{w})}(x)$ obeys the very same asymptotics (2.44) as $F(x)$ both for $x \ll 1$ and for $x \gg 1$. It is therefore tempting to conjecture that both scaling functions are identical.

4. Discussion

We studied the quantum return probability for a system of N non-interacting lattice fermions launched from N consecutive sites, either on the infinite chain or near an impenetrable wall ending a semi-infinite chain.

In each case we derived exact expressions for the return probability valid for all fermion numbers N and time t. We thus obtained two complementary kinds of expressions, namely integral formulas (see (2.15) and (3.9)), which are the natural outcome of the second-quantized formalism, and determinantal formulas (see (2.16) and (3.10)), which could have been obtained by a first-quantized approach as well. The Andréief identity provides an explicit way of checking the equivalence of the first-quantized and second-quantized approaches. We deduced the asymptotic long-time behavior of the return probability by evaluating the integral formulas by means of the saddle-point method. Even and odd values of the fermion number N yield different qualitative behaviors, as well as slightly different expressions for the decay exponents. For even N, the echo falls off monotonically as a power law. For odd N, it exhibits periodic oscillations modulated by a decaying power law. This qualitative dependence on the parity of the fermion number is a pure lattice effect, which is absent in the continuum limit. The return probability of N particles thus provides yet another example of a situation where quantum dynamics exhibits qualitatively different features on the lattice and in the continuum. The exponents characterizing the temporal decay of the return probability are gathered in table 1.

infinite chain (lattice)	infinite chain (continuum)	near a wall (lattice)	near a wall (continuum)
N even: $\frac{1}{2}N^2$ N odd: $\frac{1}{2}(N^2+1)$	N^2	N even: $N(N+1)$ N odd: N^2+N+1	$N(2N+1)$

Table 1. Exponents of the temporal decay of the return probability on the infinite chain and near a wall. Comparison between the values of the exponents for lattice fermions and the predictions of the continuum limit.

Our results also yield explicit expressions for the prefactors of the asymptotic power-law decay of the return probability, in both geometries and for all values of the fermion number N. The key point in the derivation of these results has been the use of Mehta integrals, which have been extensively used in random matrix theory [40, Ch. 17] and can be viewed as limiting cases of the Selberg integral. Reference [43] provides a historical overview of the Selberg and related integrals, whereas the recent work [44] and the references therein mention yet other connections between random matrix theory and systems of free fermions. Our expressions for the prefactors of the power-law decays involve as an essential ingredient Barnes' G-function, which is also ubiquitous in random matrix theory. Table 2 gives our predictions in factorized form up to $N = 10$.

N odd	infinite chain $\overline{R}_N(t)$ (2.32)	near a wall $\overline{R}_N^{(\mathrm{w})}(t)$ (3.19)	N even	infinite chain $R_N(t)$ (2.30)	near a wall $R_N^{(\mathrm{w})}(t)$ (3.17)
1	$\frac{1}{2\pi t}$	$\frac{1}{2\pi t^3}$	2	$\frac{1}{\pi^2 t^2}$	$\frac{2^4}{\pi^2 t^6}$
3	$\frac{2}{\pi^3 t^5}$	$\frac{2^9 3^2}{\pi^3 t^{13}}$	4	$\frac{2^4}{\pi^4 t^8}$	$\frac{2^{20} 3^4}{\pi^4 t^{20}}$
5	$\frac{2^9}{\pi^5 t^{13}}$	$\frac{2^{33} 3^6 5^2}{\pi^5 t^{31}}$	6	$\frac{2^{16}}{\pi^6 t^{18}}$	$\frac{2^{52} 3^8 5^4}{\pi^6 t^{42}}$
7	$\frac{2^{23} 3^2}{\pi^7 t^{25}}$	$\frac{2^{71} 3^{12} 5^6 7^2}{\pi^7 t^{57}}$	8	$\frac{2^{32} 3^4}{\pi^8 t^{32}}$	$\frac{2^{96} 3^{16} 5^8 7^4}{\pi^8 t^{72}}$
9	$\frac{2^{45} 3^6}{\pi^9 t^{41}}$	$\frac{2^{125} 3^{24} 5^{10} 7^6}{\pi^9 t^{91}}$	10	$\frac{2^{60} 3^8}{\pi^{10} t^{50}}$	$\frac{2^{160} 3^{32} 5^{12} 7^8}{\pi^{10} t^{110}}$

Table 2. Asymptotic temporal decay of the return probability on the infinite chain and near a wall, for fermion numbers up to $N = 10$. The exact prefactors are given in factorized form.

The behavior of our results at large fermion numbers led us to hypothesize the scaling laws (2.43) and (3.27), with arguments $x = t/N$ and $x = t/(2N)$, in the regime where t and N are both large and comparable. The form of these scaling variables reflects the ballistic nature of a free tight-binding particle. It would be worth investigating the above scaling regime in a more thorough fashion, possibly by means of more advanced techniques, in order, among other things, to prove or disprove the identity of the scaling functions F and $F^{(\mathrm{w})}$.

Finally, the framework of this study could be extended to investigate the return probability of a system of N particles launched from other localized initial states, such as compactly supported but not most compact states, or more general initial states whose many-body wavefunction is localized in the center-of-mass coordinate.

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Appendix A. The Andréief identity

The Andréief identity

$$\int_a^b \cdots \int_a^b \prod_{n=1}^N \rho(x_n)\,\mathrm{d}x_n \det(f_m(x_n))_{1\le m,n\le N}\ \det(g_m(x_n))_{1\le m,n\le N}$$
$$= N!\det\left(\int_a^b \rho(x)\,\mathrm{d}x\, f_m(x) g_n(x)\right)_{1\le m,n\le N} \qquad \text{(A.1)}$$

relates the integral of the product of the determinants built upon two families of functions to the determinant of their scalar products. This identity comes in many guises in various branches of the mathematical literature, concerning especially orthogonal polynomials and random matrix theory. Although it seems to appear for the first time in print in an article by Andréief in 1883 [45], it is also associated with other names, including Cauchy-Binet, Gram and Heine. Here it provides an explicit way of checking the equivalence of the first-quantized and second-quantized approaches, as it allows us to respectively derive (2.16) and (3.10) from (2.13) and (3.6).

Let us give an elementary proof of the above identity for the sake of completeness. Starting from the left-hand side, let us introduce the Leibniz expansions of the determinants:

$$\det(f_m(x_n))_{1\le m,n\le N} = \sum_\sigma \operatorname{sgn}\sigma \prod_{n=1}^N f_{\sigma_n}(x_n),$$
$$\det(g_m(x_n))_{1\le m,n\le N} = \sum_\tau \operatorname{sgn}\tau \prod_{n=1}^N g_{\tau_n}(x_n), \qquad \text{(A.2)}$$

where σ and τ are permutations acting on N symbols, and $\operatorname{sgn}\sigma = \pm 1$ and $\operatorname{sgn}\tau = \pm 1$ are their signatures. We thus obtain

$$I_N = \sum_{\sigma,\tau} \operatorname{sgn}\sigma \operatorname{sgn}\tau \int_a^b \cdots \int_a^b \prod_{n=1}^N \rho(x_n)\,\mathrm{d}x_n\, f_{\sigma_n}(x_n) g_{\tau_n}(x_n). \qquad \text{(A.3)}$$

The integrand is now a product, and so

$$I_N = \sum_{\sigma,\tau} \operatorname{sgn}\sigma \operatorname{sgn}\tau \prod_{n=1}^N \int_a^b \rho(x)\,\mathrm{d}x\, f_{\sigma_n}(x) g_{\tau_n}(x). \qquad \text{(A.4)}$$

For fixed permutations σ and τ, let us change the index from n to $m = \tau_n$. We have then $\sigma_n = \mu_m$, where $\mu = \sigma \cdot \tau^{-1}$, and so $\operatorname{sgn}\mu = \operatorname{sgn}\sigma \operatorname{sgn}\tau$. For fixed τ, the sum over σ can be replaced by a sum over μ. The sum over τ simply yields a factor $N!$. We thus obtain

$$I_N = N! \sum_\mu \operatorname{sgn}\mu \prod_{n=1}^N \int_a^b \rho(x)\,\mathrm{d}x\, f_{\mu_m}(x) g_m(x). \qquad \text{(A.5)}$$

The sum over μ is nothing but the Leibniz expansion of the determinant given in the right-hand side of (A.1).

On the infinite chain, the expression (2.13) of the amplitude $A_N(t)$ is proportional to the left-hand side of (A.1), with

$$x_n = q_n, \qquad \rho(x_n) = \mathrm{e}^{-2\mathrm{i}t\cos q_n},$$
$$f_m(x_n) = \mathrm{e}^{\mathrm{i}mq_n}, \qquad g_m(x_n) = \mathrm{e}^{-\mathrm{i}mq_n}. \qquad \text{(A.6)}$$

Applying the identity (A.1) yields (2.16).

On the semi-infinite chain, the expression (3.6) of the amplitude $A_N^{(\mathrm{w})}(t)$ is proportional to the left-hand side of (A.1), with the same $\rho(x_n)$ and

$$f_m(x_n) = g_m(x_n) = \sqrt{2}\,\sin mq_n. \tag{A.7}$$

Applying the identity (A.1) yields (3.10).

Appendix B. Barnes' G-function

Barnes' G-function shares many common features with Euler's Γ-function. This appendix summarizes the main properties of both functions, which can be found in the Wikipedia article [46] or in the *Digital Library of Mathematical Functions* [47, Ch. 5.17].

Euler's Γ-function and Barnes' G-function are meromorphic functions in the complex plane obeying the recursion relations

$$\Gamma(z+1) = z\Gamma(z), \qquad G(z+1) = \Gamma(z)G(z), \tag{B.1}$$

with appropriate regularity conditions.

When z is a positive integer, Euler's Γ-function becomes the usual factorial:

$$\Gamma(n+1) = n!, \tag{B.2}$$

whereas Barnes' G-function becomes the 'superfactorial':

$$G(n+2) = \prod_{k=1}^{n} k! = \prod_{\ell=1}^{n} \ell^{n+1-\ell} = \prod_{1\le i<j\le n+1} (j-i). \tag{B.3}$$

We have in particular $\Gamma(1) = \Gamma(2) = 1$ and $G(1) = G(2) = G(3) = 1$. The 'superfactorial' numbers $G(n+2)$ appear in the OEIS [48] as sequence number A000178, together with many further properties and references.

Euler's Γ-function and Barnes' G-function have the following asymptotic expansions as $z \to +\infty$:

$$\ln\Gamma(z+1) = \left(z+\frac{1}{2}\right)\ln z - z + \ln\sqrt{2\pi} + \cdots, \tag{B.4}$$

$$\ln G(z+1) = \left(\frac{z^2}{2}-\frac{1}{12}\right)\ln z - \frac{3z^2}{4} + z\ln\sqrt{2\pi} + \zeta'(-1) + \cdots, \tag{B.5}$$

where $\zeta'(-1) = -0.165\,421\,143\ldots$ (ζ being Riemann's ζ-function), and the remainders go to zero for large z.

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