Convex geometries representable by at most 5 circles on the plane

PolyMath REU-2020, Convex Geometries Team*

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Abstract

A convex geometry is a closure system satisfying the anti-exchange property. In this work we document all convex geometries on 4- and 5-element base sets with respect to their representation by circles on the plane. All 34 non-isomorphic geometries on a 4-element set can be represented by circles, and of the 672 geometries on a 5-element set, we made representations of 621. Of the 51 remaining geometries on a 5-element set, one was already shown not to be representable due to the Weak Carousel property, as articulated by Adaricheva and Bolat (Discrete Mathematics, 2019). In this paper we show that 7 more of these convex geometries cannot be represented by circles on the plane, due to what we term the Triangle Property.

Contents

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1 Introduction

This project was done during two summer months of 2020 within the framework of the Polymath REU project, which brought together undergraduate students from around the world who worked together online to make progress on 12 different mathematical problems.

In this project we address the problem raised in Adaricheva and Bolat [1]: whether all geometries with convex dimension at most 5 are representable by circles on the plane using the closure operator of convex hull for circles.

The early survey on the topic of convex geometries is given by Edelman and Jamison [7], and the theory of infinite convex geometries was developed in Adaricheva, Gorbunov, Tumanov [2]. A recent survey on the topic, which includes both finite and infinite convex geometries, is in a chapter of *Lattice Theory: Special Topics and Applications* (2016), by Adaricheva and Nation [3].

The representation of finite convex geometries by circles was introduced by G. Czédli in [6], where the usual closure operator of convex hull acting on sets of points was generalized to the convex hull operator acting on circles.

Moreover, he proved that every finite convex geometry of convex dimension 2 can be represented by circles on the plane.

The natural question of whether a finite convex geometry of any convex dimension can be represented by circles on the plane was answered in [1] *in the negative*: it turns out that the convex hull operator acting on circles in the plane satisfies a condition called the 2×3-*Weak carousel property* - and not all finite convex geometries satisfy it. An example of a geometry with a convex dimension of 6 that does not satisfy this condition was found on a 5-element set.

This left the question of whether any convex geometry on an at most 5-element set, with convex dimension at most 5, could be represented by circles on the plane.

The goal of the project was to document all convex geometries on 4- and 5-element sets, to show their representations by circles (when possible), and to describe the features of "impossible" geometries.

After generating the list of all geometries on 4- and 5-element sets, we found representations of all 34 non-isomorphic geometries on 4-element sets and of 621 geometries on 5-element sets. 51 geometries on a 5-element set resisted representation by circles.

Geometry G4 from the list of 672 non-isomorphic geometries on a 5-element set is the "impossible" geometry shown not to be representable in [1]. In this paper we prove that 7 more geometries from the list of 672 are not representable. The proof is based on the Triangle Property (Lemma 5 in section 9).

Three geometries from this new set of "impossible" geometries have a convex dimension of 4, and one has a convex dimension of 5. **This provides a solution to Problem 1 in [1] in that geometries not representable by circles exist with those convex dimensions.**

On the other hand, all geometries on 4- and 5-element sets with convex dimension 3 are represented by circles. Thus, the following question remains:

Question 1. *Does there exist a convex geometry of convex dimension 3 not representable by circles on the plane?*

The paper is organized as follows. We provide all necessary definitions related to convex geometries in section 2. In particular the parameter of *convex dimension* is explained in detail. Section 3 provides the background for the algorithm generating all non-isomorphic geometries (anti-matroids) on n-element set, for values of n up to 7. The algorithm was developed around 2013, in the framework of enumerating anti-matroids in the On-line Encyclopedia of Integer Sequences (OEIS). The existing code was enhanced for computing the *implicational basis* of each geometry and its convex dimension. Sections 4 and 5 are devoted to methods for representing geometries by circles. Some basic principles of verifying representations are discussed in section 6. Sections 7 and 8 provide an overview of different software available for enhancing this area of mathematical work, in our case GeoGebra and Google Colab toolkit. Finally, section 9 gives the proof of the Triangle Property, and its consequences for a subset of "impossible" geometries not being representable by circles.

We provide two sets of appendices: in Appendix A there is a list of 34 geometries on 4-element set, and a list of 672 geometries on 5-element set, given by their alignments (convex sets), as well as sets of implications representing the associated closure operator. The meet-irreducible elements of alignments are provided, together with their maximal anti-chains, which allows the computation of each geometry's convex dimension.

Appendix B has representations by circles for all 34 geometries on a 4-element set, as well as 621 geometries on a 5-element set. The numbering corresponds to the order of geometries in the lists featured in Appendix A.

The last part of Appendix B lists 51 geometries which are not yet represented, giving their implicational bases, also featured in Appendix A. The list of geometries which we were not able to represent is as follows:

$$G4, G7, G11, G12, G14, G15, G18, G21, G23, G26, G27, G33, G34, G35, G39, G43,$$

$$G45, G46, G47, G49, G54, G56, G57, G60, G62, G69, G70, G74, G84, G87, G88,$$

$$G89, G94, G95, G96, G105, G114, G115, G122, G129, G132, G134, G143, G147,$$

$$G153, G161, G175, G206, G211, G35, G351.$$

Among these 51 geometries, we showed in this paper that the following are not representable by circles:

$$G7, G11, G12, G18, G21, G23, G34.$$

Excluding these 7 geometries, as well as earlier example G4 which cannot be represented, we ask:

Question 2. *Are 43 remaining geometries on the list above representable by circles on the plane?*

2 Preliminaries

2.1 Basic Definitions

A convex geometry is a special case of a closure system.

Definition 2.1. *Let X be a set and $\varphi\colon 2^X \to 2^X$ be a function. Then (X, φ) is a* closure system *if and only if for all $Y, Z \in 2^X$:*

1. *$Y \subseteq \varphi(Y)$*
2. *if $Y \subseteq Z$ then $\varphi(Y) \subseteq \varphi(Z)$*
3. *$\varphi(\varphi(Y)) = \varphi(Y)$*

Given a closure system (X, φ), refer to φ as its closure operator. *A subset $Y \subseteq X$ is* closed *if $\varphi(Y) = Y$.*

Definition 2.2. *A closure system $(X, \mathcal{F})$ is called a* convex geometry *if*

1. $\varphi(\emptyset) = \emptyset$
2. *For any closed set* $Y \subseteq X$ *and any distinct points* $x, y \in X \setminus Y$*, if* $x \in \varphi(Y \cup \{y\})$ *then* $y \notin \varphi(Y \cup \{x\})$.

The second property in this definition is called the *Anti-Exchange Property.*

Definition 2.3. *Given any (finite) set* X*, an alignment on* X *is a family* $\mathcal{F}$ *of subsets of* X *which satisfies two properties:*

1. $X \in \mathcal{F}$
2. *If* $Y, Z \in \mathcal{F}$ *then* $Y \cap Z \in \mathcal{F}$.

Closure systems are dual to alignments in the following sense:

If (X, φ) is a closure system, then let $\mathcal{F} := \{Y \subseteq X : \varphi(Y) = Y\}$. Then $\mathcal{F}$ is an alignment.

If $\mathcal{F}$ is an alignment, then define $\varphi : 2^X \to 2^X$ in the following manner:
For all $Y \subseteq X$, let $\mathcal{C}_Y := \{Z \in \mathcal{F} : Y \subseteq Z\}$ and $\varphi(Y) := \bigcap_{Z \in \mathcal{C}_Y} Z$. Then (X, φ) is a closure system.

We can use this duality to provide another definition of a convex geometry.

Definition 2.4. *A closure system is a convex geometry if the corresponding alignment* $\mathcal{F}$ *satisfies the following two properties:*

1. $\emptyset \in \mathcal{F}$
2. *If* $Y \in \mathcal{F}$ *and* $Y \neq X$ *then* $\exists a \in X \setminus Y$ *s.t.* $Y \cup \{a\} \in \mathcal{F}$.

2.2 Convex Dimension

This section follows the survey on convex geometries [7].

Definition 2.5. *An alignment* $\mathcal{F}$ *is called a monotone alignment if the sets of* $\mathcal{F}$ *form a chain under the inclusion order.*

Definition 2.6. *Given two alignments* $\mathcal{F}, \mathcal{K}$*, their join* $\mathcal{F} \vee \mathcal{K}$ *is defined to be the smallest alignment that contains both* $\mathcal{F}$ *and* $\mathcal{K}$*. Explicitly, for alignments on finite set* X*, this is the collection of all intersections of sets taken from* $\mathcal{F}, \mathcal{K}$*:* $\mathcal{F} \vee \mathcal{K} = \{F \cap K : F \in \mathcal{F} \text{ and } K \in \mathcal{K}\}$.

A known result in Edelman and Jamison [7] about joins follows.

Theorem 2.1. *Every alignment can be expressed as the join of some collection of monotone alignments on the same base set.*

This motivates the following definition.

Definition 2.7. *The* convex dimension *of a convex geometry* G *is the least number* k *such that the alignment* $\mathcal{F}$ *corresponding to* G *can be expressed as the join of* k *monotone alignments.*

To compute the convex dimension of a convex geometry, we can examine antichains of meet-irreducibles.

Definition 2.8. *A meet-irreducible in an alignment $\mathcal{F}$ is a closed set $Y \in \mathcal{F}$ for which there exists $Z \in \mathcal{F}$ such that*

1. *$Y \subset Z$*
2. *If $Y \subset W$, for some closed set W, then $Z \subseteq W$.*

This is equivalent to saying that if $Y = \bigcap_{i \in I} Y_i$ where $Y_i \in \mathcal{F}$ for all i, then $Y = Y_i$ for some $i \in I$.

Note that by definition 2.4 of a convex geometry, we can additionally claim that $|Z \setminus Y| = 1$.

Definition 2.9. *In a partially ordered set, an* antichain *is a collection of pairwise non-comparable elements.*

Set inclusion forms a partial order on the elements of an alignment, and this partial order allows us to talk about antichains of elements of an alignment. The following result is due to Edelman and Saks [8].

Theorem 2.2. *Let G be a convex geometry and $\mathcal{F}$ be the corresponding alignment. The convex dimension of G is equal to the largest size antichain of meet-irreducibles.*

2.3 Convex hull operator

A particular example of a closure operator on a set is the convex hull operator, where the base set X is a set of points in Euclidean space $\mathbb{R}^n$. For the goals of this paper we use only the plane, i.e. space $\mathbb{R}^2$.

Definition 2.10. *A set S in $\mathbb{R}^2$ is called a convex set if for any two points $p, q \in S$, the line segment connecting p and q is also contained in S.*

See Figure 1 for illustration.

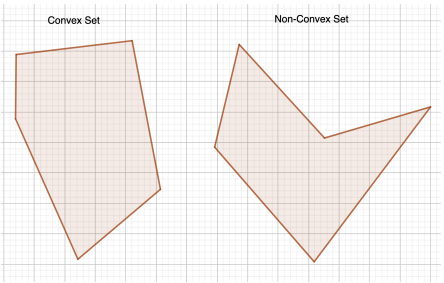


Figure 1: Example of a set which is convex and a set which is not convex.

Definition 2.11. *Given a set S of points in $\mathbb{R}^2$, the convex hull of S, in notation* $\mathrm{CH}(S)$*, is the intersection of all convex sets in $\mathbb{R}^2$ which contain S.*

From the illustration, if $S = \{A, B, C, D, E, F, G\}$, a set of of 7 points on the plane, $\mathrm{CH}(S)$ is a convex shape with vertices A, C, D, F, G on Figure 2.

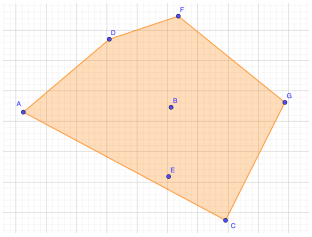


Figure 2: The convex hull $\mathrm{CH}(ABCDEFG)$

The following definition allows us to introduce a closure operator induced by the convex hull, on any set of points X on the plane.

Definition 2.12. *Let X be a finite set of points in $\mathbb{R}^2$. Define operator* $\mathrm{ch} : 2^X \to 2^X$ *as follows:*

$$\mathrm{ch}(Y) = \mathrm{CH}(Y) \cap X,$$

for any $Y \in 2^X$.

It is straightforward to check that ch is a closure operator. For example, $\mathrm{ch}(ACDFG) = S$ from Example on Figure 2. Moreover, ch satisfies the Anti-Exchange Property. Therefore, (X, ch) is a convex geometry.

Finally, for the purposes of the current paper we want to recall the definition of the convex hull operator for circles introduced in Cédli [6].

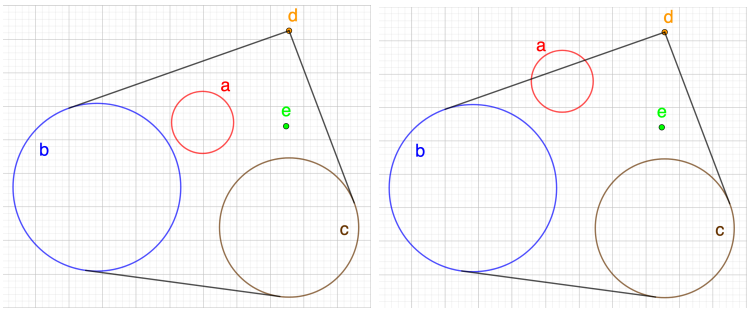
If x is a circle on the plane, then by $\tilde{x}$ we denote a set of points belonging to x. We allow a circle to have a radius 0, thus, a point is a circle that includes a single point.

Definition 2.13. *Let X be a finite set of circles in $\mathbb{R}^2$. Define operator* $\mathrm{ch}_c : 2^X \to 2^X$*, a convex hull operator for circles, as follows:*

$$\mathrm{ch}_c(Y) = \{x \in X : \tilde{x} \subseteq \mathrm{CH}(\bigcup_{y \in Y} \tilde{y})\},$$

for any $Y \in 2^X$.

For example, $a, e \in \mathrm{ch}_c(b, c, d)$ on Figure 3a, while $a \notin \mathrm{ch}_c(b, c, d)$, $e \in \mathrm{ch}_c(b, c, d)$ in Figure 3b.



(a) Circle a is in the convex hull of b, c, d

(b) Circle a is not in the convex hull of b, c, d

3 Creating the list of geometries on 4- and 5-element sets

An OEIS (On-line Encyclopedia of Integer Sequences) submission on the number of non-isomorphic antimatroids by Przemysław Uznański [10] served as the starting point for our project. From a combinatorial point of view, a family $\mathcal{F}$ of subsets of (finite) X is called an *antimatroid* if and only if:

1. $\mathcal{F}$ is closed under unions, i.e. $\forall Z, Y \in \mathcal{F}, Z \cup Y \in \mathcal{F}$
2. $\mathcal{F}$ is non-empty accessible, i.e $\forall Y \in \mathcal{F}$ either $Y = \emptyset$ or $\exists\, x \in Y$ such that $Y \setminus x \in \mathcal{F}$.

A close look at the properties reveals that there exists a bijective map f, between antimatroids and convex geometries given by:

$$f(\mathcal{F}) = \{Y^c \colon Y \in \mathcal{F}\} \tag{1}$$

Here $Y^c = X \setminus Y$, a set-theoretical complement of subset $Y \subseteq X$.
Uznański in [10] generated a list of all non-isomorphic antimatroids on base sets of order 3, 4, 5 and 6 using an extension property on antimatroids combined with a reverse tree traversal algorithm. The theoretical background of the algorithm was developed in Kempner and Levit [9].

Adapting Uznański's original codebase, all convex geometries can be easily generated using the bijection noted above. The antimatroids and their corresponding geometries are represented as elements of 2^{2^X} using their associated alignments as described by Definition 2.3. For efficiency and portability however, the alignments are physically stored, and listed in the Appendices A, as 32-bit integers through a programming technique known as *bitmasking* [12].

Given a 32-bit representation $b_{31}b_{30}\ldots b_1b_0$ and $X = \{x_1 \ldots x_n\}$ where $n = 3, 4, 5$, one may obtain the convex sets, F, by writing $S = \{i \,:\, b_i = 1\}$. The convex sets shall then be given by

$$F_k = \{x_i \,:\, c_i = 1,\ c_5 \dots c_1 = S_k \in S\} \tag{2}$$

where c_i are the bits of S_k.

Example 1.

For a base set of order 3, represented as $\{a, b, c\}$, assign the zeroth, first and second bits to a, b and c respectively. Then $\{a\}$ is represented as 001(1), $\{b\}$ is represented as 010(2), $\{a, b, c\}$ is represented as 111(7), $\emptyset$ as 000(0) and so on... It is easy to check that the decimal equivalents of the binary representation of subsets form the set $\{0, 1, 2 \dots 2^n - 1\}$ where n is the order of the base set. A family F has its n^{th} bit set (value 1), if and only if the subset is represented by $n \in F$.

Example 2.

For example, for a base set of order 3, the family $F = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}\}$ is represented as 10001011(139) because $\emptyset \to 0, \{a\} \to 1, \{a, b\} \to 3, \{a, b, c\} \to 7$, and hence the zeroth, first, third and seventh bits are set.

A useful perspective on a convex geometry is provided by its *implications*. Given a convex geometry $(X, \mathcal{F})$, $A \to B$, where $A, B \in 2^X$, is an implication if and only if:

$$B \subseteq \varphi(A) = \bigcap \{Y : A \subseteq Y,\ Y \in F\} \tag{3}$$

where φ is the closure operator associated with $(X, \mathcal{F})$ as defined in Definition 2.1. A set of implications fully defining closure operator φ is often referred to as *implicational basis* of a closure system. The recent exposition on implicational bases is given in Adaricheva and J.B.Nation [4].
Because implications are very useful for representing geometries, they were generated for each geometry in addition to its convex sets. However, some implications are redundant with respect to others. For example, $ab \to c$ is redundant if $a \to c$. Hence work was also done in attempt to remove such redundancies.

In practice we only removed pairwise redundancies, but an *optimum* implication list could be found using a brute force approach (in general the problem of finding such an optimum implication list is NP-hard, see [5, 11]). The pairwise reduction was achieved by declaring implication $A \to B$ redundant if there exists an implication $C \to D$ such that $C \subseteq A$ and $B \subseteq D$.

To obtain a list of only convex geometries with convex dimension less than or equal to 5, as per Problem 1 in [1], calculation of each geometry's dimension was also necessary. As described in Section 2.2, one may compute convex dimension by starting with the meet-irreducible convex sets, which can be identified by the convex sets Y that have only one convex superset Z such that $|Z \setminus Y| = 1$. From there, one can build a maximal antichain of said irreducibles using recursive techniques. And by Theorem 2.2, the convex dimension is the cardinality of this maximal antichain.

4 Conquering the list

Once we had a complete list of geometries on 4- and 5-element sets, we began working on finding representations for them. The 34 4-element convex geometries helped us all gain some intuition and practice; some of us represented (or even came up without looking at the list) all 34 independently, while others represented the easier ones first and then worked together on harder representations. As the list was only 34 geometries, this work took no longer than a week. The work was posted and discussed on the social site Discord, and images were tagged with spoilers so that we all had a chance to try problems in order to practice our visual problem solving.

In contrast, the list of 672 5-element geometries needed much more time and organization. Each team member was initially assigned 10 geometries, and since it was already known that at least some convex geometries of dimension 6 could not be represented by circles and points [1], we were only assigned convex geometries of dimension less than 6 and left those of dimension 6 for later analysis. After a representation was made by one team member and before it was added to the final paper, it was posted to Discord where another team member would check it. We used a spreadsheet, originally organized by team member but later also organized by geometry number, to keep track of who was representing each geometry, who had checked it, and whether it had been added into the final file.

In the first week of working on the 5-set geometries, we realized some geometries resisted attempts at representation by circles and points. One Team member started the discussion by posting a geometry they could not represent, and other Team members also began asking each other to check difficult geometries.

At the next group meeting it was agreed that there seemed to be convex geometries of dimension less than 6 that could not be represented by circles and points, and a list was started to keep track of difficult convex geometries. Some Team members were able to represent the more difficult convex geometries, but some geometries stayed on the "Impossible Problems" list.

Some team members who had encountered such suspected impossible problems had several meetings where they examined these problems closely and began trying to articulate why they were not representable and formulate precise hypotheses. These efforts, and additional individual work, led to more formal conjectures and proofs of the impossibility of certain geometries.

Meanwhile, we continued working through the list by self-assigning geometries beyond our original assignment. Some students focused on one 5-element geometry at a time, while others worked on larger "chunks" by using various strategies to obtain 5-element geometry representations from representations of the 4-element convex geometries. As the list of represented geometries grew, so did the list of geometries we believed were impossible to represent.

Our final list of "Impossible" geometries came up with 51 cases. It is interesting to point out that majority of cases occurred within the first 100 geometries on the list of 672. The list of 672 is ordered by decreasing number of convex

sets in alignment, thus, geometries with more convex sets tend to have greater probability to appear on the list of "Impossible".

5 Representation of geometries on 5-element set based on representations on 4-element or 3-element set

In addition to building our intuition, creating representations for the 34 order 4 convex geometries served another purpose in that several "chunks" of representations of geometries on a 5-element base set could be generated from these 4-element representations simply by strategically adding another circle.

The first chunk of representations we generated were for those geometries with a unique atom (a, b, c, d, or e). To do this, we modified the representation on a 4-element base set so that all of the circles a, b, c, and d had a nontrivial intersection and then placed a point e in this intersection so that it would be in the convex hull of any other circle, adding the implications $a \to e$, $b \to e$, $c \to e$, and $d \to e$ to those of the previous 4-element geometry. This generated 34 previously unrepresented convex geometries on a base set of order 5 that were later identified, using a list of 5-element convex geometries with a unique atom generated by a coded search function, and added to the main collection of representations for 5-element geometries.

The next chunk of geometries we represented were those with a unique coatom ($abcd, abce, abde$, or $bcde$). To do this, we kept the representation of the 4-element convex geometry and simply added a fifth circle e that contained all of the other circles, adding the implication $e \to abcd$ to the previous list of implications for the 4-element geometry. This generated another 28 previously unrepresented geometries that were later identified and added to the master list in the same way as the first group.

Next we represented a chunk of geometries on a 5-element base set by essentially doubling a circle representing an element that was on the left side of all of its implications. To do this, we made a copy e of the circle as close to the original circle as possible, so as not to add any unwanted implications. This added copies of each implication that had previously involved the original circle but with e substituting for the element represented by the original circle in each implication. Sometimes only one circle could be duplicated in this way per geometry, but other times multiple circles could be doubled, allowing us to generate multiple 5-element representations from a single 4-element representation. Because there was not an easy way to code a search for the new geometries we had represented, it was necessary to identify them manually by scanning through the 5-element list.

The final chunk of order 5 representations we generated from those of order 4 were created by placing a fifth element e in one of the other four elements. This was inspired by an observation that after a certain point in our project, almost all of the remaining geometries included binary implications (of the form

$x \to y$). We tried to generate implications in as many different ways as we could by placing this fifth element within one of the other four while also avoiding duplicating a geometry in one of the chunks we had already generated. This allowed us to generate several order 5 representations from a single order 4 representation, although some were isomorphic to each other and others were isomorphic to existing representations. We identified these using an automated search program for geometries isomorphic to the one given.

An additional chunk of 5-element geometries could be represented based on representations on a base set of order 3, of which there are only 6. Each of these 6 representations could generate 2 unique 5-element representations, one by creating a non-trivial intersection between the 3 elements and then placing two new elements d and e in the middle, and one by keeping the 3-element representation the same and enclosing it in two large overlapping circles d and e. These newly generated representations were identified using a coded search function and added to the master list of representations for geometries on a base set of order 5.

6 Checking correctness of representations

Originally, the method for checking the correctness of representations required a tedious procedure of making sure that a circle representation satisfied all implications of its associated convex geometry, and no additional implications were forced by the configuration of any two, three, or four circles of the representation. Eventually we came up with a simplified procedure based on statements that would imply the isomorphism of two geometries.

We recall from section 3 that the closure operator φ can be recorded in the set Σ of implications, where $Y \to Z$ means $Z \subseteq \varphi(Y)$. Then the corresponding alignment of φ-closed sets comprises all subsets of X that *respect* all implications in Σ: if $(Y \to Z) \in \Sigma$ and $Y \subseteq W$ for set W in alignment, this would imply $Z \subseteq W$.

A discussion of the nature of our representations by circles is necessary here to further explain the reasoning behind our checking method. Convex geometries on our list were given by both an alignment $\mathcal{F}$ on set X and a set of implications Σ. In order to produce a representation by circles, each element of X was represented by a circle, and the positioning of circles in the plane would define a closure operator ch_c, the convex hull operator for circles. A geometry generated by these circles would be isomorphic to $(X, \mathcal{F})$ if the alignments match, i.e. if the alignment $\mathcal{G}$ generated by ch_c consists of the same subsets as $\mathcal{F}$. The following two statements formed the basis for such matching.

Proposition 6.1. *Let $(X, \mathcal{F})$ be an alignment defined by a set of implications Σ. Let ψ be any closure operator defined on X and the corresponding alignment of ψ-closed sets be $\mathcal{G}$. If for every $(Y \to Z) \in \Sigma$ one has $Z \subseteq \psi(Y)$, then $\mathcal{G} \subseteq \mathcal{F}$.*

Proof. Indeed, take any set $W \in \mathcal{G}$. We need to show that $W \in \mathcal{F}$, i.e., it respects all implications from Σ. So take $Y \to Z$ in Σ and let $Y \subseteq W$. Since

by assumption we have $Z \subseteq \psi(Y)$, and W is ψ-closed, we have $Z \subseteq W$ as desired. □

In the practical procedure of checking representation, we would check whether for any implication $Y \to Z$ from the set of implications defining the given convex geometry, all circles in Z are in the convex hull of circles from Y.

Proposition 6.2. *If all meet-irreducible sets from alignment $\mathcal{F}$ belong to $\mathcal{G}$, then $\mathcal{F} \subseteq \mathcal{G}$.*

Proof. Indeed, every set in $\mathcal{F}$ is the intersection of meet-irreducible sets. Therefore, they all belong to $\mathcal{G}$. □

In practice, we would check that all circles belonging to the meet-irreducible set of a given convex geometry form a convex set of circles. Combining the two practical procedures following from the propositions above, we could conclude that the given convex geometry is isomorphic to our convex geometry of circles.

7 GeoGebra as a toolkit for representations

At the beginning of this project, creating a representation for a geometry entailed drawing out the circles by hand, and then manually representing the drawing on Overleaf, the Latex processing website. One had to take care to preserve the convex hulls as precisely as possible and not generate unwanted implications.

This became especially cumbersome on certain geometries that were possible but required a very precarious setup, where even the slightest shift in the position of a circle would make the representation incorrect. A few weeks into the project, several team members highlighted the usefulness of GeoGebra as a tool for representing Geometries, which allowed for the easy rearrangement of circles and points to one's liking. Eventually a Team member came up with a template GeoGebra file [13] that allowed users to adjust not only the position and radii of the circles, but also to draw the convex hulls of all possible combinations of circles. This made it easier to check if a representation was actually correct or not.

Using GeoGebra helped in checking if a representation of a geometry was correct, but once represented, they still had to be remade in LaTeXusing the TikZ package. It was soon discovered that GeoGebra had a built in way to export images to TikZ code, but this code was improved upon by members of our team to generate the desired representation.

8 Google Colab toolkit

In order to streamline the process of exporting representations of geometries to Overleaf, a few team members worked to make a Python script to automatically generate TikZ code to paste into Overleaf. GeoGebra already had a feature that

allowed this, but the generated code had unnecessary elements that needed to be deleted, making the process less than optimal.

Midway through the project, a Team member wrote a Python script where the user would input the centers and radii of each circle, and the program would generate TikZ code that was readily available to be pasted into Overleaf. The centers and radii of the circles were shown in GeoGebra, so the user only had to copy those numbers into the script.

Simultaneously, another Team member had created a script in a Google Colab notebook that would parse through the TikZ code generated by GeoGebra, extract all of the necessary information (such as circle location and radii) and then output useable TikZ code. This new code, however, had issues with normalizing the size of the circles to fit nicely on the page, and so features of both scripts were combined in the Google Colab notebook. The end result was code that would accept a text file from GeoGebra and reliably output useable TikZ code for the Overleaf [14].

9 Triangle Property and Impossible geometries

In this section we will be proving that several geometries on a 5 element set are not representable by circles on the plane.

9.1 Tight implication on circles

The following statement is a modification of Lemma 5.1 in [1].

Lemma 1. *Let a, b, c be three circles on the plane. Then there are four different types of configurations of three circles a, b, c:*

(1) $x \in \mathrm{ch}_c(y, z)$, $\{x, y, z\} = \{a, b, c\}$;

(2) $\mathrm{ch}_c(x, y, z) = \mathrm{ch}_c(x, y) \cup \mathrm{ch}_c(y, z)$ *as in Figure 4 with* $a = x$, $y = b$, $z = c$ *or in Figure 5 with* $b = x$, $c = y$, $a = z$.

(3) $\mathrm{CH}(\tilde{a} \cup \tilde{b} \cup \tilde{c})$ *is inscribed into a triangle as in Figure 6;*

(4) $\mathrm{CH}(\tilde{a} \cup \tilde{b} \cup \tilde{c})$ *forms a shape as in Figure 7.*

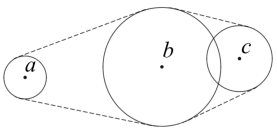


Figure 4

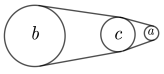


Figure 5

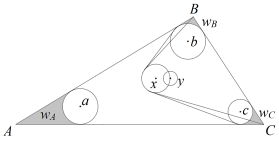


Figure 6

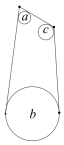


Figure 7

Proof. The argument can be corroborated by Lemma 4 that describes all possible borders of the convex hull of three circles.

Case (1) is distinguished from others by the property $\mathrm{ch}_c(x, y, z) = \mathrm{ch}_c(y, z)$.

In case (2) one of the circles crosses both outer tangent lines of two other circles and has two points of intersection with at least one of those outer tangents.

Case (4) is similar to (3), but it is distinguished by having an arc larger than π from one of the circles as a part of the border of $\mathrm{CH}(\tilde{a} \cup \tilde{b} \cup \tilde{c})$. □

Definition 9.1. *An implication $Y \to u$, $Y \subseteq X, u \in X$, is called* tight*, if implications $(Y \setminus z) \to u$ do not hold, for all $z \in Y$.*

Lemma 2. *For an implication $Y \to u$, $Y \subseteq X, u \in X$, if there exists $y \in Y$ such that $(Y \setminus y) \to y$, then implication $Y \to u$ cannot be tight.*

Proof. Let φ be the closure operator. If there exists $y \in Y$ such that $(Y \setminus y) \to y$, then $y \in \varphi((Y \setminus y))$. Also since for any A, $A \subseteq \varphi(A)$, we have $Y \subseteq \varphi((Y \setminus y))$. Now applying the closure operator to both sides, $\varphi(Y) \subseteq \varphi((Y \setminus y))$, as $\varphi(\varphi(A)) = \varphi(A)$. Since $Y \to u$, $u \in \varphi(Y)$ and $u \in \varphi((Y \setminus y))$. Thus $(Y \setminus y) \to u$, and hence $Y \to u$ cannot be tight. □

Lemma 3. *If $abc \to d$ is any tight implication that holds for circles a, b, c, d, then a, b, c cannot form a configuration in (1) or (2) of Lemma 1, for any assignment of x, y, z to a, b, c.*

Proof. Indeed, if a, b, c form a configuration in (1), then the implication $abc \to d$ cannot be tight due to Lemma 2. For configuration (2) where $\mathrm{CH}(\tilde{x} \cup \tilde{y} \cup \tilde{z}) = \mathrm{CH}(\tilde{x} \cup \tilde{y}) \cup \mathrm{CH}(\tilde{y} \cup \tilde{z})$, we note that for any circle d in $\mathrm{ch}_c(x, y, z)$ we have either $d \in \mathrm{ch}_c(x, y)$ or $d \in \mathrm{ch}_c(y, z)$. Note that the same may not be true for a shape d different from circle. □

Lemma 4. *The border of $\mathrm{CH}(\tilde{x} \cup \tilde{y} \cup \tilde{z})$ for any three circles a, b, c comprises*

(I) two tangent segments and two arcs of circles, when configuration (1) from Lemma 1 occurs;

(II) four tangent segments and four arcs, as in Figure 4, for configurations in (2) from Lemma 1;

(III) three tangent segments and three arcs, as in Figure 6 and Figure 7, for configurations in (3) and (4) from Lemma 1.

Corollary 9.1. *If $abc \to e$ is any tight implication that holds for circles a, b, c, e, then the border of $\mathrm{CH}(\tilde{a} \cup \tilde{b} \cup \tilde{c})$ comprises three tangent segments and three arcs.*

9.2 Triangle Property

Given that the statement of Lemma 5 below is rather straightforward and its proof is elementary, we assume it is a part of mathematical folklore.

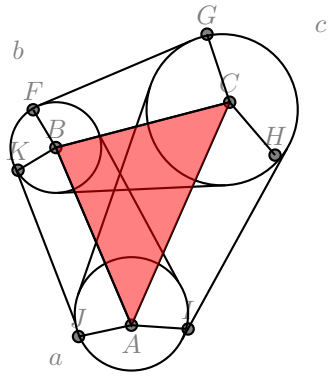


Figure 8: Triangle Property

Lemma 5 (The Triangle Property)**.** *If a, b, c, and e are circles in the plane with centers A, B, C, and E respectively, and $abc \to e$ is a tight implication, then E must lie in the interior of triangle formed by A,B and C.*

Proof. If implication $abc \to e$ is tight, then by Lemma 3 circles a, b, c cannot form any configuration from (1) and (2) of Lemma 1.

Moreover, since $abc \to e$ is tight, the centers of a, b, c cannot be on a line, because, in that case one obtains one of configurations (1) or (2) in Lemma 1. Therefore, the centers form a triangle.

By Corollary 9.1, the border of $\mathrm{CH}(\tilde{a} \cup \tilde{b} \cup \tilde{c})$ comprises three tangent segments and three arcs, as in Figure 8. The convex hull of three circles a, b, c splits into two parts:

(I) the interior $int(\triangle ABC)$ of triangle formed by the centers of the circles;

(II) the shape $S = \mathrm{CH}(\tilde{a} \cup \tilde{b} \cup \tilde{c}) \setminus int(\triangle ABC)$.

Moreover, S splits into the union of the following shapes:

(I) trapezoids formed between pairs of circles, each of which includes a segment connecting the centers of the two circles, two radii, and the circles' common tangent segment;

(II) sectors of circles bordered by the radii which are the borders of trapezoids from (I).

If circle e is inside of the convex hull of a, b, c, and its center E is not in the interior of $\triangle ABC$, then it is in S. Thus it is either in one of the trapezoids or in one of the sectors.

Without loss of generality, suppose that E is inside of one of sectors of circle a. Then the distance σ from E to the border of circle is $\leq r_a$. Therefore, $r_e \leq r_a$ and $e \in \mathrm{ch}_c(a)$.

Suppose now that E is in one of the trapezoids. Without loss of generality, suppose that E is in $BCGF$ in Figure 8, where FG is the part of the border of $\mathrm{CH}(\tilde{a} \cup \tilde{b} \cup \tilde{c})$, and σ is the distance from E to segment FG. Then $r_e \leq \sigma$ and $e \in \mathrm{ch}_c(b, c)$ thusly.

Therefore, the center of e for which $abc \to e$ is tight cannot be in either the trapezoids or sectors, and thus it must be in interior of $\triangle ABC$. □

The following statement is a generalization of the Triangle Property.

Corollary 9.2. *If $n > 2$, $a_1, a_2, a_3, \cdots, a_n, d$ are circles in the plane with centers $C_1, C_2, \cdots, C_n, D$ respectively, and $a_1 a_2 a_3 \cdots a_n \to d$ is tight then $D \in \mathrm{CH}(C_1, C_2, \cdots, C_n)$.*

Proof. Let $S = \mathrm{CH}(\tilde{a_1} \cup \tilde{a_2} \cup \cdot \cup \tilde{a_n})$ and let $S' = \mathrm{CH}(C_1, C_2, \cdots, C_n)$. Obviously, D, as well as every other point in $\mathrm{CH}(d)$, must be contained wholly within S in order for the implication $a_1 a_2 a_3 \cdots a_n \to d$ to hold. Since $n > 2$, the centers of the circles cannot all be colinear as this would force every subset of three circles into configuration (1) or (2) as in Lemma 1.

The boundary of S consists of a cyclic sequence of circles connected by mutual tangents. If a_i and a_j, for $i, j \in \{1, 2, \cdots, n\}$, are consecutive circles along the boundary of S then let ℓ be the line connecting C_i to C_j. The line ℓ divides the plane into two half-planes, we define $H_{i,j}$ to be the outward facing half-plane, that is, it is the half plane that does not contain S'. The set of points in $S \cap H_{i,j}$ can be split as the union of three regions,

(I) A trapezoidal region which includes a segment connecting C_i to C_j, the mutual tangent segment connecting a_i to a_j on the boundary of S, and two radii of a_i and a_j connecting to endpoints of the mutual tangent.

(II) The sectors of the circles a_i and a_j in $H_{i,j}$ but outside of the trapezoid described in (I).

If $D \in S \setminus S'$, then D must be in one of the half-planes associated to a pair of consecutive circles on the boundary of S. Suppose that $D \in H_{i,j}$.

If D is inside one of the sectors of circle a_i (without loss of generality), let δ be the distance between the circumference of d and the point x on the circumference of a_i which is closest to d. If the radius of d is less than or equal to δ then $d \subseteq \mathrm{CH}(c)$, which violates the assumption that $a_1 a_2 a_3 \cdots a_n \to d$ is a tight implication, and thus the the radius of d must be greater than δ. A ray, r, centered at C_i which passes through D will intersect a_i at the point x which is closest to D. Since C_i is on ℓ and D is in $H_{i,j}$, every point on r is in $H_{i,j}$, furthermore, since D is not inside the trapezoid described in (I), every

point on on the ray r beyond x must be outside of S. Hence, if the radius of d is greater than δ then $\mathrm{CH}(\tilde{d})$ must contain some points outside of S implying $a_1a_2a_3\cdots a_n \not\to d$.

Now, suppose that D is inside the trapezoid described in (I). Let s_1 be the mutual outer tangent between a_i and a_j in $H_{i,j}$, let s_2 be the inner tangent, outside of $H_{i,j}$, and let δ be the distance between D and s_1. Because the two circles, a_i and a_j are symmetric about the line ℓ, the distance between ℓ and s_1 is equal to the distance between ℓ and s_2. Since $D \in H_{i,j}$, it follows that δ is less than or equal to the distance between D and s_2. If the radius of d is less than δ then d is contained entirely between s_1 and s_2 implying that $d \subseteq \mathrm{CH}(\tilde{a_i} \cup \tilde{a_j})$, which violates the assumption that the implication $a_1a_2a_3\cdots a_n \to d$ is tight. Alternatively, if the radius of d exceeds δ then, since s_1 is a boundary of S, $\mathrm{CH}(d)$ will contain at least one point not in S, implying $a_1a_2a_3\cdots a_n \not\to d$. □

Corollary 9.3. *Convex geometries representable by circles cannot have three tight implications $abc \to e$, $abd \to e$, $acd \to e$.*

Proof. Indeed, consider A, B, C, D, E, the centers of circles that might represent this geometry. Connect A with B, C and D. Then the plane splits into three angles at vertex A. Choose two of smallest angles among these three. Without loss of generality, assume that the smallest are $\angle CAB$ and $\angle BAD$. Then each of the two smallest angles are less than π, thus, line (AB) splits the plane into two semi-planes, so that C and D are in distinct semi-planes (see illustration in Figure 9).

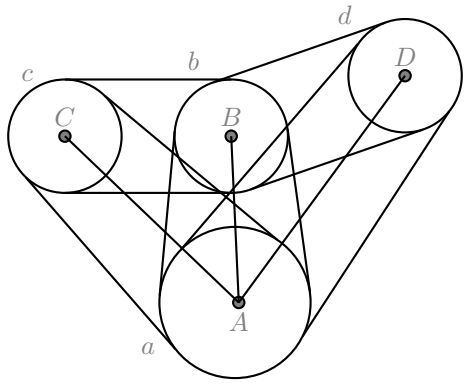


Figure 9: Corollary 9.3

By Lemma 5, E must be in the interior of $\triangle ABC$ as well as in the interior of $\triangle ABD$, and since those interiors do not intersect, there is no possible location for E. □

This gives us six geometries on the list that are not representable: G7, G11, G12, G18, G21 and G34. Note that G7, G21 and G34 have cdim=4.

Corollary 9.4. *Convex geometries representable by circles cannot have three tight implications $abc \to d$, $acd \to e$, $bcd \to e$.*

Proof. Assume that the geometry does have a representation as a geometry of circles. The proof then follows from the use of Lemma 5 two times. First, since $abc \to d$ is a tight implication, D must lie in the interior of triangle formed by vertices A, B and C (which are the centers of a, b, c). Moreover, we have two tight implications $acd \to e$, $bcd \to e$. However, by Lemma 5 again, we would need E to be in the interior of both $\triangle ACD$ and $\triangle BCD$, which is not possible. □

This gives us that the additional geometry G23 cannot be represented by circles on the plane.

10 Acknowledgments.

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[14] Google Colab notebook `https://colab.research.google.com/drive/1wocYWp8zzt_qNe5vvPyR-6ho0i0dlWOn?usp=sharing#scrollTo=OCIxsoO49TWn`

Appendices

A

This appendix lists the output of the process described in §3, i.e., it lists every non-isomorphic convex geometry of sizes 4 and 5. The id tag represents the corresponding convex geometry's 32-bit integer representation as also described in §3.

A.1 Description of size 4 geometries

```
1.
id: 65535
meet-irreducibles: abc abd acd bcd
max antichain: abc abd acd bcd
dimension: 4

2.
id: 65407
abc -> d
meet-irreducibles: ab ac bc abd acd bcd
max antichain: ab ac bc
dimension: 3

3.
id: 65399
ab -> d
meet-irreducibles: ac bc abd acd bcd
max antichain: ac bc abd
dimension: 3

4.
id: 65367
ac -> d
ab -> d
meet-irreducibles: a bc abd acd bcd
max antichain: bc abd acd
dimension: 3

5.
id: 63351
ab -> cd
meet-irreducibles: ac bc ad bd acd bcd
max antichain: ac bc ad bd
dimension: 4
```

```
6.
id: 65365
a -> d
meet-irreducibles: bc abd acd bcd
max antichain: bc abd acd
dimension: 3

7.
id: 65303
bc -> d
ac -> d
ab -> d
meet-irreducibles: a b c abd acd bcd
max antichain: a b c
dimension: 3

8.
id: 63319
ac -> d
ab -> cd
meet-irreducibles: a bc ad bd acd bcd
max antichain: a bc bd
dimension: 3

9.
id: 65301
bc -> d
a -> d
meet-irreducibles: b c abd acd bcd
max antichain: abd acd bcd
dimension: 3

10.
id: 63317
ab -> cd
a -> d
meet-irreducibles: bc ad bd acd bcd
max antichain: bc ad bd
dimension: 3

11.
id: 63255
bc -> d
ac -> d
ab -> cd
```

```
meet-irreducibles: a b c ad bd acd bcd
max antichain: a b c
dimension: 3

12.
id: 62775
ad -> c
bc -> d
ab -> cd
meet-irreducibles: a b ac bd acd bcd
max antichain: a b
dimension: 2

13.
id: 65297
b -> d
a -> d
meet-irreducibles: c abd acd bcd
max antichain: abd acd bcd
dimension: 3

14.
id: 63253
bc -> d
ab -> cd
a -> d
meet-irreducibles: b c ad bd acd bcd
max antichain: b c ad
dimension: 3

15.
id: 62805
a -> cd
meet-irreducibles: bc bd acd bcd
max antichain: bc bd acd
dimension: 3

16.
id: 62773
bc -> d
ab -> cd
a -> c
meet-irreducibles: b ac bd acd bcd
max antichain: b ac
dimension: 2
```

```
17.
id: 63239
c -> d
ab -> cd
meet-irreducibles: a b ad bd acd bcd
max antichain: a b
dimension: 2

18.
id: 65281
c -> d
b -> d
a -> d
meet-irreducibles: abd acd bcd
max antichain: abd acd bcd
dimension: 3

19.
id: 63249
ab -> cd
b -> d
a -> d
meet-irreducibles: c ad bd acd bcd
max antichain: c ad bd
dimension: 3

20.
id: 62769
ab -> cd
b -> d
a -> c
meet-irreducibles: ac bd acd bcd
max antichain: ac bd
dimension: 2

21.
id: 63237
c -> d
ab -> cd
a -> d
meet-irreducibles: b ad bd acd bcd
max antichain: b ad
dimension: 2

22.
id: 62741
```

```
bc -> d
a -> cd
meet-irreducibles: b c bd acd bcd
max antichain: b c
dimension: 2

23.
id: 54613
a -> bcd
meet-irreducibles: bc bd cd bcd
max antichain: bc bd cd
dimension: 3

24.
id: 63233
c -> d
ab -> cd
b -> d
a -> d
meet-irreducibles: ad bd acd bcd
max antichain: ad bd
dimension: 2

25.
id: 62737
b -> d
a -> cd
meet-irreducibles: c bd acd bcd
max antichain: c bd
dimension: 2

26.
id: 62725
c -> d
a -> cd
meet-irreducibles: b bd acd bcd
max antichain: b acd
dimension: 2

27.
id: 54549
bc -> d
a -> bcd
meet-irreducibles: b c bd cd bcd
max antichain: b c
dimension: 2
```

```
28.
id: 62721
c -> d
b -> d
a -> cd
meet-irreducibles: bd acd bcd
max antichain: bd acd
dimension: 2

29.
id: 61713
b -> cd
a -> cd
meet-irreducibles: c d acd bcd
max antichain: c d
dimension: 2

30.
id: 54545
b -> d
a -> bcd
meet-irreducibles: c bd cd bcd
max antichain: c bd
dimension: 2

31.
id: 61697
c -> d
b -> cd
a -> cd
meet-irreducibles: d acd bcd
max antichain: acd bcd
dimension: 2

32.
id: 54529
c -> d
b -> d
a -> bcd
meet-irreducibles: bd cd bcd
max antichain: bd cd
dimension: 2

33.
id: 53521
```

```
b -> cd
a -> bcd
meet-irreducibles: c d cd bcd
max antichain: c d
dimension: 2

34.
id: 53505
c -> d
b -> cd
a -> bcd
meet-irreducibles: d cd bcd
max antichain: d
dimension: 1
```

A.2 Description of size 5 geometries

```
1.
id: 4294967295
meet-irreducibles: abcd abce abde acde bcde
max antichain: abcd abce abde acde bcde
dimension: 5

2.
id: 4294934527
abcd -> e
meet-irreducibles: abc abd acd bcd abce abde acde bcde
max antichain: abc abd acd bcd
dimension: 4

3.
id: 4294934399
abc -> e
meet-irreducibles: abd acd bcd abce abde acde bcde
max antichain: abd acd bcd abce
dimension: 4

4.
id: 4286545791
abc -> de
meet-irreducibles: abd acd bcd abe ace bce abde acde bcde
max antichain: abd acd bcd abe ace bce
dimension: 6
```

Appendix A.2: Description of size 5 geometries

```
5.
id: 4294932351
abd -> e
abc -> e
meet-irreducibles: ab acd bcd abce abde acde bcde
max antichain: acd bcd abce abde
dimension: 4

6.
id: 4294932343
ab -> e
meet-irreducibles: acd bcd abce abde acde bcde
max antichain: acd bcd abce abde
dimension: 4

7.
id: 4294924159
acd -> e
abd -> e
abc -> e
meet-irreducibles: ab ac ad bcd abce abde acde bcde
max antichain: ab ac ad bcd
dimension: 4

8.
id: 4286543743
abd -> e
abc -> de
meet-irreducibles: ab acd bcd abe ace bce abde acde bcde
max antichain: ab acd bcd ace bce
dimension: 5

9.
id: 4294924151
acd -> e
ab -> e
meet-irreducibles: ac ad bcd abce abde acde bcde
max antichain: bcd abce abde acde
dimension: 4

10.
id: 4286543735
abc -> de
ab -> e
meet-irreducibles: acd bcd abe ace bce abde acde bcde
max antichain: acd bcd abe ace bce
```

```
dimension: 5

11.
id: 4294907775
bcd -> e
acd -> e
abd -> e
abc -> e
meet-irreducibles: ab ac bc ad bd cd abce abde acde bcde
max antichain: ab ac bc ad bd cd
dimension: 6

12.
id: 4286535551
acd -> e
abd -> e
abc -> de
meet-irreducibles: ab ac ad bcd abe ace bce abde acde bcde
max antichain: ab ac ad bcd bce
dimension: 5

13.
id: 4286013311
abe -> d
acd -> e
abc -> de
meet-irreducibles: ab ac abd bcd ace bce abde acde bcde
max antichain: ab ac bcd bce
dimension: 4

14.
id: 4294907767
bcd -> e
acd -> e
ab -> e
meet-irreducibles: ac bc ad bd cd abce abde acde bcde
max antichain: ac bc ad bd cd
dimension: 5

15.
id: 4286535543
acd -> e
abc -> de
ab -> e
meet-irreducibles: ac ad bcd abe ace bce abde acde bcde
max antichain: ac ad bcd abe bce
```

```
dimension: 5

16.
id: 4286535039
ad -> e
abc -> de
meet-irreducibles: ab ac bcd abe ace bce abde acde bcde
max antichain: ab ac bcd bce
dimension: 4

17.
id: 4286019447
ab -> de
meet-irreducibles: acd bcd ace bce abde acde bcde
max antichain: acd bcd ace bce abde
dimension: 5

18.
id: 4286519167
bcd -> e
acd -> e
abd -> e
abc -> de
meet-irreducibles: ab ac bc ad bd cd abe ace bce abde acde bcde
max antichain: ab ac bc ad bd cd
dimension: 6

19.
id: 4294924119
ac -> e
ab -> e
meet-irreducibles: ad bcd abce abde acde bcde
max antichain: bcd abce abde acde
dimension: 4

20.
id: 4286013303
acd -> e
abc -> de
ab -> d
meet-irreducibles: ac abd bcd ace bce abde acde bcde
max antichain: ac abd bcd bce
dimension: 4

21.
id: 4285996927
```

```
abe -> d
bcd -> e
acd -> e
abc -> de
meet-irreducibles: ab ac bc abd cd ace bce abde acde bcde
max antichain: ab ac bc cd
dimension: 4

22.
id: 4286535031
ad -> e
abc -> de
ab -> e
meet-irreducibles: ac bcd abe ace bce abde acde bcde
max antichain: ac bcd abe bce
dimension: 4

23.
id: 4286519159
bcd -> e
acd -> e
abc -> de
ab -> e
meet-irreducibles: ac bc ad bd cd abe ace bce abde acde bcde
max antichain: ac bc ad bd cd abe
dimension: 6

24.
id: 4286013271
abc -> de
ac -> e
ab -> d
meet-irreducibles: abd bcd ace bce abde acde bcde
max antichain: abd bcd ace bce
dimension: 4

25.
id: 4286011255
acd -> e
ab -> de
meet-irreducibles: ac ad bcd ace bce abde acde bcde
max antichain: ac ad bcd bce
dimension: 4

26.
id: 4286518655
```

```
bcd -> e
ad -> e
abc -> de
meet-irreducibles: ab ac bc bd cd abe ace bce abde acde bcde
max antichain: ab ac bc bd cd
dimension: 5

27.
id: 4285996895
abe -> d
bcd -> e
abc -> de
ac -> e
meet-irreducibles: ab bc abd cd ace bce abde acde bcde
max antichain: ab bc cd ace
dimension: 4

28.
id: 4151801719
ab -> cde
meet-irreducibles: acd bcd ace bce ade bde acde bcde
max antichain: acd bcd ace bce ade bde
dimension: 6

29.
id: 4294907199
ad -> e
bc -> e
meet-irreducibles: ab ac bd cd abce abde acde bcde
max antichain: ab ac bd cd
dimension: 4

30.
id: 4294923607
ad -> e
ac -> e
ab -> e
meet-irreducibles: a bcd abce abde acde bcde
max antichain: bcd abce abde acde
dimension: 4

31.
id: 4294907735
bcd -> e
ac -> e
ab -> e
```

```
meet-irreducibles: bc ad bd cd abce abde acde bcde
max antichain: bc ad bd cd
dimension: 4

32.
id: 4285992831
abe -> d
cd -> e
abc -> de
meet-irreducibles: ab ac bc abd ace bce abde acde bcde
max antichain: ab ac bc
dimension: 3

33.
id: 4286535511
abc -> de
ac -> e
ab -> e
meet-irreducibles: ad bcd abe ace bce abde acde bcde
max antichain: ad bcd abe ace bce
dimension: 5

34.
id: 4285996919
bcd -> e
acd -> e
abc -> de
ab -> d
meet-irreducibles: ac bc abd cd ace bce abde acde bcde
max antichain: ac bc abd cd
dimension: 4

35.
id: 4286518647
bcd -> e
ad -> e
abc -> de
ab -> e
meet-irreducibles: ac bc bd cd abe ace bce abde acde bcde
max antichain: ac bc bd cd abe
dimension: 5

36.
id: 4285992799
abe -> d
cd -> e
```

```
abc -> de
ac -> e
meet-irreducibles: ab bc abd ace bce abde acde bcde
max antichain: ab bc ace
dimension: 3

37.
id: 4286010743
ad -> e
ab -> de
meet-irreducibles: ac bcd ace bce abde acde bcde
max antichain: ac bcd bce abde
dimension: 4

38.
id: 4285996887
bcd -> e
abc -> de
ac -> e
ab -> d
meet-irreducibles: bc abd cd ace bce abde acde bcde
max antichain: bc abd cd ace
dimension: 4

39.
id: 4285994871
bcd -> e
acd -> e
ab -> de
meet-irreducibles: ac bc ad bd cd ace bce abde acde bcde
max antichain: ac bc ad bd cd
dimension: 5

40.
id: 4151793527
acd -> e
ab -> cde
meet-irreducibles: ac ad bcd ace bce ade bde acde bcde
max antichain: ac ad bcd bce bde
dimension: 5

41.
id: 4286517631
bd -> e
ad -> e
abc -> de
```

```
meet-irreducibles: ab ac bc cd abe ace bce abde acde bcde
max antichain: ab ac bc cd
dimension: 4

42.
id: 4294907191
ad -> e
bc -> e
ab -> e
meet-irreducibles: ac bd cd abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

43.
id: 4283905911
ace -> d
bcd -> e
ab -> de
meet-irreducibles: ac bc bd acd ae bce abde acde bcde
max antichain: ac bc bd ae
dimension: 4

44.
id: 4285992823
cd -> e
abc -> de
ab -> d
meet-irreducibles: ac bc abd ace bce abde acde bcde
max antichain: ac bc abd
dimension: 3

45.
id: 4286518591
ad -> e
abc -> de
bc -> e
meet-irreducibles: ab ac bd cd abe ace bce abde acde bcde
max antichain: ab ac bd cd bce
dimension: 5

46.
id: 4294907223
bcd -> e
ad -> e
ac -> e
ab -> e
```

```
meet-irreducibles: a bc bd cd abce abde acde bcde
max antichain: a bc bd cd
dimension: 4

47.
id: 4285996831
abe -> d
abc -> de
bc -> e
ac -> e
meet-irreducibles: ab abd cd ace bce abde acde bcde
max antichain: ab cd ace bce
dimension: 4

48.
id: 4286534999
ad -> e
abc -> de
ac -> e
ab -> e
meet-irreducibles: a bcd abe ace bce abde acde bcde
max antichain: bcd abe ace bce
dimension: 4

49.
id: 4286519127
bcd -> e
abc -> de
ac -> e
ab -> e
meet-irreducibles: bc ad bd cd abe ace bce abde acde bcde
max antichain: bc ad bd cd abe ace
dimension: 6

50.
id: 4286011223
ac -> e
ab -> de
meet-irreducibles: ad bcd ace bce abde acde bcde
max antichain: ad bcd ace bce
dimension: 4

51.
id: 4294923605
a -> e
meet-irreducibles: bcd abce abde acde bcde
```

```
max antichain: bcd abce abde acde
dimension: 4

52.
id: 4294907671
bc -> e
ac -> e
ab -> e
meet-irreducibles: ad bd cd abce abde acde bcde
max antichain: ad bd cd abce
dimension: 4

53.
id: 4285994359
bcd -> e
ad -> e
ab -> de
meet-irreducibles: ac bc bd cd ace bce abde acde bcde
max antichain: ac bc bd cd
dimension: 4

54.
id: 4151777143
bcd -> e
acd -> e
ab -> cde
meet-irreducibles: ac bc ad bd cd ace bce ade bde acde bcde
max antichain: ac bc ad bd cd
dimension: 5

55.
id: 4283904887
ace -> d
bd -> e
ab -> de
meet-irreducibles: ac bc acd ae bce abde acde bcde
max antichain: ac bc ae
dimension: 3

56.
id: 4286518583
ad -> e
abc -> de
bc -> e
ab -> e
meet-irreducibles: ac bd cd abe ace bce abde acde bcde
```

```
max antichain: ac bd cd abe bce
dimension: 5

57.
id: 4149688183
ace -> d
bcd -> e
ab -> cde
meet-irreducibles: ac bc bd acd ae bce ade bde acde bcde
max antichain: ac bc bd ae
dimension: 4

58.
id: 4285992735
abe -> d
cd -> e
abc -> de
bc -> e
ac -> e
meet-irreducibles: ab c abd ace bce abde acde bcde
max antichain: ab ace bce
dimension: 3

59.
id: 4285996823
abc -> de
bc -> e
ac -> e
ab -> d
meet-irreducibles: abd cd ace bce abde acde bcde
max antichain: abd cd ace bce
dimension: 4

60.
id: 4286518615
bcd -> e
ad -> e
abc -> de
ac -> e
ab -> e
meet-irreducibles: a bc bd cd abe ace bce abde acde bcde
max antichain: bc bd cd abe ace
dimension: 5

61.
id: 4286010711
```

```
ad -> e
ac -> e
ab -> de
meet-irreducibles: a bcd ace bce abde acde bcde
max antichain: bcd ace bce abde
dimension: 4

62.
id: 4285994839
bcd -> e
ac -> e
ab -> de
meet-irreducibles: bc ad bd cd ace bce abde acde bcde
max antichain: bc ad bd cd ace
dimension: 5

63.
id: 4294907221
bcd -> e
a -> e
meet-irreducibles: bc bd cd abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

64.
id: 4283914071
ac -> de
ab -> de
meet-irreducibles: ad bcd ae bce abde acde bcde
max antichain: ad bcd ae bce
dimension: 4

65.
id: 4151793495
ac -> e
ab -> cde
meet-irreducibles: ad bcd ace bce ade bde acde bcde
max antichain: ad bcd ace bce bde
dimension: 5

66.
id: 4286517599
bd -> e
ad -> e
abc -> de
ac -> e
```

```
meet-irreducibles: ab bc cd abe ace bce abde acde bcde
max antichain: ab bc cd ace
dimension: 4

67.
id: 4286534997
abc -> de
a -> e
meet-irreducibles: bcd abe ace bce abde acde bcde
max antichain: bcd abe ace bce
dimension: 4

68.
id: 4294907159
ad -> e
bc -> e
ac -> e
ab -> e
meet-irreducibles: a bd cd abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

69.
id: 4283905879
bcd -> e
ac -> d
ab -> de
meet-irreducibles: bc bd acd ae bce abde acde bcde
max antichain: bc bd acd ae
dimension: 4

70.
id: 4286519063
abc -> de
bc -> e
ac -> e
ab -> e
meet-irreducibles: ad bd cd abe ace bce abde acde bcde
max antichain: ad bd cd abe ace bce
dimension: 6

71.
id: 4285992791
cd -> e
abc -> de
ac -> e
```

```
ab -> d
meet-irreducibles: bc abd ace bce abde acde bcde
max antichain: bc abd ace
dimension: 3

72.
id: 4285990775
cd -> e
ab -> de
meet-irreducibles: ac bc ad bd ace bce abde acde bcde
max antichain: ac bc ad bd
dimension: 4

73.
id: 4294906143
bd -> e
ad -> e
bc -> e
ac -> e
meet-irreducibles: ab cd abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

74.
id: 4286513535
cd -> e
bd -> e
ad -> e
abc -> de
meet-irreducibles: ab ac bc d abe ace bce abde acde bcde
max antichain: ab ac bc d
dimension: 4

75.
id: 4286517623
bd -> e
ad -> e
abc -> de
ab -> e
meet-irreducibles: ac bc cd abe ace bce abde acde bcde
max antichain: ac bc cd abe
dimension: 4

76.
id: 4285992727
cd -> e
```

```
abc -> de
bc -> e
ac -> e
ab -> d
meet-irreducibles: c abd ace bce abde acde bcde
max antichain: abd ace bce
dimension: 3

77.
id: 4285992719
abe -> d
abc -> de
c -> e
meet-irreducibles: ab abd ace bce abde acde bcde
max antichain: ab ace bce
dimension: 3

78.
id: 4283773815
ae -> d
bd -> e
ab -> de
meet-irreducibles: ac bc acd bce abde acde bcde
max antichain: ac bc abde
dimension: 3

79.
id: 4149687159
ace -> d
bd -> e
ab -> cde
meet-irreducibles: ac bc acd ae bce ade bde acde bcde
max antichain: ac bc ae bde
dimension: 4

80.
id: 4285994295
ad -> e
bc -> e
ab -> de
meet-irreducibles: ac bd cd ace bce abde acde bcde
max antichain: ac bd cd bce
dimension: 4

81.
id: 4285990743
```

```
cd -> e
ac -> e
ab -> de
meet-irreducibles: bc ad bd ace bce abde acde bcde
max antichain: bc ad bd ace
dimension: 4

82.
id: 4286517591
bd -> e
ad -> e
abc -> de
ac -> e
ab -> e
meet-irreducibles: a bc cd abe ace bce abde acde bcde
max antichain: bc cd abe ace
dimension: 4

83.
id: 4283904823
ace -> d
bd -> e
bc -> e
ab -> de
meet-irreducibles: b ac acd ae bce abde acde bcde
max antichain: b ac ae
dimension: 3

84.
id: 4285994327
bcd -> e
ad -> e
ac -> e
ab -> de
meet-irreducibles: a bc bd cd ace bce abde acde bcde
max antichain: a bc bd cd
dimension: 4

85.
id: 4283913559
ad -> e
ac -> de
ab -> de
meet-irreducibles: a bcd ae bce abde acde bcde
max antichain: bcd bce abde acde
dimension: 4
```

```
86.
id: 4151792983
ad -> e
ac -> e
ab -> cde
meet-irreducibles: a bcd ace bce ade bde acde bcde
max antichain: bcd ace bce ade bde
dimension: 5

87.
id: 4283897687
bcd -> e
ac -> de
ab -> de
meet-irreducibles: bc ad bd cd ae bce abde acde bcde
max antichain: bc ad bd cd ae
dimension: 5

88.
id: 4151777111
bcd -> e
ac -> e
ab -> cde
meet-irreducibles: bc ad bd cd ace bce ade bde acde bcde
max antichain: bc ad bd cd ace
dimension: 5

89.
id: 4286518613
bcd -> e
abc -> de
a -> e
meet-irreducibles: bc bd cd abe ace bce abde acde bcde
max antichain: bc bd cd abe ace
dimension: 5

90.
id: 4149696343
ac -> de
ab -> cde
meet-irreducibles: ad bcd ae bce ade bde acde bcde
max antichain: ad bcd ae bce bde
dimension: 5

91.
```

```
id: 4294906135
bd -> e
ad -> e
bc -> e
ac -> e
ab -> e
meet-irreducibles: a b cd abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

92.
id: 4283904855
bd -> e
ac -> d
ab -> de
meet-irreducibles: bc acd ae bce abde acde bcde
max antichain: bc acd ae
dimension: 3

93.
id: 4286010709
ab -> de
a -> e
meet-irreducibles: bcd ace bce abde acde bcde
max antichain: bcd ace bce abde
dimension: 4

94.
id: 4286518551
ad -> e
abc -> de
bc -> e
ac -> e
ab -> e
meet-irreducibles: a bd cd abe ace bce abde acde bcde
max antichain: bd cd abe ace bce
dimension: 5

95.
id: 4149688151
bcd -> e
ac -> d
ab -> cde
meet-irreducibles: bc bd acd ae bce ade bde acde bcde
max antichain: bc bd acd ae
dimension: 4
```

```
96.
id: 4286513527
cd -> e
bd -> e
ad -> e
abc -> de
ab -> e
meet-irreducibles: ac bc d abe ace bce abde acde bcde
max antichain: ac bc d abe
dimension: 4

97.
id: 4286513279
d -> e
abc -> de
meet-irreducibles: ab ac bc abe ace bce abde acde bcde
max antichain: ab ac bc
dimension: 3

98.
id: 4285990263
cd -> e
ad -> e
ab -> de
meet-irreducibles: ac bc bd ace bce abde acde bcde
max antichain: ac bc bd
dimension: 3

99.
id: 4285994775
bc -> e
ac -> e
ab -> de
meet-irreducibles: ad bd cd ace bce abde acde bcde
max antichain: ad bd cd ace bce
dimension: 5

100.
id: 4294907157
bc -> e
a -> e
meet-irreducibles: bd cd abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4
```

```
101.
id: 4151773047
cd -> e
ab -> cde
meet-irreducibles: ac bc ad bd ace bce ade bde acde bcde
max antichain: ac bc ad bd
dimension: 4

102.
id: 4286517535
bd -> e
ad -> e
abc -> de
bc -> e
ac -> e
meet-irreducibles: ab cd abe ace bce abde acde bcde
max antichain: ab cd ace bce
dimension: 4

103.
id: 4283905815
bc -> e
ac -> d
ab -> de
meet-irreducibles: bd acd ae bce abde acde bcde
max antichain: bd acd ae bce
dimension: 4

104.
id: 4285993335
bd -> e
ad -> e
ab -> de
meet-irreducibles: ac bc cd ace bce abde acde bcde
max antichain: ac bc cd abde
dimension: 4

105.
id: 4286513495
cd -> e
bd -> e
ad -> e
abc -> de
ac -> e
ab -> e
meet-irreducibles: a bc d abe ace bce abde acde bcde
```

```
max antichain: bc d abe ace
dimension: 4

106.
id: 4149556087
ae -> d
bd -> e
ab -> cde
meet-irreducibles: ac bc acd bce ade bde acde bcde
max antichain: ac bc ade bde
dimension: 4

107.
id: 4285990231
cd -> e
ad -> e
ac -> e
ab -> de
meet-irreducibles: a bc bd ace bce abde acde bcde
max antichain: a bc bd
dimension: 3

108.
id: 4116002679
ae -> cd
bcd -> e
ab -> cde
meet-irreducibles: ac bc ad bd acd bce bde acde bcde
max antichain: ac bc ad bd
dimension: 4

109.
id: 4151776567
ad -> e
bc -> e
ab -> cde
meet-irreducibles: ac bd cd ace bce ade bde acde bcde
max antichain: ac bd cd bce ade
dimension: 5

110.
id: 4151773015
cd -> e
ac -> e
ab -> cde
meet-irreducibles: bc ad bd ace bce ade bde acde bcde
```

```
max antichain: bc ad bd ace
dimension: 4

111.
id: 4294902039
cd -> e
bd -> e
ad -> e
bc -> e
ac -> e
ab -> e
meet-irreducibles: a b c d abce abde acde bcde
max antichain: a b c d
dimension: 4

112.
id: 4285993303
bd -> e
ad -> e
ac -> e
ab -> de
meet-irreducibles: a bc cd ace bce abde acde bcde
max antichain: bc cd ace abde
dimension: 4

113.
id: 4149687095
ace -> d
bd -> e
bc -> e
ab -> cde
meet-irreducibles: b ac acd ae bce ade bde acde bcde
max antichain: ac ae bce bde
dimension: 4

114.
id: 4283897175
bcd -> e
ad -> e
ac -> de
ab -> de
meet-irreducibles: a bc bd cd ae bce abde acde bcde
max antichain: a bc bd cd
dimension: 4

115.
```

```
id: 4151776599
bcd -> e
ad -> e
ac -> e
ab -> cde
meet-irreducibles: a bc bd cd ace bce ade bde acde bcde
max antichain: bc bd cd ace ade
dimension: 5

116.
id: 4283896663
bd -> e
ac -> de
ab -> de
meet-irreducibles: bc ad cd ae bce abde acde bcde
max antichain: bc ad cd ae
dimension: 4

117.
id: 4286513271
d -> e
abc -> de
ab -> e
meet-irreducibles: ac bc abe ace bce abde acde bcde
max antichain: ac bc abe
dimension: 3

118.
id: 4285990679
cd -> e
bc -> e
ac -> e
ab -> de
meet-irreducibles: c ad bd ace bce abde acde bcde
max antichain: ad bd ace bce
dimension: 4

119.
id: 4149695831
ad -> e
ac -> de
ab -> cde
meet-irreducibles: a bcd ae bce ade bde acde bcde
max antichain: a bcd bce bde
dimension: 4
```

```
120.
id: 4286517589
bd -> e
abc -> de
a -> e
meet-irreducibles: bc cd abe ace bce abde acde bcde
max antichain: bc cd abe ace
dimension: 4

121.
id: 4283766615
ae -> d
bcd -> e
ac -> de
ab -> de
meet-irreducibles: a bc ad bd cd bce abde acde bcde
max antichain: a bc bd cd
dimension: 4

122.
id: 4149679959
bcd -> e
ac -> de
ab -> cde
meet-irreducibles: bc ad bd cd ae bce ade bde acde bcde
max antichain: bc ad bd cd ae
dimension: 5

123.
id: 4285994325
bcd -> e
ab -> de
a -> e
meet-irreducibles: bc bd cd ace bce abde acde bcde
max antichain: bc bd cd ace
dimension: 4

124.
id: 4283773783
ae -> d
bd -> e
ac -> d
ab -> de
meet-irreducibles: a bc acd bce abde acde bcde
max antichain: bc acd abde
dimension: 3
```

```
125.
id: 4286517527
bd -> e
ad -> e
abc -> de
bc -> e
ac -> e
ab -> e
meet-irreducibles: a b cd abe ace bce abde acde bcde
max antichain: cd abe ace bce
dimension: 4

126.
id: 4149687127
bd -> e
ac -> d
ab -> cde
meet-irreducibles: bc acd ae bce ade bde acde bcde
max antichain: bc acd ae bde
dimension: 4

127.
id: 4283913557
ac -> de
ab -> de
a -> e
meet-irreducibles: bcd ae bce abde acde bcde
max antichain: bcd bce abde acde
dimension: 4

128.
id: 4151792981
ab -> cde
a -> e
meet-irreducibles: bcd ace bce ade bde acde bcde
max antichain: bcd ace bce ade bde
dimension: 5

129.
id: 4285994263
ad -> e
bc -> e
ac -> e
ab -> de
meet-irreducibles: a bd cd ace bce abde acde bcde
```

```
max antichain: a bd cd bce
dimension: 4

130.
id: 4285989239
cd -> e
bd -> e
ad -> e
ab -> de
meet-irreducibles: ac bc d ace bce abde acde bcde
max antichain: ac bc d
dimension: 3

131.
id: 4294906133
bd -> e
bc -> e
a -> e
meet-irreducibles: b cd abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

132.
id: 4283897623
bc -> e
ac -> de
ab -> de
meet-irreducibles: ad bd cd ae bce abde acde bcde
max antichain: ad bd cd ae bce
dimension: 5

133.
id: 4151777047
bc -> e
ac -> e
ab -> cde
meet-irreducibles: ad bd cd ace bce ade bde acde bcde
max antichain: ad bd cd ace bce
dimension: 5

134.
id: 4286518549
abc -> de
bc -> e
a -> e
meet-irreducibles: bd cd abe ace bce abde acde bcde
```

```
max antichain: bd cd abe ace bce
dimension: 5

135.
id: 4283904791
bd -> e
bc -> e
ac -> d
ab -> de
meet-irreducibles: b acd ae bce abde acde bcde
max antichain: b acd ae
dimension: 3

136.
id: 4149688087
bc -> e
ac -> d
ab -> cde
meet-irreducibles: bd acd ae bce ade bde acde bcde
max antichain: bd acd ae bce
dimension: 4

137.
id: 4285992711
abc -> de
c -> e
ab -> d
meet-irreducibles: abd ace bce abde acde bcde
max antichain: abd ace bce
dimension: 3

138.
id: 4285990199
cd -> e
ad -> e
bc -> e
ab -> de
meet-irreducibles: ac bd ace bce abde acde bcde
max antichain: ac bd bce
dimension: 3

139.
id: 4283904819
ace -> d
ab -> de
b -> e
```

```
meet-irreducibles: ac acd ae bce abde acde bcde
max antichain: ac ae bce
dimension: 3

140.
id: 4285989207
cd -> e
bd -> e
ad -> e
ac -> e
ab -> de
meet-irreducibles: a bc d ace bce abde acde bcde
max antichain: a bc d
dimension: 3

141.
id: 4151772503
cd -> e
ad -> e
ac -> e
ab -> cde
meet-irreducibles: a bc bd ace bce ade bde acde bcde
max antichain: bc bd ace ade
dimension: 4

142.
id: 4283892567
cd -> e
bd -> e
ac -> de
ab -> de
meet-irreducibles: bc ad ae bce abde acde bcde
max antichain: bc ad ae
dimension: 3

143.
id: 4286513493
cd -> e
bd -> e
abc -> de
a -> e
meet-irreducibles: bc d abe ace bce abde acde bcde
max antichain: bc d abe ace
dimension: 4

144.
```

```
id: 4149675863
cd -> e
ac -> de
ab -> cde
meet-irreducibles: bc ad bd ae bce ade bde acde bcde
max antichain: bc ad bd ae
dimension: 4

145.
id: 4285990229
cd -> e
ab -> de
a -> e
meet-irreducibles: bc bd ace bce abde acde bcde
max antichain: bc bd ace
dimension: 3

146.
id: 4294906129
b -> e
a -> e
meet-irreducibles: cd abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

147.
id: 4286513431
cd -> e
bd -> e
ad -> e
abc -> de
bc -> e
ac -> e
ab -> e
meet-irreducibles: a b c d abe ace bce abde acde bcde
max antichain: a b c d
dimension: 4

148.
id: 4285990663
c -> e
ab -> de
meet-irreducibles: ad bd ace bce abde acde bcde
max antichain: ad bd ace bce
dimension: 4
```

```
149.
id: 4283896151
bd -> e
ad -> e
ac -> de
ab -> de
meet-irreducibles: a bc cd ae bce abde acde bcde
max antichain: a bc cd
dimension: 3

150.
id: 4285990167
cd -> e
ad -> e
bc -> e
ac -> e
ab -> de
meet-irreducibles: a c bd ace bce abde acde bcde
max antichain: a c bd
dimension: 3

151.
id: 4283904787
ac -> d
ab -> de
b -> e
meet-irreducibles: acd ae bce abde acde bcde
max antichain: acd ae bce
dimension: 3

152.
id: 4116002615
ae -> cd
bc -> e
ab -> cde
meet-irreducibles: ac ad bd acd bce bde acde bcde
max antichain: ac ad bd bce
dimension: 4

153.
id: 4149679447
bcd -> e
ad -> e
ac -> de
ab -> cde
meet-irreducibles: a bc bd cd ae bce ade bde acde bcde
```

```
max antichain: a bc bd cd
dimension: 4

154.
id: 4283765591
ae -> d
bd -> e
ac -> de
ab -> de
meet-irreducibles: a bc ad cd bce abde acde bcde
max antichain: a bc cd
dimension: 3

155.
id: 4294902037
cd -> e
bd -> e
bc -> e
a -> e
meet-irreducibles: b c d abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

156.
id: 4149678935
bd -> e
ac -> de
ab -> cde
meet-irreducibles: bc ad cd ae bce ade bde acde bcde
max antichain: bc ad cd ae bde
dimension: 5

157.
id: 4285988983
d -> e
ab -> de
meet-irreducibles: ac bc ace bce abde acde bcde
max antichain: ac bc abde
dimension: 3

158.
id: 4151772951
cd -> e
bc -> e
ac -> e
ab -> cde
```

```
meet-irreducibles: c ad bd ace bce ade bde acde bcde
max antichain: ad bd ace bce
dimension: 4

159.
id: 4285993301
bd -> e
ab -> de
a -> e
meet-irreducibles: bc cd ace bce abde acde bcde
max antichain: bc cd ace abde
dimension: 4

160.
id: 4149548887
ae -> d
bcd -> e
ac -> de
ab -> cde
meet-irreducibles: a bc ad bd cd bce ade bde acde bcde
max antichain: a bc bd cd
dimension: 4

161.
id: 4283897173
bcd -> e
ac -> de
ab -> de
a -> e
meet-irreducibles: bc bd cd ae bce abde acde bcde
max antichain: bc bd cd ae
dimension: 4

162.
id: 4151776597
bcd -> e
ab -> cde
a -> e
meet-irreducibles: bc bd cd ace bce ade bde acde bcde
max antichain: bc bd cd ace ade
dimension: 5

163.
id: 4149556055
ae -> d
bd -> e
```

```
ac -> d
ab -> cde
meet-irreducibles: a bc acd bce ade bde acde bcde
max antichain: bc acd ade bde
dimension: 4

164.
id: 4285993239
bd -> e
ad -> e
bc -> e
ac -> e
ab -> de
meet-irreducibles: a b cd ace bce abde acde bcde
max antichain: cd ace bce abde
dimension: 4

165.
id: 4283782485
a -> de
meet-irreducibles: bcd bce abde acde bcde
max antichain: bcd bce abde acde
dimension: 4

166.
id: 4149695829
ac -> de
ab -> cde
a -> e
meet-irreducibles: bcd ae bce ade bde acde bcde
max antichain: bcd ae bce bde
dimension: 4

167.
id: 4283766613
bcd -> e
ac -> de
ab -> de
a -> d
meet-irreducibles: bc ad bd cd bce abde acde bcde
max antichain: bc ad bd cd
dimension: 4

168.
id: 4116002647
ae -> cd
```

```
bcd -> e
ac -> d
ab -> cde
meet-irreducibles: a bc ad bd acd bce bde acde bcde
max antichain: a bc bd
dimension: 3

169.
id: 4283897111
ad -> e
bc -> e
ac -> de
ab -> de
meet-irreducibles: a bd cd ae bce abde acde bcde
max antichain: a bd cd bce
dimension: 4

170.
id: 4151776535
ad -> e
bc -> e
ac -> e
ab -> cde
meet-irreducibles: a bd cd ace bce ade bde acde bcde
max antichain: bd cd ace bce ade
dimension: 5

171.
id: 4283896599
bd -> e
bc -> e
ac -> de
ab -> de
meet-irreducibles: b ad cd ae bce abde acde bcde
max antichain: b ad cd ae
dimension: 4

172.
id: 4283773781
bd -> e
ab -> de
a -> d
meet-irreducibles: bc acd bce abde acde bcde
max antichain: bc acd abde
dimension: 3
```

```
173.
id: 4286517525
bd -> e
abc -> de
bc -> e
a -> e
meet-irreducibles: b cd abe ace bce abde acde bcde
max antichain: cd abe ace bce
dimension: 4

174.
id: 4283766551
ae -> d
bc -> e
ac -> de
ab -> de
meet-irreducibles: a ad bd cd bce abde acde bcde
max antichain: a bd cd bce
dimension: 4

175.
id: 4279703319
bc -> de
ac -> de
ab -> de
meet-irreducibles: ad bd cd ae be ce abde acde bcde
max antichain: ad bd cd ae be ce
dimension: 6

176.
id: 4149679895
bc -> e
ac -> de
ab -> cde
meet-irreducibles: ad bd cd ae bce ade bde acde bcde
max antichain: ad bd cd ae bce
dimension: 5

177.
id: 4285994261
bc -> e
ab -> de
a -> e
meet-irreducibles: bd cd ace bce abde acde bcde
max antichain: bd cd ace bce
dimension: 4
```

```
178.
id: 4283773719
ae -> d
bd -> e
bc -> e
ac -> d
ab -> de
meet-irreducibles: a b acd bce abde acde bcde
max antichain: acd bce abde
dimension: 3

179.
id: 4149687063
bd -> e
bc -> e
ac -> d
ab -> cde
meet-irreducibles: b acd ae bce ade bde acde bcde
max antichain: acd ae bce bde
dimension: 4

180.
id: 4286513239
d -> e
abc -> de
ac -> e
ab -> e
meet-irreducibles: a bc abe ace bce abde acde bcde
max antichain: bc abe ace
dimension: 3

181.
id: 4151772471
cd -> e
ad -> e
bc -> e
ab -> cde
meet-irreducibles: ac bd ace bce ade bde acde bcde
max antichain: ac bd bce ade
dimension: 4

182.
id: 4149687091
ace -> d
ab -> cde
```

```
b -> e
meet-irreducibles: ac acd ae bce ade bde acde bcde
max antichain: ac ae bce bde
dimension: 4

183.
id: 4294902033
cd -> e
b -> e
a -> e
meet-irreducibles: c d abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

184.
id: 4286513237
d -> e
abc -> de
a -> e
meet-irreducibles: bc abe ace bce abde acde bcde
max antichain: bc abe ace
dimension: 3

185.
id: 4283892055
cd -> e
bd -> e
ad -> e
ac -> de
ab -> de
meet-irreducibles: a bc d ae bce abde acde bcde
max antichain: a bc d
dimension: 3

186.
id: 4286513175
d -> e
abc -> de
bc -> e
ac -> e
ab -> e
meet-irreducibles: a b c abe ace bce abde acde bcde
max antichain: a b c
dimension: 3

187.
```

```
id: 4149675351
cd -> e
ad -> e
ac -> de
ab -> cde
meet-irreducibles: a bc bd ae bce ade bde acde bcde
max antichain: a bc bd
dimension: 3

188.
id: 4114027831
ad -> ce
bc -> de
ab -> cde
meet-irreducibles: ac bd cd ae be ace bde acde bcde
max antichain: ac bd cd ae be
dimension: 5

189.
id: 4283761495
ae -> d
cd -> e
bd -> e
ac -> de
ab -> de
meet-irreducibles: a bc ad bce abde acde bcde
max antichain: bc abde acde
dimension: 3

190.
id: 4149674839
cd -> e
bd -> e
ac -> de
ab -> cde
meet-irreducibles: bc ad ae bce ade bde acde bcde
max antichain: bc ad ae bde
dimension: 4

191.
id: 4285989205
cd -> e
bd -> e
ab -> de
a -> e
meet-irreducibles: bc d ace bce abde acde bcde
```

```
max antichain: bc d ace
dimension: 3

192.
id: 4149544791
ae -> d
cd -> e
ac -> de
ab -> cde
meet-irreducibles: a bc ad bd bce ade bde acde bcde
max antichain: a bc bd
dimension: 3

193.
id: 4283896595
ac -> de
ab -> de
b -> e
meet-irreducibles: ad cd ae bce abde acde bcde
max antichain: ad cd ae bce
dimension: 4

194.
id: 4285990151
ad -> e
c -> e
ab -> de
meet-irreducibles: a bd ace bce abde acde bcde
max antichain: a bd bce
dimension: 3

195.
id: 4151772501
cd -> e
ab -> cde
a -> e
meet-irreducibles: bc bd ace bce ade bde acde bcde
max antichain: bc bd ace ade
dimension: 4

196.
id: 4286517521
abc -> de
b -> e
a -> e
meet-irreducibles: cd abe ace bce abde acde bcde
```

```
max antichain: cd abe ace bce
dimension: 4

197.
id: 4116002103
ae -> cd
ad -> c
bc -> e
ab -> cde
meet-irreducibles: a ac bd acd bce bde acde bcde
max antichain: a bd bce
dimension: 3

198.
id: 4285989143
cd -> e
bd -> e
ad -> e
bc -> e
ac -> e
ab -> de
meet-irreducibles: a b c d ace bce abde acde bcde
max antichain: a b c d
dimension: 4

199.
id: 4116001591
ae -> cd
bd -> e
bc -> e
ab -> cde
meet-irreducibles: b ac ad acd bce bde acde bcde
max antichain: ac ad bce bde
dimension: 4

200.
id: 4151772935
c -> e
ab -> cde
meet-irreducibles: ad bd ace bce ade bde acde bcde
max antichain: ad bd ace bce
dimension: 4

201.
id: 4149678423
bd -> e
```

```
ad -> e
ac -> de
ab -> cde
meet-irreducibles: a bc cd ae bce ade bde acde bcde
max antichain: a bc cd bde
dimension: 4

202.
id: 4151772439
cd -> e
ad -> e
bc -> e
ac -> e
ab -> cde
meet-irreducibles: a c bd ace bce ade bde acde bcde
max antichain: bd ace bce ade
dimension: 4

203.
id: 4149687059
ac -> d
ab -> cde
b -> e
meet-irreducibles: acd ae bce ade bde acde bcde
max antichain: acd ae bce bde
dimension: 4

204.
id: 4283892503
cd -> e
bd -> e
bc -> e
ac -> de
ab -> de
meet-irreducibles: b c ad ae bce abde acde bcde
max antichain: b c ad ae
dimension: 4

205.
id: 4149547863
ae -> d
bd -> e
ac -> de
ab -> cde
meet-irreducibles: a bc ad cd bce ade bde acde bcde
max antichain: a bc cd bde
```

```
dimension: 4

206.
id: 4286513429
cd -> e
bd -> e
abc -> de
bc -> e
a -> e
meet-irreducibles: b c d abe ace bce abde acde bcde
max antichain: d abe ace bce
dimension: 4

207.
id: 4149675799
cd -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: c ad bd ae bce ade bde acde bcde
max antichain: c ad bd ae
dimension: 4

208.
id: 4283896149
bd -> e
ac -> de
ab -> de
a -> e
meet-irreducibles: bc cd ae bce abde acde bcde
max antichain: bc cd ae
dimension: 3

209.
id: 4285990165
cd -> e
bc -> e
ab -> de
a -> e
meet-irreducibles: c bd ace bce abde acde bcde
max antichain: bd ace bce
dimension: 3

210.
id: 4283766101
bcd -> e
```

```
a -> de
meet-irreducibles: bc bd cd bce abde acde bcde
max antichain: bc bd cd
dimension: 3

211.
id: 4149679445
bcd -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: bc bd cd ae bce ade bde acde bcde
max antichain: bc bd cd ae
dimension: 4

212.
id: 4283765589
bd -> e
ac -> de
ab -> de
a -> d
meet-irreducibles: bc ad cd bce abde acde bcde
max antichain: bc ad cd
dimension: 3

213.
id: 4283896087
bd -> e
ad -> e
bc -> e
ac -> de
ab -> de
meet-irreducibles: a b cd ae bce abde acde bcde
max antichain: a b cd
dimension: 3

214.
id: 4151775511
bd -> e
ad -> e
bc -> e
ac -> e
ab -> cde
meet-irreducibles: a b cd ace bce ade bde acde bcde
max antichain: cd ace bce ade bde
dimension: 5
```

```
215.
id: 4149564757
ab -> cde
a -> de
meet-irreducibles: bcd bce ade bde acde bcde
max antichain: bcd bce ade bde
dimension: 4

216.
id: 4149548885
bcd -> e
ac -> de
ab -> cde
a -> d
meet-irreducibles: bc ad bd cd bce ade bde acde bcde
max antichain: bc ad bd cd
dimension: 4

217.
id: 4279702807
ad -> e
bc -> de
ac -> de
ab -> de
meet-irreducibles: a bd cd ae be ce abde acde bcde
max antichain: a bd cd be ce
dimension: 5

218.
id: 4149679383
ad -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a bd cd ae bce ade bde acde bcde
max antichain: a bd cd bce
dimension: 4

219.
id: 4283765527
ae -> d
bd -> e
bc -> e
ac -> de
ab -> de
```

```
meet-irreducibles: a b ad cd bce abde acde bcde
max antichain: a b cd
dimension: 3

220.
id: 4149678871
bd -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: b ad cd ae bce ade bde acde bcde
max antichain: ad cd ae bce bde
dimension: 5

221.
id: 4149556053
bd -> e
ab -> cde
a -> d
meet-irreducibles: bc acd bce ade bde acde bcde
max antichain: bc acd ade bde
dimension: 4

222.
id: 4285993237
bd -> e
bc -> e
ab -> de
a -> e
meet-irreducibles: b cd ace bce abde acde bcde
max antichain: cd ace bce abde
dimension: 4

223.
id: 4149548823
ae -> d
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a ad bd cd bce ade bde acde bcde
max antichain: a bd cd bce
dimension: 4

224.
id: 4145485591
bc -> de
```

```
ac -> de
ab -> cde
meet-irreducibles: ad bd cd ae be ce ade bde acde bcde
max antichain: ad bd cd ae be ce
dimension: 6

225.
id: 4116002645
ae -> cd
bcd -> e
ab -> cde
a -> d
meet-irreducibles: bc ad bd acd bce bde acde bcde
max antichain: bc ad bd
dimension: 3

226.
id: 4283897109
bc -> e
ac -> de
ab -> de
a -> e
meet-irreducibles: bd cd ae bce abde acde bcde
max antichain: bd cd ae bce
dimension: 4

227.
id: 4151776533
bc -> e
ab -> cde
a -> e
meet-irreducibles: bd cd ace bce ade bde acde bcde
max antichain: bd cd ace bce ade
dimension: 5

228.
id: 4149555991
ae -> d
bd -> e
bc -> e
ac -> d
ab -> cde
meet-irreducibles: a b acd bce ade bde acde bcde
max antichain: acd bce ade bde
dimension: 4
```

```
229.
id: 4283766549
bc -> e
ac -> de
ab -> de
a -> d
meet-irreducibles: ad bd cd bce abde acde bcde
max antichain: ad bd cd bce
dimension: 4

230.
id: 4116002583
ae -> cd
bc -> e
ac -> d
ab -> cde
meet-irreducibles: a ad bd acd bce bde acde bcde
max antichain: a bd bce
dimension: 3

231.
id: 4285988951
d -> e
ac -> e
ab -> de
meet-irreducibles: a bc ace bce abde acde bcde
max antichain: bc ace abde
dimension: 3

232.
id: 4283773717
bd -> e
bc -> e
ab -> de
a -> d
meet-irreducibles: b acd bce abde acde bcde
max antichain: acd bce abde
dimension: 3

233.
id: 4116002581
ae -> cd
bc -> e
ab -> cde
a -> d
meet-irreducibles: ad bd acd bce bde acde bcde
```

```
max antichain: ad bd bce
dimension: 3

234.
id: 4283892499
cd -> e
ac -> de
ab -> de
b -> e
meet-irreducibles: c ad ae bce abde acde bcde
max antichain: c ad ae
dimension: 3

235.
id: 4286513425
cd -> e
abc -> de
b -> e
a -> e
meet-irreducibles: c d abe ace bce abde acde bcde
max antichain: d abe ace bce
dimension: 4

236.
id: 4285988949
d -> e
ab -> de
a -> e
meet-irreducibles: bc ace bce abde acde bcde
max antichain: bc ace abde
dimension: 3

237.
id: 4149674327
cd -> e
bd -> e
ad -> e
ac -> de
ab -> cde
meet-irreducibles: a bc d ae bce ade bde acde bcde
max antichain: a bc d
dimension: 3

238.
id: 4285988887
d -> e
```

```
bc -> e
ac -> e
ab -> de
meet-irreducibles: a b c ace bce abde acde bcde
max antichain: a b c
dimension: 3

239.
id: 4283896083
ad -> e
ac -> de
ab -> de
b -> e
meet-irreducibles: a cd ae bce abde acde bcde
max antichain: a cd bce
dimension: 3

240.
id: 4113896759
ae -> c
ad -> ce
bc -> de
ab -> cde
meet-irreducibles: a ac bd cd be ace bde acde bcde
max antichain: a bd cd be
dimension: 4

241.
id: 4149543767
ae -> d
cd -> e
bd -> e
ac -> de
ab -> cde
meet-irreducibles: a bc ad bce ade bde acde bcde
max antichain: a bc bde
dimension: 3

242.
id: 4285989127
bd -> e
ad -> e
c -> e
ab -> de
meet-irreducibles: a b d ace bce abde acde bcde
max antichain: a b d
```

```
dimension: 3

243.
id: 4283892053
cd -> e
bd -> e
ac -> de
ab -> de
a -> e
meet-irreducibles: bc d ae bce abde acde bcde
max antichain: bc d ae
dimension: 3

244.
id: 4283765523
ae -> d
ac -> de
ab -> de
b -> e
meet-irreducibles: a ad cd bce abde acde bcde
max antichain: a cd bce
dimension: 3

245.
id: 4149678867
ac -> de
ab -> cde
b -> e
meet-irreducibles: ad cd ae bce ade bde acde bcde
max antichain: ad cd ae bce bde
dimension: 5

246.
id: 4286513173
d -> e
abc -> de
bc -> e
a -> e
meet-irreducibles: b c abe ace bce abde acde bcde
max antichain: abe ace bce
dimension: 3

247.
id: 4151772423
ad -> e
c -> e
```

```
ab -> cde
meet-irreducibles: a bd ace bce ade bde acde bcde
max antichain: bd ace bce ade
dimension: 4

248.
id: 4149675349
cd -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: bc bd ae bce ade bde acde bcde
max antichain: bc bd ae
dimension: 3

249.
id: 4285993233
ab -> de
b -> e
a -> e
meet-irreducibles: cd ace bce abde acde bcde
max antichain: cd ace bce abde
dimension: 4

250.
id: 4283761493
cd -> e
bd -> e
ac -> de
ab -> de
a -> d
meet-irreducibles: bc ad bce abde acde bcde
max antichain: bc abde acde
dimension: 3

251.
id: 4283891991
cd -> e
bd -> e
ad -> e
bc -> e
ac -> de
ab -> de
meet-irreducibles: a b c d ae bce abde acde bcde
max antichain: a b c d
dimension: 4
```

252.
id: 4151771415
cd -> e
bd -> e
ad -> e
bc -> e
ac -> e
ab -> cde
meet-irreducibles: a b c d ace bce ade bde acde bcde
max antichain: a b c d
dimension: 4

253.
id: 4149675783
ac -> de
c -> e
ab -> cde
meet-irreducibles: ad bd ae bce ade bde acde bcde
max antichain: ad bd ae bce
dimension: 4

254.
id: 4149544789
cd -> e
ac -> de
ab -> cde
a -> d
meet-irreducibles: bc ad bd bce ade bde acde bcde
max antichain: bc ad bd
dimension: 3

255.
id: 4149675287
cd -> e
ad -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a c bd ae bce ade bde acde bcde
max antichain: a c bd
dimension: 3

256.
id: 4285990149
c -> e

```
ab -> de
a -> e
meet-irreducibles: bd ace bce abde acde bcde
max antichain: bd ace bce
dimension: 3

257.
id: 4283761431
ae -> d
cd -> e
bd -> e
bc -> e
ac -> de
ab -> de
meet-irreducibles: a b c ad bce abde acde bcde
max antichain: a b c
dimension: 3

258.
id: 4149674775
cd -> e
bd -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: b c ad ae bce ade bde acde bcde
max antichain: b c ad ae
dimension: 4

259.
id: 4116002101
ae -> cd
bc -> e
ab -> cde
a -> c
meet-irreducibles: ac bd acd bce bde acde bcde
max antichain: ac bd bce
dimension: 3

260.
id: 4285989141
cd -> e
bd -> e
bc -> e
ab -> de
a -> e
```

```
meet-irreducibles: b c d ace bce abde acde bcde
max antichain: b c d
dimension: 3

261.
id: 4149544727
ae -> d
cd -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a c ad bd bce ade bde acde bcde
max antichain: a c bd
dimension: 3

262.
id: 4145481495
cd -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: c ad bd ae be ce ade bde acde bcde
max antichain: c ad bd ae be
dimension: 5

263.
id: 4283765077
bd -> e
a -> de
meet-irreducibles: bc cd bce abde acde bcde
max antichain: bc cd abde
dimension: 3

264.
id: 4149678421
bd -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: bc cd ae bce ade bde acde bcde
max antichain: bc cd ae bde
dimension: 4

265.
id: 4283773713
ab -> de
```

```
b -> e
a -> d
meet-irreducibles: acd bce abde acde bcde
max antichain: acd bce abde
dimension: 3

266.
id: 4151772437
cd -> e
bc -> e
ab -> cde
a -> e
meet-irreducibles: c bd ace bce ade bde acde bcde
max antichain: bd ace bce ade
dimension: 4

267.
id: 4149548373
bcd -> e
ab -> cde
a -> de
meet-irreducibles: bc bd cd bce ade bde acde bcde
max antichain: bc bd cd ade
dimension: 4

268.
id: 4149547861
bd -> e
ac -> de
ab -> cde
a -> d
meet-irreducibles: bc ad cd bce ade bde acde bcde
max antichain: bc ad cd bde
dimension: 4

269.
id: 4279701783
bd -> e
ad -> e
bc -> de
ac -> de
ab -> de
meet-irreducibles: a b cd ae be ce abde acde bcde
max antichain: a b cd ce
dimension: 4
```

```
270.
id: 4149678359
bd -> e
ad -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a b cd ae bce ade bde acde bcde
max antichain: a cd bce bde
dimension: 4

271.
id: 4116010325
a -> cde
meet-irreducibles: bcd bce bde acde bcde
max antichain: bcd bce bde acde
dimension: 4

272.
id: 4145485079
ad -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: a bd cd ae be ce ade bde acde bcde
max antichain: a bd cd be ce
dimension: 5

273.
id: 4114027799
ad -> ce
bc -> de
ac -> e
ab -> cde
meet-irreducibles: a bd cd ae be ace bde acde bcde
max antichain: a bd cd be
dimension: 4

274.
id: 4116002133
bcd -> e
ab -> cde
a -> cd
meet-irreducibles: bc bd acd bce bde acde bcde
max antichain: bc bd acd
dimension: 3
```

```
275.
id: 4279571223
ae -> d
bd -> e
bc -> de
ac -> de
ab -> de
meet-irreducibles: a b ad cd be ce abde acde bcde
max antichain: a b cd ce
dimension: 4

276.
id: 4149547799
ae -> d
bd -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a b ad cd bce ade bde acde bcde
max antichain: a cd bce bde
dimension: 4

277.
id: 4283896085
bd -> e
bc -> e
ac -> de
ab -> de
a -> e
meet-irreducibles: b cd ae bce abde acde bcde
max antichain: b cd ae
dimension: 3

278.
id: 4151775509
bd -> e
bc -> e
ab -> cde
a -> e
meet-irreducibles: b cd ace bce ade bde acde bcde
max antichain: cd ace bce ade bde
dimension: 5

279.
id: 4283766037
```

```
bc -> e
a -> de
meet-irreducibles: bd cd bce abde acde bcde
max antichain: bd cd bce
dimension: 3

280.
id: 4279702805
bc -> de
ac -> de
ab -> de
a -> e
meet-irreducibles: bd cd ae be ce abde acde bcde
max antichain: bd cd ae be ce
dimension: 5

281.
id: 4149679381
bc -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: bd cd ae bce ade bde acde bcde
max antichain: bd cd ae bce
dimension: 4

282.
id: 4283765525
bd -> e
bc -> e
ac -> de
ab -> de
a -> d
meet-irreducibles: b ad cd bce abde acde bcde
max antichain: b ad cd
dimension: 3

283.
id: 4116001559
ae -> cd
bd -> e
bc -> e
ac -> d
ab -> cde
meet-irreducibles: a b ad acd bce bde acde bcde
max antichain: a bce bde
```

```
dimension: 3

284.
id: 4149548821
bc -> e
ac -> de
ab -> cde
a -> d
meet-irreducibles: ad bd cd bce ade bde acde bcde
max antichain: ad bd cd bce
dimension: 4

285.
id: 4283891799
d -> e
ac -> de
ab -> de
meet-irreducibles: a bc ae bce abde acde bcde
max antichain: bc abde acde
dimension: 3

286.
id: 4294902017
c -> e
b -> e
a -> e
meet-irreducibles: d abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

287.
id: 4116001587
ae -> cd
ab -> cde
b -> e
meet-irreducibles: ac ad acd bce bde acde bcde
max antichain: ac ad bce bde
dimension: 4

288.
id: 4114023735
cd -> e
ad -> ce
bc -> de
ab -> cde
meet-irreducibles: ac bd ae be ace bde acde bcde
```

```
max antichain: ac bd ae be
dimension: 4

289.
id: 4149555989
bd -> e
bc -> e
ab -> cde
a -> d
meet-irreducibles: b acd bce ade bde acde bcde
max antichain: acd bce ade bde
dimension: 4

290.
id: 4283761427
ae -> d
cd -> e
ac -> de
ab -> de
b -> e
meet-irreducibles: a c ad bce abde acde bcde
max antichain: bce abde acde
dimension: 3

291.
id: 4149674771
cd -> e
ac -> de
ab -> cde
b -> e
meet-irreducibles: c ad ae bce ade bde acde bcde
max antichain: c ad ae bde
dimension: 4

292.
id: 4285989137
cd -> e
ab -> de
b -> e
a -> e
meet-irreducibles: c d ace bce abde acde bcde
max antichain: d ace bce
dimension: 3

293.
id: 4285988871
```

```
d -> e
c -> e
ab -> de
meet-irreducibles: a b ace bce abde acde bcde
max antichain: ace bce abde
dimension: 3

294.
id: 4283891797
d -> e
ac -> de
ab -> de
a -> e
meet-irreducibles: bc ae bce abde acde bcde
max antichain: bc abde acde
dimension: 3

295.
id: 4283891735
d -> e
bc -> e
ac -> de
ab -> de
meet-irreducibles: a b c ae bce abde acde bcde
max antichain: a b c
dimension: 3

296.
id: 4113895735
ae -> c
bd -> e
ad -> ce
bc -> de
ab -> cde
meet-irreducibles: a b ac cd be ace bde acde bcde
max antichain: a b cd
dimension: 3

297.
id: 4149678355
ad -> e
ac -> de
ab -> cde
b -> e
meet-irreducibles: a cd ae bce ade bde acde bcde
max antichain: a cd bce bde
```

```
dimension: 4

298.
id: 4113634615
be -> d
ae -> c
ad -> ce
bc -> de
ab -> cde
meet-irreducibles: a b ac bd cd ace bde acde bcde
max antichain: a b cd
dimension: 3

299.
id: 4151771399
bd -> e
ad -> e
c -> e
ab -> cde
meet-irreducibles: a b d ace bce ade bde acde bcde
max antichain: ace bce ade bde
dimension: 4

300.
id: 4283760981
cd -> e
bd -> e
a -> de
meet-irreducibles: bc d bce abde acde bcde
max antichain: bc abde acde
dimension: 3

301.
id: 4149674325
cd -> e
bd -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: bc d ae bce ade bde acde bcde
max antichain: bc d ae
dimension: 3

302.
id: 4149547795
ae -> d
```

```
ac -> de
ab -> cde
b -> e
meet-irreducibles: a ad cd bce ade bde acde bcde
max antichain: a cd bce bde
dimension: 4

303.
id: 4285988885
d -> e
bc -> e
ab -> de
a -> e
meet-irreducibles: b c ace bce abde acde bcde
max antichain: ace bce abde
dimension: 3

304.
id: 4149675271
ad -> e
ac -> de
c -> e
ab -> cde
meet-irreducibles: a bd ae bce ade bde acde bcde
max antichain: a bd bce
dimension: 3

305.
id: 4149544277
cd -> e
ab -> cde
a -> de
meet-irreducibles: bc bd bce ade bde acde bcde
max antichain: bc bd ade
dimension: 3

306.
id: 4283896081
ac -> de
ab -> de
b -> e
a -> e
meet-irreducibles: cd ae bce abde acde bcde
max antichain: cd ae bce
dimension: 3
```

```
307.
id: 4151775505
ab -> cde
b -> e
a -> e
meet-irreducibles: cd ace bce ade bde acde bcde
max antichain: cd ace bce ade bde
dimension: 5

308.
id: 4113896757
ad -> ce
bc -> de
ab -> cde
a -> c
meet-irreducibles: ac bd cd be ace bde acde bcde
max antichain: ac bd cd be
dimension: 4

309.
id: 4149674759
bd -> e
ac -> de
c -> e
ab -> cde
meet-irreducibles: b ad ae bce ade bde acde bcde
max antichain: ad ae bce bde
dimension: 4

310.
id: 4149543765
cd -> e
bd -> e
ac -> de
ab -> cde
a -> d
meet-irreducibles: bc ad bce ade bde acde bcde
max antichain: bc ad bde
dimension: 3

311.
id: 4279697687
cd -> e
bd -> e
ad -> e
bc -> de
```

```
ac -> de
ab -> de
meet-irreducibles: a b c d ae be ce abde acde bcde
max antichain: a b c d
dimension: 4

312.
id: 4149674263
cd -> e
bd -> e
ad -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a b c d ae bce ade bde acde bcde
max antichain: a b c d
dimension: 4

313.
id: 4285989125
bd -> e
c -> e
ab -> de
a -> e
meet-irreducibles: b d ace bce abde acde bcde
max antichain: b d ace
dimension: 3

314.
id: 4149544711
ae -> d
ac -> de
c -> e
ab -> cde
meet-irreducibles: a ad bd bce ade bde acde bcde
max antichain: a bd bce
dimension: 3

315.
id: 4145481479
bc -> de
ac -> de
c -> e
ab -> cde
meet-irreducibles: ad bd ae be ce ade bde acde bcde
max antichain: ad bd ae be ce
```

```
dimension: 5

316.
id: 4283765521
ac -> de
ab -> de
b -> e
a -> d
meet-irreducibles: ad cd bce abde acde bcde
max antichain: ad cd bce
dimension: 3

317.
id: 4116001555
ae -> cd
ac -> d
ab -> cde
b -> e
meet-irreducibles: a ad acd bce bde acde bcde
max antichain: a bce bde
dimension: 3

318.
id: 4145480983
cd -> e
ad -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: a c bd ae be ce ade bde acde bcde
max antichain: a c bd be
dimension: 4

319.
id: 4114023703
cd -> e
ad -> ce
bc -> de
ac -> e
ab -> cde
meet-irreducibles: a c bd ae be ace bde acde bcde
max antichain: a c bd be
dimension: 4

320.
id: 4151772421
```

```
c -> e
ab -> cde
a -> e
meet-irreducibles: bd ace bce ade bde acde bcde
max antichain: bd ace bce ade
dimension: 4

321.
id: 4279567127
ae -> d
cd -> e
bd -> e
bc -> de
ac -> de
ab -> de
meet-irreducibles: a b c ad be ce abde acde bcde
max antichain: a b c
dimension: 3

322.
id: 4149543703
ae -> d
cd -> e
bd -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a b c ad bce ade bde acde bcde
max antichain: a b c
dimension: 3

323.
id: 4283891989
cd -> e
bd -> e
bc -> e
ac -> de
ab -> de
a -> e
meet-irreducibles: b c d ae bce abde acde bcde
max antichain: b c d ae
dimension: 4

324.
id: 4151771413
cd -> e
```

```
bd -> e
bc -> e
ab -> cde
a -> e
meet-irreducibles: b c d ace bce ade bde acde bcde
max antichain: ace bce ade bde
dimension: 4

325.
id: 4145350423
ae -> d
cd -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: a c ad bd be ce ade bde acde bcde
max antichain: a c bd be
dimension: 4

326.
id: 4149547349
bd -> e
ab -> cde
a -> de
meet-irreducibles: bc cd bce ade bde acde bcde
max antichain: bc cd ade bde
dimension: 4

327.
id: 4149555985
ab -> cde
b -> e
a -> d
meet-irreducibles: acd bce ade bde acde bcde
max antichain: acd bce ade bde
dimension: 4

328.
id: 4149675285
cd -> e
bc -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: c bd ae bce ade bde acde bcde
max antichain: c bd ae
```

```
dimension: 3

329.
id: 4115993941
bcd -> e
a -> cde
meet-irreducibles: bc bd cd bce bde acde bcde
max antichain: bc bd cd
dimension: 3

330.
id: 4283761429
cd -> e
bd -> e
bc -> e
ac -> de
ab -> de
a -> d
meet-irreducibles: b c ad bce abde acde bcde
max antichain: b c ad
dimension: 3

331.
id: 4145484055
bd -> e
ad -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: a b cd ae be ce ade bde acde bcde
max antichain: a b cd ce
dimension: 4

332.
id: 4114026775
bd -> e
ad -> ce
bc -> de
ac -> e
ab -> cde
meet-irreducibles: a b cd ae be ace bde acde bcde
max antichain: a b cd
dimension: 3

333.
id: 4149544725
```

```
cd -> e
bc -> e
ac -> de
ab -> cde
a -> d
meet-irreducibles: c ad bd bce ade bde acde bcde
max antichain: c ad bd
dimension: 3

334.
id: 3579139413
a -> bcde
meet-irreducibles: bcd bce bde cde bcde
max antichain: bcd bce bde cde
dimension: 4

335.
id: 4145353495
ae -> d
bd -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: a b ad cd be ce ade bde acde bcde
max antichain: a b cd ce
dimension: 4

336.
id: 4283765013
bd -> e
bc -> e
a -> de
meet-irreducibles: b cd bce abde acde bcde
max antichain: cd bce abde
dimension: 3

337.
id: 4279701781
bd -> e
bc -> de
ac -> de
ab -> de
a -> e
meet-irreducibles: b cd ae be ce abde acde bcde
max antichain: b cd ae ce
dimension: 4
```

```
338.
id: 4149678357
bd -> e
bc -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: b cd ae bce ade bde acde bcde
max antichain: cd ae bce bde
dimension: 4

339.
id: 4294901761
d -> e
c -> e
b -> e
a -> e
meet-irreducibles: abce abde acde bcde
max antichain: abce abde acde bcde
dimension: 4

340.
id: 4279571733
bc -> de
a -> de
meet-irreducibles: bd cd be ce abde acde bcde
max antichain: bd cd be ce
dimension: 4

341.
id: 4149548309
bc -> e
ab -> cde
a -> de
meet-irreducibles: bd cd bce ade bde acde bcde
max antichain: bd cd bce ade
dimension: 4

342.
id: 4145485077
bc -> de
ac -> de
ab -> cde
a -> e
meet-irreducibles: bd cd ae be ce ade bde acde bcde
```

```
max antichain: bd cd ae be ce
dimension: 5

343.
id: 4114027797
ad -> ce
bc -> de
ab -> cde
a -> e
meet-irreducibles: bd cd ae be ace bde acde bcde
max antichain: bd cd ae be
dimension: 4

344.
id: 4279571221
bd -> e
bc -> de
ac -> de
ab -> de
a -> d
meet-irreducibles: b ad cd be ce abde acde bcde
max antichain: b ad cd ce
dimension: 4

345.
id: 4149547797
bd -> e
bc -> e
ac -> de
ab -> cde
a -> d
meet-irreducibles: b ad cd bce ade bde acde bcde
max antichain: ad cd bce bde
dimension: 4

346.
id: 4286513169
d -> e
abc -> de
b -> e
a -> e
meet-irreducibles: c abe ace bce abde acde bcde
max antichain: abe ace bce
dimension: 3

347.
```

```
id: 4283892483
ac -> de
c -> e
ab -> de
b -> e
meet-irreducibles: ad ae bce abde acde bcde
max antichain: ad ae bce
dimension: 3

348.
id: 4116002069
bc -> e
ab -> cde
a -> cd
meet-irreducibles: bd acd bce bde acde bcde
max antichain: bd acd bce
dimension: 3

349.
id: 4283891987
cd -> e
ad -> e
ac -> de
ab -> de
b -> e
meet-irreducibles: a c d ae bce abde acde bcde
max antichain: a c d
dimension: 3

350.
id: 4149674071
d -> e
ac -> de
ab -> cde
meet-irreducibles: a bc ae bce ade bde acde bcde
max antichain: a bc bde
dimension: 3

351.
id: 4286513409
abc -> de
c -> e
b -> e
a -> e
meet-irreducibles: d abe ace bce abde acde bcde
max antichain: d abe ace bce
```

```
dimension: 4

352.
id: 4113892663
ae -> c
cd -> e
ad -> ce
bc -> de
ab -> cde
meet-irreducibles: a ac bd be ace bde acde bcde
max antichain: a bd be
dimension: 3

353.
id: 4116001557
ae -> cd
bd -> e
bc -> e
ab -> cde
a -> d
meet-irreducibles: b ad acd bce bde acde bcde
max antichain: ad bce bde
dimension: 3

354.
id: 4279567123
ae -> d
cd -> e
bc -> de
ac -> de
ab -> de
b -> e
meet-irreducibles: a c ad be ce abde acde bcde
max antichain: a c be
dimension: 3

355.
id: 4149543699
ae -> d
cd -> e
ac -> de
ab -> cde
b -> e
meet-irreducibles: a c ad bce ade bde acde bcde
max antichain: a c bde
dimension: 3
```

```
356.
id: 4283891985
cd -> e
ac -> de
ab -> de
b -> e
a -> e
meet-irreducibles: c d ae bce abde acde bcde
max antichain: c d ae
dimension: 3

357.
id: 4151771409
cd -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: c d ace bce ade bde acde bcde
max antichain: ace bce ade bde
dimension: 4

358.
id: 4151771143
d -> e
c -> e
ab -> cde
meet-irreducibles: a b ace bce ade bde acde bcde
max antichain: ace bce ade bde
dimension: 4

359.
id: 4283760725
d -> e
a -> de
meet-irreducibles: bc bce abde acde bcde
max antichain: bc abde acde
dimension: 3

360.
id: 4149674069
d -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: bc ae bce ade bde acde bcde
```

```
max antichain: bc ae bde
dimension: 3

361.
id: 4113892661
cd -> e
ad -> ce
bc -> de
ab -> cde
a -> c
meet-irreducibles: ac bd be ace bde acde bcde
max antichain: ac bd be
dimension: 3

362.
id: 4283761425
cd -> e
ac -> de
ab -> de
b -> e
a -> d
meet-irreducibles: c ad bce abde acde bcde
max antichain: bce abde acde
dimension: 3

363.
id: 4279697431
d -> e
bc -> de
ac -> de
ab -> de
meet-irreducibles: a b c ae be ce abde acde bcde
max antichain: a b c
dimension: 3

364.
id: 4149674007
d -> e
bc -> e
ac -> de
ab -> cde
meet-irreducibles: a b c ae bce ade bde acde bcde
max antichain: a b c
dimension: 3

365.
```

```
id: 4285988869
d -> e
c -> e
ab -> de
a -> e
meet-irreducibles: b ace bce abde acde bcde
max antichain: ace bce abde
dimension: 3

366.
id: 4149674247
bd -> e
ad -> e
ac -> de
c -> e
ab -> cde
meet-irreducibles: a b d ae bce ade bde acde bcde
max antichain: a b d
dimension: 3

367.
id: 4149543253
cd -> e
bd -> e
ab -> cde
a -> de
meet-irreducibles: bc d bce ade bde acde bcde
max antichain: bc ade bde
dimension: 3

368.
id: 4283891733
d -> e
bc -> e
ac -> de
ab -> de
a -> e
meet-irreducibles: b c ae bce abde acde bcde
max antichain: b c ae
dimension: 3

369.
id: 4113895733
bd -> e
ad -> ce
bc -> de
```

```
ab -> cde
a -> c
meet-irreducibles: b ac cd be ace bde acde bcde
max antichain: b ac cd
dimension: 3

370.
id: 4145480967
ad -> e
bc -> de
ac -> de
c -> e
ab -> cde
meet-irreducibles: a bd ae be ce ade bde acde bcde
max antichain: a bd be ce
dimension: 4

371.
id: 4114023687
ad -> ce
bc -> de
c -> e
ab -> cde
meet-irreducibles: a bd ae be ace bde acde bcde
max antichain: a bd be
dimension: 3

372.
id: 4115989845
cd -> e
a -> cde
meet-irreducibles: bc bd bce bde acde bcde
max antichain: bc bd acde
dimension: 3

373.
id: 4283765009
b -> e
a -> de
meet-irreducibles: cd bce abde acde bcde
max antichain: cd bce abde
dimension: 3

374.
id: 4279701777
bc -> de
```

```
ac -> de
ab -> de
b -> e
a -> e
meet-irreducibles: cd ae be ce abde acde bcde
max antichain: cd ae be ce
dimension: 4

375.
id: 4149678353
ac -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: cd ae bce ade bde acde bcde
max antichain: cd ae bce bde
dimension: 4

376.
id: 4149543687
ae -> d
bd -> e
ac -> de
c -> e
ab -> cde
meet-irreducibles: a b ad bce ade bde acde bcde
max antichain: a bce bde
dimension: 3

377.
id: 4113634613
be -> d
ad -> ce
bc -> de
ab -> cde
a -> c
meet-irreducibles: b ac bd cd ace bde acde bcde
max antichain: b ac cd
dimension: 3

378.
id: 4145479959
cd -> e
bd -> e
ad -> e
bc -> de
```

```
ac -> de
ab -> cde
meet-irreducibles: a b c d ae be ce ade bde acde bcde
max antichain: a b c d
dimension: 4

379.
id: 4114022679
cd -> e
bd -> e
ad -> ce
bc -> de
ac -> e
ab -> cde
meet-irreducibles: a b c d ae be ace bde acde bcde
max antichain: a b c d
dimension: 4

380.
id: 4151771397
bd -> e
c -> e
ab -> cde
a -> e
meet-irreducibles: b d ace bce ade bde acde bcde
max antichain: ace bce ade bde
dimension: 4

381.
id: 4145350407
ae -> d
bc -> de
ac -> de
c -> e
ab -> cde
meet-irreducibles: a ad bd be ce ade bde acde bcde
max antichain: a bd be ce
dimension: 4

382.
id: 4144432903
c -> de
ab -> cde
meet-irreducibles: ad bd ae be ade bde acde bcde
max antichain: ad bd ae be
dimension: 4
```

383.
id: 4279571217
bc -> de
ac -> de
ab -> de
b -> e
a -> d
meet-irreducibles: ad cd be ce abde acde bcde
max antichain: ad cd be ce
dimension: 4

384.
id: 4149547793
ac -> de
ab -> cde
b -> e
a -> d
meet-irreducibles: ad cd bce ade bde acde bcde
max antichain: ad cd bce bde
dimension: 4

385.
id: 4149675269
ac -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: bd ae bce ade bde acde bcde
max antichain: bd ae bce
dimension: 3

386.
id: 4145349399
ae -> d
cd -> e
bd -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: a b c ad be ce ade bde acde bcde
max antichain: a b c
dimension: 3

387.
id: 4283760917

```
cd -> e
bd -> e
bc -> e
a -> de
meet-irreducibles: b c d bce abde acde bcde
max antichain: b c d
dimension: 3

388.
id: 4279697685
cd -> e
bd -> e
bc -> de
ac -> de
ab -> de
a -> e
meet-irreducibles: b c d ae be ce abde acde bcde
max antichain: b c d ae
dimension: 4

389.
id: 4149674261
cd -> e
bd -> e
bc -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: b c d ae bce ade bde acde bcde
max antichain: b c d ae
dimension: 4

390.
id: 4145088279
be -> d
ae -> d
cd -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: a b c ad bd ce ade bde acde bcde
max antichain: a b c
dimension: 3

391.
id: 4149544709
```

```
ac -> de
c -> e
ab -> cde
a -> d
meet-irreducibles: ad bd bce ade bde acde bcde
max antichain: ad bd bce
dimension: 3

392.
id: 4149544213
cd -> e
bc -> e
ab -> cde
a -> de
meet-irreducibles: c bd bce ade bde acde bcde
max antichain: c bd ade
dimension: 3

393.
id: 4116001553
ae -> cd
ab -> cde
b -> e
a -> d
meet-irreducibles: ad acd bce bde acde bcde
max antichain: ad bce bde
dimension: 3

394.
id: 4145480981
cd -> e
bc -> de
ac -> de
ab -> cde
a -> e
meet-irreducibles: c bd ae be ce ade bde acde bcde
max antichain: c bd ae be
dimension: 4

395.
id: 4114023701
cd -> e
ad -> ce
bc -> de
ab -> cde
a -> e
```

```
meet-irreducibles: c bd ae be ace bde acde bcde
max antichain: c bd ae be
dimension: 4

396.
id: 3579123029
bcd -> e
a -> bcde
meet-irreducibles: bc bd cd bce bde cde bcde
max antichain: bc bd cd
dimension: 3

397.
id: 4279567125
cd -> e
bd -> e
bc -> de
ac -> de
ab -> de
a -> d
meet-irreducibles: b c ad be ce abde acde bcde
max antichain: b c ad
dimension: 3

398.
id: 4149543701
cd -> e
bd -> e
bc -> e
ac -> de
ab -> cde
a -> d
meet-irreducibles: b c ad bce ade bde acde bcde
max antichain: b c ad
dimension: 3

399.
id: 4145350421
cd -> e
bc -> de
ac -> de
ab -> cde
a -> d
meet-irreducibles: c ad bd be ce ade bde acde bcde
max antichain: c ad bd be
dimension: 4
```

```
400.
id: 4283891731
d -> e
ac -> de
ab -> de
b -> e
meet-irreducibles: a c ae bce abde acde bcde
max antichain: bce abde acde
dimension: 3

401.
id: 4113895731
ae -> c
ad -> ce
bc -> de
ab -> cde
b -> e
meet-irreducibles: a ac cd be ace bde acde bcde
max antichain: a cd be
dimension: 3

402.
id: 4279570709
bd -> e
bc -> de
a -> de
meet-irreducibles: b cd be ce abde acde bcde
max antichain: b cd ce
dimension: 3

403.
id: 4149547285
bd -> e
bc -> e
ab -> cde
a -> de
meet-irreducibles: b cd bce ade bde acde bcde
max antichain: cd bce ade bde
dimension: 4

404.
id: 4145484053
bd -> e
bc -> de
ac -> de
```

```
ab -> cde
a -> e
meet-irreducibles: b cd ae be ce ade bde acde bcde
max antichain: b cd ae ce
dimension: 4

405.
id: 4114026773
bd -> e
ad -> ce
bc -> de
ab -> cde
a -> e
meet-irreducibles: b cd ae be ace bde acde bcde
max antichain: b cd ae
dimension: 3

406.
id: 4286513153
d -> e
abc -> de
c -> e
b -> e
a -> e
meet-irreducibles: abe ace bce abde acde bcde
max antichain: abe ace bce
dimension: 3

407.
id: 4145354005
bc -> de
ab -> cde
a -> de
meet-irreducibles: bd cd be ce ade bde acde bcde
max antichain: bd cd be ce ade
dimension: 5

408.
id: 4115993877
bc -> e
a -> cde
meet-irreducibles: bd cd bce bde acde bcde
max antichain: bd cd bce
dimension: 3

409.
```

```
id: 4113896725
bc -> de
ab -> cde
a -> ce
meet-irreducibles: bd cd be ace bde acde bcde
max antichain: bd cd be ace
dimension: 4

410.
id: 4283891971
ad -> e
ac -> de
c -> e
ab -> de
b -> e
meet-irreducibles: a d ae bce abde acde bcde
max antichain: a d bce
dimension: 3

411.
id: 4145353493
bd -> e
bc -> de
ac -> de
ab -> cde
a -> d
meet-irreducibles: b ad cd be ce ade bde acde bcde
max antichain: b ad cd ce
dimension: 4

412.
id: 4285988881
d -> e
ab -> de
b -> e
a -> e
meet-irreducibles: c ace bce abde acde bcde
max antichain: ace bce abde
dimension: 3

413.
id: 4116001045
bd -> e
bc -> e
ab -> cde
a -> cd
```

```
meet-irreducibles: b acd bce bde acde bcde
max antichain: acd bce bde
dimension: 3

414.
id: 4283761411
ae -> d
ac -> de
c -> e
ab -> de
b -> e
meet-irreducibles: a ad bce abde acde bcde
max antichain: bce abde acde
dimension: 3

415.
id: 4113891639
ae -> c
cd -> e
bd -> e
ad -> ce
bc -> de
ab -> cde
meet-irreducibles: a b ac d be ace bde acde bcde
max antichain: a b d
dimension: 3

416.
id: 4149674755
ac -> de
c -> e
ab -> cde
b -> e
meet-irreducibles: ad ae bce ade bde acde bcde
max antichain: ad ae bce bde
dimension: 4

417.
id: 4149674259
cd -> e
ad -> e
ac -> de
ab -> cde
b -> e
meet-irreducibles: a c d ae bce ade bde acde bcde
max antichain: a c d
```

```
dimension: 3

418.
id: 4285989121
c -> e
ab -> de
b -> e
a -> e
meet-irreducibles: d ace bce abde acde bcde
max antichain: d ace bce
dimension: 3

419.
id: 4113630519
be -> d
ae -> c
cd -> e
ad -> ce
bc -> de
ab -> cde
meet-irreducibles: a b ac bd ace bde acde bcde
max antichain: a b
dimension: 2

420.
id: 4145349395
ae -> d
cd -> e
bc -> de
ac -> de
ab -> cde
b -> e
meet-irreducibles: a c ad be ce ade bde acde bcde
max antichain: a c be
dimension: 3

421.
id: 4283761409
ac -> de
c -> e
ab -> de
b -> e
a -> d
meet-irreducibles: ad bce abde acde bcde
max antichain: bce abde acde
dimension: 3
```

422.
id: 4113891637
cd -> e
bd -> e
ad -> ce
bc -> de
ab -> cde
a -> c
meet-irreducibles: b ac d be ace bde acde bcde
max antichain: b ac d
dimension: 3

423.
id: 4283760913
cd -> e
b -> e
a -> de
meet-irreducibles: c d bce abde acde bcde
max antichain: bce abde acde
dimension: 3

424.
id: 4279697681
cd -> e
bc -> de
ac -> de
ab -> de
b -> e
a -> e
meet-irreducibles: c d ae be ce abde acde bcde
max antichain: c d ae be
dimension: 4

425.
id: 4149674257
cd -> e
ac -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: c d ae bce ade bde acde bcde
max antichain: c d ae
dimension: 3

426.

```
id: 4149673991
d -> e
ac -> de
c -> e
ab -> cde
meet-irreducibles: a b ae bce ade bde acde bcde
max antichain: a bce bde
dimension: 3

427.
id: 4149542997
d -> e
ab -> cde
a -> de
meet-irreducibles: bc bce ade bde acde bcde
max antichain: bc ade bde
dimension: 3

428.
id: 4113630517
be -> d
cd -> e
ad -> ce
bc -> de
ab -> cde
a -> c
meet-irreducibles: b ac bd ace bde acde bcde
max antichain: b ac
dimension: 2

429.
id: 4279567121
cd -> e
bc -> de
ac -> de
ab -> de
b -> e
a -> d
meet-irreducibles: c ad be ce abde acde bcde
max antichain: c ad be
dimension: 3

430.
id: 4149543697
cd -> e
ac -> de
```

```
ab -> cde
b -> e
a -> d
meet-irreducibles: c ad bce ade bde acde bcde
max antichain: c ad bde
dimension: 3

431.
id: 4145479703
d -> e
bc -> de
ac -> de
ab -> cde
meet-irreducibles: a b c ae be ce ade bde acde bcde
max antichain: a b c
dimension: 3

432.
id: 4151771141
d -> e
c -> e
ab -> cde
a -> e
meet-irreducibles: b ace bce ade bde acde bcde
max antichain: ace bce ade bde
dimension: 4

433.
id: 4145479943
bd -> e
ad -> e
bc -> de
ac -> de
c -> e
ab -> cde
meet-irreducibles: a b d ae be ce ade bde acde bcde
max antichain: a b d ce
dimension: 4

434.
id: 4114022663
bd -> e
ad -> ce
bc -> de
c -> e
ab -> cde
```

```
meet-irreducibles: a b d ae be ace bde acde bcde
max antichain: a b d
dimension: 3

435.
id: 4283760661
d -> e
bc -> e
a -> de
meet-irreducibles: b c bce abde acde bcde
max antichain: bce abde acde
dimension: 3

436.
id: 4279697429
d -> e
bc -> de
ac -> de
ab -> de
a -> e
meet-irreducibles: b c ae be ce abde acde bcde
max antichain: b c ae
dimension: 3

437.
id: 4149674005
d -> e
bc -> e
ac -> de
ab -> cde
a -> e
meet-irreducibles: b c ae bce ade bde acde bcde
max antichain: b c ae
dimension: 3

438.
id: 4144432391
ad -> e
c -> de
ab -> cde
meet-irreducibles: a bd ae be ade bde acde bcde
max antichain: a bd be
dimension: 3

439.
id: 4279570705
```

```
bc -> de
b -> e
a -> de
meet-irreducibles: cd be ce abde acde bcde
max antichain: cd be ce
dimension: 3

440.
id: 4149547281
ab -> cde
b -> e
a -> de
meet-irreducibles: cd bce ade bde acde bcde
max antichain: cd bce ade bde
dimension: 4

441.
id: 4145484049
bc -> de
ac -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: cd ae be ce ade bde acde bcde
max antichain: cd ae be ce
dimension: 4

442.
id: 4114026769
ad -> ce
bc -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: cd ae be ace bde acde bcde
max antichain: cd ae be
dimension: 3

443.
id: 4145349383
ae -> d
bd -> e
bc -> de
ac -> de
c -> e
ab -> cde
```

```
meet-irreducibles: a b ad be ce ade bde acde bcde
max antichain: a b ce
dimension: 3

444.
id: 4149674245
bd -> e
ac -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: b d ae bce ade bde acde bcde
max antichain: b d ae
dimension: 3

445.
id: 4145088263
be -> d
ae -> d
bc -> de
ac -> de
c -> e
ab -> cde
meet-irreducibles: a b ad bd ce ade bde acde bcde
max antichain: a b ce
dimension: 3

446.
id: 4145353489
bc -> de
ac -> de
ab -> cde
b -> e
a -> d
meet-irreducibles: ad cd be ce ade bde acde bcde
max antichain: ad cd be ce
dimension: 4

447.
id: 4149544197
c -> e
ab -> cde
a -> de
meet-irreducibles: bd bce ade bde acde bcde
max antichain: bd bce ade
dimension: 3
```

```
448.
id: 4145480965
bc -> de
ac -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: bd ae be ce ade bde acde bcde
max antichain: bd ae be ce
dimension: 4

449.
id: 4114023685
ad -> ce
bc -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: bd ae be ace bde acde bcde
max antichain: bd ae be
dimension: 3

450.
id: 4116001041
ab -> cde
b -> e
a -> cd
meet-irreducibles: acd bce bde acde bcde
max antichain: acd bce bde
dimension: 3

451.
id: 4149543685
bd -> e
ac -> de
c -> e
ab -> cde
a -> d
meet-irreducibles: b ad bce ade bde acde bcde
max antichain: ad bce bde
dimension: 3

452.
id: 4113891635
ae -> c
```

```
cd -> e
ad -> ce
bc -> de
ab -> cde
b -> e
meet-irreducibles: a ac d be ace bde acde bcde
max antichain: a d be
dimension: 3

453.
id: 4279566613
cd -> e
bd -> e
bc -> de
a -> de
meet-irreducibles: b c d be ce abde acde bcde
max antichain: b c d
dimension: 3

454.
id: 4149543189
cd -> e
bd -> e
bc -> e
ab -> cde
a -> de
meet-irreducibles: b c d bce ade bde acde bcde
max antichain: b c d
dimension: 3

455.
id: 4145479957
cd -> e
bd -> e
bc -> de
ac -> de
ab -> cde
a -> e
meet-irreducibles: b c d ae be ce ade bde acde bcde
max antichain: b c d ae
dimension: 4

456.
id: 4114022677
cd -> e
bd -> e
```

```
ad -> ce
bc -> de
ab -> cde
a -> e
meet-irreducibles: b c d ae be ace bde acde bcde
max antichain: b c d ae
dimension: 4

457.
id: 4145350405
bc -> de
ac -> de
c -> e
ab -> cde
a -> d
meet-irreducibles: ad bd be ce ade bde acde bcde
max antichain: ad bd be ce
dimension: 4

458.
id: 4279305493
be -> d
cd -> e
bc -> de
a -> de
meet-irreducibles: b c bd ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

459.
id: 4145349909
cd -> e
bc -> de
ab -> cde
a -> de
meet-irreducibles: c bd be ce ade bde acde bcde
max antichain: c bd be ade
dimension: 4

460.
id: 4115989781
cd -> e
bc -> e
a -> cde
meet-irreducibles: c bd bce bde acde bcde
max antichain: bd bce acde
```

```
dimension: 3

461.
id: 4113892629
cd -> e
bc -> de
ab -> cde
a -> ce
meet-irreducibles: c bd be ace bde acde bcde
max antichain: c bd be
dimension: 3

462.
id: 4145349397
cd -> e
bd -> e
bc -> de
ac -> de
ab -> cde
a -> d
meet-irreducibles: b c ad be ce ade bde acde bcde
max antichain: b c ad
dimension: 3

463.
id: 4283891715
d -> e
ac -> de
c -> e
ab -> de
b -> e
meet-irreducibles: a ae bce abde acde bcde
max antichain: bce abde acde
dimension: 3

464.
id: 4145088277
be -> d
cd -> e
bc -> de
ac -> de
ab -> cde
a -> d
meet-irreducibles: b c ad bd ce ade bde acde bcde
max antichain: b c ad
dimension: 3
```

```
465.
id: 4113891383
ae -> c
ad -> ce
d -> e
bc -> de
ab -> cde
meet-irreducibles: a b ac be ace bde acde bcde
max antichain: a b
dimension: 2

466.
id: 4149674003
d -> e
ac -> de
ab -> cde
b -> e
meet-irreducibles: a c ae bce ade bde acde bcde
max antichain: a c bde
dimension: 3

467.
id: 4145352981
bd -> e
bc -> de
ab -> cde
a -> de
meet-irreducibles: b cd be ce ade bde acde bcde
max antichain: b cd ce ade
dimension: 4

468.
id: 4115992853
bd -> e
bc -> e
a -> cde
meet-irreducibles: b cd bce bde acde bcde
max antichain: cd bce bde
dimension: 3

469.
id: 4113895701
bd -> e
bc -> de
ab -> cde
```

```
a -> ce
meet-irreducibles: b cd be ace bde acde bcde
max antichain: b cd ace
dimension: 3

470.
id: 4285988865
d -> e
c -> e
ab -> de
b -> e
a -> e
meet-irreducibles: ace bce abde acde bcde
max antichain: ace bce abde
dimension: 3

471.
id: 4113634581
be -> d
bc -> de
ab -> cde
a -> ce
meet-irreducibles: b bd cd ace bde acde bcde
max antichain: b cd ace
dimension: 3

472.
id: 4111799573
bc -> de
a -> cde
meet-irreducibles: bd cd be ce bde acde bcde
max antichain: bd cd be ce
dimension: 4

473.
id: 3579122965
bc -> e
a -> bcde
meet-irreducibles: bd cd bce bde cde bcde
max antichain: bd cd bce
dimension: 3

474.
id: 4149674243
ad -> e
ac -> de
```

```
c -> e
ab -> cde
b -> e
meet-irreducibles: a d ae bce ade bde acde bcde
max antichain: a d bce
dimension: 3

475.
id: 4283891729
d -> e
ac -> de
ab -> de
b -> e
a -> e
meet-irreducibles: c ae bce abde acde bcde
max antichain: bce abde acde
dimension: 3

476.
id: 4113895729
ad -> ce
bc -> de
ab -> cde
b -> e
a -> c
meet-irreducibles: ac cd be ace bde acde bcde
max antichain: ac cd be
dimension: 3

477.
id: 4279567107
ae -> d
bc -> de
ac -> de
c -> e
ab -> de
b -> e
meet-irreducibles: a ad be ce abde acde bcde
max antichain: a be ce
dimension: 3

478.
id: 4149543683
ae -> d
ac -> de
c -> e
```

```
ab -> cde
b -> e
meet-irreducibles: a ad bce ade bde acde bcde
max antichain: a bce bde
dimension: 3

479.
id: 4113634609
ad -> ce
bc -> de
ab -> cde
b -> d
a -> c
meet-irreducibles: ac bd cd ace bde acde bcde
max antichain: ac bd cd
dimension: 3

480.
id: 4283891969
ac -> de
c -> e
ab -> de
b -> e
a -> e
meet-irreducibles: d ae bce abde acde bcde
max antichain: d ae bce
dimension: 3

481.
id: 4151771393
c -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: d ace bce ade bde acde bcde
max antichain: ace bce ade bde
dimension: 4

482.
id: 4279567105
bc -> de
ac -> de
c -> e
ab -> de
b -> e
a -> d
```

```
meet-irreducibles: ad be ce abde acde bcde
max antichain: ad be ce
dimension: 3

483.
id: 4149543681
ac -> de
c -> e
ab -> cde
b -> e
a -> d
meet-irreducibles: ad bce ade bde acde bcde
max antichain: ad bce bde
dimension: 3

484.
id: 4279566609
cd -> e
bc -> de
b -> e
a -> de
meet-irreducibles: c d be ce abde acde bcde
max antichain: c d be
dimension: 3

485.
id: 4149543185
cd -> e
ab -> cde
b -> e
a -> de
meet-irreducibles: c d bce ade bde acde bcde
max antichain: c ade bde
dimension: 3

486.
id: 4145479953
cd -> e
bc -> de
ac -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: c d ae be ce ade bde acde bcde
max antichain: c d ae be
dimension: 4
```

```
487.
id: 4114022673
cd -> e
ad -> ce
bc -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: c d ae be ace bde acde bcde
max antichain: c d ae be
dimension: 4

488.
id: 4145479687
d -> e
bc -> de
ac -> de
c -> e
ab -> cde
meet-irreducibles: a b ae be ce ade bde acde bcde
max antichain: a b ce
dimension: 3

489.
id: 4114022407
ad -> ce
d -> e
bc -> de
c -> e
ab -> cde
meet-irreducibles: a b ae be ace bde acde bcde
max antichain: a b
dimension: 2

490.
id: 4279305489
cd -> e
bc -> de
b -> d
a -> de
meet-irreducibles: c bd ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

491.
```

```
id: 4145349393
cd -> e
bc -> de
ac -> de
ab -> cde
b -> e
a -> d
meet-irreducibles: c ad be ce ade bde acde bcde
max antichain: c ad be
dimension: 3

492.
id: 4149673989
d -> e
ac -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: b ae bce ade bde acde bcde
max antichain: ae bce bde
dimension: 3

493.
id: 4145088273
cd -> e
bc -> de
ac -> de
ab -> cde
b -> d
a -> d
meet-irreducibles: c ad bd ce ade bde acde bcde
max antichain: c ad bd
dimension: 3

494.
id: 4144431367
bd -> e
ad -> e
c -> de
ab -> cde
meet-irreducibles: a b d ae be ade bde acde bcde
max antichain: a b d
dimension: 3

495.
id: 4113891379
```

```
ae -> c
ad -> ce
d -> e
bc -> de
ab -> cde
b -> e
meet-irreducibles: a ac be ace bde acde bcde
max antichain: a be
dimension: 2

496.
id: 4279566357
d -> e
bc -> de
a -> de
meet-irreducibles: b c be ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

497.
id: 4149542933
d -> e
bc -> e
ab -> cde
a -> de
meet-irreducibles: b c bce ade bde acde bcde
max antichain: b c ade
dimension: 3

498.
id: 4145479701
d -> e
bc -> de
ac -> de
ab -> cde
a -> e
meet-irreducibles: b c ae be ce ade bde acde bcde
max antichain: b c ae
dimension: 3

499.
id: 4279308561
b -> de
a -> de
meet-irreducibles: cd ce abde acde bcde
max antichain: cd ce abde
```

```
dimension: 3

500.
id: 4145352977
bc -> de
ab -> cde
b -> e
a -> de
meet-irreducibles: cd be ce ade bde acde bcde
max antichain: cd be ce ade
dimension: 4

501.
id: 4115992849
b -> e
a -> cde
meet-irreducibles: cd bce bde acde bcde
max antichain: cd bce bde
dimension: 3

502.
id: 4113895697
bc -> de
ab -> cde
b -> e
a -> ce
meet-irreducibles: cd be ace bde acde bcde
max antichain: cd be ace
dimension: 3

503.
id: 4144300807
ae -> d
bd -> e
c -> de
ab -> cde
meet-irreducibles: a b ad be ade bde acde bcde
max antichain: a b
dimension: 2

504.
id: 4113634577
bc -> de
ab -> cde
b -> d
a -> ce
```

```
meet-irreducibles: bd cd ace bde acde bcde
max antichain: bd cd ace
dimension: 3

505.
id: 4149543173
bd -> e
c -> e
ab -> cde
a -> de
meet-irreducibles: b d bce ade bde acde bcde
max antichain: bce ade bde
dimension: 3

506.
id: 4145479941
bd -> e
bc -> de
ac -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: b d ae be ce ade bde acde bcde
max antichain: b d ae ce
dimension: 4

507.
id: 4114022661
bd -> e
ad -> ce
bc -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: b d ae be ace bde acde bcde
max antichain: b d ae
dimension: 3

508.
id: 4145349893
bc -> de
c -> e
ab -> cde
a -> de
meet-irreducibles: bd be ce ade bde acde bcde
max antichain: bd be ce ade
```

```
dimension: 4

509.
id: 4115989765
c -> e
a -> cde
meet-irreducibles: bd bce bde acde bcde
max antichain: bd bce acde
dimension: 3

510.
id: 4113892613
bc -> de
c -> e
ab -> cde
a -> ce
meet-irreducibles: bd be ace bde acde bcde
max antichain: bd be ace
dimension: 3

511.
id: 4144432389
c -> de
ab -> cde
a -> e
meet-irreducibles: bd ae be ade bde acde bcde
max antichain: bd ae be
dimension: 3

512.
id: 4145349381
bd -> e
bc -> de
ac -> de
c -> e
ab -> cde
a -> d
meet-irreducibles: b ad be ce ade bde acde bcde
max antichain: b ad ce
dimension: 3

513.
id: 4145348885
cd -> e
bd -> e
bc -> de
```

```
ab -> cde
a -> de
meet-irreducibles: b c d be ce ade bde acde bcde
max antichain: b c d
dimension: 3

514.
id: 4115988757
cd -> e
bd -> e
bc -> e
a -> cde
meet-irreducibles: b c d bce bde acde bcde
max antichain: b c d
dimension: 3

515.
id: 4113891605
cd -> e
bd -> e
bc -> de
ab -> cde
a -> ce
meet-irreducibles: b c d be ace bde acde bcde
max antichain: b c d
dimension: 3

516.
id: 4145088261
be -> d
bc -> de
ac -> de
c -> e
ab -> cde
a -> d
meet-irreducibles: b ad bd ce ade bde acde bcde
max antichain: b ad ce
dimension: 3

517.
id: 4145087765
be -> d
cd -> e
bc -> de
ab -> cde
a -> de
```

```
meet-irreducibles: b c bd ce ade bde acde bcde
max antichain: b c ade
dimension: 3

518.
id: 4113630485
be -> d
cd -> e
bc -> de
ab -> cde
a -> ce
meet-irreducibles: b c bd ace bde acde bcde
max antichain: b c
dimension: 2

519.
id: 4111795477
cd -> e
bc -> de
a -> cde
meet-irreducibles: c bd be ce bde acde bcde
max antichain: c bd be
dimension: 3

520.
id: 4113891633
cd -> e
ad -> ce
bc -> de
ab -> cde
b -> e
a -> c
meet-irreducibles: ac d be ace bde acde bcde
max antichain: ac d be
dimension: 3

521.
id: 4149673987
d -> e
ac -> de
c -> e
ab -> cde
b -> e
meet-irreducibles: a ae bce ade bde acde bcde
max antichain: a bce bde
dimension: 3
```

```
522.
id: 4113630513
cd -> e
ad -> ce
bc -> de
ab -> cde
b -> d
a -> c
meet-irreducibles: ac bd ace bde acde bcde
max antichain: ac bd
dimension: 2

523.
id: 4111798549
bd -> e
bc -> de
a -> cde
meet-irreducibles: b cd be ce bde acde bcde
max antichain: b cd ce
dimension: 3

524.
id: 3579121941
bd -> e
bc -> e
a -> bcde
meet-irreducibles: b cd bce bde cde bcde
max antichain: cd bce bde
dimension: 3

525.
id: 4283891713
d -> e
ac -> de
c -> e
ab -> de
b -> e
a -> e
meet-irreducibles: ae bce abde acde bcde
max antichain: bce abde acde
dimension: 3

526.
id: 4151771137
d -> e
```

```
c -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: ace bce ade bde acde bcde
max antichain: ace bce ade bde
dimension: 4

527.
id: 3574928661
bc -> de
a -> bcde
meet-irreducibles: bd cd be ce bde cde bcde
max antichain: bd cd be ce
dimension: 4

528.
id: 4114022659
ad -> ce
bc -> de
c -> e
ab -> cde
b -> e
meet-irreducibles: a d ae be ace bde acde bcde
max antichain: a d be
dimension: 3

529.
id: 4113891381
ad -> ce
d -> e
bc -> de
ab -> cde
a -> c
meet-irreducibles: b ac be ace bde acde bcde
max antichain: b ac
dimension: 2

530.
id: 4283760657
d -> e
b -> e
a -> de
meet-irreducibles: c bce abde acde bcde
max antichain: bce abde acde
dimension: 3
```

```
531.
id: 4279697425
d -> e
bc -> de
ac -> de
ab -> de
b -> e
a -> e
meet-irreducibles: c ae be ce abde acde bcde
max antichain: c ae be
dimension: 3

532.
id: 4149674001
d -> e
ac -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: c ae bce ade bde acde bcde
max antichain: c ae bde
dimension: 3

533.
id: 4145349379
ae -> d
bc -> de
ac -> de
c -> e
ab -> cde
b -> e
meet-irreducibles: a ad be ce ade bde acde bcde
max antichain: a be ce
dimension: 3

534.
id: 4283760897
c -> e
b -> e
a -> de
meet-irreducibles: d bce abde acde bcde
max antichain: bce abde acde
dimension: 3

535.
```

```
id: 4279697665
bc -> de
ac -> de
c -> e
ab -> de
b -> e
a -> e
meet-irreducibles: d ae be ce abde acde bcde
max antichain: d ae be ce
dimension: 4

536.
id: 4149674241
ac -> de
c -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: d ae bce ade bde acde bcde
max antichain: d ae bce
dimension: 3

537.
id: 4279305473
bc -> de
c -> e
b -> d
a -> de
meet-irreducibles: bd ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

538.
id: 4145349377
bc -> de
ac -> de
c -> e
ab -> cde
b -> e
a -> d
meet-irreducibles: ad be ce ade bde acde bcde
max antichain: ad be ce
dimension: 3

539.
id: 4279304465
```

```
cd -> e
b -> de
a -> de
meet-irreducibles: c d ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

540.
id: 4145348881
cd -> e
bc -> de
ab -> cde
b -> e
a -> de
meet-irreducibles: c d be ce ade bde acde bcde
max antichain: c d be
dimension: 3

541.
id: 4115988753
cd -> e
b -> e
a -> cde
meet-irreducibles: c d bce bde acde bcde
max antichain: bce bde acde
dimension: 3

542.
id: 4113891601
cd -> e
bc -> de
ab -> cde
b -> e
a -> ce
meet-irreducibles: c d be ace bde acde bcde
max antichain: c d be
dimension: 3

543.
id: 4144431111
d -> e
c -> de
ab -> cde
meet-irreducibles: a b ae be ade bde acde bcde
max antichain: a b
dimension: 2
```

544.
id: 4145088257
bc -> de
ac -> de
c -> e
ab -> cde
b -> d
a -> d
meet-irreducibles: ad bd ce ade bde acde bcde
max antichain: ad bd ce
dimension: 3

545.
id: 4145087761
cd -> e
bc -> de
ab -> cde
b -> d
a -> de
meet-irreducibles: c bd ce ade bde acde bcde
max antichain: c bd ade
dimension: 3

546.
id: 4113630481
cd -> e
bc -> de
ab -> cde
b -> d
a -> ce
meet-irreducibles: c bd ace bde acde bcde
max antichain: c bd
dimension: 2

547.
id: 4149542917
d -> e
c -> e
ab -> cde
a -> de
meet-irreducibles: b bce ade bde acde bcde
max antichain: bce ade bde
dimension: 3

548.

```
id: 4145479685
d -> e
bc -> de
ac -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: b ae be ce ade bde acde bcde
max antichain: b ae ce
dimension: 3

549.
id: 4114022405
ad -> ce
d -> e
bc -> de
c -> e
ab -> cde
a -> e
meet-irreducibles: b ae be ace bde acde bcde
max antichain: b ae
dimension: 2

550.
id: 4145348629
d -> e
bc -> de
ab -> cde
a -> de
meet-irreducibles: b c be ce ade bde acde bcde
max antichain: b c ade
dimension: 3

551.
id: 4113891349
d -> e
bc -> de
ab -> cde
a -> ce
meet-irreducibles: b c be ace bde acde bcde
max antichain: b c
dimension: 2

552.
id: 4145090833
ab -> cde
```

```
b -> de
a -> de
meet-irreducibles: cd ce ade bde acde bcde
max antichain: cd ce ade bde
dimension: 4

553.
id: 4113633553
ab -> cde
b -> de
a -> ce
meet-irreducibles: cd ace bde acde bcde
max antichain: cd ace bde
dimension: 3

554.
id: 4111798545
bc -> de
b -> e
a -> cde
meet-irreducibles: cd be ce bde acde bcde
max antichain: cd be ce
dimension: 3

555.
id: 3579121937
b -> e
a -> bcde
meet-irreducibles: cd bce bde cde bcde
max antichain: cd bce bde
dimension: 3

556.
id: 4145348869
bd -> e
bc -> de
c -> e
ab -> cde
a -> de
meet-irreducibles: b d be ce ade bde acde bcde
max antichain: b d ce
dimension: 3

557.
id: 4115988741
bd -> e
```

```
c -> e
a -> cde
meet-irreducibles: b d bce bde acde bcde
max antichain: bce bde acde
dimension: 3

558.
id: 4113891589
bd -> e
bc -> de
c -> e
ab -> cde
a -> ce
meet-irreducibles: b d be ace bde acde bcde
max antichain: b d ace
dimension: 3

559.
id: 4144431365
bd -> e
c -> de
ab -> cde
a -> e
meet-irreducibles: b d ae be ade bde acde bcde
max antichain: b d ae
dimension: 3

560.
id: 4113891377
ad -> ce
d -> e
bc -> de
ab -> cde
b -> e
a -> c
meet-irreducibles: ac be ace bde acde bcde
max antichain: ac be
dimension: 2

561.
id: 4145087749
be -> d
bc -> de
c -> e
ab -> cde
a -> de
```

```
meet-irreducibles: b bd ce ade bde acde bcde
max antichain: b ce ade
dimension: 3

562.
id: 4113630469
be -> d
bc -> de
c -> e
ab -> cde
a -> ce
meet-irreducibles: b bd ace bde acde bcde
max antichain: b ace
dimension: 2

563.
id: 4144301317
c -> de
ab -> cde
a -> de
meet-irreducibles: bd be ade bde acde bcde
max antichain: bd be ade
dimension: 3

564.
id: 4111795461
bc -> de
c -> e
a -> cde
meet-irreducibles: bd be ce bde acde bcde
max antichain: bd be ce
dimension: 3

565.
id: 4144300805
bd -> e
c -> de
ab -> cde
a -> d
meet-irreducibles: b ad be ade bde acde bcde
max antichain: b ad
dimension: 2

566.
id: 4111794453
cd -> e
```

```
bd -> e
bc -> de
a -> cde
meet-irreducibles: b c d be ce bde acde bcde
max antichain: b c d
dimension: 3

567.
id: 3579117845
cd -> e
bd -> e
bc -> e
a -> bcde
meet-irreducibles: b c d bce bde cde bcde
max antichain: b c d
dimension: 3

568.
id: 4111533333
be -> d
cd -> e
bc -> de
a -> cde
meet-irreducibles: b c bd ce bde acde bcde
max antichain: b c
dimension: 2

569.
id: 3574927637
bd -> e
bc -> de
a -> bcde
meet-irreducibles: b cd be ce bde cde bcde
max antichain: b cd ce
dimension: 3

570.
id: 4283760641
d -> e
c -> e
b -> e
a -> de
meet-irreducibles: bce abde acde bcde
max antichain: bce abde acde
dimension: 3
```

```
571.
id: 4279697409
d -> e
bc -> de
ac -> de
c -> e
ab -> de
b -> e
a -> e
meet-irreducibles: ae be ce abde acde bcde
max antichain: ae be ce
dimension: 3

572.
id: 4149673985
d -> e
ac -> de
c -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: ae bce ade bde acde bcde
max antichain: ae bce bde
dimension: 3

573.
id: 4279566353
d -> e
bc -> de
b -> e
a -> de
meet-irreducibles: c be ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

574.
id: 4149542929
d -> e
ab -> cde
b -> e
a -> de
meet-irreducibles: c bce ade bde acde bcde
max antichain: c ade bde
dimension: 3

575.
```

```
id: 4145479697
d -> e
bc -> de
ac -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: c ae be ce ade bde acde bcde
max antichain: c ae be
dimension: 3

576.
id: 4279566593
bc -> de
c -> e
b -> e
a -> de
meet-irreducibles: d be ce abde acde bcde
max antichain: d be ce
dimension: 3

577.
id: 4149543169
c -> e
ab -> cde
b -> e
a -> de
meet-irreducibles: d bce ade bde acde bcde
max antichain: bce ade bde
dimension: 3

578.
id: 4145479937
bc -> de
ac -> de
c -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: d ae be ce ade bde acde bcde
max antichain: d ae be ce
dimension: 4

579.
id: 4114022657
ad -> ce
```

```
bc -> de
c -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: d ae be ace bde acde bcde
max antichain: d ae be
dimension: 3

580.
id: 4145087745
bc -> de
c -> e
ab -> cde
b -> d
a -> de
meet-irreducibles: bd ce ade bde acde bcde
max antichain: bd ce ade
dimension: 3

581.
id: 4113630465
bc -> de
c -> e
ab -> cde
b -> d
a -> ce
meet-irreducibles: bd ace bde acde bcde
max antichain: bd ace
dimension: 2

582.
id: 4144300801
c -> de
ab -> cde
b -> e
a -> d
meet-irreducibles: ad be ade bde acde bcde
max antichain: ad be
dimension: 2

583.
id: 4145086737
cd -> e
ab -> cde
b -> de
```

```
a -> de
meet-irreducibles: c d ce ade bde acde bcde
max antichain: c ade bde
dimension: 3

584.
id: 4113629457
cd -> e
ab -> cde
b -> de
a -> ce
meet-irreducibles: c d ace bde acde bcde
max antichain: c d
dimension: 2

585.
id: 4111794449
cd -> e
bc -> de
b -> e
a -> cde
meet-irreducibles: c d be ce bde acde bcde
max antichain: c d be
dimension: 3

586.
id: 3579117841
cd -> e
b -> e
a -> bcde
meet-irreducibles: c d bce bde cde bcde
max antichain: bce bde cde
dimension: 3

587.
id: 4111533329
cd -> e
bc -> de
b -> d
a -> cde
meet-irreducibles: c bd ce bde acde bcde
max antichain: c bd
dimension: 2

588.
id: 4145348613
```

```
d -> e
bc -> de
c -> e
ab -> cde
a -> de
meet-irreducibles: b be ce ade bde acde bcde
max antichain: b ce ade
dimension: 3

589.
id: 4115988485
d -> e
c -> e
a -> cde
meet-irreducibles: b bce bde acde bcde
max antichain: bce bde acde
dimension: 3

590.
id: 4113891333
d -> e
bc -> de
c -> e
ab -> cde
a -> ce
meet-irreducibles: b be ace bde acde bcde
max antichain: b ace
dimension: 2

591.
id: 4144431109
d -> e
c -> de
ab -> cde
a -> e
meet-irreducibles: b ae be ade bde acde bcde
max antichain: b ae
dimension: 2

592.
id: 4111794197
d -> e
bc -> de
a -> cde
meet-irreducibles: b c be ce bde acde bcde
max antichain: b c
```

```
dimension: 2

593.
id: 4111536401
b -> de
a -> cde
meet-irreducibles: cd ce bde acde bcde
max antichain: cd ce bde
dimension: 3

594.
id: 3574927633
bc -> de
b -> e
a -> bcde
meet-irreducibles: cd be ce bde cde bcde
max antichain: cd be ce
dimension: 3

595.
id: 4144300293
bd -> e
c -> de
ab -> cde
a -> de
meet-irreducibles: b d be ade bde acde bcde
max antichain: b d
dimension: 2

596.
id: 4111794437
bd -> e
bc -> de
c -> e
a -> cde
meet-irreducibles: b d be ce bde acde bcde
max antichain: b d ce
dimension: 3

597.
id: 4111533317
be -> d
bc -> de
c -> e
a -> cde
meet-irreducibles: b bd ce bde acde bcde
```

```
max antichain: b ce
dimension: 2

598.
id: 4110746885
c -> de
a -> cde
meet-irreducibles: bd be bde acde bcde
max antichain: bd be acde
dimension: 3

599.
id: 3574923541
cd -> e
bd -> e
bc -> de
a -> bcde
meet-irreducibles: b c d be ce bde cde bcde
max antichain: b c d
dimension: 3

600.
id: 3574662421
be -> d
cd -> e
bc -> de
a -> bcde
meet-irreducibles: b c bd ce bde cde bcde
max antichain: b c
dimension: 2

601.
id: 4279566337
d -> e
bc -> de
c -> e
b -> e
a -> de
meet-irreducibles: be ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

602.
id: 4149542913
d -> e
c -> e
```

```
ab -> cde
b -> e
a -> de
meet-irreducibles: bce ade bde acde bcde
max antichain: bce ade bde
dimension: 3

603.
id: 4145479681
d -> e
bc -> de
ac -> de
c -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: ae be ce ade bde acde bcde
max antichain: ae be ce
dimension: 3

604.
id: 4114022401
ad -> ce
d -> e
bc -> de
c -> e
ab -> cde
b -> e
a -> e
meet-irreducibles: ae be ace bde acde bcde
max antichain: ae be
dimension: 2

605.
id: 4279304209
d -> e
b -> de
a -> de
meet-irreducibles: c ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

606.
id: 4145348625
d -> e
bc -> de
```

```
ab -> cde
b -> e
a -> de
meet-irreducibles: c be ce ade bde acde bcde
max antichain: c be ade
dimension: 3

607.
id: 4113891345
d -> e
bc -> de
ab -> cde
b -> e
a -> ce
meet-irreducibles: c be ace bde acde bcde
max antichain: c be
dimension: 2

608.
id: 4279304449
c -> e
b -> de
a -> de
meet-irreducibles: d ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

609.
id: 4145348865
bc -> de
c -> e
ab -> cde
b -> e
a -> de
meet-irreducibles: d be ce ade bde acde bcde
max antichain: d be ce
dimension: 3

610.
id: 4115988737
c -> e
b -> e
a -> cde
meet-irreducibles: d bce bde acde bcde
max antichain: bce bde acde
dimension: 3
```

611.
id: 4113891585
bc -> de
c -> e
ab -> cde
b -> e
a -> ce
meet-irreducibles: d be ace bde acde bcde
max antichain: d be ace
dimension: 3

612.
id: 4144431361
c -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: d ae be ade bde acde bcde
max antichain: d ae be
dimension: 3

613.
id: 4111533313
bc -> de
c -> e
b -> d
a -> cde
meet-irreducibles: bd ce bde acde bcde
max antichain: bd ce
dimension: 2

614.
id: 4111532305
cd -> e
b -> de
a -> cde
meet-irreducibles: c d ce bde acde bcde
max antichain: c d
dimension: 2

615.
id: 3574923537
cd -> e
bc -> de
b -> e

```
a -> bcde
meet-irreducibles: c d be ce bde cde bcde
max antichain: c d be
dimension: 3

616.
id: 3574662417
cd -> e
bc -> de
b -> d
a -> bcde
meet-irreducibles: c bd ce bde cde bcde
max antichain: c bd
dimension: 2

617.
id: 4144300037
d -> e
c -> de
ab -> cde
a -> de
meet-irreducibles: b be ade bde acde bcde
max antichain: b ade
dimension: 2

618.
id: 4111794181
d -> e
bc -> de
c -> e
a -> cde
meet-irreducibles: b be ce bde acde bcde
max antichain: b ce
dimension: 2

619.
id: 3574923285
d -> e
bc -> de
a -> bcde
meet-irreducibles: b c be ce bde cde bcde
max antichain: b c
dimension: 2

620.
id: 4044427537
```

```
b -> cde
a -> cde
meet-irreducibles: cd ce de acde bcde
max antichain: cd ce de
dimension: 3

621.
id: 3574665489
b -> de
a -> bcde
meet-irreducibles: cd ce bde cde bcde
max antichain: cd ce bde
dimension: 3

622.
id: 4110745861
bd -> e
c -> de
a -> cde
meet-irreducibles: b d be bde acde bcde
max antichain: b d
dimension: 2

623.
id: 4279304193
d -> e
c -> e
b -> de
a -> de
meet-irreducibles: ce abde acde bcde
max antichain: abde acde bcde
dimension: 3

624.
id: 4145348609
d -> e
bc -> de
c -> e
ab -> cde
b -> e
a -> de
meet-irreducibles: be ce ade bde acde bcde
max antichain: be ce ade
dimension: 3

625.
```

```
id: 4115988481
d -> e
c -> e
b -> e
a -> cde
meet-irreducibles: bce bde acde bcde
max antichain: bce bde acde
dimension: 3

626.
id: 4113891329
d -> e
bc -> de
c -> e
ab -> cde
b -> e
a -> ce
meet-irreducibles: be ace bde acde bcde
max antichain: be ace
dimension: 2

627.
id: 4144431105
d -> e
c -> de
ab -> cde
b -> e
a -> e
meet-irreducibles: ae be ade bde acde bcde
max antichain: ae be
dimension: 2

628.
id: 4145086481
d -> e
ab -> cde
b -> de
a -> de
meet-irreducibles: c ce ade bde acde bcde
max antichain: c ade bde
dimension: 3

629.
id: 4111794193
d -> e
bc -> de
```

```
b -> e
a -> cde
meet-irreducibles: c be ce bde acde bcde
max antichain: c be
dimension: 2

630.
id: 4278255873
c -> de
b -> de
a -> de
meet-irreducibles: d e abde acde bcde
max antichain: abde acde bcde
dimension: 3

631.
id: 4145086721
c -> e
ab -> cde
b -> de
a -> de
meet-irreducibles: d ce ade bde acde bcde
max antichain: ce ade bde
dimension: 3

632.
id: 4113629441
c -> e
ab -> cde
b -> de
a -> ce
meet-irreducibles: d ace bde acde bcde
max antichain: d ace
dimension: 2

633.
id: 4144300289
c -> de
ab -> cde
b -> e
a -> de
meet-irreducibles: d be ade bde acde bcde
max antichain: d be
dimension: 2

634.
```

```
id: 4111794433
bc -> de
c -> e
b -> e
a -> cde
meet-irreducibles: d be ce bde acde bcde
max antichain: d be ce
dimension: 3

635.
id: 3579117825
c -> e
b -> e
a -> bcde
meet-irreducibles: d bce bde cde bcde
max antichain: bce bde cde
dimension: 3

636.
id: 3574662401
bc -> de
c -> e
b -> d
a -> bcde
meet-irreducibles: bd ce bde cde bcde
max antichain: bd ce
dimension: 2

637.
id: 4044423441
cd -> e
b -> cde
a -> cde
meet-irreducibles: c d ce de acde bcde
max antichain: c d
dimension: 2

638.
id: 3574661393
cd -> e
b -> de
a -> bcde
meet-irreducibles: c d ce bde cde bcde
max antichain: c d
dimension: 2
```

```
639.
id: 4110745605
d -> e
c -> de
a -> cde
meet-irreducibles: b be bde acde bcde
max antichain: b acde
dimension: 2

640.
id: 3507556625
b -> cde
a -> bcde
meet-irreducibles: cd ce de cde bcde
max antichain: cd ce de
dimension: 3

641.
id: 4278255617
d -> e
c -> de
b -> de
a -> de
meet-irreducibles: e abde acde bcde
max antichain: abde acde bcde
dimension: 3

642.
id: 4145086465
d -> e
c -> e
ab -> cde
b -> de
a -> de
meet-irreducibles: ce ade bde acde bcde
max antichain: ce ade bde
dimension: 3

643.
id: 4113629185
d -> e
c -> e
ab -> cde
b -> de
a -> ce
meet-irreducibles: ace bde acde bcde
```

```
max antichain: ace bde
dimension: 2

644.
id: 4144300033
d -> e
c -> de
ab -> cde
b -> e
a -> de
meet-irreducibles: be ade bde acde bcde
max antichain: be ade
dimension: 2

645.
id: 4111794177
d -> e
bc -> de
c -> e
b -> e
a -> cde
meet-irreducibles: be ce bde acde bcde
max antichain: be ce
dimension: 2

646.
id: 3579117569
d -> e
c -> e
b -> e
a -> bcde
meet-irreducibles: bce bde cde bcde
max antichain: bce bde cde
dimension: 3

647.
id: 4111532049
d -> e
b -> de
a -> cde
meet-irreducibles: c ce bde acde bcde
max antichain: c bde
dimension: 2

648.
id: 3574923281
```

```
d -> e
bc -> de
b -> e
a -> bcde
meet-irreducibles: c be ce bde cde bcde
max antichain: c be
dimension: 2

649.
id: 4144038145
c -> de
ab -> cde
b -> de
a -> de
meet-irreducibles: d e ade bde acde bcde
max antichain: d e
dimension: 2

650.
id: 4111532289
c -> e
b -> de
a -> cde
meet-irreducibles: d ce bde acde bcde
max antichain: d ce
dimension: 2

651.
id: 4110745857
c -> de
b -> e
a -> cde
meet-irreducibles: d be bde acde bcde
max antichain: d be
dimension: 2

652.
id: 3574923521
bc -> de
c -> e
b -> e
a -> bcde
meet-irreducibles: d be ce bde cde bcde
max antichain: d be ce
dimension: 3
```

```
653.
id: 3507552529
cd -> e
b -> cde
a -> bcde
meet-irreducibles: c d ce de cde bcde
max antichain: c d
dimension: 2

654.
id: 4144037889
d -> e
c -> de
ab -> cde
b -> de
a -> de
meet-irreducibles: e ade bde acde bcde
max antichain: ade bde
dimension: 2

655.
id: 4111532033
d -> e
c -> e
b -> de
a -> cde
meet-irreducibles: ce bde acde bcde
max antichain: ce bde
dimension: 2

656.
id: 4110745601
d -> e
c -> de
b -> e
a -> cde
meet-irreducibles: be bde acde bcde
max antichain: be acde
dimension: 2

657.
id: 3574923265
d -> e
bc -> de
c -> e
b -> e
```

```
a -> bcde
meet-irreducibles: be ce bde cde bcde
max antichain: be ce
dimension: 2

658.
id: 3574661137
d -> e
b -> de
a -> bcde
meet-irreducibles: c ce bde cde bcde
max antichain: c bde
dimension: 2

659.
id: 4110483713
c -> de
b -> de
a -> cde
meet-irreducibles: d e bde acde bcde
max antichain: d e
dimension: 2

660.
id: 4044423425
c -> e
b -> cde
a -> cde
meet-irreducibles: d ce de acde bcde
max antichain: d ce
dimension: 2

661.
id: 3574661377
c -> e
b -> de
a -> bcde
meet-irreducibles: d ce bde cde bcde
max antichain: d ce
dimension: 2

662.
id: 4110483457
d -> e
c -> de
b -> de
```

```
a -> cde
meet-irreducibles: e bde acde bcde
max antichain: bde acde
dimension: 2

663.
id: 4044423169
d -> e
c -> e
b -> cde
a -> cde
meet-irreducibles: ce de acde bcde
max antichain: ce de
dimension: 2

664.
id: 3574661121
d -> e
c -> e
b -> de
a -> bcde
meet-irreducibles: ce bde cde bcde
max antichain: ce bde
dimension: 2

665.
id: 4043374849
c -> de
b -> cde
a -> cde
meet-irreducibles: d e de acde bcde
max antichain: d e
dimension: 2

666.
id: 3573612801
c -> de
b -> de
a -> bcde
meet-irreducibles: d e bde cde bcde
max antichain: d e
dimension: 2

667.
id: 3507552513
c -> e
```

```
b -> cde
a -> bcde
meet-irreducibles: d ce de cde bcde
max antichain: d ce
dimension: 2

668.
id: 4043374593
d -> e
c -> de
b -> cde
a -> cde
meet-irreducibles: e de acde bcde
max antichain: acde bcde
dimension: 2

669.
id: 3573612545
d -> e
c -> de
b -> de
a -> bcde
meet-irreducibles: e bde cde bcde
max antichain: bde cde
dimension: 2

670.
id: 3507552257
d -> e
c -> e
b -> cde
a -> bcde
meet-irreducibles: ce de cde bcde
max antichain: ce de
dimension: 2

671.
id: 3506503937
c -> de
b -> cde
a -> bcde
meet-irreducibles: d e de cde bcde
max antichain: d e
dimension: 2

672.
```

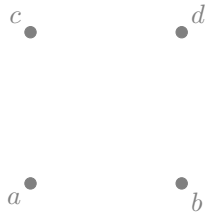
```
id: 3506503681
d -> e
c -> de
b -> cde
a -> bcde
meet-irreducibles: e de cde bcde
max antichain: e
dimension: 1
```

B

This appendix includes the representations of all 34 size 4 geometries (§B.1) and 621 of 672 size 5 geometries (§B.2).

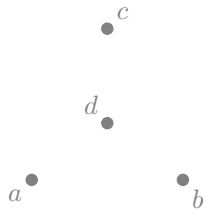
B.1 Representation of size 4 geometries

1



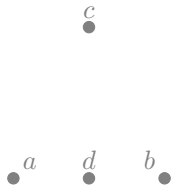
Boolean

2



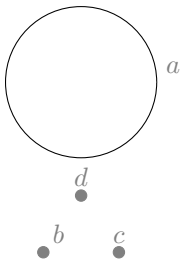
$abc \to d$

3



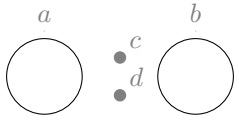
$ab \to d$

4



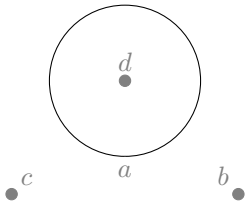
$ab \to d$, $ac \to d$

5



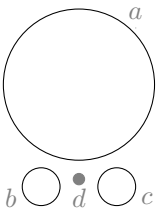
$ab \to cd$

6



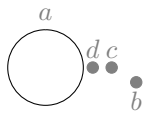
$a \to d$

7



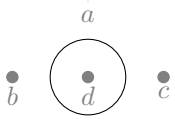
$ab \to d$, $ac \to d$, $bc \to d$

8



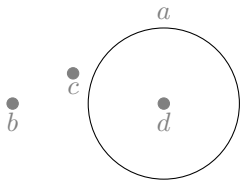
$ab \to cd, ac \to d$

9



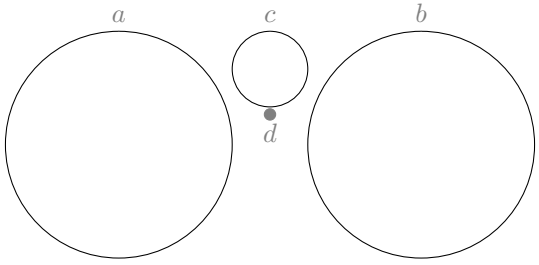
$a \to d, bc \to d$

10



$a \to d, ab \to cd$

11

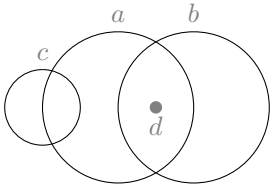


$ac \to d,\ bc \to d,\ ab \to cd$

12

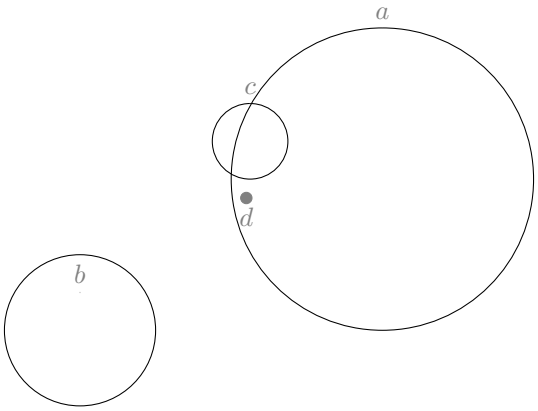
$ad \to c,\ bc \to d,\ ab \to cd$

13



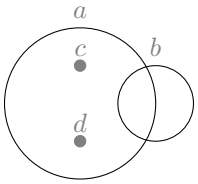
$a \to d,\ b \to d$

14



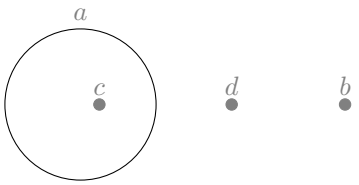
$a \to d, bc \to d, ab \to cd$

15



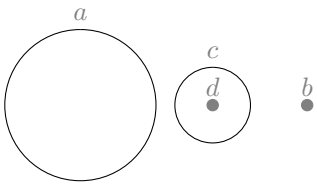
$a \to cd$

16



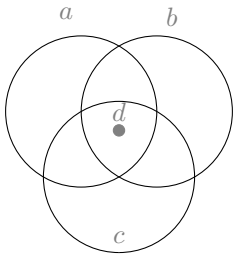
$a \to c, bc \to d, ab \to cd$

17



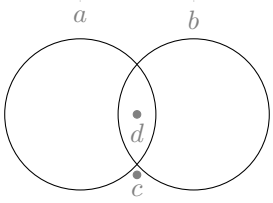
$c \to d, ab \to cd$

18



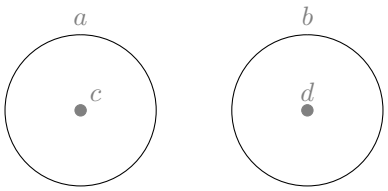
$a \to d, b \to d, c \to d$

19



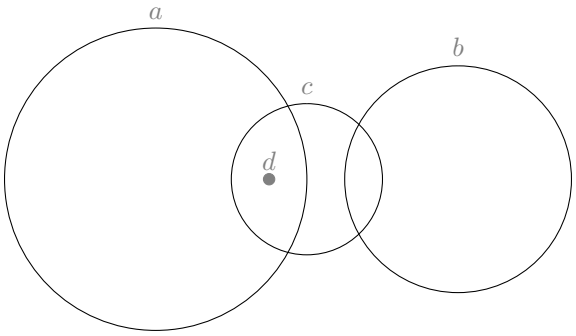
$a \to d, b \to d, ab \to cd$

20



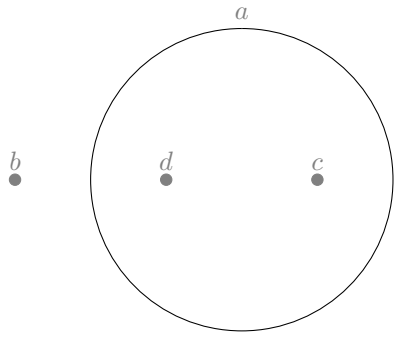
$a \to c$, $b \to d$

21



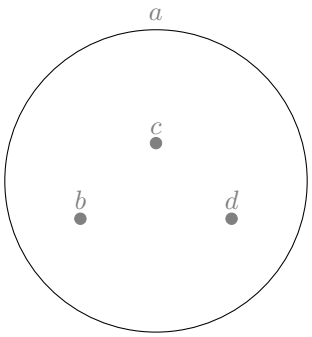
$a \to d$, $c \to d$, $ab \to cd$

22



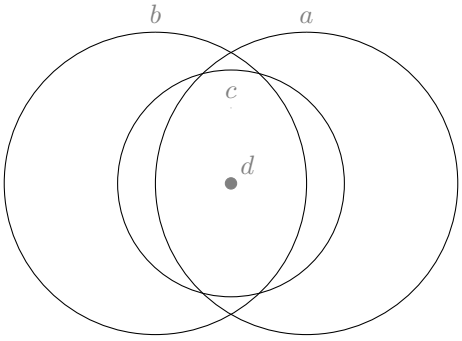
$a \to cd$, $bc \to d$

23



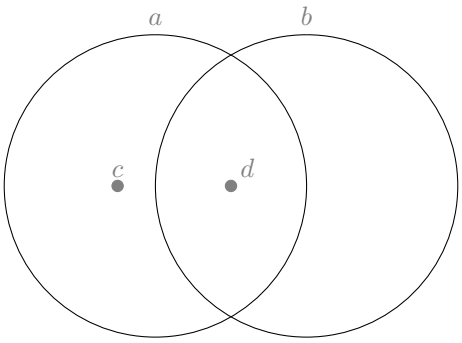
$a \to bcd$

24



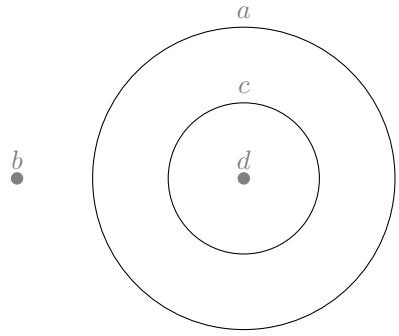
$a \to d$, $b \to d$, $c \to d$, $ab \to cd$

25



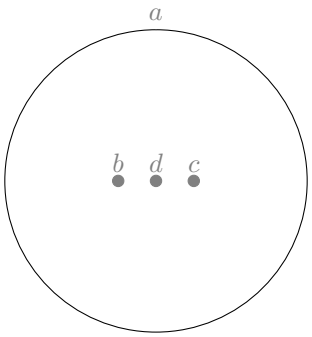
$a \to cd$, $b \to d$

26



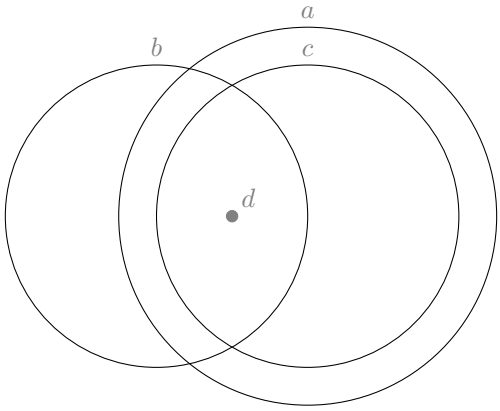
$a \to cd$, $c \to d$

27



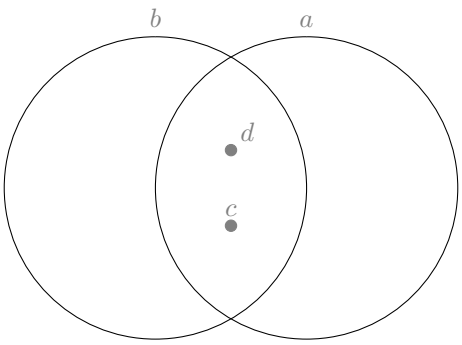
$a \to bcd$, $bc \to d$

28



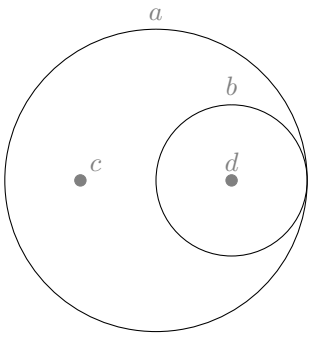
$a \to cd$, $c \to d$, $b \to d$

29



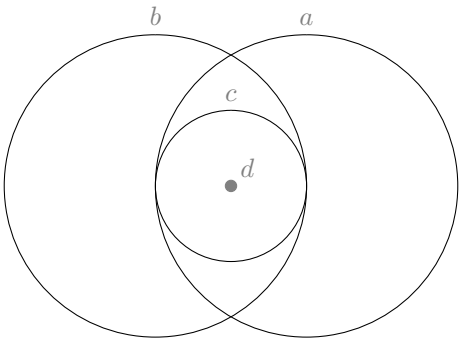
$b \to cd$, $a \to cd$

30



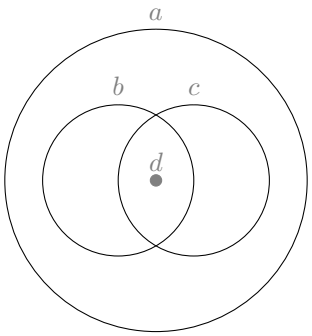
$b \to d$, $a \to bcd$

31



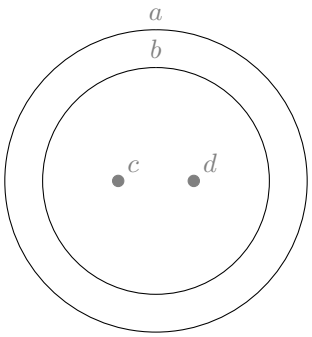
$a \to cd$, $c \to d$, $b \to cd$

32



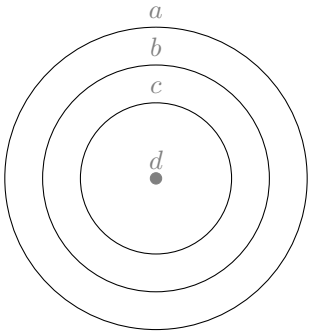
$a \to bcd$, $c \to d$, $b \to d$

33



$b \to cd$, $a \to bcd$

34



$c \to d$, $b \to cd$, $a \to bcd$

B.2 Representation of size 5 geometries

1

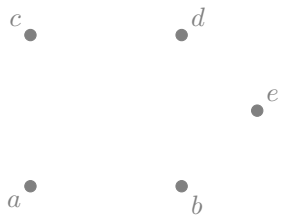


Figure 1: Boolean

2

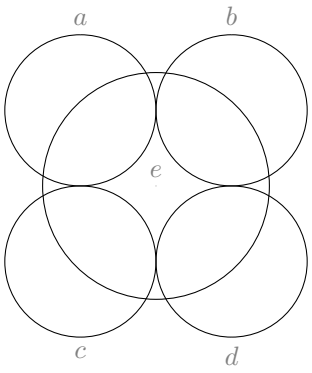


Figure 2: $abcd \to e$

3

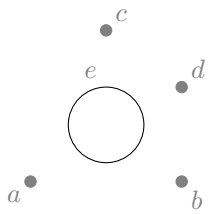


Figure 3: $abc \to e$

5

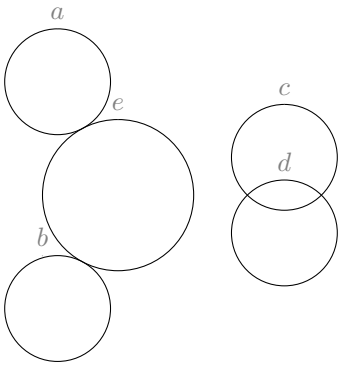


Figure 5: $abd \to e, abc \to e$

6

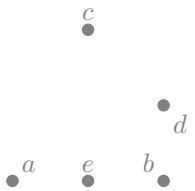


Figure 6: $ab \to e$

8

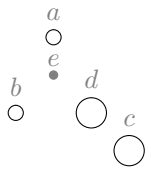


Figure 8: $abd \to e$, $abc \to de$

9

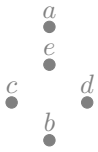


Figure 9: $ab \to e$, $acd \to e$

10

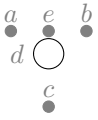


Figure 10: $ab \to e$, $abc \to de$

13

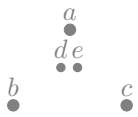


Figure 13: $abe \to d$, $acd \to e$, $abc \to de$

16

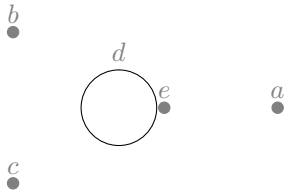


Figure 16: $ad \to e$, $abc \to de$

17

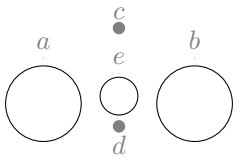


Figure 17: $ab \to de$

19

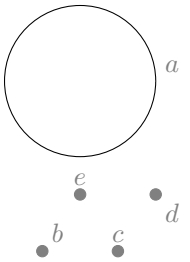


Figure 19: $ab \to e, ac \to e$

20

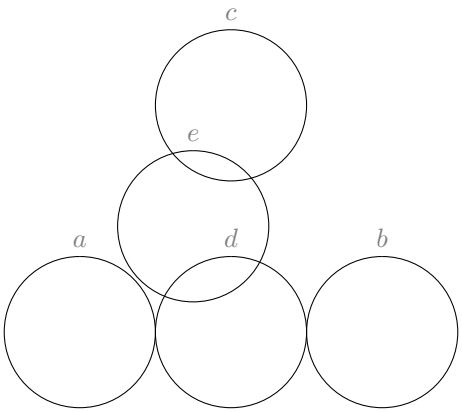


Figure 20: $acd \to e, abc \to de, ab \to d$

22

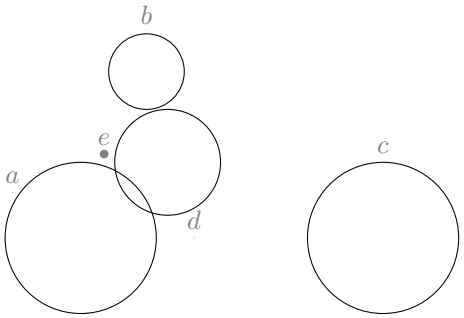


Figure 22: $ad \to e, abc \to de, ab \to e$

24

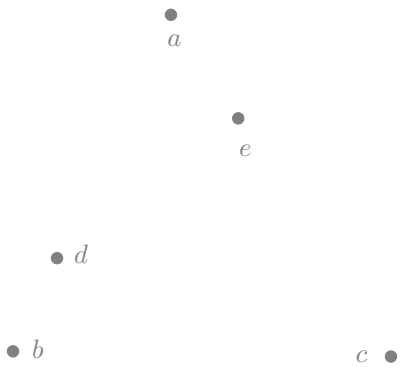


Figure 24: $abc \to de, ac \to e, ab \to d$

25

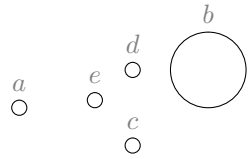


Figure 25: $acd \to e$, $ab \to de$

28

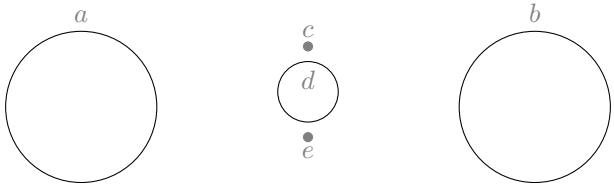


Figure 28: $ab \to cde$

29

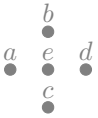


Figure 29: $ad \to e$, $bc \to e$

30

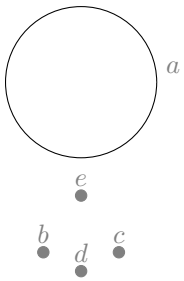


Figure 30: $ab \to e$, $ac \to e$, $ad \to e$

31

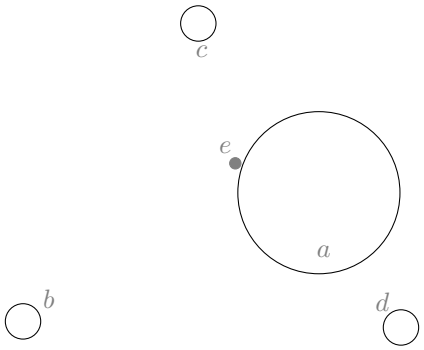


Figure 31: $bcd \to e, ac \to e, ab \to e$

32

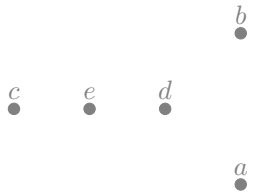


Figure 32: $abe \to e, cd \to e, abc \to de$.

36

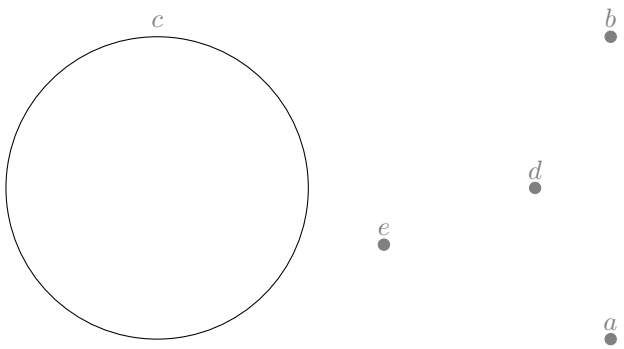


Figure 36: $abe \to d, cd \to e, abc \to de, ac \to e$.

37

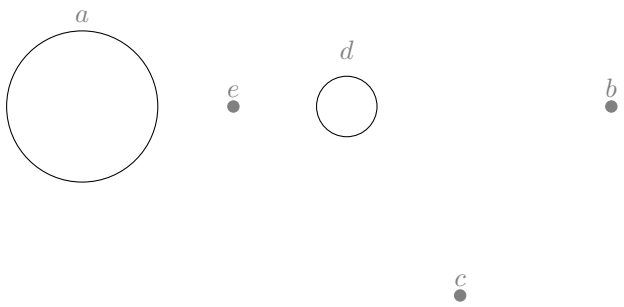


Figure 37: $ad \to e, ab \to de$.

38

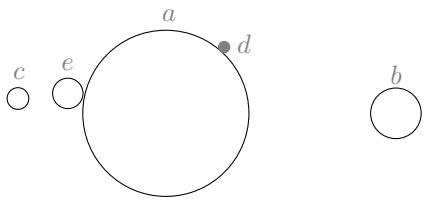


Figure 38: $bcd \to e, abc \to de, ac \to e, ab \to d$

40

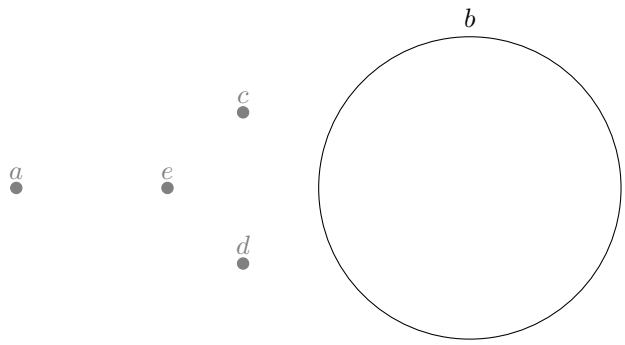


Figure 40: $acd \to e, ab \to cde$.

41

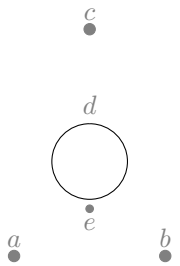


Figure 41: $ad \to e, bd \to e, abc \to de$

42

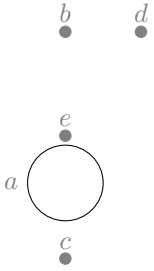


Figure 42: $ab \to e, ad \to e, bc \to e$.

44

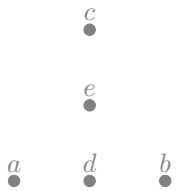


Figure 44: $ab \to d, cd \to e, abc \to de$

48

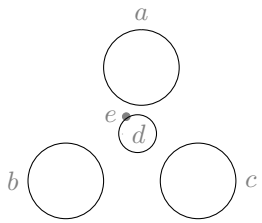


Figure 48: $ab \to e, ad \to e, ac \to e, abc \to de$.

50

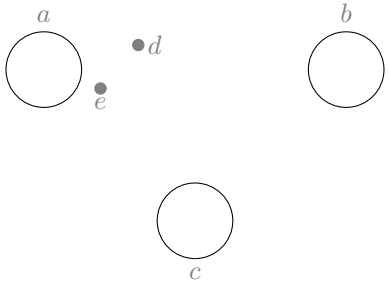


Figure 50: $ac \to e, ab \to de$

51

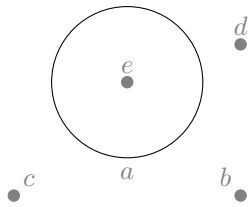


Figure 51: $a \to e$

52

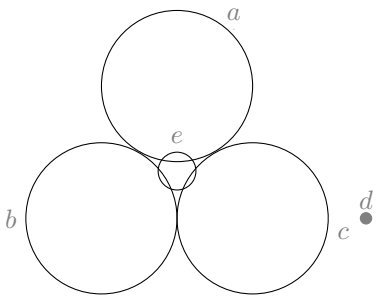


Figure 52: $ab \to e$, $ac \to e$, $bc \to e$

53

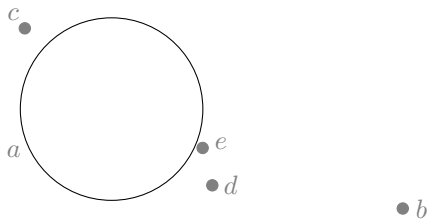


Figure 53: $bcd \to e, ad \to e, ab \to de$

55

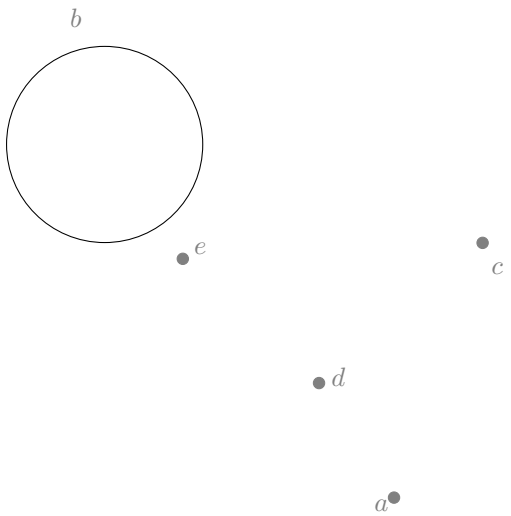


Figure 55: $ace \to d, bd \to e, ab \to de$

58

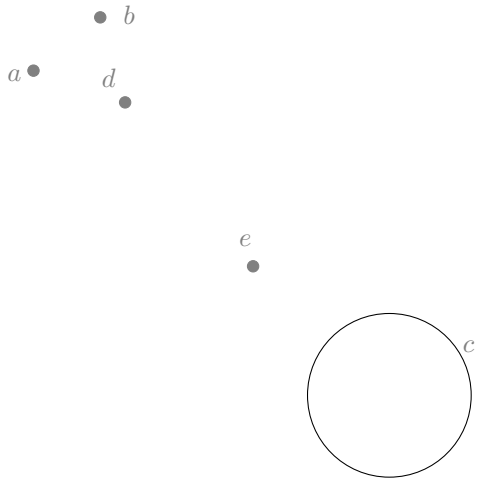


Figure 58: $abe \to d, cd \to e, abc \to de, bc \to e, ac \to e$

59

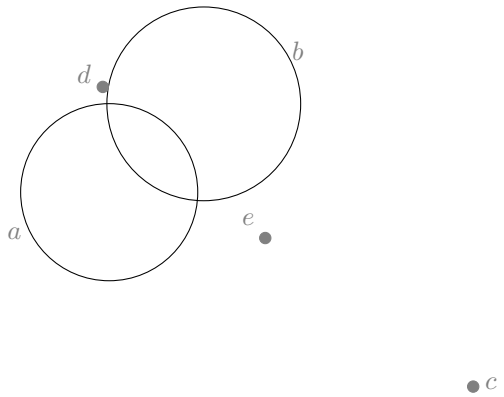


Figure 59: $abc \to de, bc \to e, ac \to e, ab \to d$

61

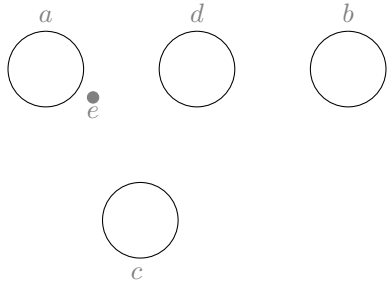


Figure 61: $ac \to e, ab \to de, ad \to e$

63

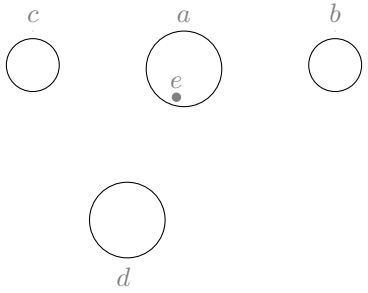


Figure 63: $a \to e, bcd \to e,$

64

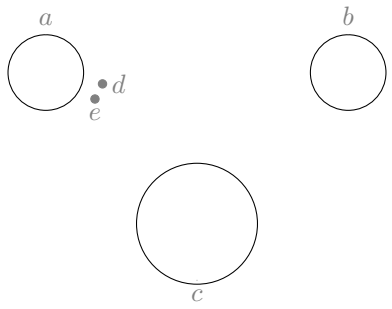


Figure 64: $ac \to de, ab \to de$

65

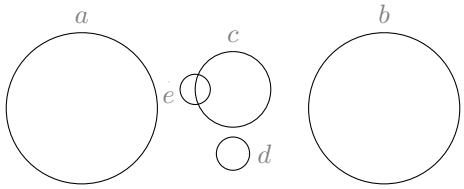


Figure 65: $ab \to cde, ac \to e$

66

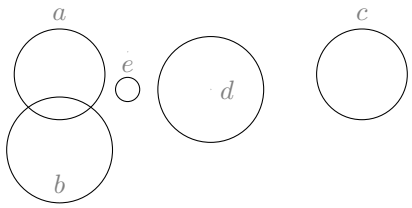


Figure 66: $abc \to de, ac \to e, ad \to e, bd \to e$

67

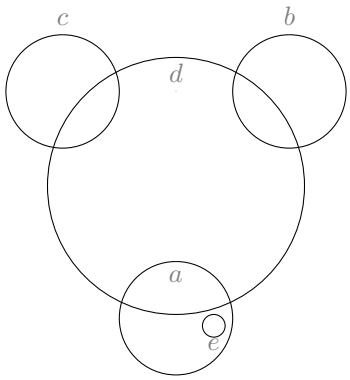


Figure 67: $a \to e, abc \to de$

68

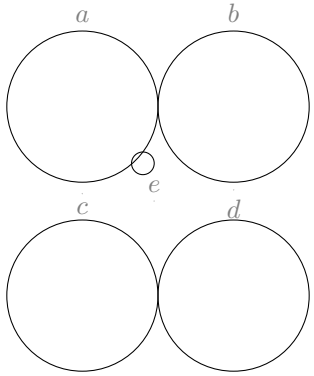


Figure 68: $ab \to e, ac \to e, bc \to e, ad \to e$

71

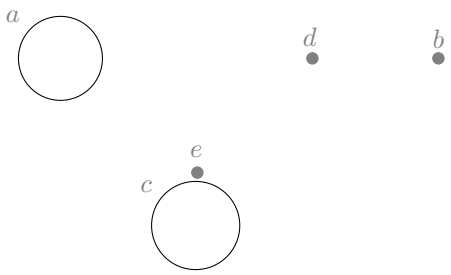


Figure 71: $cd \to e, abc \to de, ac \to e, ab \to d$

72

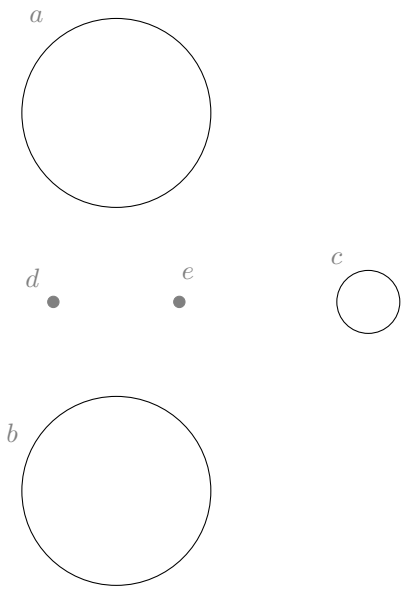


Figure 72: $cd \to e, ab \to de$

73

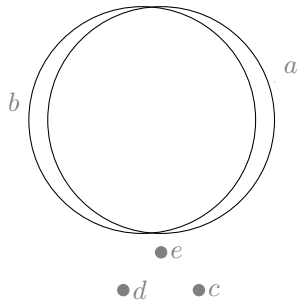


Figure 73: $ac \to e$, $ad \to e$, $bc \to e$, $bd \to e$

75

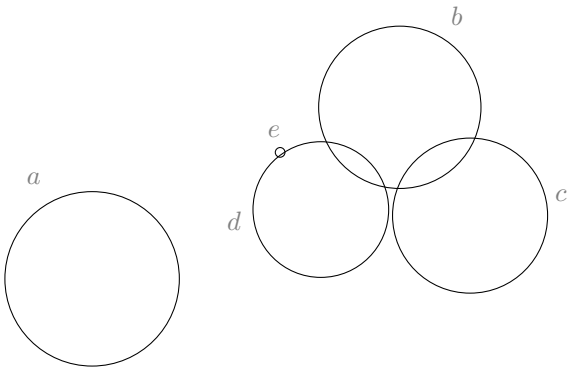


Figure 75: $abc \to de, ad \to e, bd \to e, ab \to e$

76

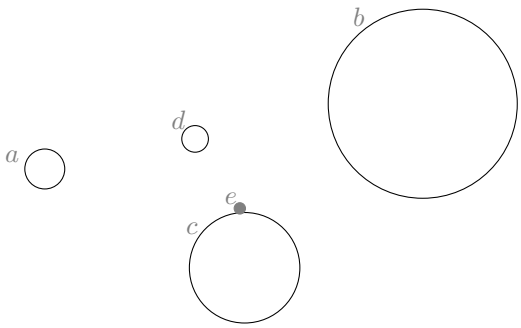


Figure 76: $cd \to e, abc \to de, bc \to e, ac \to e, ab \to d$

77

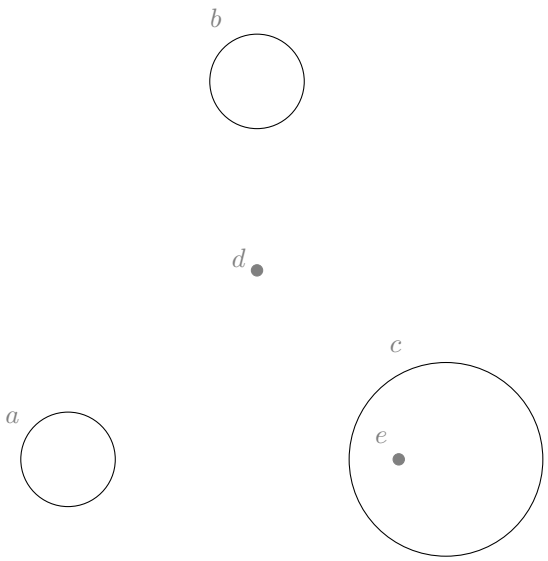


Figure 77: $abe \to d, abc \to de, c \to e$

78

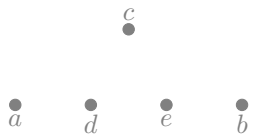


Figure 78: $ae \to d$, $bd \to e$, $ab \to de$

79

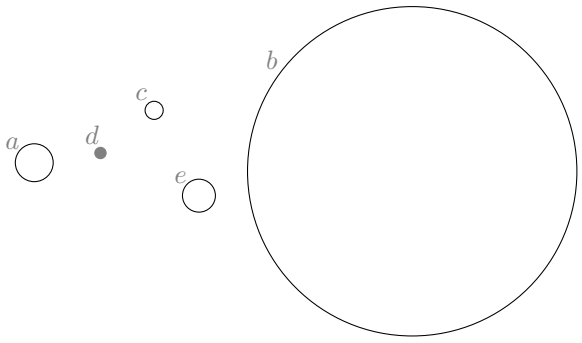


Figure 79: $ace \to d, bd \to e, ab \to cde$

80

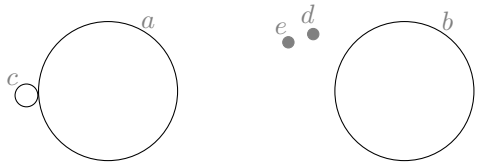


Figure 80: $ad \to e, bc \to e, ab \to de$

81

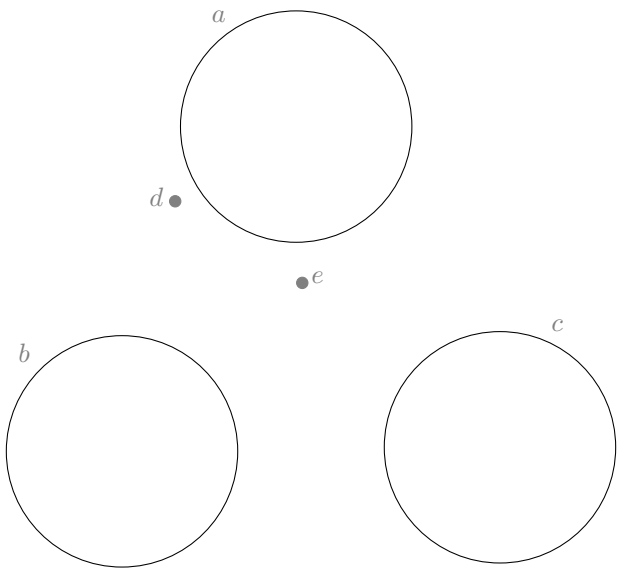


Figure 81: $ab \to de, ac \to e, cd \to e$

82

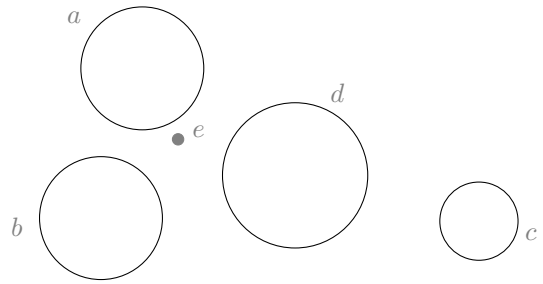


Figure 82: $ab \to e, ac \to e, ad \to e, bd \to e, abc \to de$

83

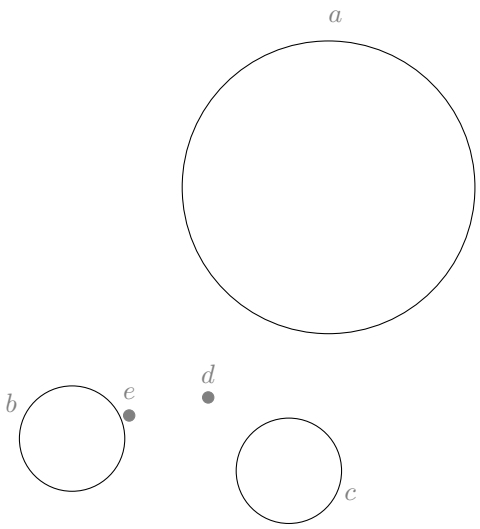


Figure 83: $ab \to de, bc \to e, bd \to e, ace \to d$

85

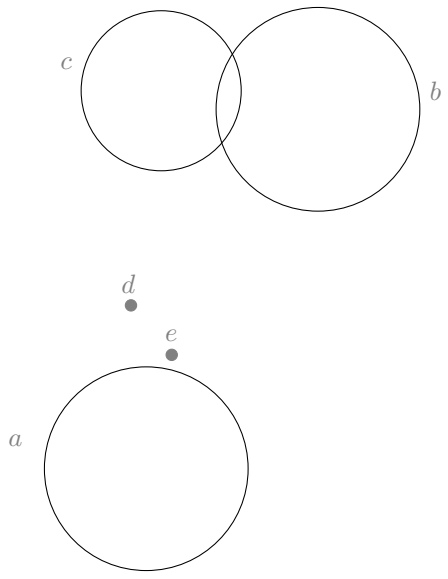


Figure 85: $ac \to de, ab \to de, ad \to e$

86

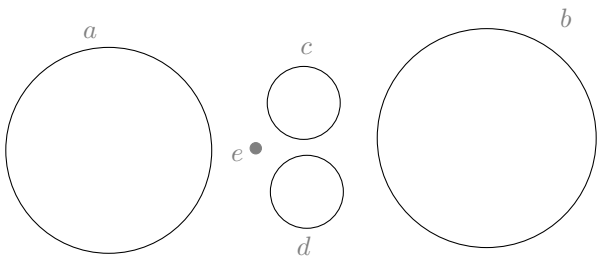


Figure 86: $ab \to cde, ac \to e, ad \to e$

90

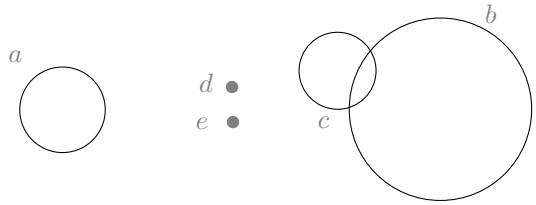


Figure 90: $ab \to cde, ac \to de$

91

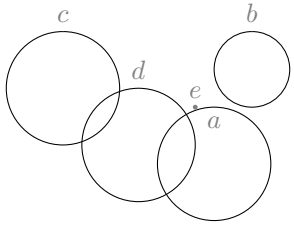


Figure 91: $bd \to e, ad \to e, ab \to e, ac \to e, bc \to e$

92

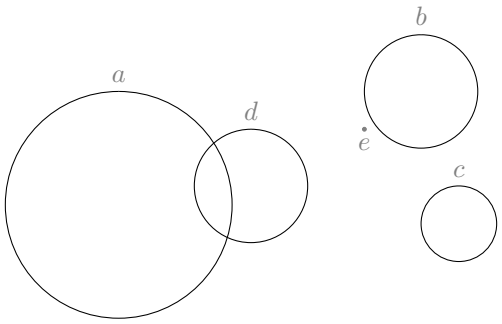


Figure 92: $bd \to e, ac \to d, ab \to de$

93

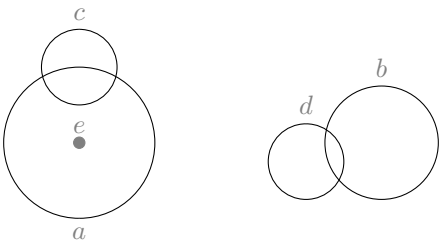


Figure 93: $ab \to de, a \to e$

97

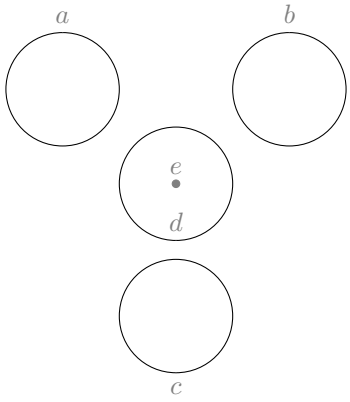


Figure 97: $d \to e, abc \to de$

98

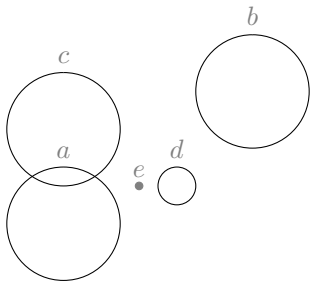


Figure 98: $cd \to e, ad \to e, ab \to de$

99

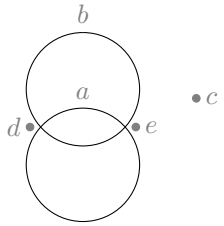


Figure 99: $bc \to e, ac \to e, ab \to de$

100

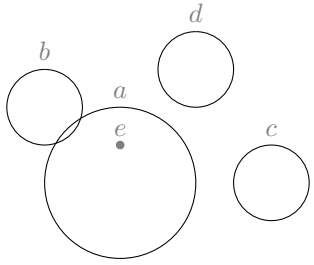


Figure 100: $bc \to e, a \to e$

101

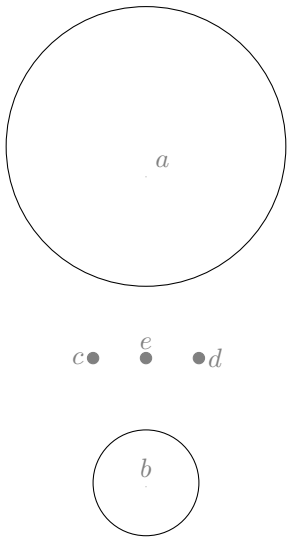


Figure 101: $cd \to e$, $ab \to cde$

102

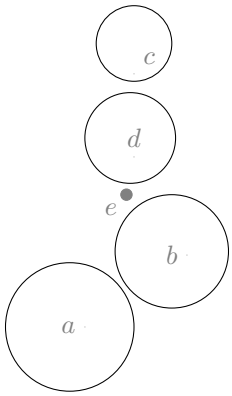


Figure 102: $bd \to e$, $ad \to e$, $ac \to e$, $bc \to e$, $abc \to de$

103

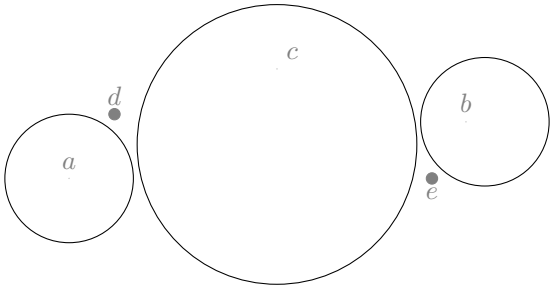


Figure 103: $bc \to e$, $ac \to d$, $ab \to de$

104

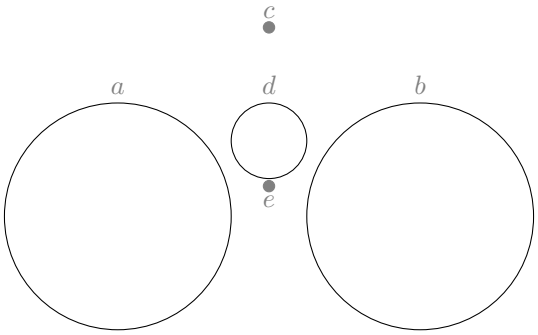


Figure 104: $ad \to e$, $bd \to e$, $ab \to de$

106

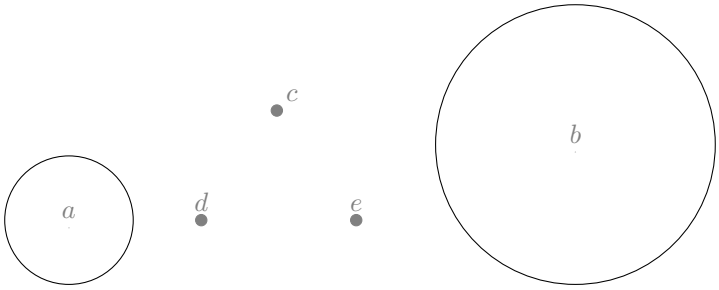


Figure 106: $ae \to d$, $bd \to e$, $ab \to cde$

107

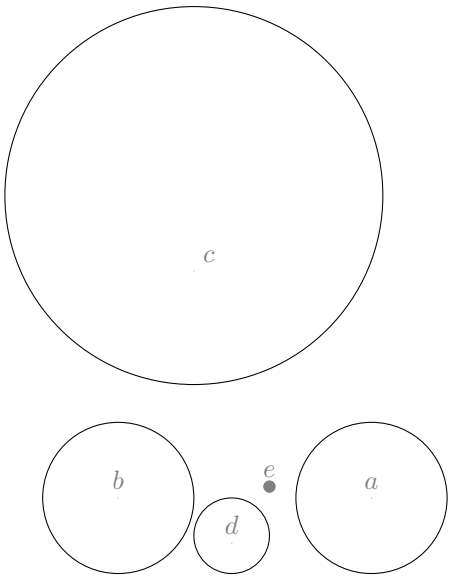


Figure 107: $cd \to e$, $ad \to e$, $ac \to e$, $ab \to de$

108

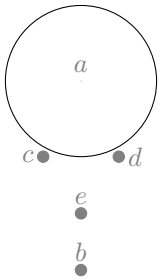


Figure 108: $ae \to cd$, $bcd \to e$, $ab \to cde$

109

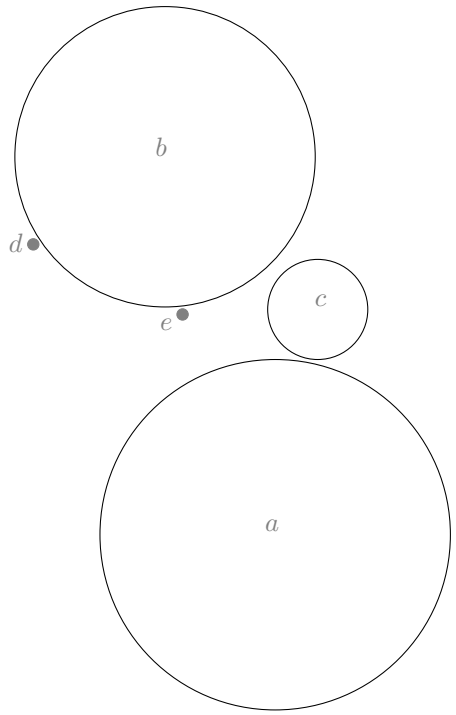


Figure 109: $ad \to e, bc \to e, ab \to cde$

110

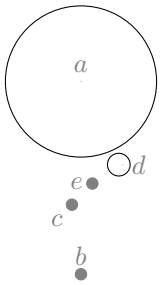


Figure 110: $cd \to e$, $ac \to e$, $ab \to cde$

111

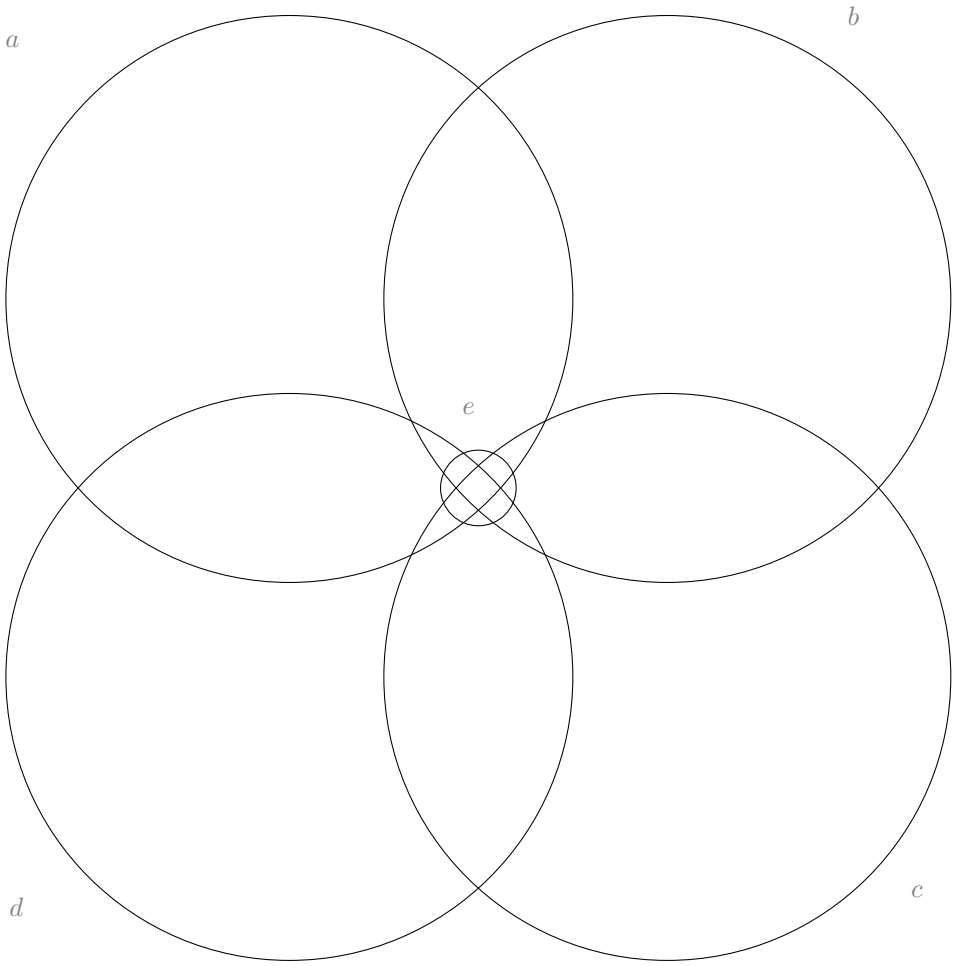


Figure 111: $ab \to e, ac \to e, ad \to e, bc \to e, bd \to e, cd \to e$

112

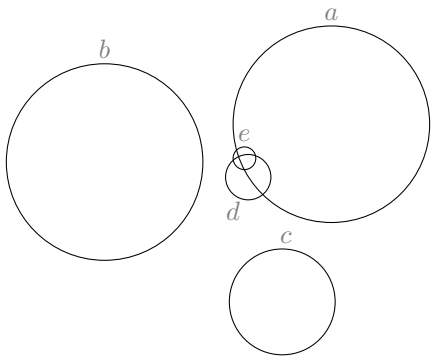


Figure 112: $bd \to e$, $ad \to e$, $ac \to e$, $ab \to de$

113

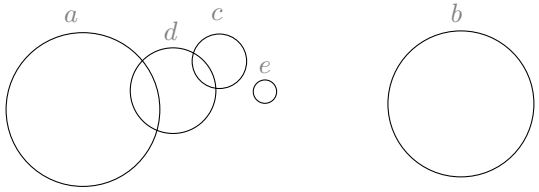


Figure 113: $ace \to d, bd \to e, bc \to e, ab \to cde$

116

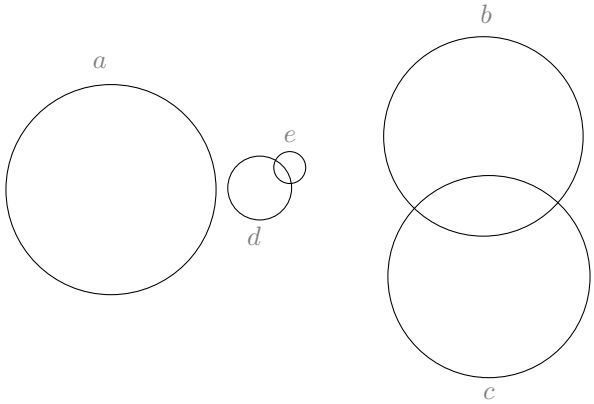


Figure 114: $bd \to e, ac \to de, ab \to de$

117

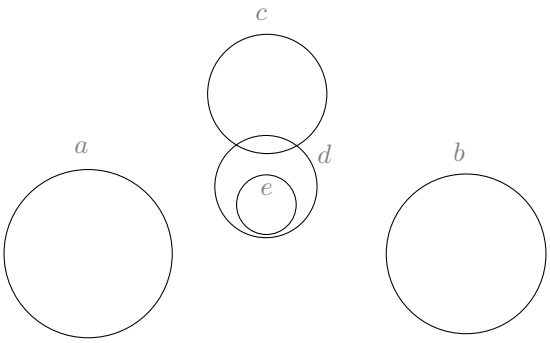


Figure 115: $d \to e, abc \to de, ab \to e$

118

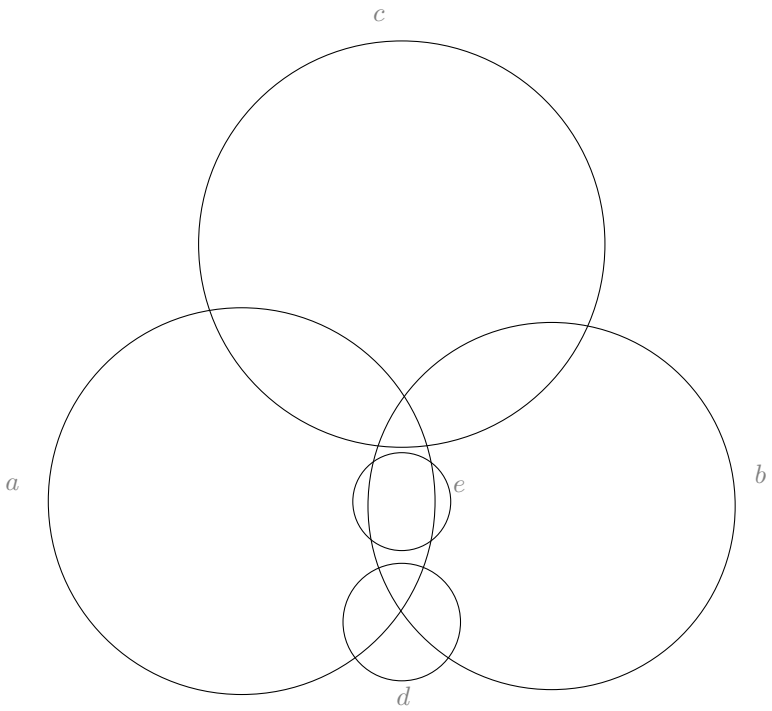


Figure 116: $cd \to e, bc \to e, ac \to e, ab \to de$

119

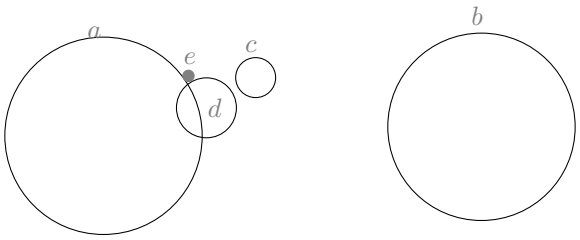


Figure 117: $ad \to e, ac \to de, ab \to cde$

120

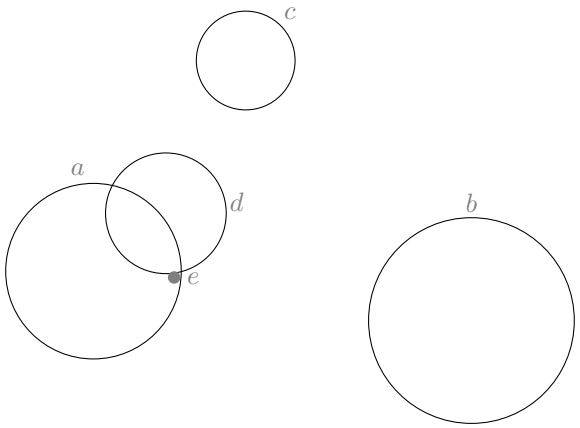


Figure 118: $bd \to e, abc \to de, a \to e$

121

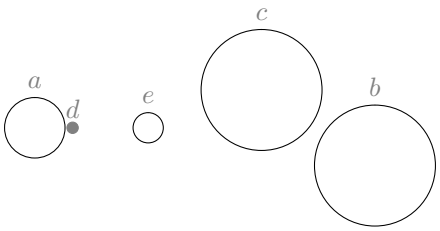


Figure 119: $ae \to d$, $bcd \to e$, $ac \to de$, $ab \to de$

123

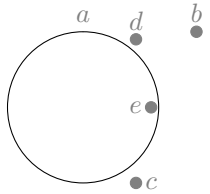


Figure 123: $a \to e$, $ab \to de$, $bcd \to e$

124

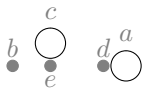


Figure 124: $ae \to d$, $bd \to e$, $ac \to d$, $ab \to de$

125

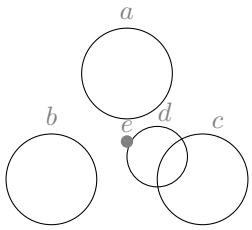


Figure 125: $bd \to e$, $ad \to e$, $abc \to de$, $bc \to e$, $ac \to e$, $ab \to e$

126

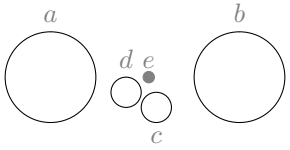


Figure 126: $bd \to e$, $ac \to d$, $ab \to cde$

127

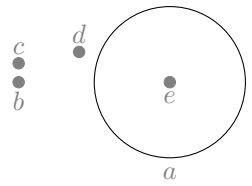


Figure 127: $a \to e, ab \to de, ac \to de$

128

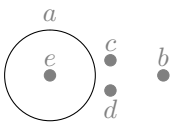


Figure 128: $a \to e$, $ab \to cde$

130

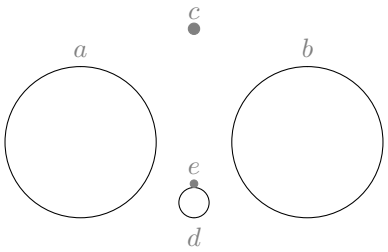


Figure 130: $cd \to e$, $bd \to e$, $ad \to e$, $ab \to de$

131

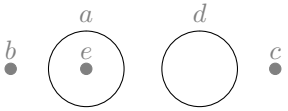


Figure 131: $a \to e, bc \to e, bd \to e$

133

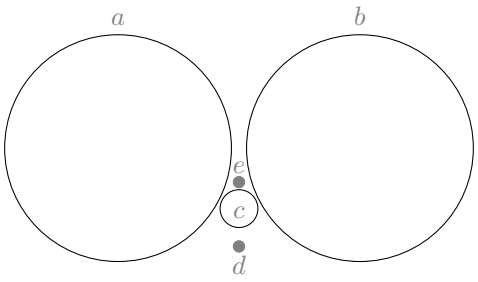


Figure 133: $ac \to e$, $bc \to e$, $ab \to cde$

135

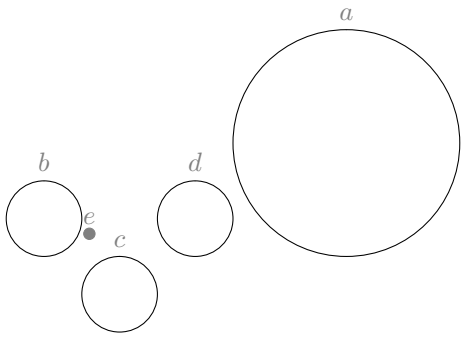


Figure 135: $ab \to de$, $ac \to d$, $bc \to e$, $bd \to e$

136

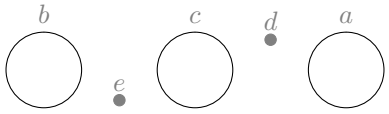


Figure 136: $ab \to cde$, $ac \to d$, $bc \to e$

137

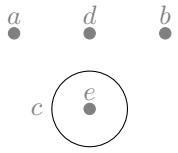


Figure 137: $ab \to d$, $c \to e$

138

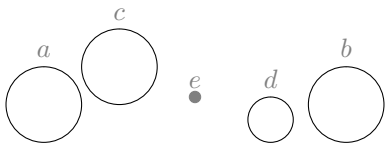


Figure 138: $ab \to de$, $bc \to e$, $ad \to e$, $cd \to e$

139

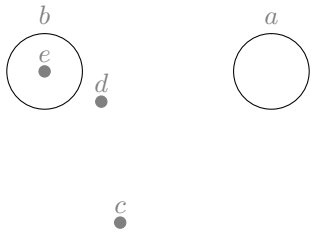


Figure 139: $b \to e$, $ab \to de$, $ace \to d$

140

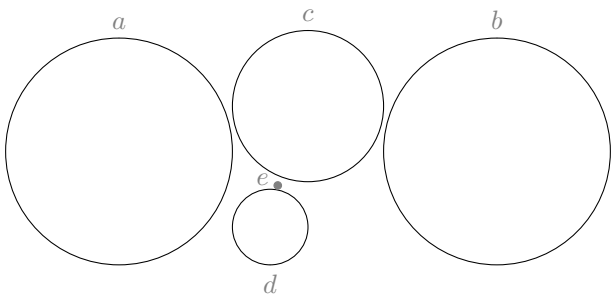


Figure 140: $ab \to de$, $ac \to e$, $ad \to e$, $bd \to e$, $cd \to e$

141

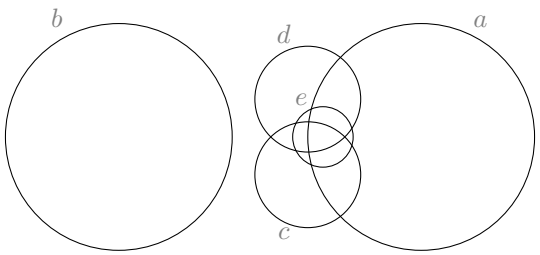


Figure 141: $cd \to e$, $ad \to e$, $ac \to e$, $ab \to cde$

142

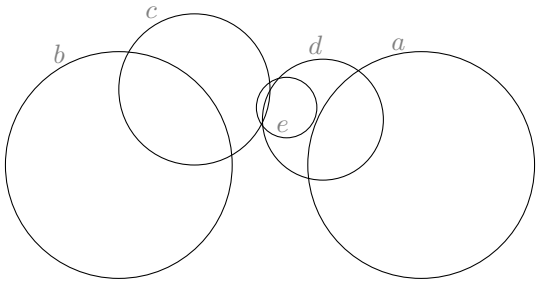


Figure 142: $cd \to e$, $bd \to e$, $ac \to de$, $ab \to de$

144

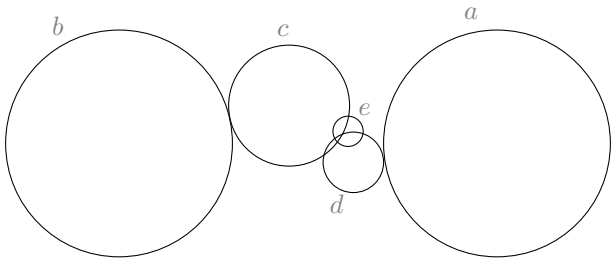


Figure 144: $cd \to e$, $ac \to de$, $ab \to cde$

145

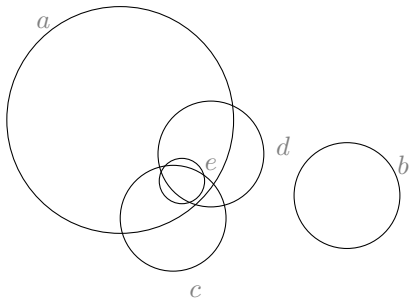


Figure 145: $a \to e, ab \to de, cd \to e$

146

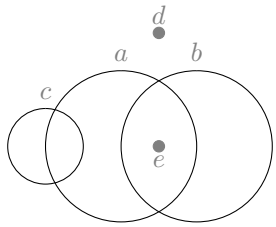


Figure 146: $a \to e$, $b \to e$

148

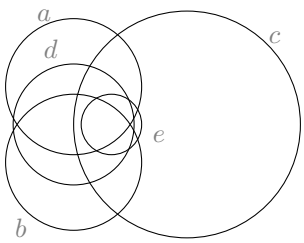


Figure 148: $c \to e, ab \to de$

149

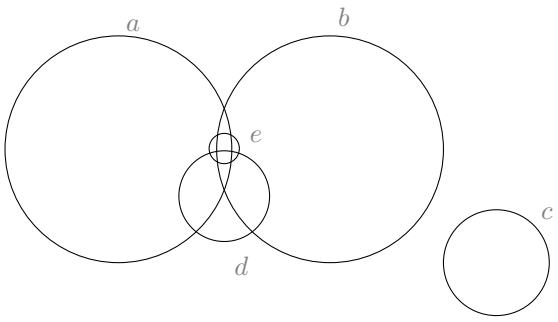


Figure 149: $ab \to de, ac \to de, ad \to e, bd \to e$

150

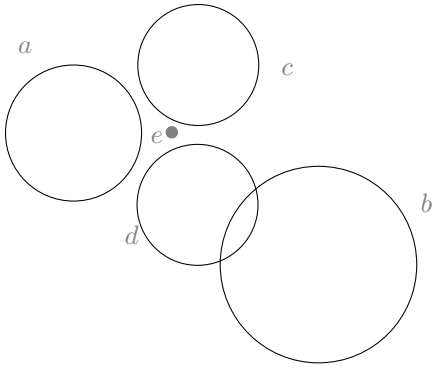


Figure 150: $cd \to e, ad \to e, bc \to e, ac \to e, ab \to de$

151

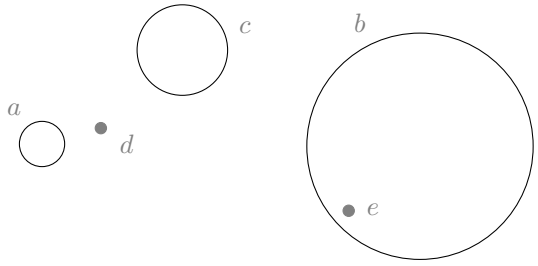


Figure 151: $ac \to d, ab \to de, b \to e$

152

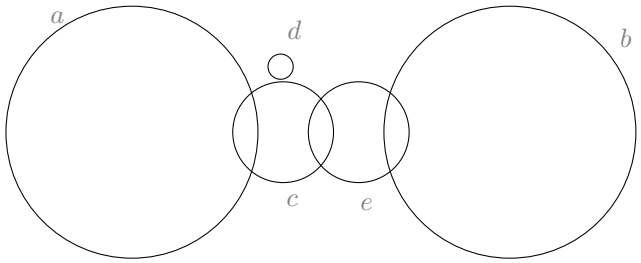


Figure 152: $ae \to cd, bc \to e, ab \to cde$

154

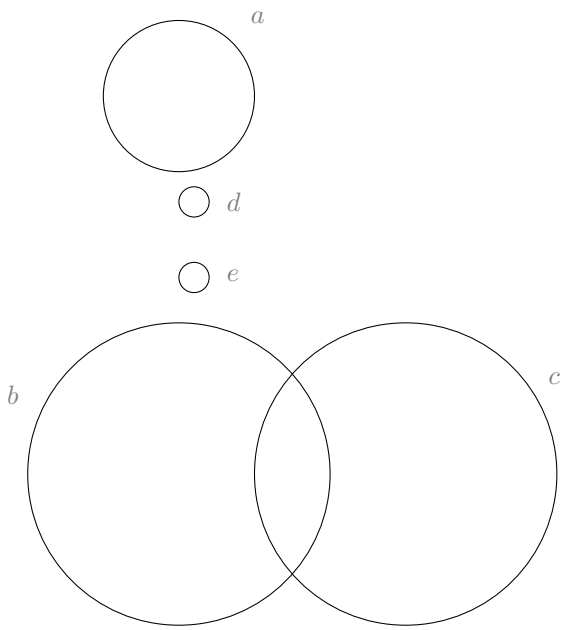


Figure 153: $ae \to d, bd \to e, ac \to de, ab \to de$

155

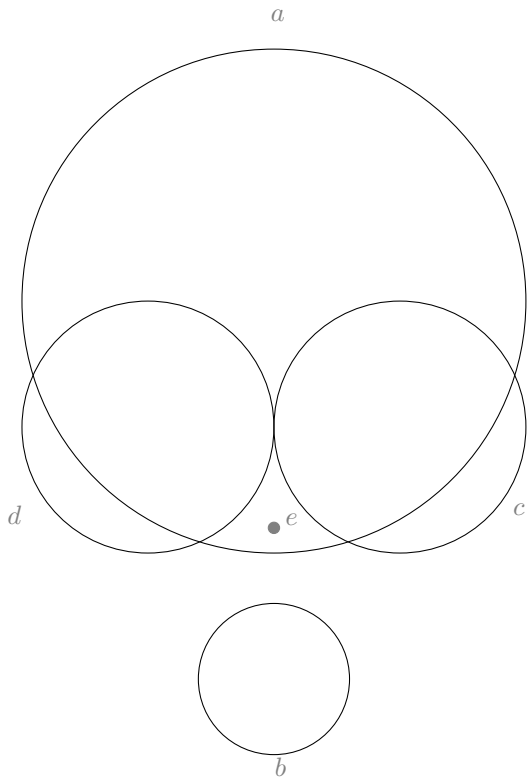


Figure 154: $cd \to e, bd \to e, bc \to e, a \to e$

156

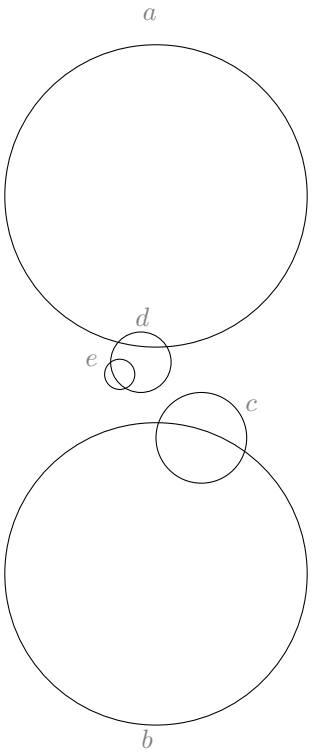


Figure 155: $bd \to e, ac \to de, ab \to cde$

157

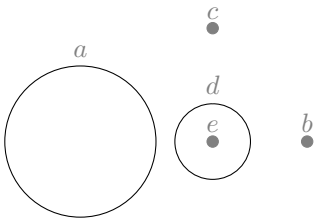


Figure 156: $d \to e, ab \to de$

158

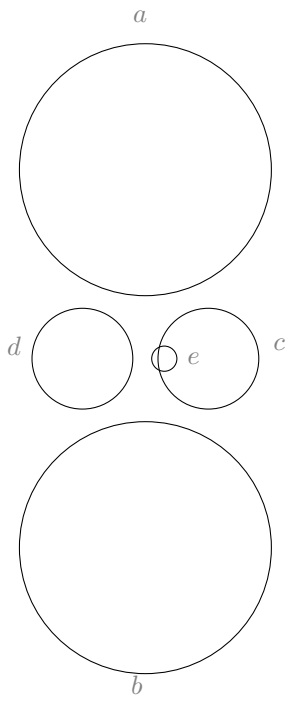


Figure 157: $cd \to e, ac \to e, bc \to e, ab \to cde$

159

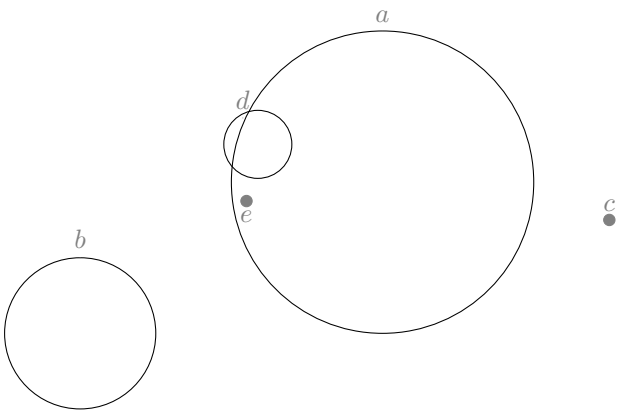


Figure 158: $a \to e, bd \to e, ab \to de$

160

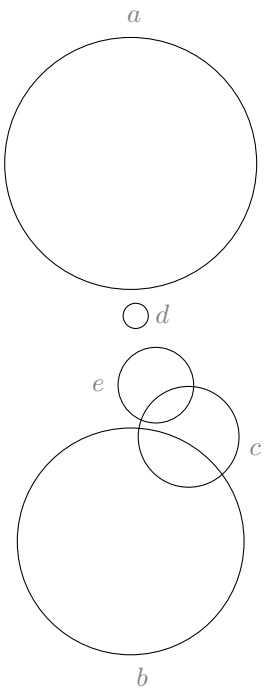


Figure 159: $ae \to d, bcd \to e, ac \to de, ab \to cde$

162

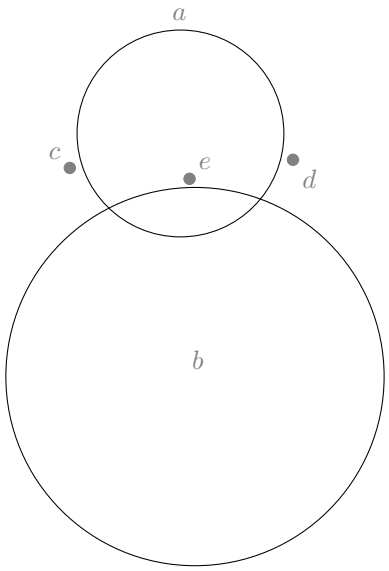


Figure 162: $a \to e$, $ab \to cde$, $bcd \to e$

163

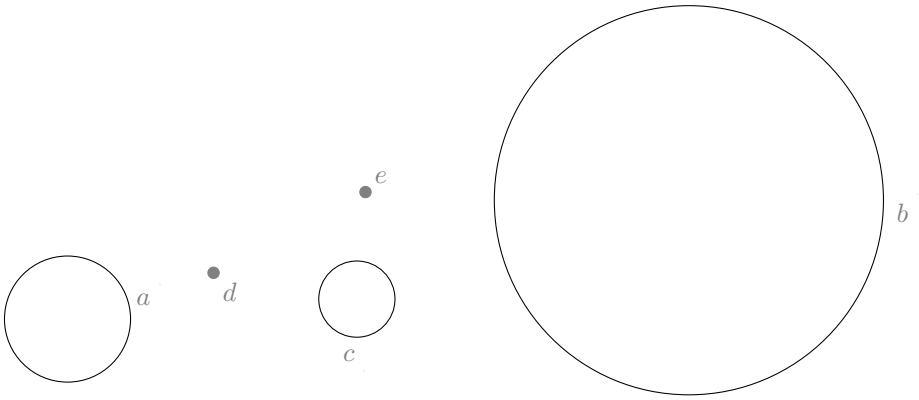


Figure 163: $ae \to d, bd \to e, ac \to d, ab \to cde$

164

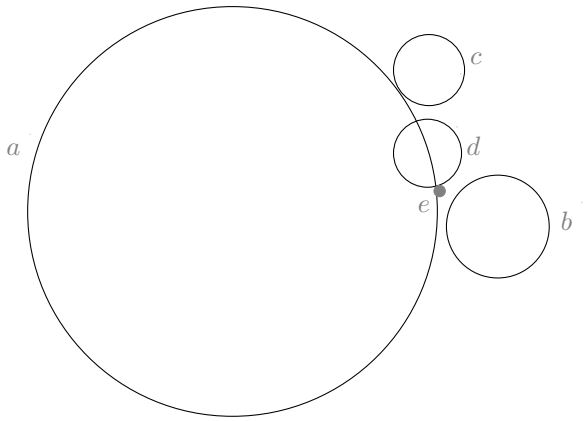


Figure 164: $ab \to de, ac \to e, ad \to e, bc \to e, bd \to e$

165

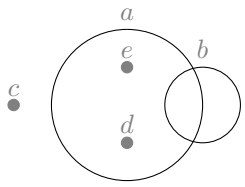


Figure 165: $a \to de$

166

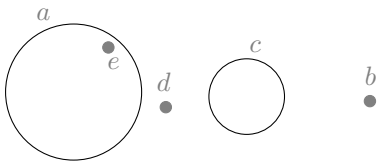


Figure 166: $a \to e, ab \to cde, ac \to de$

167

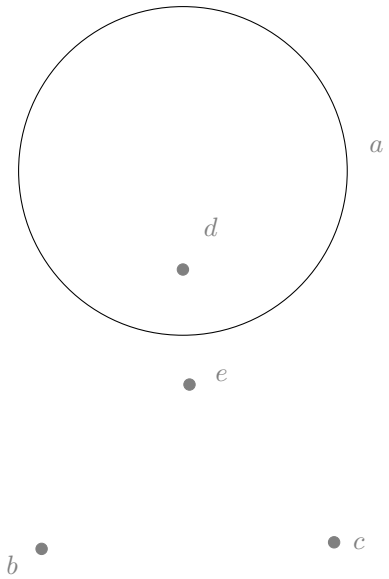


Figure 167: $a->d$,$ab->de$,$ac->de$,$bcd->e$

168

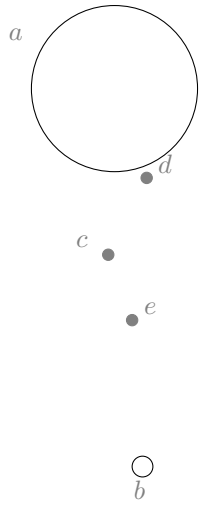


Figure 168: $ab \to cde, ac \to d, ae \to cd, bcd \to e$

169

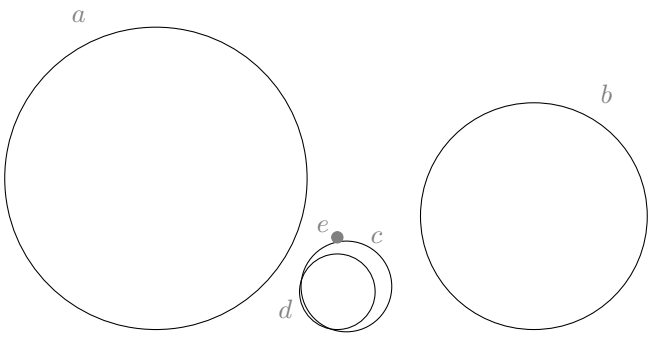


Figure 169: $ab \to de, ac \to de, ad \to e, bc \to e$

170

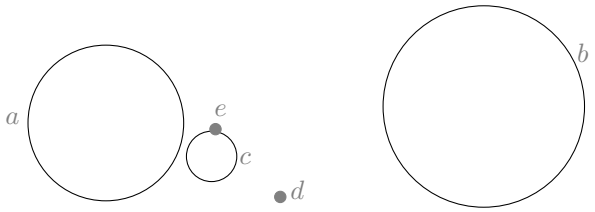


Figure 170: $ab \to cde, ac \to e, ad \to e, bc \to e$

171

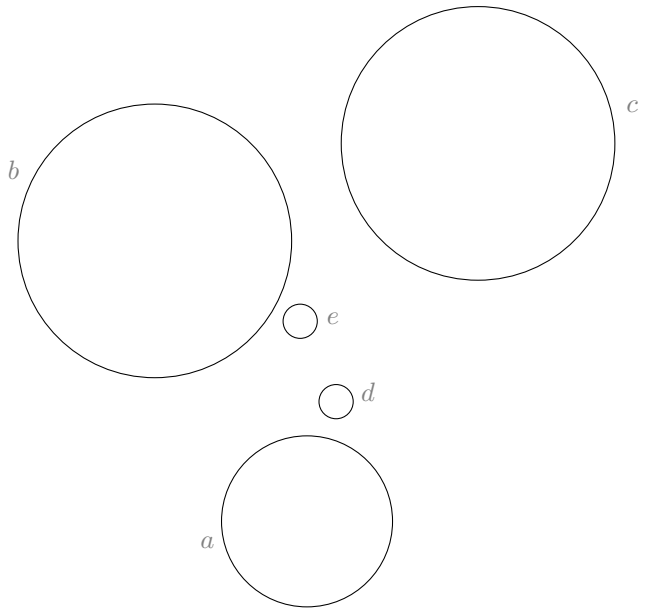


Figure 171: $ab \to de, ac \to de, bc \to e, bd \to e$

172

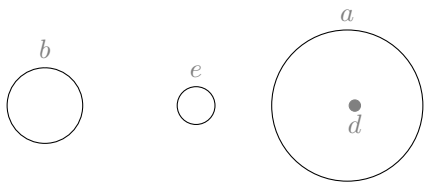


Figure 172: $bd \to e$, $ab \to de$, $a \to d$

173

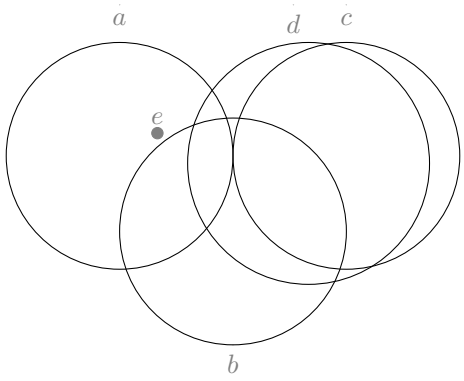


Figure 173: $abc \to d$, $bc \to e$, $bd \to e$, $a \to e$

174

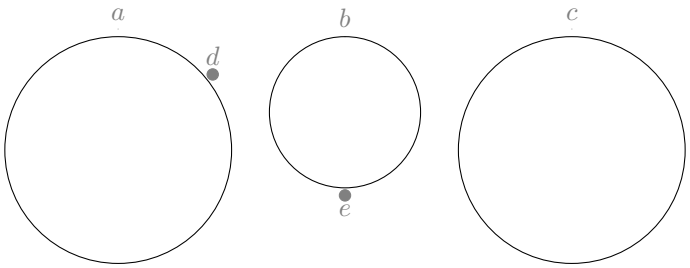


Figure 174: $ab \to de$, $ac \to de$, $bc \to e$, $ae \to d$

176

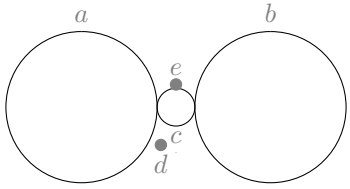


Figure 176: $bc \to e$, $ac \to de$, $ab \to cde$

177

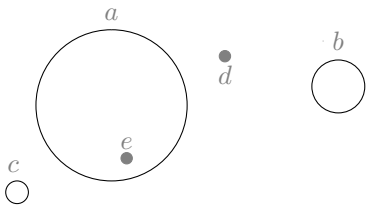


Figure 177: $bc \to e$, $ab \to de$, $a \to e$

178

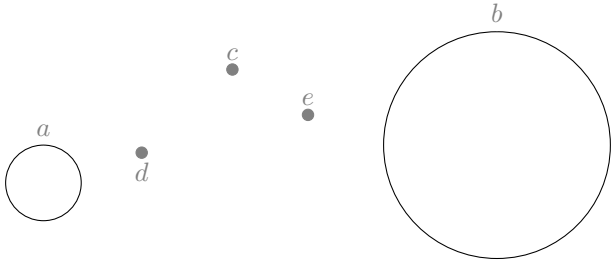


Figure 178: $ae \to d$, $bd \to e$, $bc \to e$,$ac \to d$, $ab \to de$

179

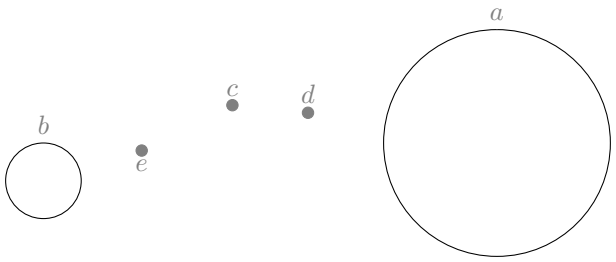


Figure 179: $bd \to e$, $bc \to e$, $ac \to d$, $ab \to cde$

180

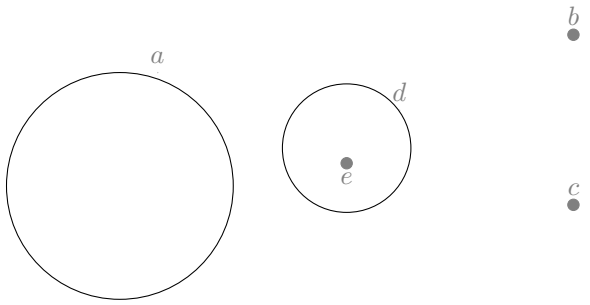


Figure 180: $d \to e$, $abc \to de$, $ac \to e$, $ab \to e$

181

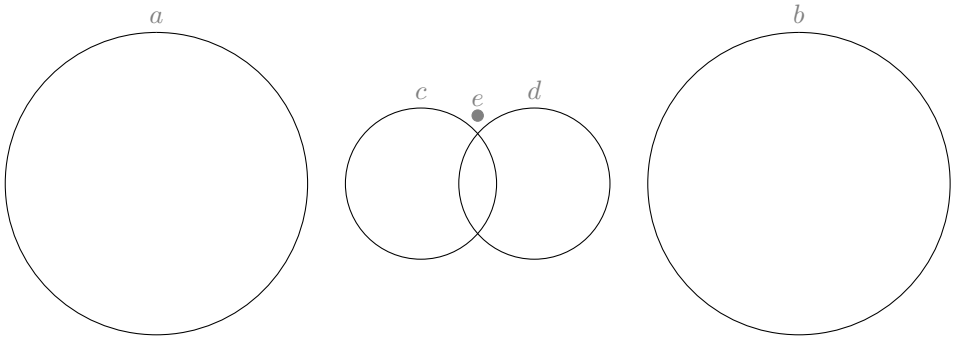


Figure 181: $cd \to e$, $ad \to e$, $bc \to e$, $ab \to cde$

182

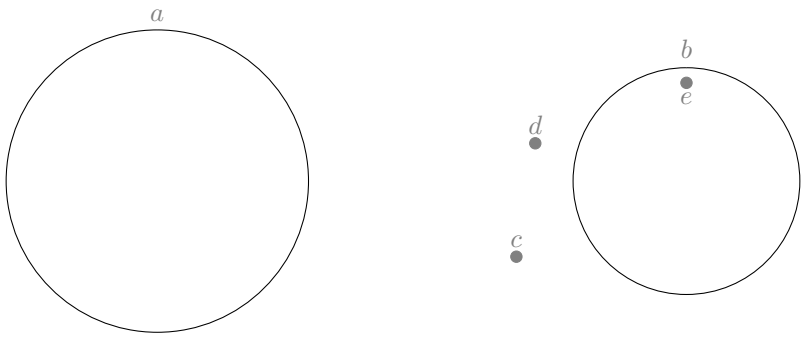


Figure 182: $ace \to d$, $ab \to cde$, $b \to e$

183

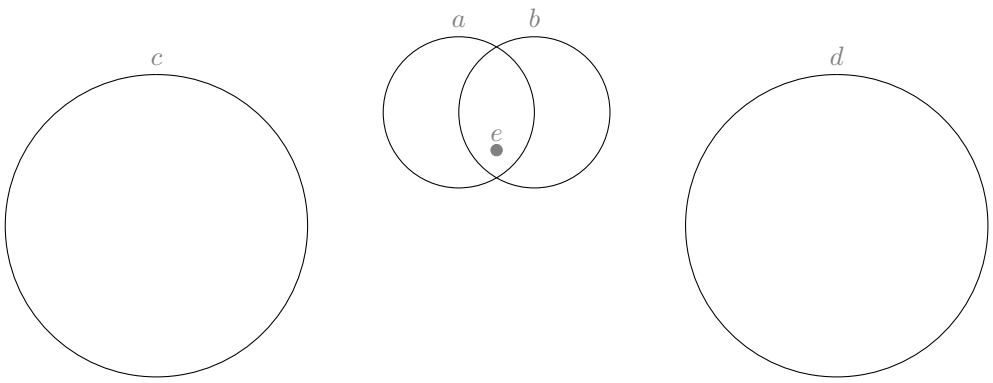


Figure 183: $cd \to e$, $b \to e$, $a \to e$

184

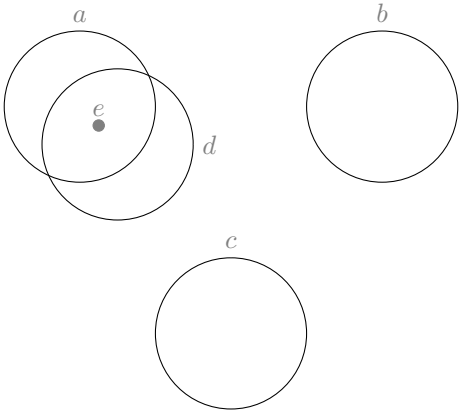


Figure 184: $d \to e$, $abc \to de$, $a \to e$

185

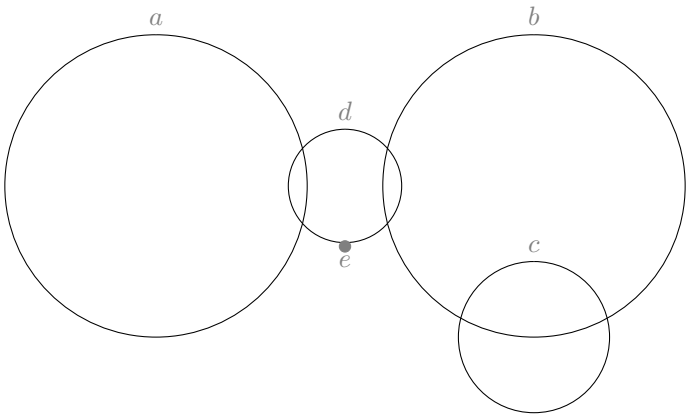


Figure 185: $cd \to e$, $bd \to e$, $ad \to e$, $ac \to de$, $ab \to de$

186

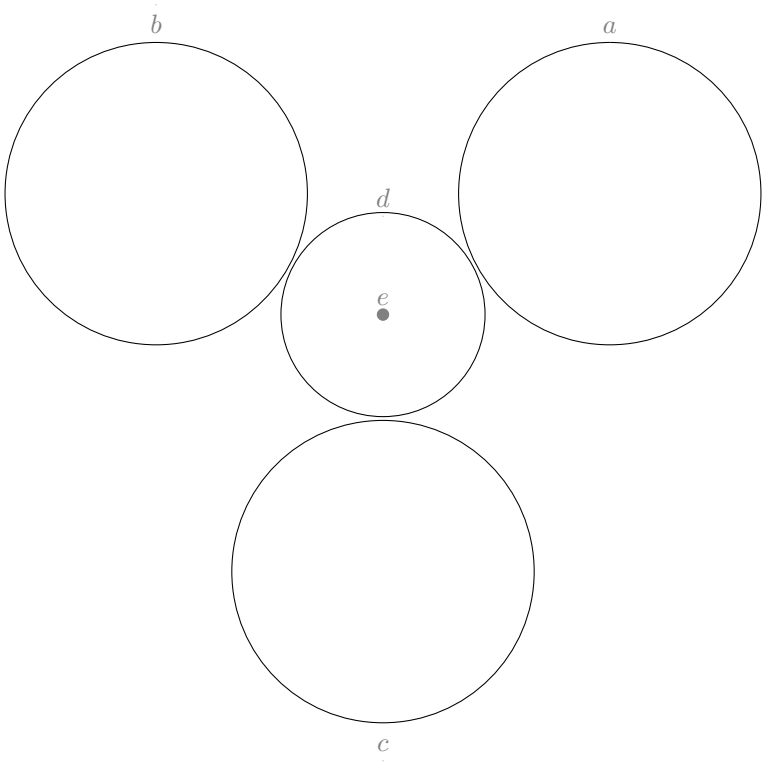


Figure 186: $d \to e$, $abc \to de$, $bc \to e$, $ac \to e$, $ab \to e$

187

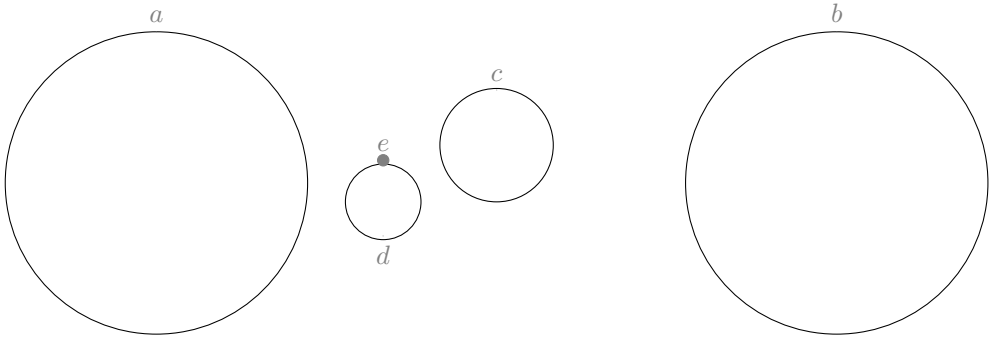


Figure 187: $cd \to e$, $ad \to e$, $ac \to de$, $ab \to cde$

188

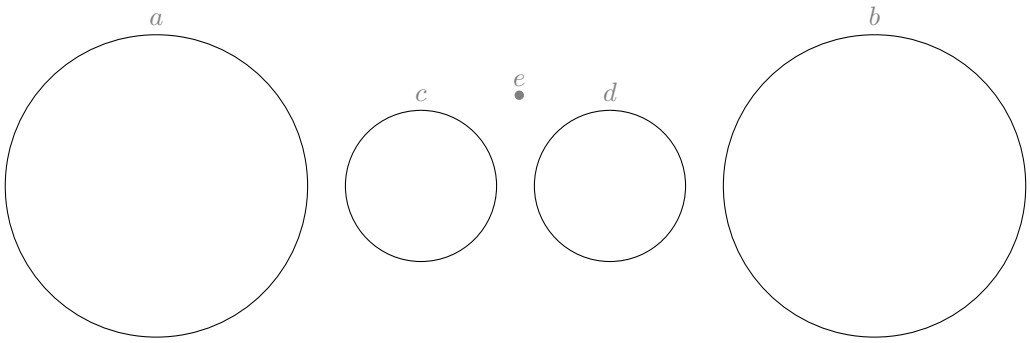


Figure 188: $ad \to ce$, $bc \to de$, $ab \to cde$

189

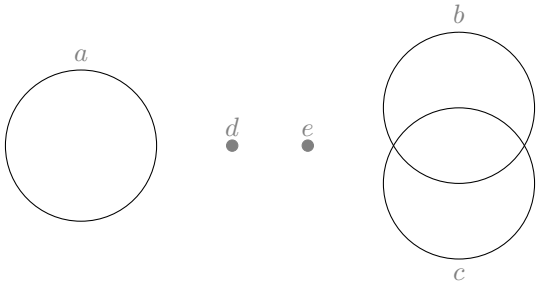


Figure 189: $ae \to d$, $cd \to e$, $bd \to e$, $ac \to de$, $ab \to de$

190

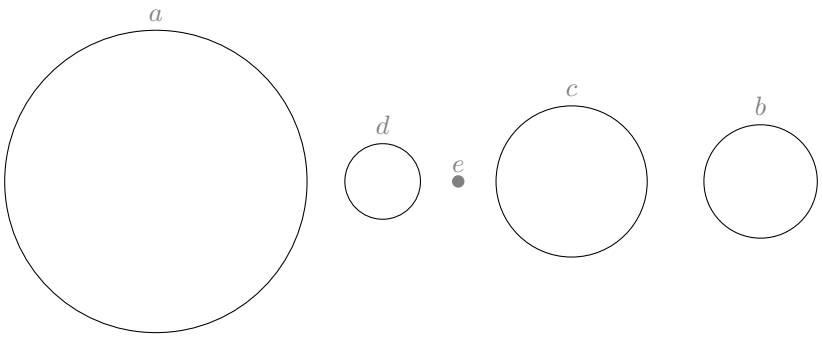


Figure 190: $cd \to e$, $bd \to e$, $ac \to de$, $ab \to cde$

191

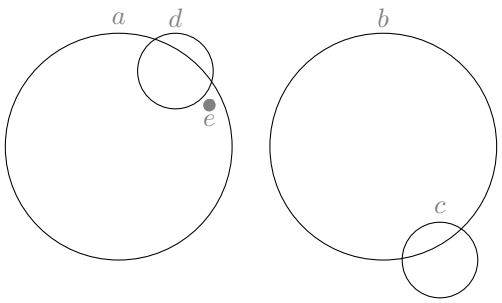


Figure 191: $cd \to e$, $bd \to e$, $ab \to de$, $a \to e$

192

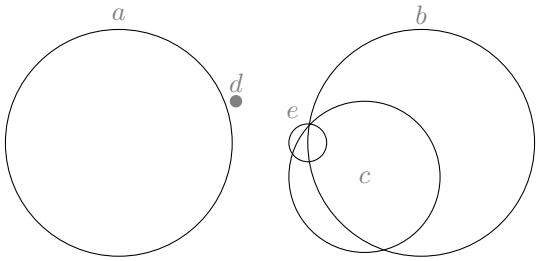


Figure 192: $ae \to d$, $cd \to e$, $ac \to de$, $ab \to cde$

193

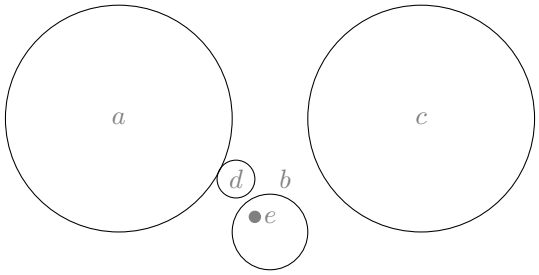


Figure 193: $ac \to de$,$ab \to de$, $b \to e$

194

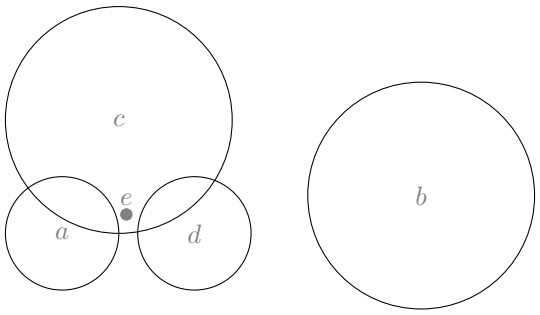


Figure 194: $ad \to e$, $c \to e$, $ab \to de$

195

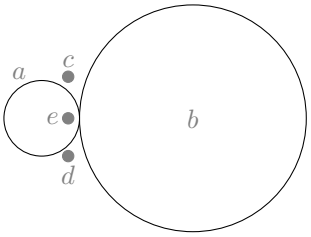


Figure 195: $cd \to e$, $ab \to cde$, $a \to e$

196

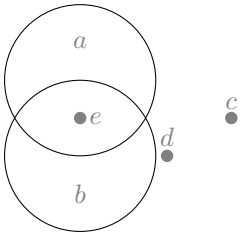


Figure 196: $abc \to de$, $b \to e$, $a \to e$

197

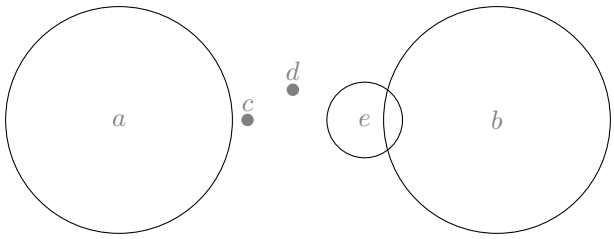


Figure 197: $ae \to cd$, $ad \to c$, $bc \to e$, $ab \to cde$

198

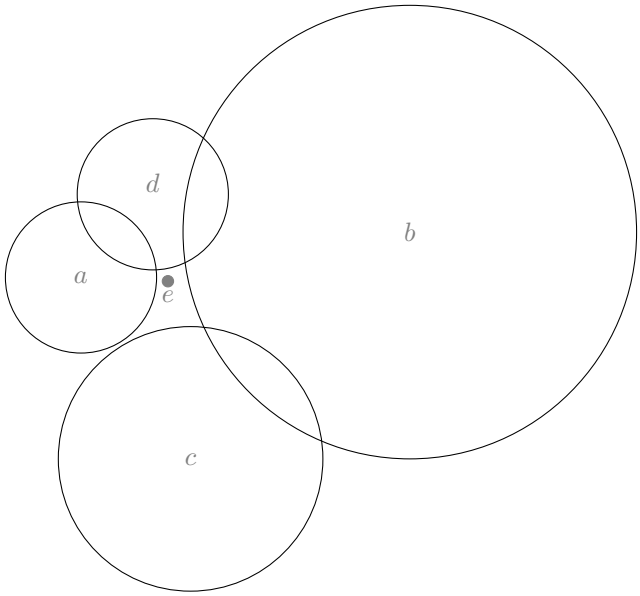


Figure 198: $cd \to e$, $bd \to e$, $ad \to e$, $bc \to e$, $ac \to e$, $ab \to de$

199

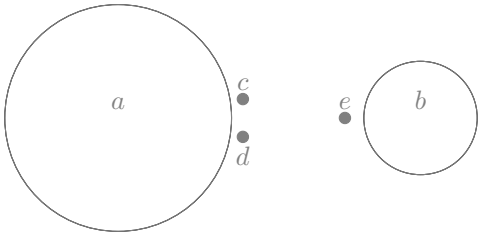


Figure 199: $ae \to cd$, $bd \to e$, $bc \to e$, $ab \to cde$

200

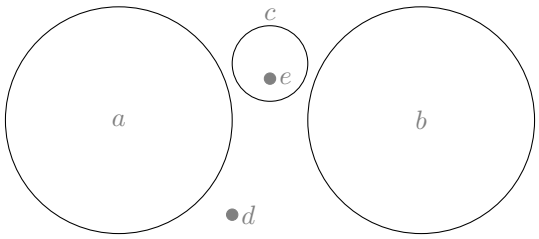


Figure 200: $c \to e$, $ab \to cde$

201

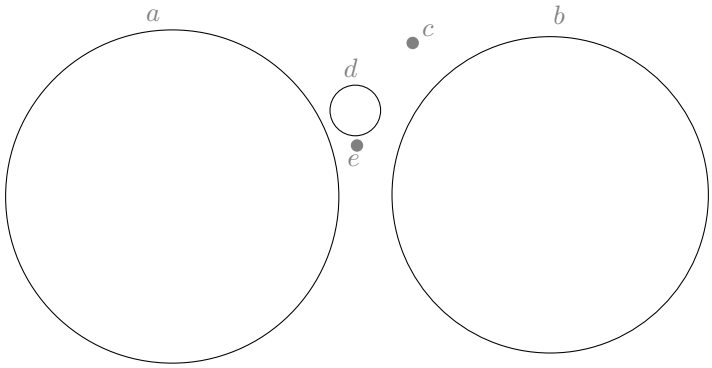


Figure 201: $bd \to e, ad \to e, ac \to de, ab \to cde$

202

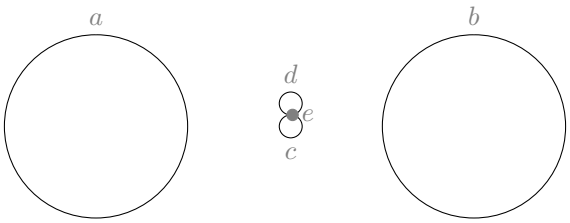


Figure 202: $cd \to e$, $ad \to e$, $bc \to e$, $ac \to e$, $ab \to cde$

203

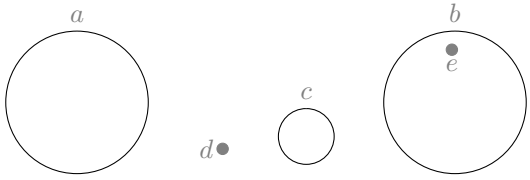


Figure 203: $ac \to d$, $ab \to cde$, $b \to e$

204

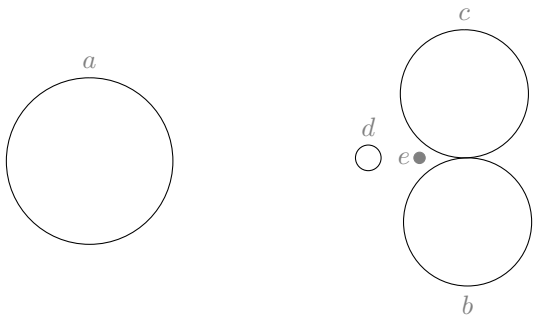


Figure 204: $cd \to e$, $bd \to e$, $bc \to e$, $ac \to de$, $ab \to de$

205

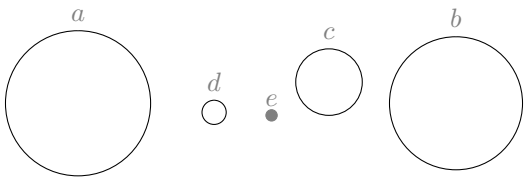


Figure 205: $ae \to d$, $bd \to e$, $ac \to de$, $ab \to cde$

207

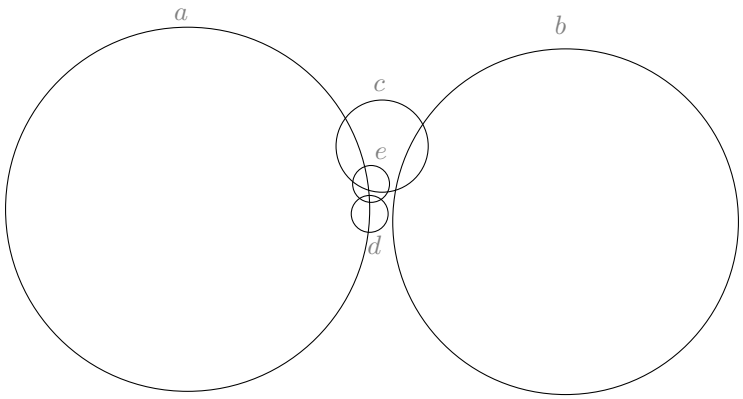


Figure 207: $cd \to e, bc \to e, ac \to de, ab \to cde$

208

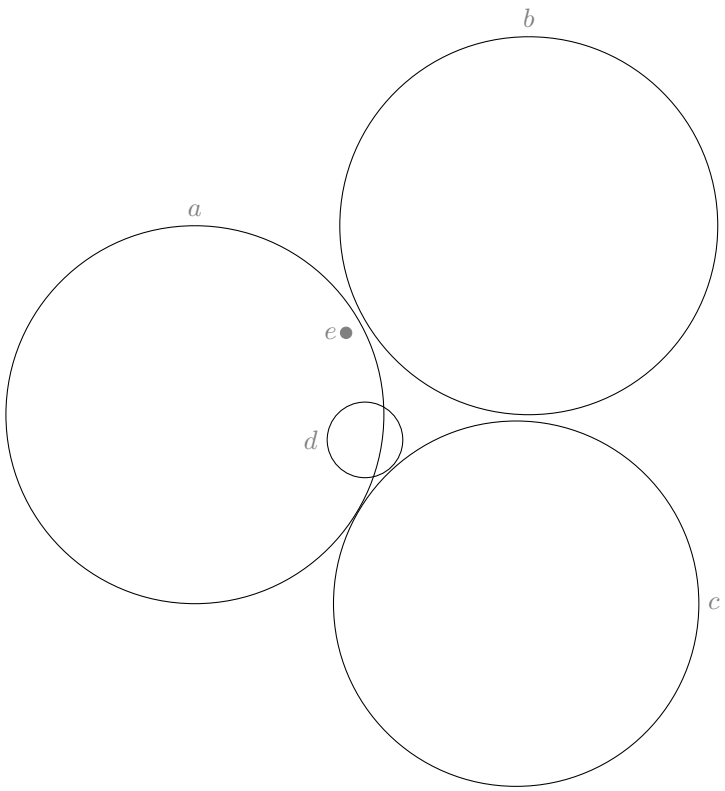


Figure 208: $bd \to e$, $ac \to de$, $ab \to de$, $a \to e$

209

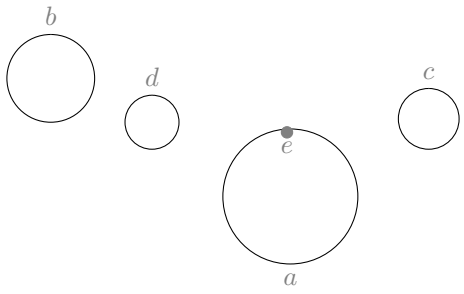


Figure 209: $cd \to e$, $bc \to e$, $ab \to de$, $a \to e$

210

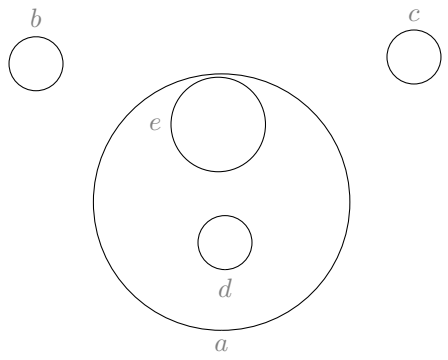


Figure 210: $bcd \to e$, $a \to de$

212

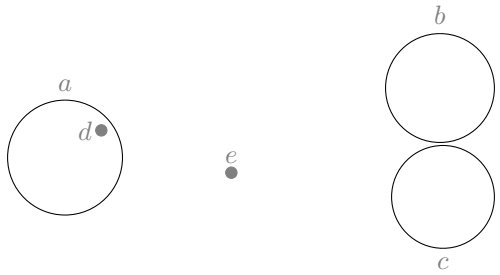


Figure 212: $bd \to e$, $ac \to de$, $ab \to de$, $a \to d$

213

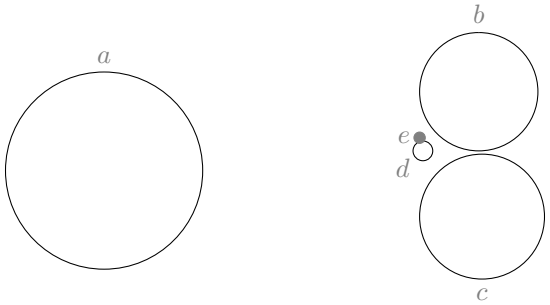


Figure 213: $bd \to e$, $ad \to e$, $bc \to e$, $ac \to de$, $ab \to de$

214

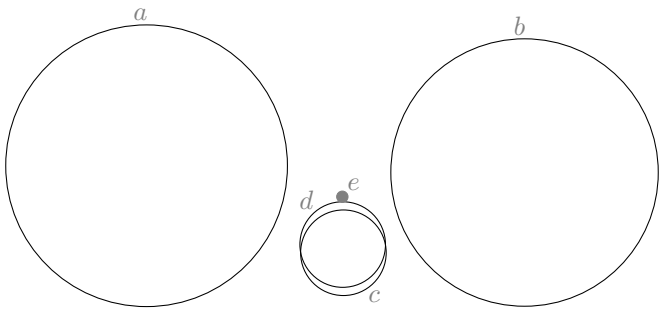


Figure 214: $bd \to e, ad \to e, bc \to e, ac \to e, ab \to cde$

215

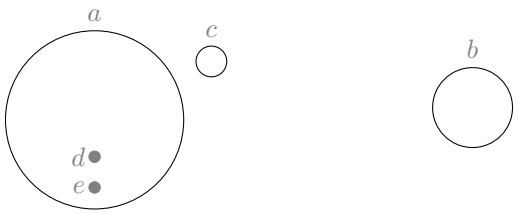


Figure 215: $ab \to cde$, $a \to de$

216

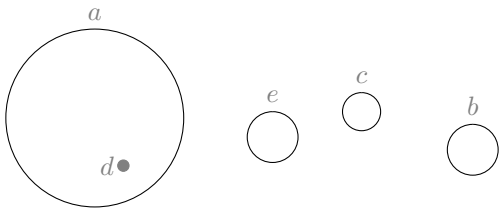


Figure 216: $bcd \to e$, $ac \to de$, $ab \to cde$, $a \to d$

217

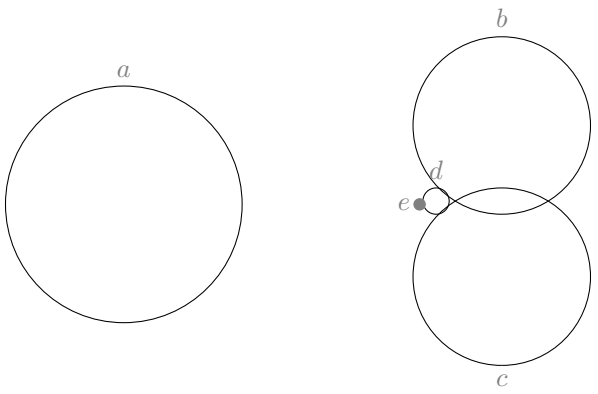


Figure 217: $ad \to e$, $bc \to de$, $ac \to de$, $ab \to de$

218

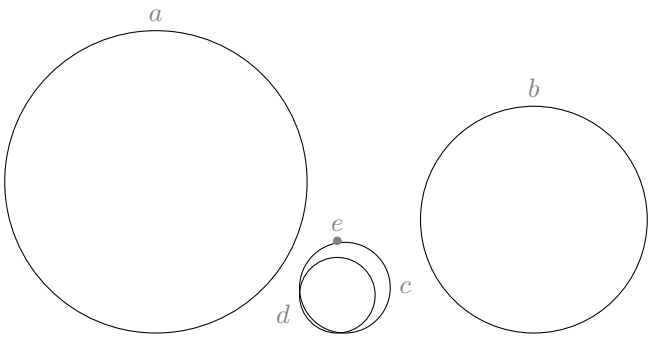


Figure 218: $ad \to e$, $bc \to e$, $ac \to de$, $ab \to cde$

219

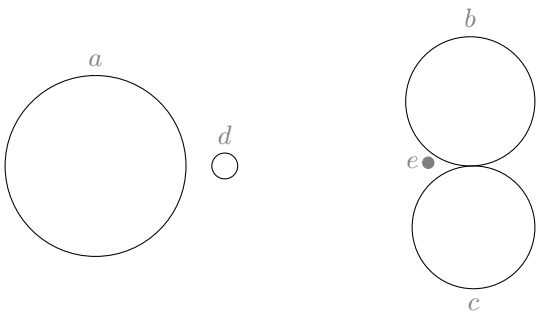


Figure 219: $ae \to d$, $bd \to e$, $bc \to e$, $ac \to de$, $ab \to de$

220

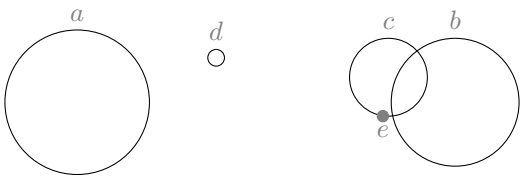


Figure 220: $bd \to e$, $bc \to e$, $ac \to de$, $ab \to cde$

221

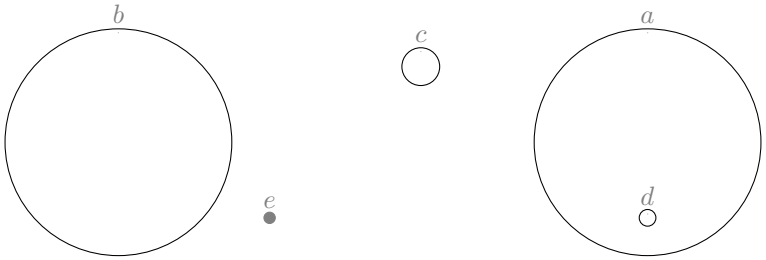


Figure 221: $bd \to e$, $ab \to cde$, $a \to d$

222

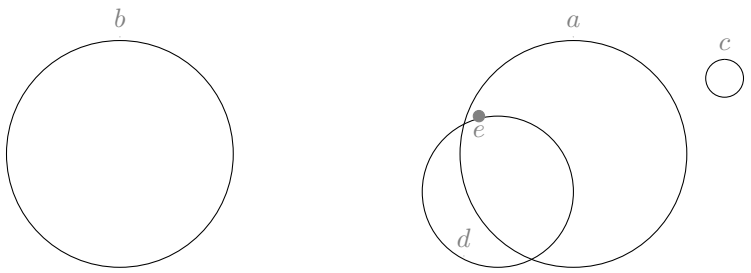


Figure 222: $bd \to e$, $bc \to e$, $ab \to de$, $a \to e$

223

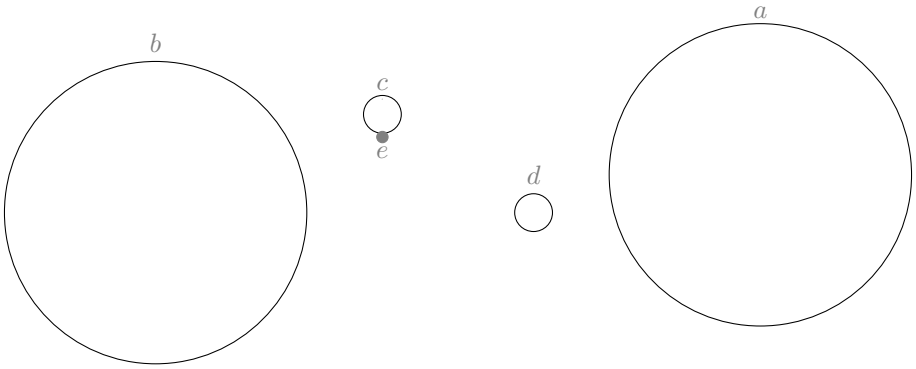


Figure 223: $ae \to d$, $bc \to e$, $ac \to de$, $ab \to cde$

224

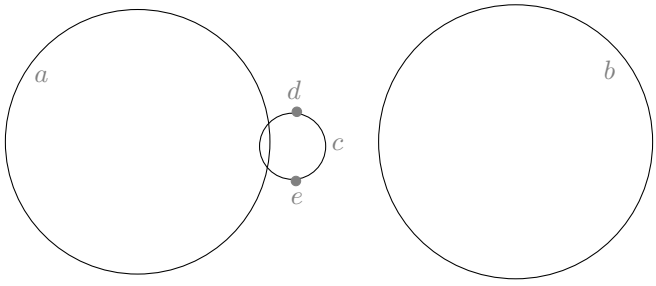


Figure 224: $ab \to cde, ac \to de, bc \to de$

225

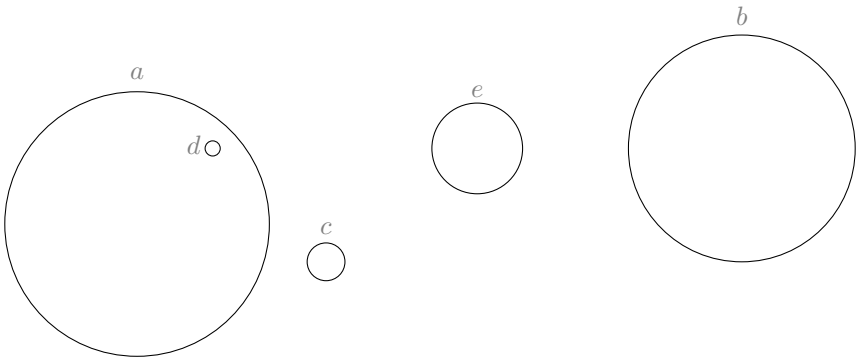


Figure 225: $ae \to cd, bcd \to e, ab \to cde,\ a \to d$

226

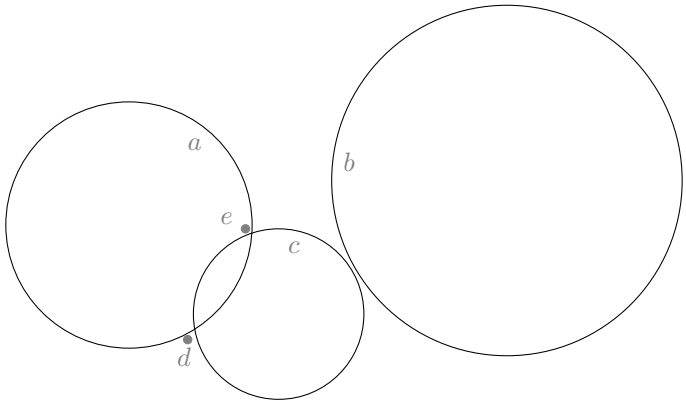


Figure 226: $bc \to e, ac \to de, ab \to de, a \to e$

227

Figure 227: $bc \to e, ab \to cde, a \to e$

228

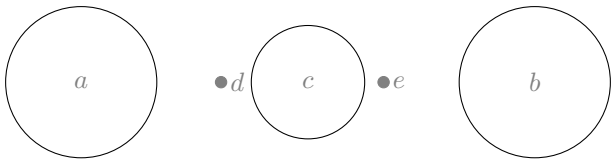


Figure 228: $ae \to d$, $bd \to e$, $ac \to d$, $ab \to cde$

229

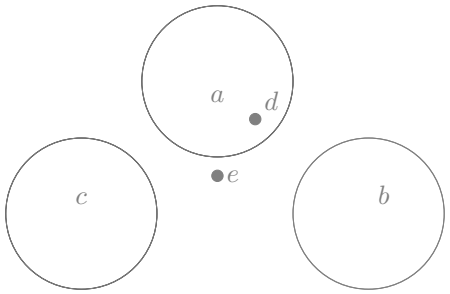


Figure 229: $bc \to e$, $ac \to de$, $ab \to de$, $a \to d$

230

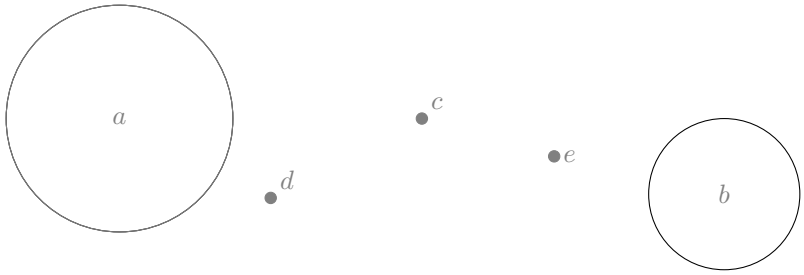


Figure 230: $ae \to cd$, $bc \to e$, $ac \to d$, $ab \to cde$

231

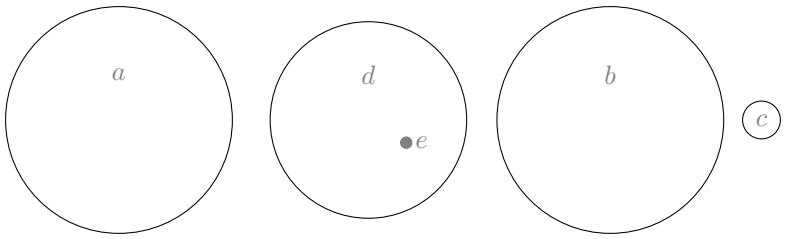


Figure 231: $d \to e$, $ac \to e$, $ab \to de$

232

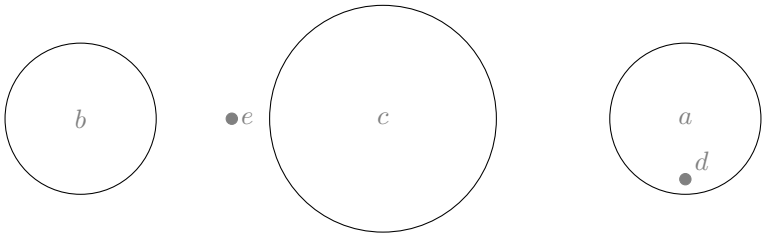


Figure 232: $bd \to e$, $bc \to e$, $ab \to de$, $a \to d$

233

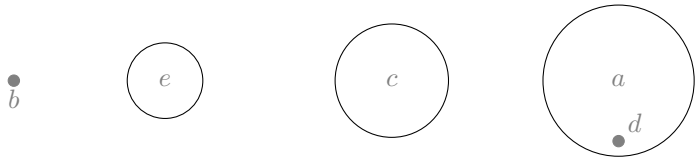


Figure 233: $ae \to cd$, $bc \to e$, $ab \to cde$, $a \to d$

234

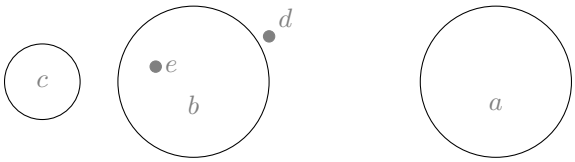


Figure 234: $cd \to e$, $ac \to de$, $ab \to de$, $b \to e$

236

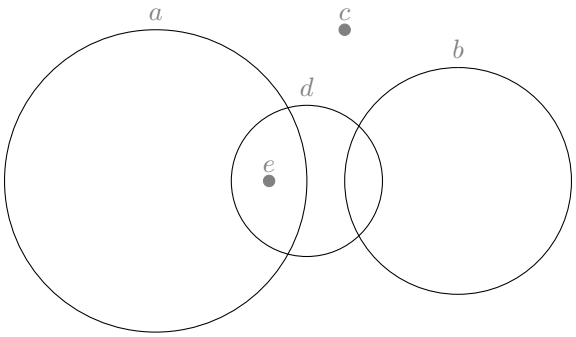


Figure 236: $a \to e$, $d \to e$, $ab \to de$

237

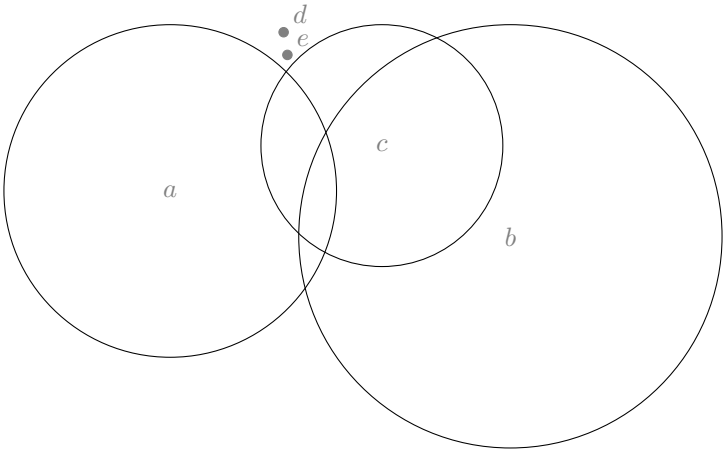


Figure 237: $cd \to e$, $bd \to e$, $ad \to e$, $ac \to de$, $ab \to cde$

238

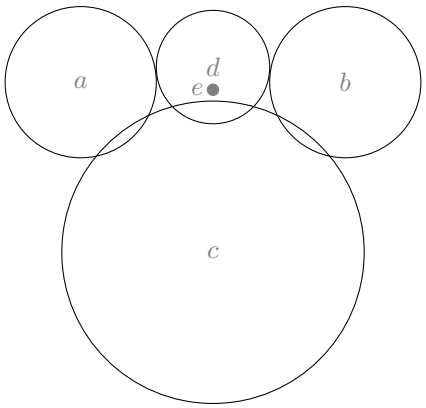


Figure 238: $d \to e$, $bc \to e$, $ac \to e$, $ab \to de$

239

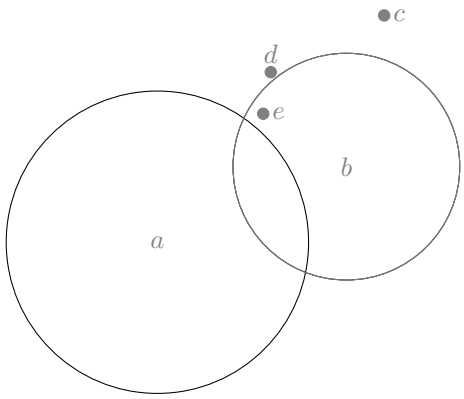


Figure 239: $ad \to e$, $ac \to de$, $ab \to de$, $b \to e$

240

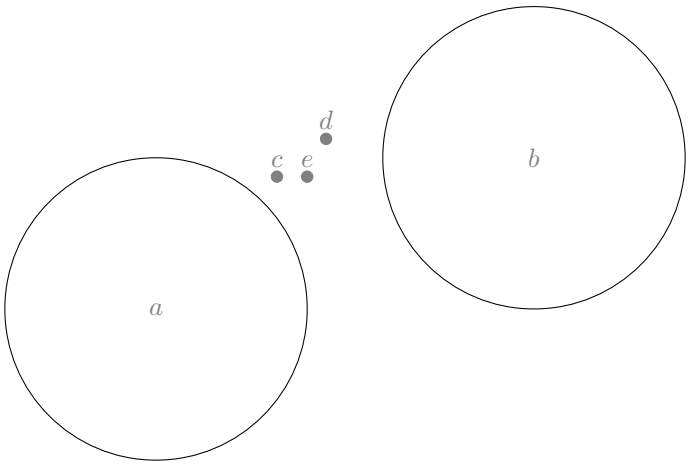


Figure 240: $ae \to c$, $ad \to ce$, $bc \to de$, $ab \to cde$

241

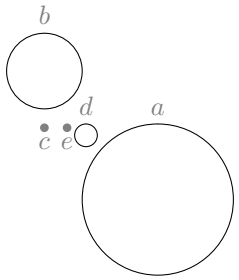


Figure 241: $ae \to d$, $cd \to e$, $bd \to e$, $ac \to de$, $ab \to cde$

242

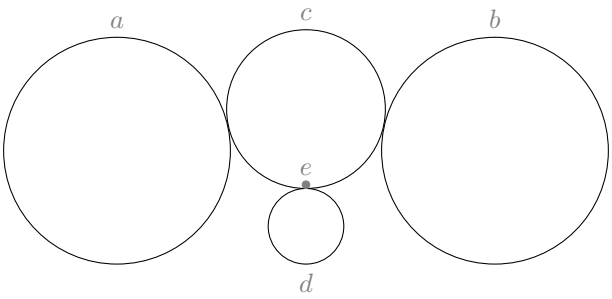


Figure 242: $c \to e$, $ad \to e$, $bd \to e$, $ab \to de$

243

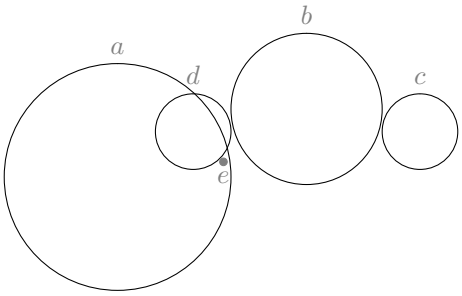


Figure 243: $a \to e$, $ab \to de$, $ac \to de$, $bd \to e$, $cd \to e$

244

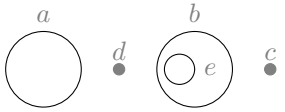


Figure 244: $b \to e$, $ab \to de$, $ac \to de$, $ae \to d$

245

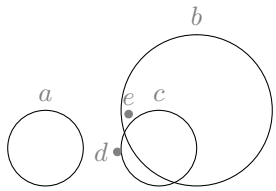


Figure 245: $b \to e$, $ab \to cde$, $ac \to de$

246

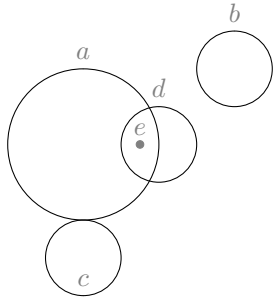


Figure 246: $a \to e$, $d \to e$, $bc \to e$, $abc \to d$

247

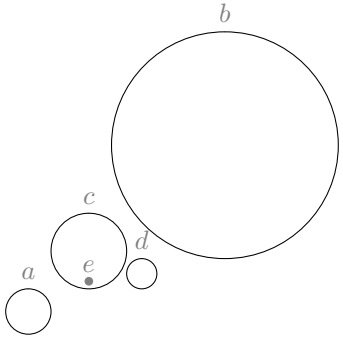


Figure 247: $c \to e$, $ad \to e$, $ab \to cd$

248

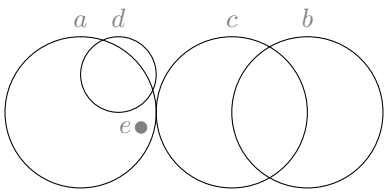


Figure 248: $a \to e$, $cd \to e$, $ac \to de$, $ab \to cde$

249

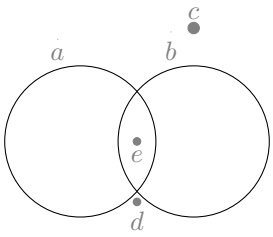


Figure 249: $a \to e, b \to e, ab \to de$

250

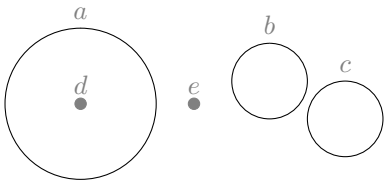


Figure 250: $a \to d$, $ab \to de$, $ac \to de$, $bd \to e$, $cd \to e$

251

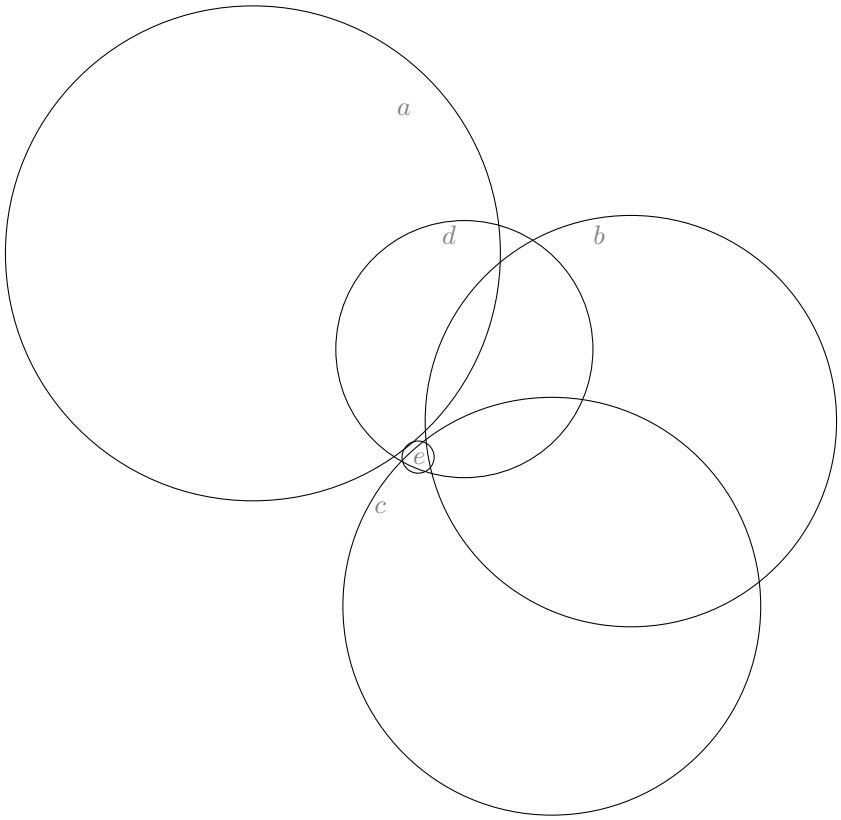


Figure 251: $cd \to e, bd \to e, ad \to e, bc \to e, ac \to de, ab \to de$

252

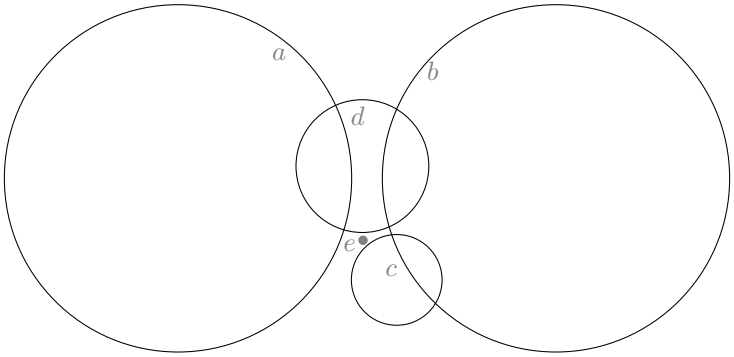


Figure 252: $cd \to e$, $bd \to e$, $ad \to e$, $bc \to e$, $ac \to e$, $ab \to cde$

253

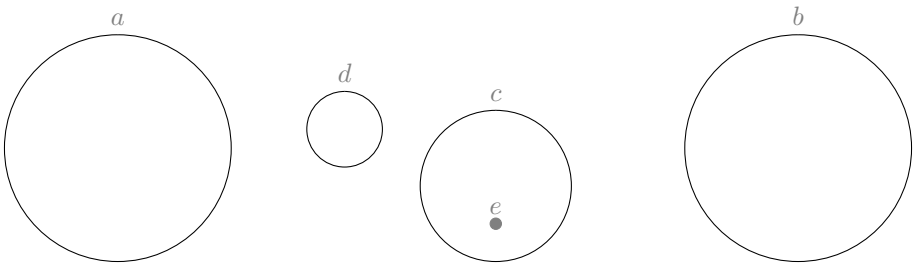


Figure 253: $ab \to cde$, $ac \to de$, $c \to e$

254

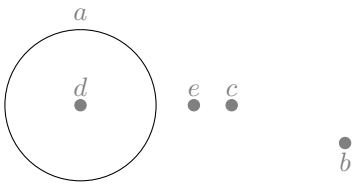


Figure 254: $ab \to cde$, $ac \to de$, $a \to d$, $cd \to e$

255

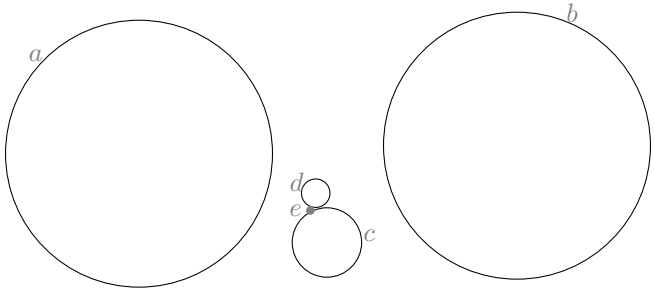


Figure 255: $ab \to cde, ac \to de, bc \to e, ad \to e, cd \to e$

256

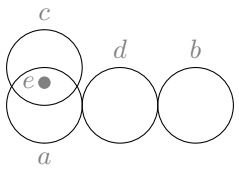


Figure 256: $ab \to de$, $a \to e$, $c \to e$

257

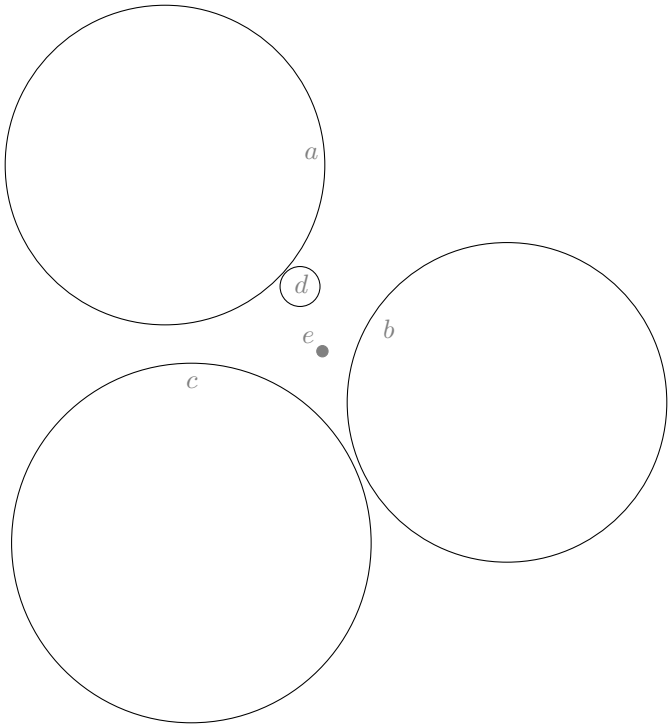


Figure 257: $ae \to d, cd \to e, bd \to e, bc \to e, ac \to de, ab \to de$

258

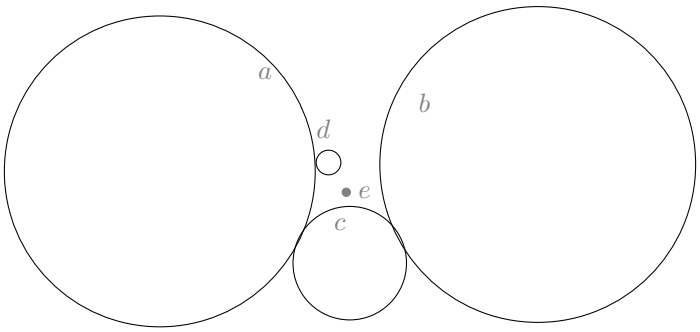


Figure 258: $cd \to e, bd \to e, bc \to e, ac \to de, ab \to cde$

259

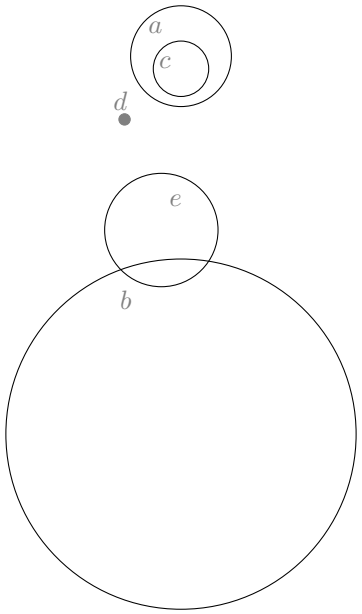


Figure 259: $ae \to cd$, $bc \to e$, $ab \to cde$, $a \to c$

260

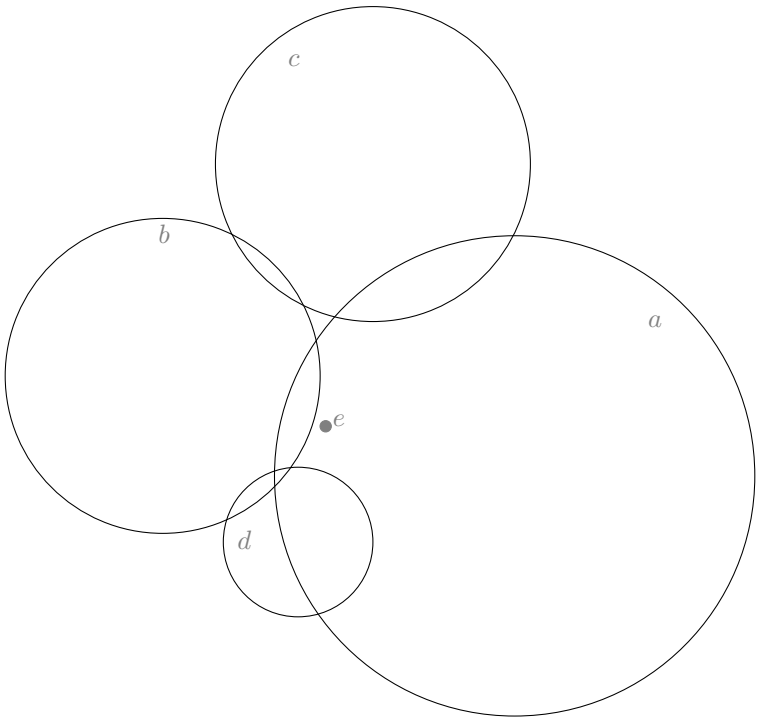


Figure 260: $cd \to e, bd \to e, bc \to e, ab \to de, a \to e$

261

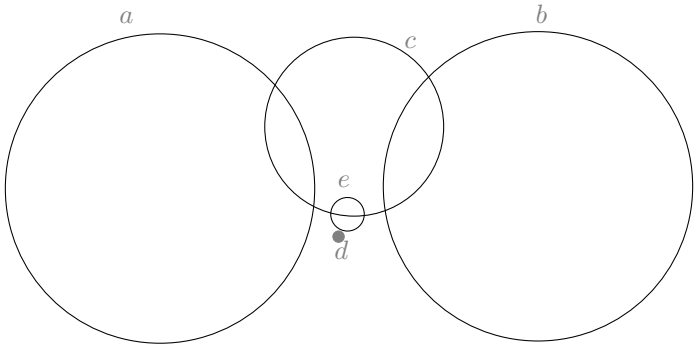


Figure 261: $ab \to cde, ac \to de, bc \to e, cd \to e, ae \to d$

262

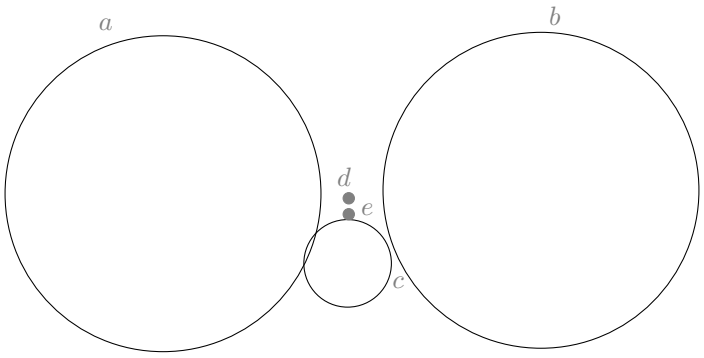


Figure 262: $ab \to cde, ac \to de, bc \to de, cd \to e$

263

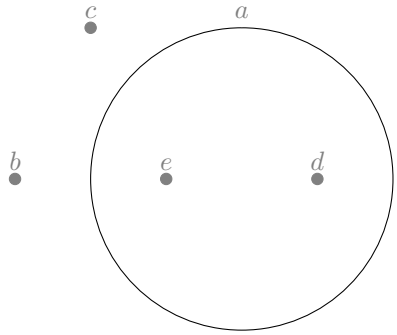


Figure 263: $a \to de$, $bd \to e$

264

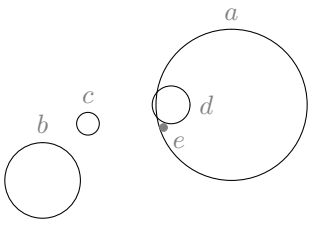


Figure 264: $a \to e$, $bd \to e$, $ac \to de$, $ab \to cde$

265

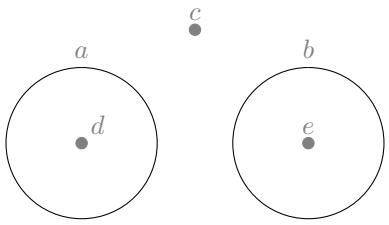


Figure 265: $a \to d$, $b \to e$

266

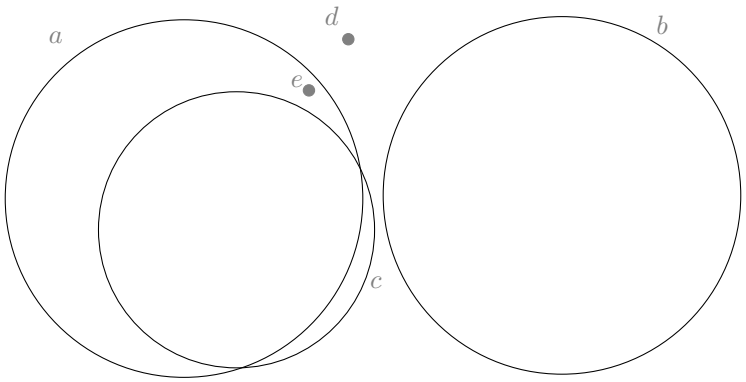


Figure 266: $ab \to cde, cd \to e, bc \to e, a \to e$

267

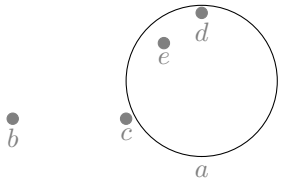


Figure 267: $a \to de$, $bcd \to e$, $ab \to cde$

268

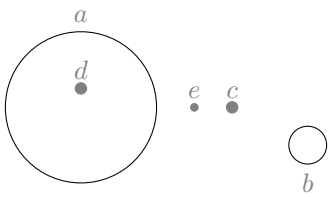


Figure 268: $ab \to cde$, $ac \to de$, $a \to d$, $bd \to e$

269

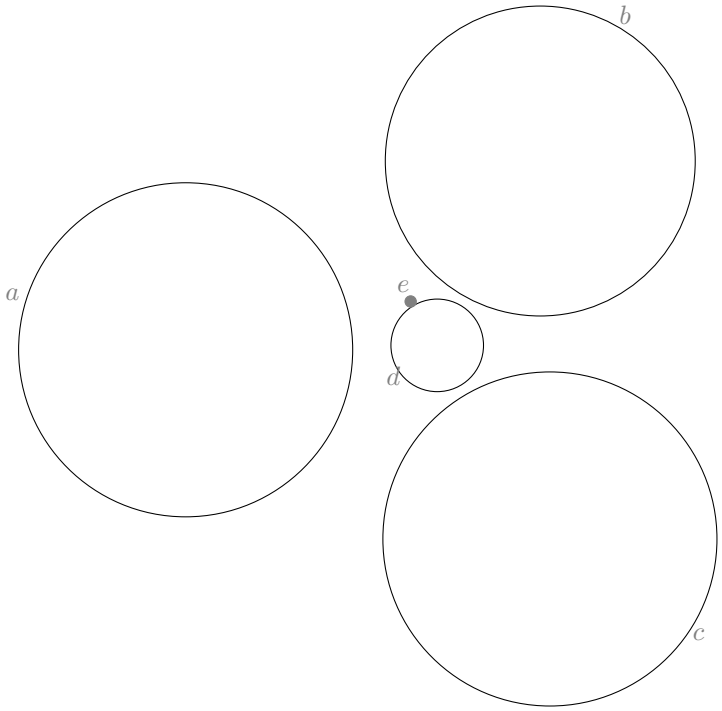


Figure 269: $bd \to e, ad \to e, bc \to de, ac \to de, ab \to de$

270

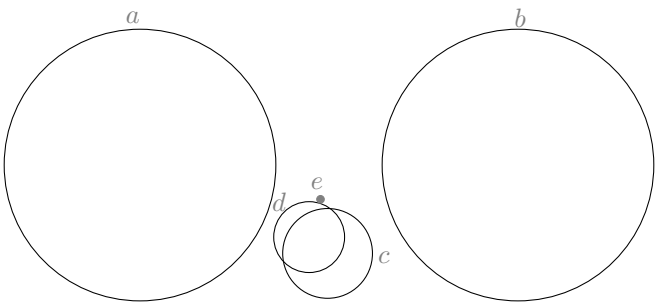


Figure 270: $bd \to e, bc \to e, ad \to e, ac \to de, ab \to cde$

271

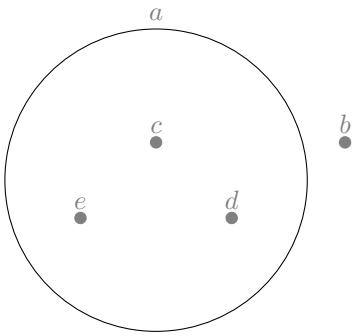


Figure 271: $a \to cde$

272

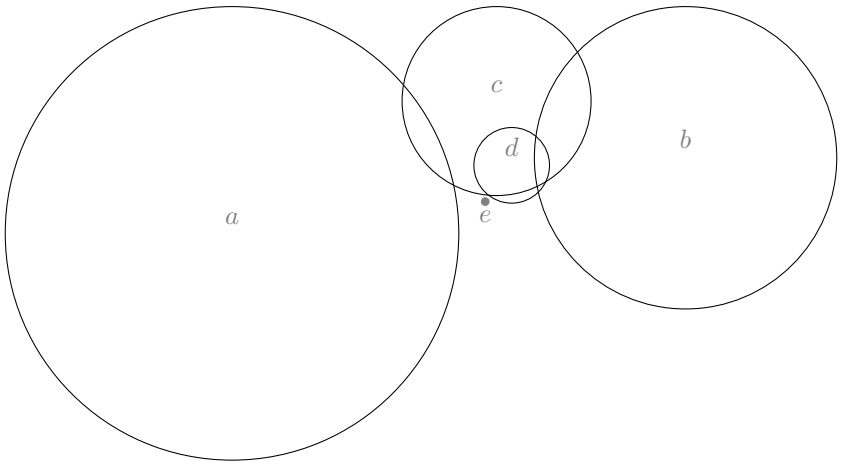


Figure 272: $ad \to e$, $bc \to de$, $ac \to de$, $ab \to cde$

273

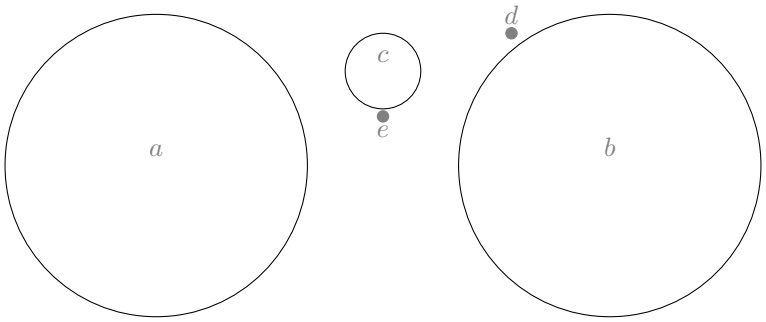


Figure 273: $ad \to ce$, $bc \to de$, $ac \to e$, $ab \to cde$

274

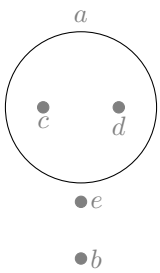


Figure 274: $bcd \to e$, $ab \to cde$, $a \to cd$

275

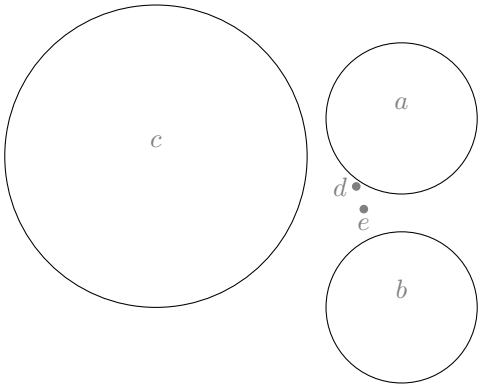


Figure 275: $ae \to d$, $bd \to e$, $bc \to de$, $ac \to de$, $ab \to de$

276

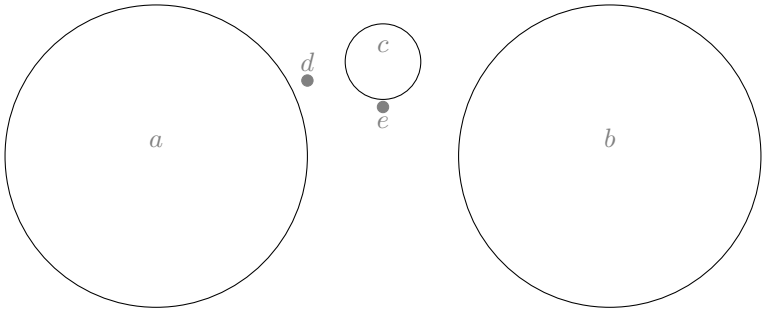


Figure 276: $ae \to d$, $bd \to e$, $bc \to e$, $ac \to de$, $ab \to cde$

277

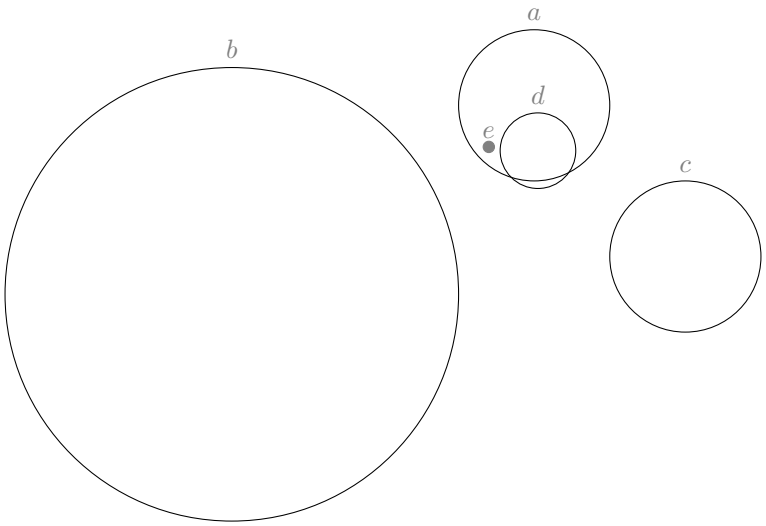


Figure 277: $bd \to e$, $bc \to e$, $ac \to de$, $ab \to de$, $a \to e$

278

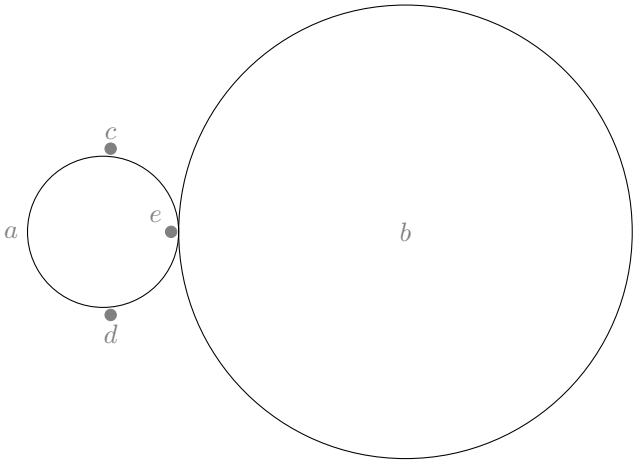


Figure 278: $bd \to e$, $bc \to e$, $ab \to cde$, $a \to e$

279

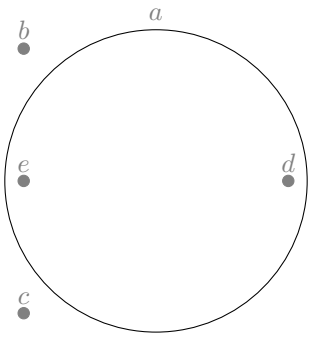


Figure 279: $bc \to e$, $a \to de$

280

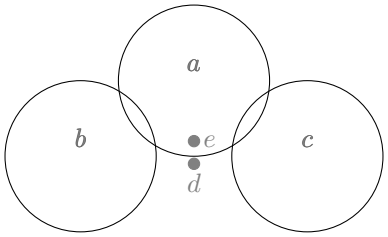


Figure 280: $bc \to de$, $ac \to de$, $ab \to de$, $a \to e$

281

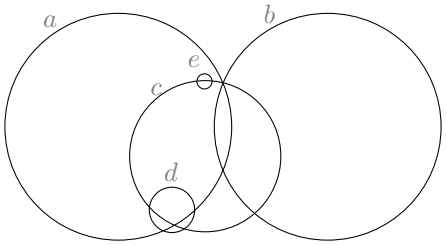


Figure 281: $ab \to cde$, $ac \to de$, $bc \to e$, $a \to e$

282

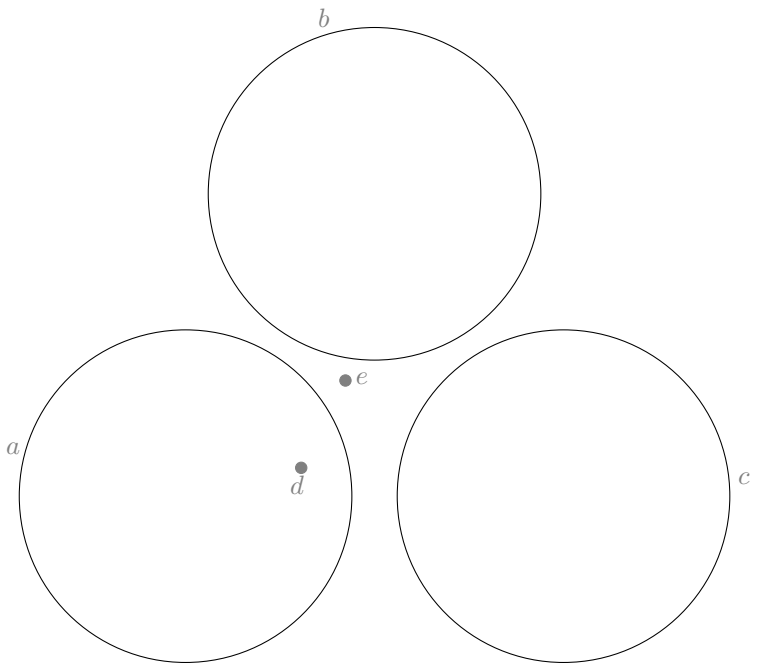


Figure 282: $bd \to e, bc \to e, ac \to de, ab \to de, a \to d$

283

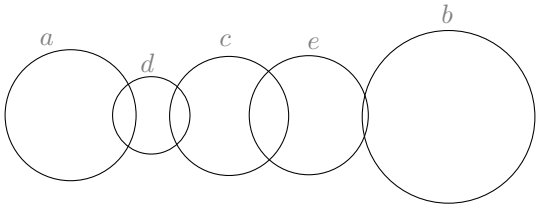


Figure 283: $ae \to cd, bd \to e, bc \to e, ac \to d, ab \to cde$

284

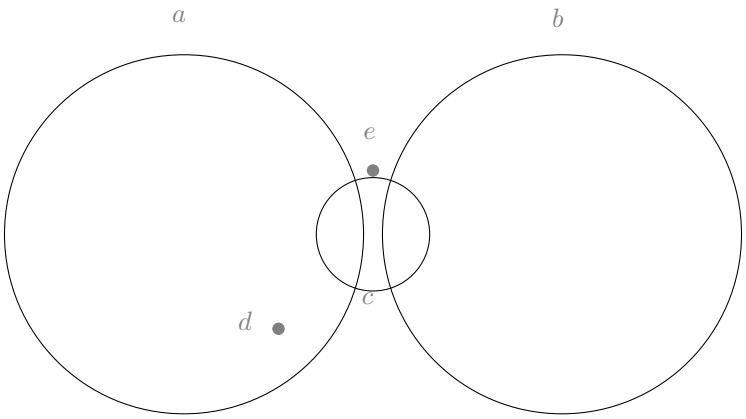


Figure 284: $a \to d, ac \to de, ab \to cde, bc \to e$

285

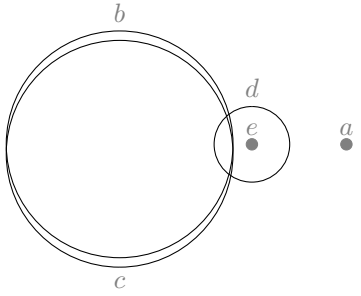


Figure 285: $d \to e, ab \to de, ac \to de$

286

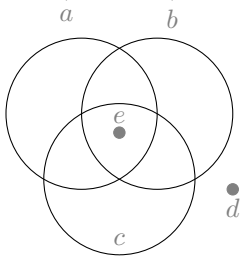


Figure 286: $a \to e, b \to e, c \to e$

287

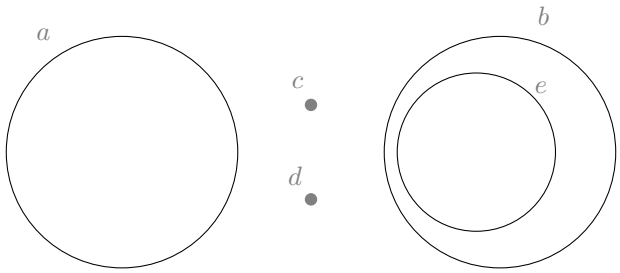


Figure 287: $b \to e, ab \to cde, ae \to cd$

288

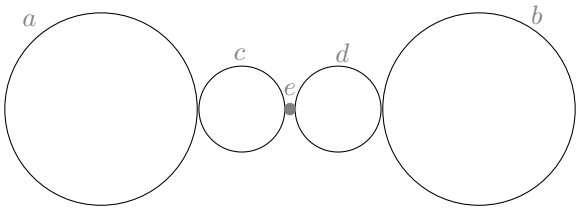


Figure 288: $cd \to e, ad \to ce, bc \to de, ab \to cde$

289

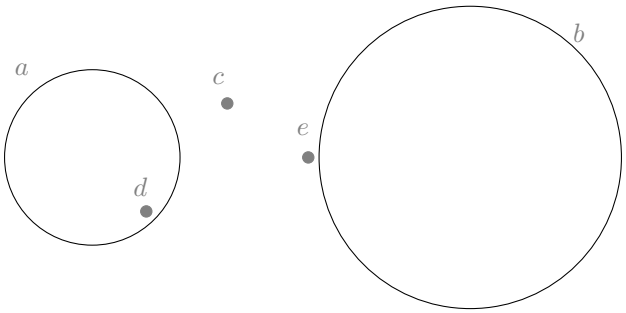


Figure 289: $bd \to e, bc \to e, ab \to cde, a \to d$

290

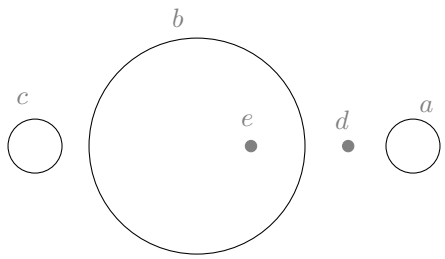


Figure 290: $ae \to d, cd \to e, ac \to de, ab \to de, b \to e$

291

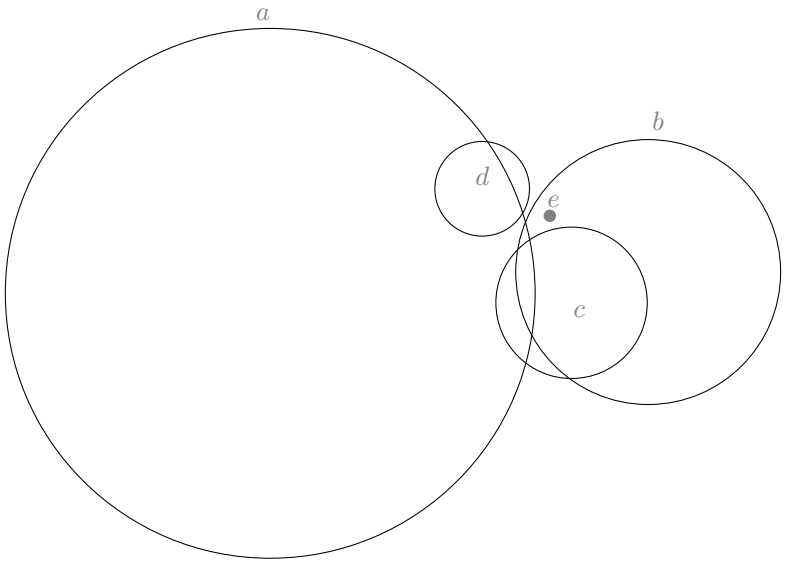


Figure 291: $cd \to e, ac \to de, ab \to cde, b \to e$

292

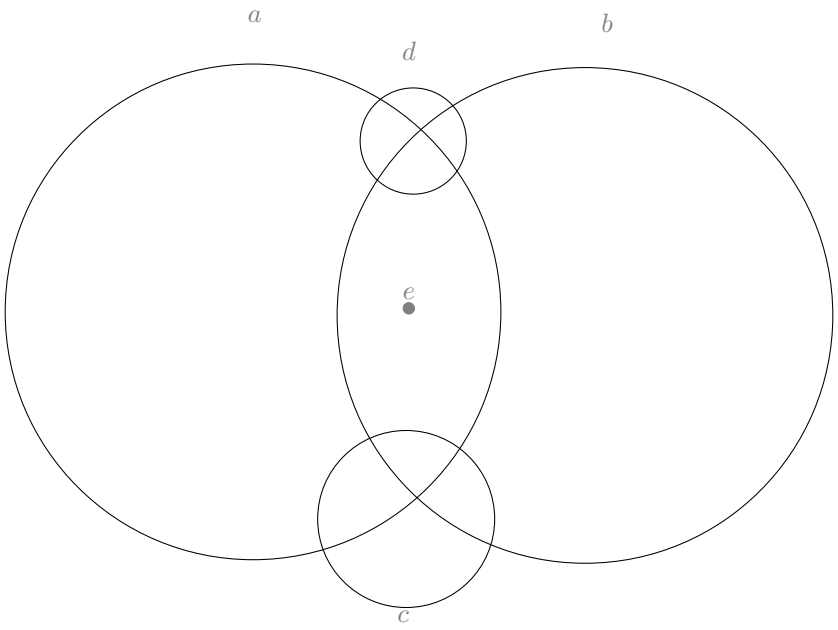


Figure 292: $cd \to e, a \to e, ab \to de, b \to e$

293

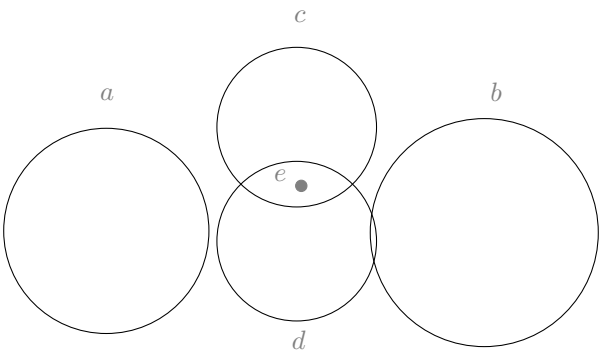


Figure 293: $d \to e, c \to e, ab \to de$

294

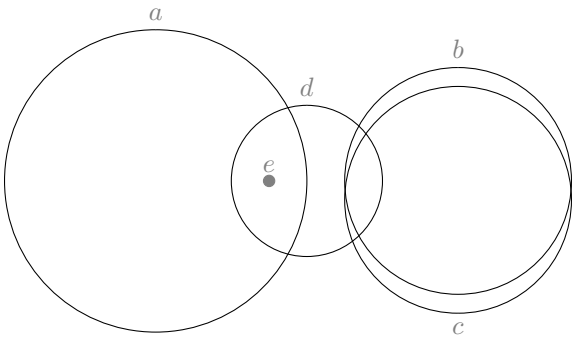


Figure 294: $a \to e$, $d \to e$, $ab \to de$, $ac \to de$

295

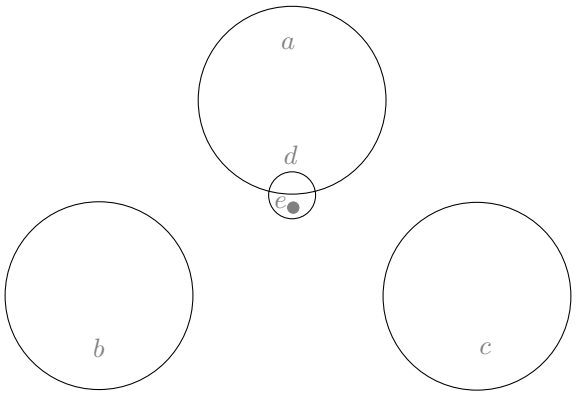


Figure 295: $d \to e, bc \to e, ac \to de, ab \to de$

296

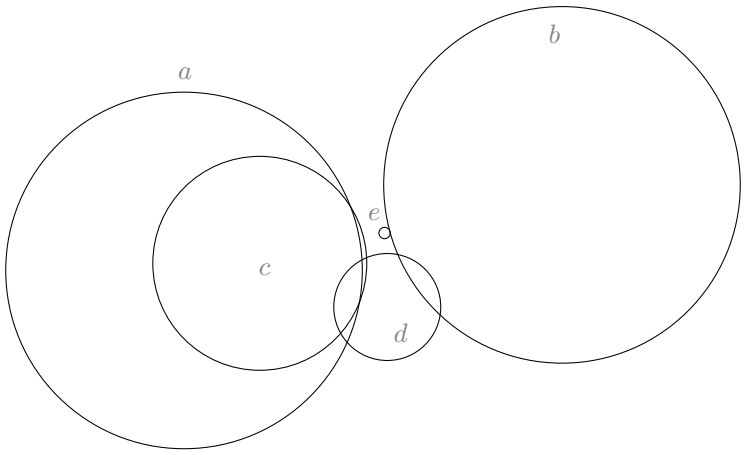


Figure 296: $bd \to e, ae \to c, ab \to cde, ad \to ce, bc \to de,$

297

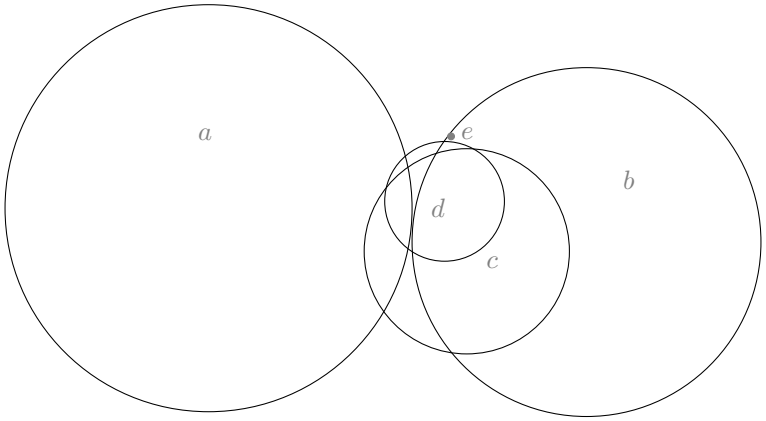


Figure 297: $ad \to e, ac \to de, ab \to cde, b \to e$

298

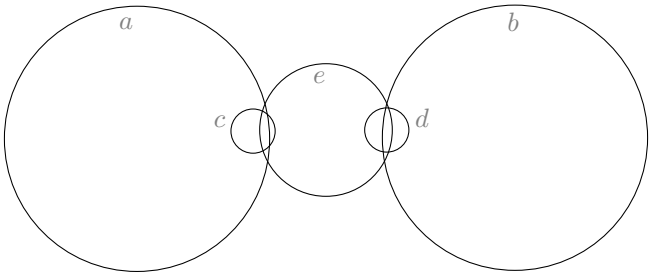


Figure 298: $ae \to c, be \to d, ad \to ce, bc \to de, ab \to cde$

299

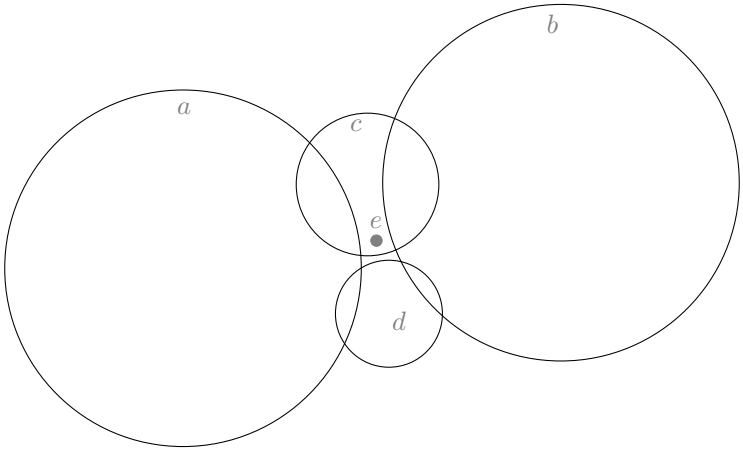


Figure 299: $bd \to e, ad \to e, ab \to cde, c \to e$

300

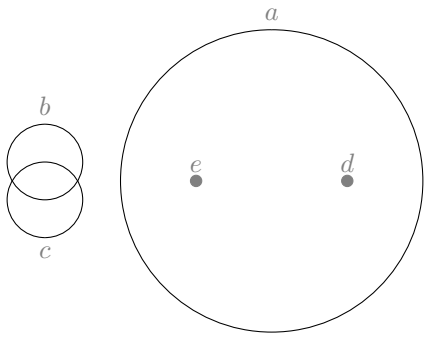


Figure 300: $a \to de$, $bd \to e$, $cd \to e$

301

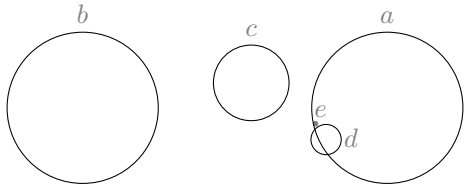


Figure 301: $cd \to e$, $bd \to e$, $ac \to de$, $ab \to cde$, $a \to e$

302

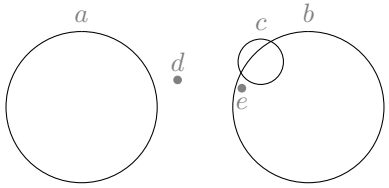


Figure 302: $ae \to d$, $ac \to de$, $ab \to cde$, $b \to e$

303

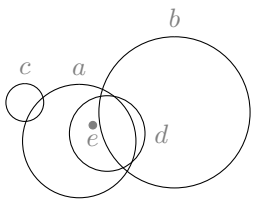


Figure 303: $a \to e$, $bc \to e$, $ab \to de$, $d \to e$

304

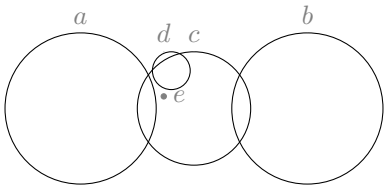


Figure 304: $ad \to e, ac \to de, c \to e, ab \to cde$

305

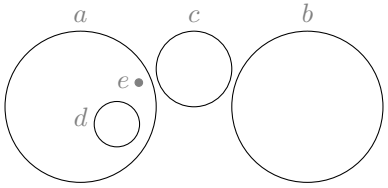


Figure 305: $cd \to e, ab \to cde, a \to de$

306

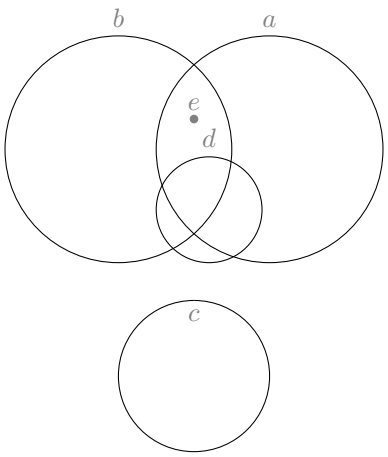


Figure 306: $ac \to de, ab \to de, b \to e, a \to e$

307

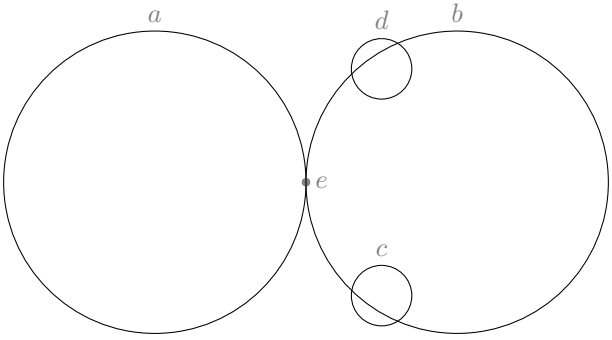


Figure 307: $ab \to cde, b \to e, a \to e$

308

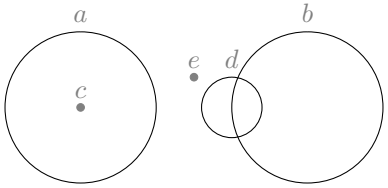


Figure 308: $ad \to ce, bc \to de, ab \to cde, a \to c$

309

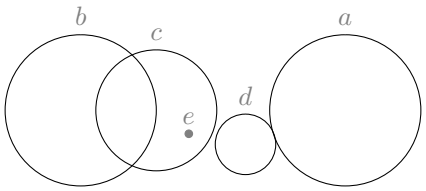


Figure 309: $bd \to e, ac \to de, c \to e, ab \to cde$

310

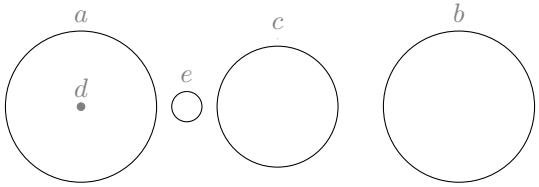


Figure 310: $cd \to e, bd \to e, ac \to de, ab \to cde, a \to d$

311

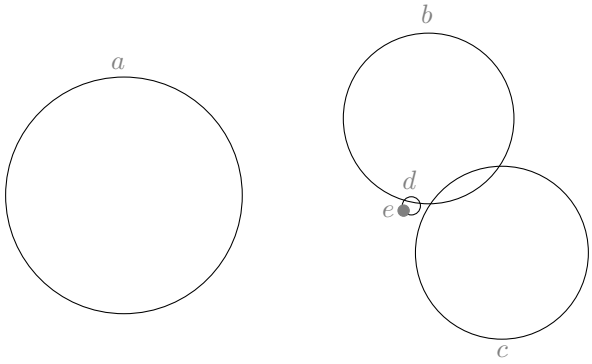


Figure 311: $cd \to e, bd \to e, ad \to e, bc \to de, ac \to de, ab \to de$

312

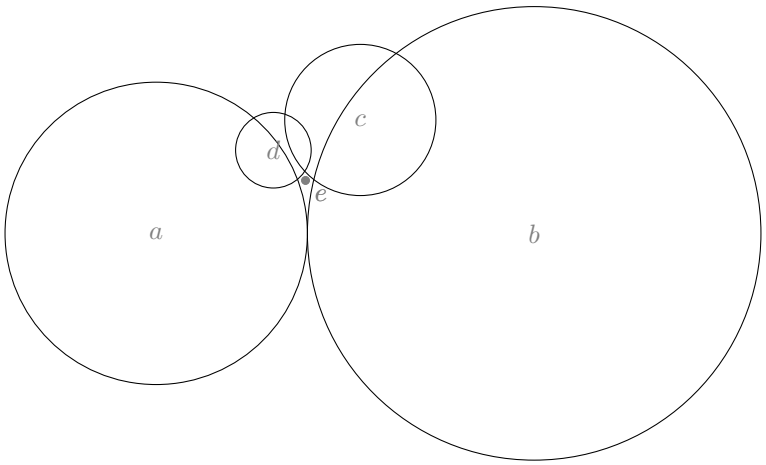


Figure 312: $cd \to e$, $bd \to e$, $ad \to e$, $bc \to e$, $ac \to de$, $ab \to cde$

313

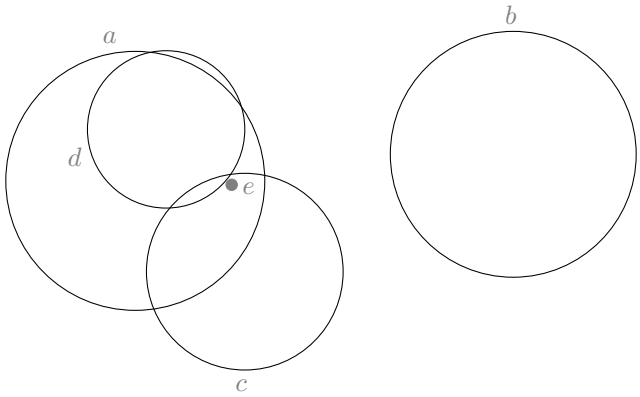


Figure 313: $bd \to e, c \to e, ab \to de, a \to e$

314

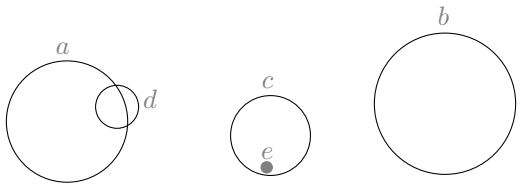


Figure 314: $ae \to d, ac \to de, c \to e, ab \to cde$

315

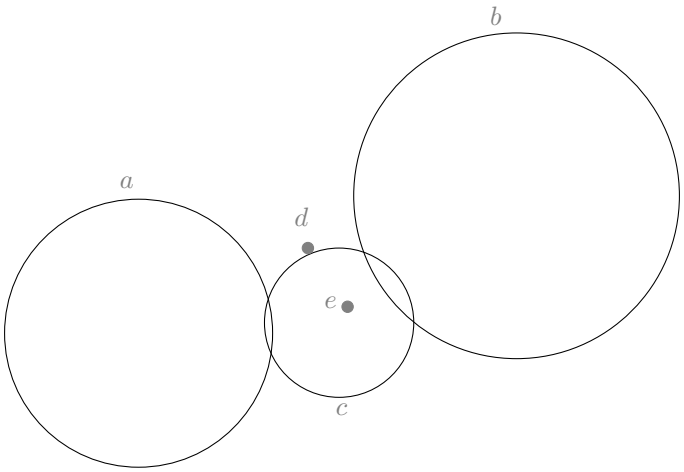


Figure 315: $bc \to de, ac \to de, c \to e, ab \to cde$

316

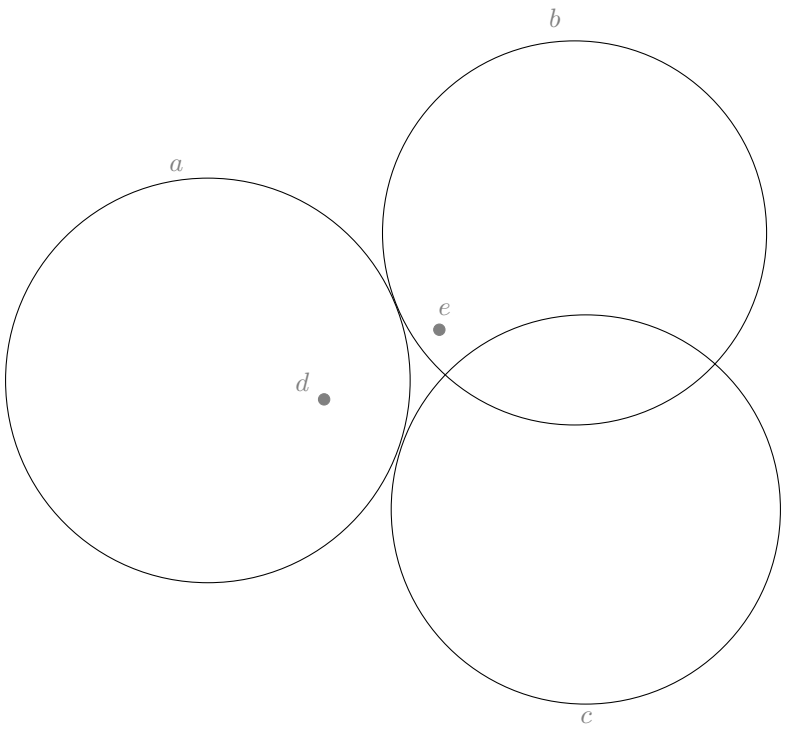


Figure 316: $ac \to de, ab \to de, b \to e, a \to d$

317

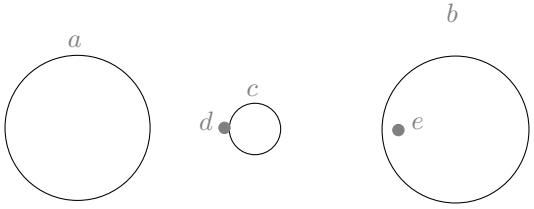


Figure 317: $ae \to cd, ac \to d, ab \to cde, b \to e$

318

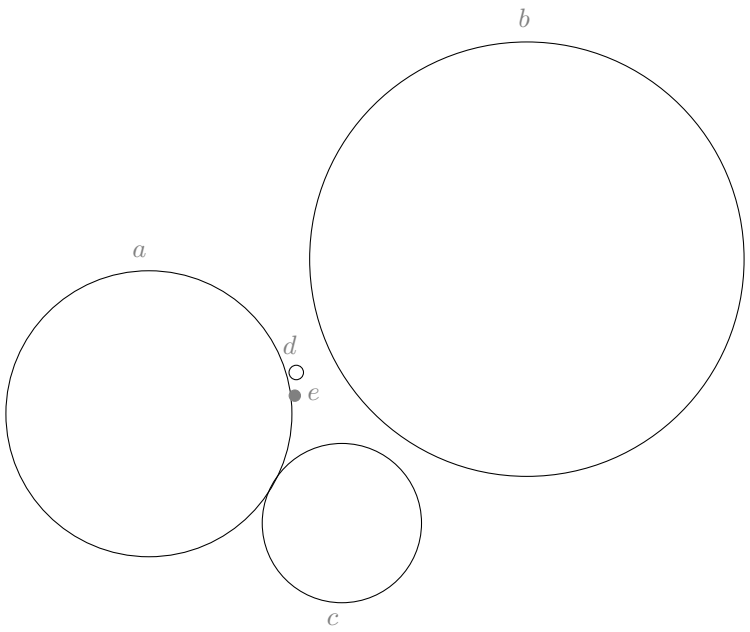


Figure 318: $cd \to e, ad \to e, bc \to de, ac \to de, ab \to cde$

319

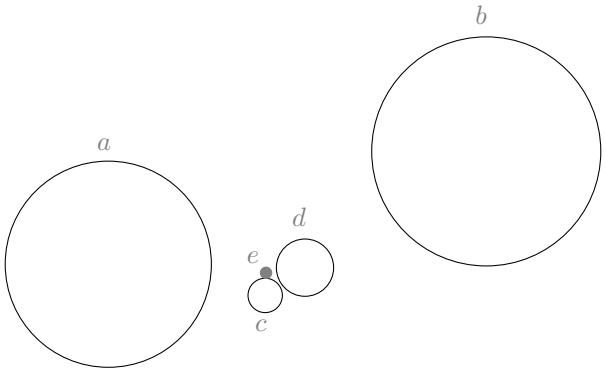


Figure 319: $cd \to e, ad \to ce, bc \to de, ac \to e, ab \to cde$

320

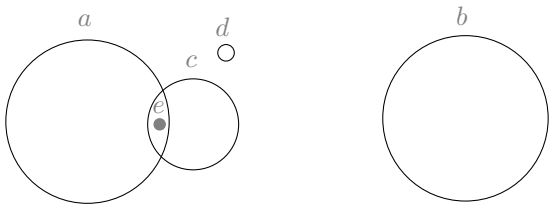


Figure 320: $c \to e, ab \to cde, a \to e$

321

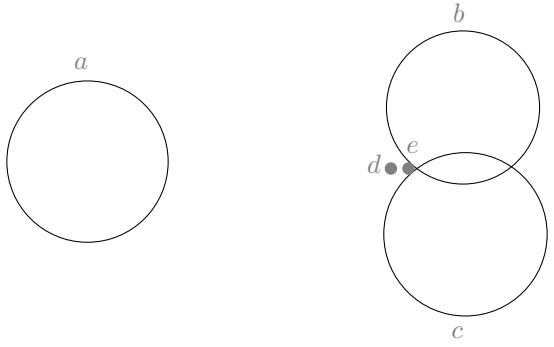


Figure 321: $ae \to d, cd \to e, bd \to e, bc \to de, ac \to de, ab \to de$

322

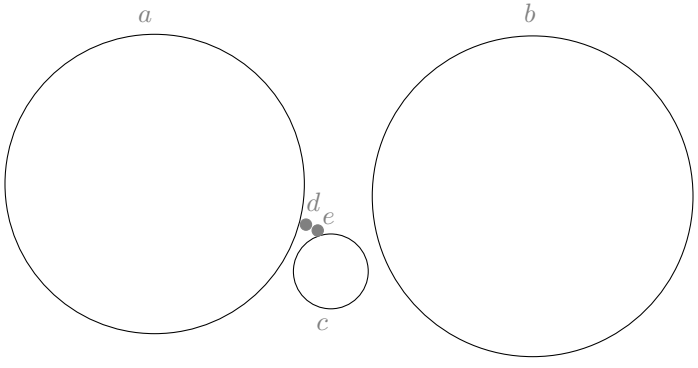


Figure 322: $ae \to d, cd \to e, bd \to e, bc \to e, ac \to de, ab \to cde$

323

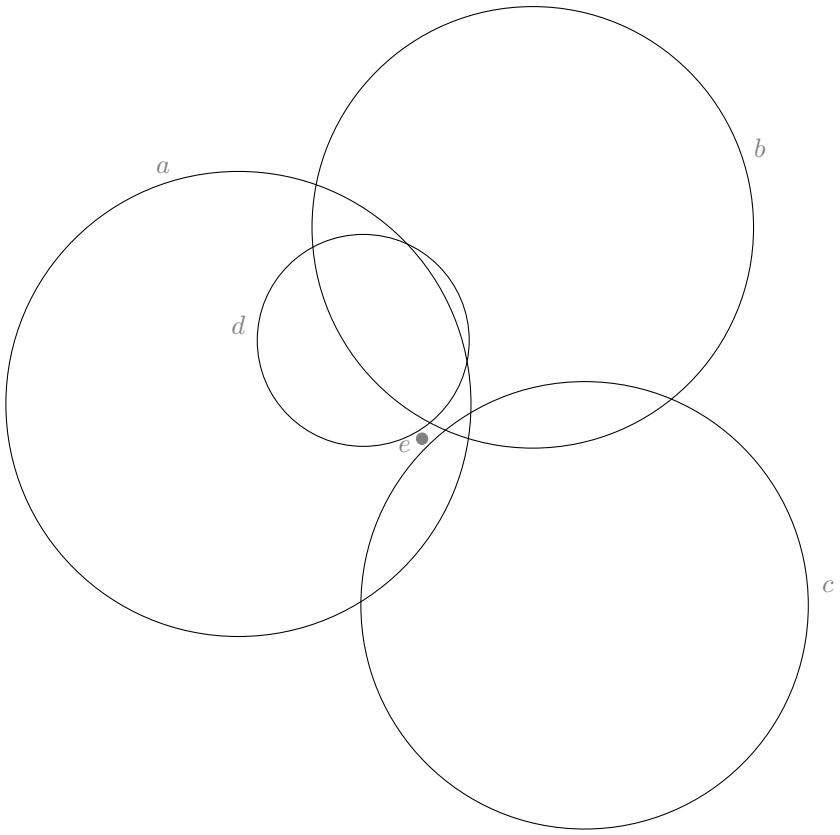


Figure 323: $cd \to e, bd \to e, bc \to e, ac \to de, ab \to de, a \to e$

324

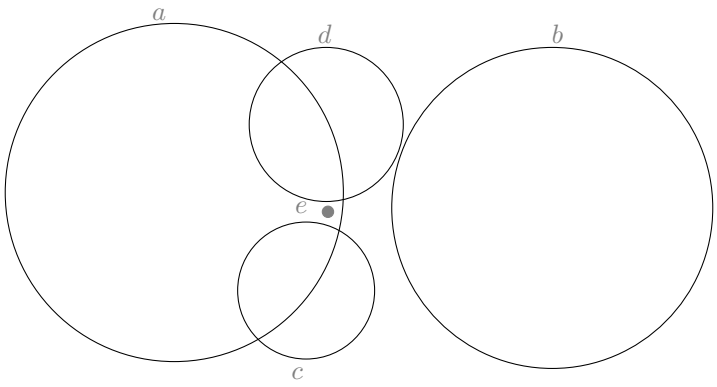


Figure 324: $cd \to e, bd \to e, bc \to e, ab \to cde, a \to e$

325

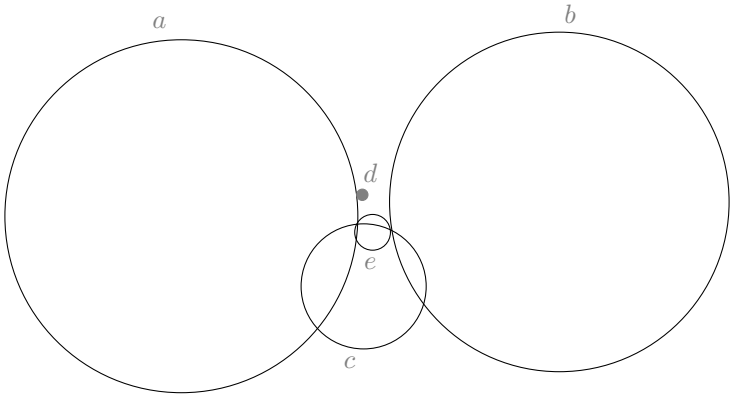


Figure 325: $ae \to d, cd \to e, bc \to de, ac \to de, ab \to cde$

326

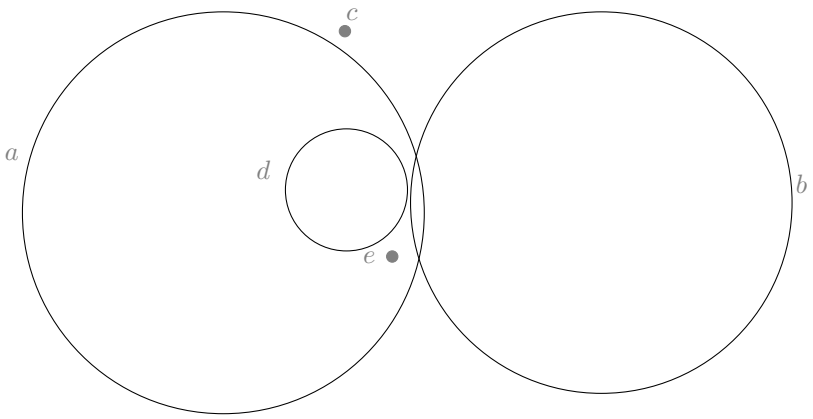


Figure 326: $bd \to e, ab \to cde, a \to de$

327

Figure 327: $ab \to cde, b \to e, a \to d$

328

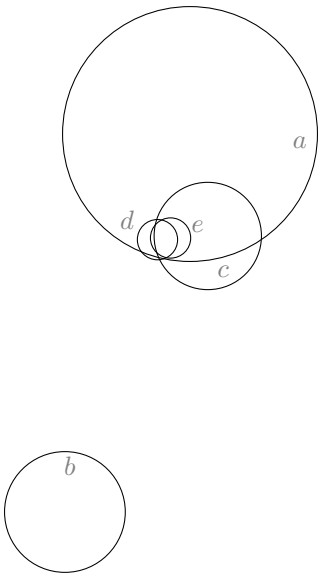


Figure 328: $cd \to e, bc \to e, ac \to de, ab \to cde, a \to e$

329

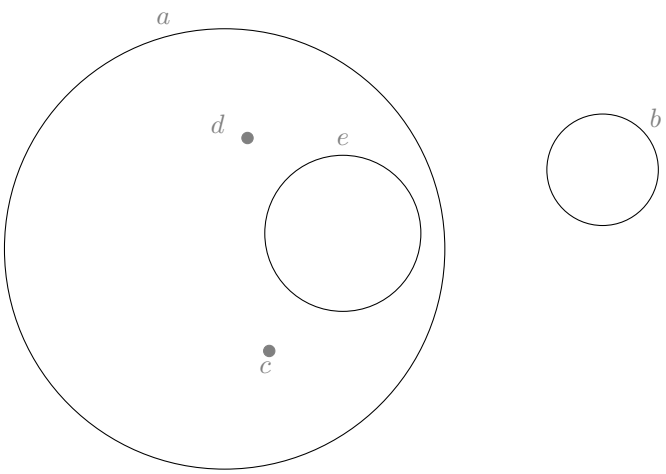


Figure 329: $bcd \to e, a \to cde$

330

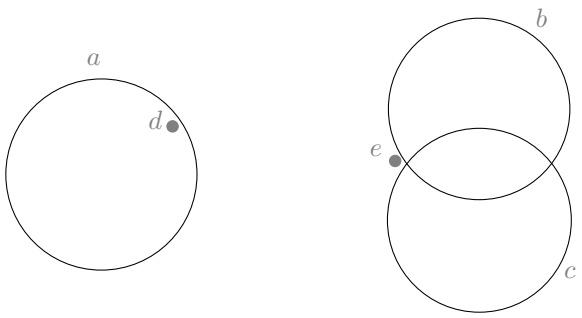


Figure 330: $cd \to e, bd \to e, bc \to e, ac \to de, ab \to de, a \to d$

331

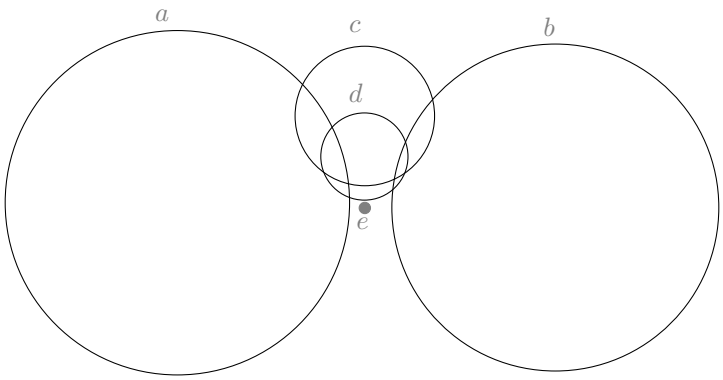


Figure 331: $bd \to e, ad \to e, bc \to de, ac \to de, ab \to cde$

332

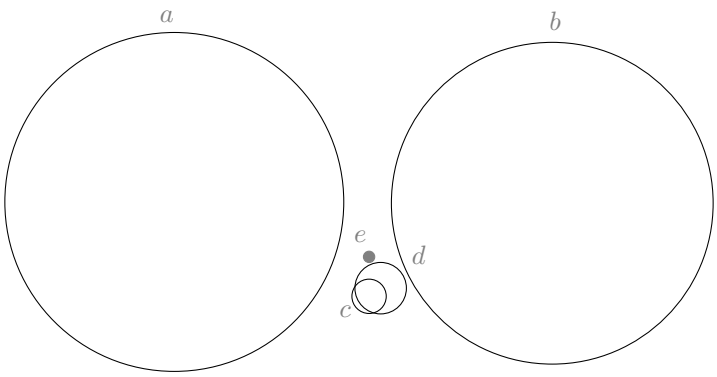


Figure 332: $bd \to e, ad \to ce, bc \to de, ac \to e, ab \to cde$

333

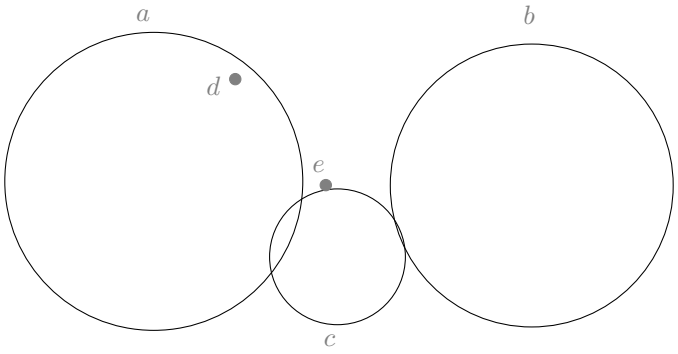


Figure 333: $cd \to e, bc \to e, ac \to de, ab \to cde, a \to d$

334

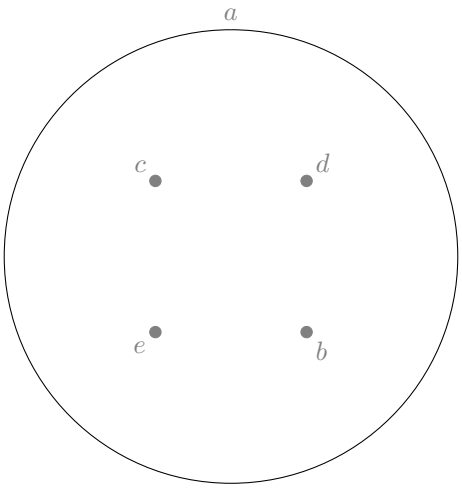


Figure 334: $a \to bcde$

335

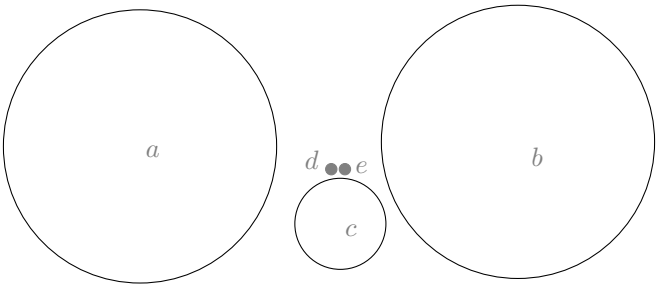


Figure 335: $ae \to d, bd \to e, bc \to de, ac \to de, ab \to cde$

336

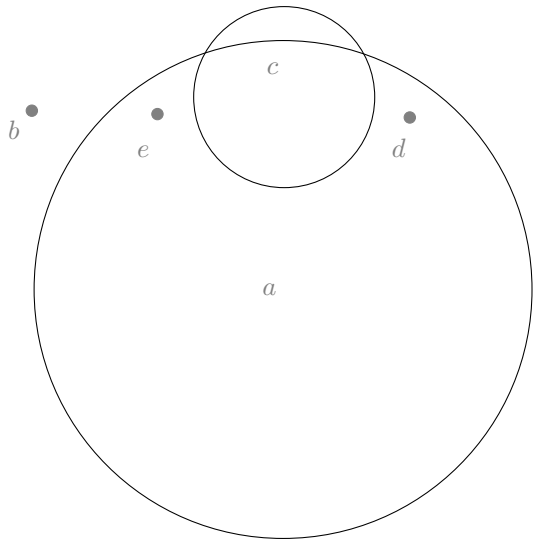


Figure 336: $bd \to e, bc \to e, a \to de$

337

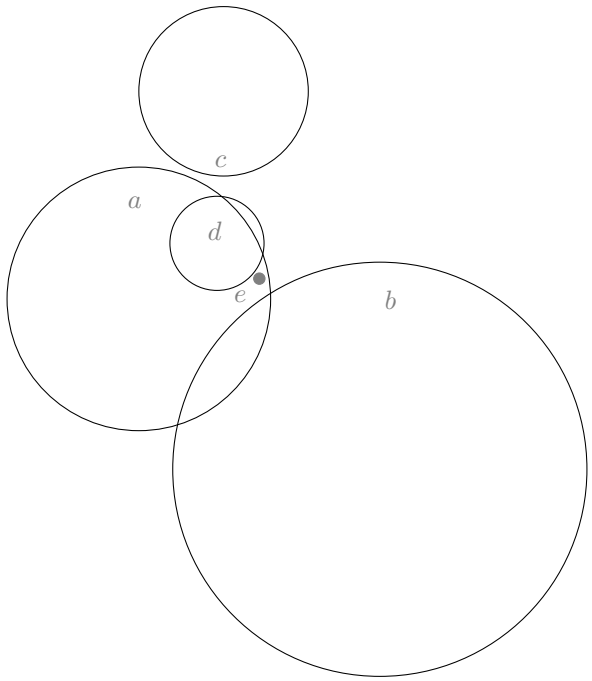


Figure 337: $bd \to e, bc \to de, ac \to de, ab \to de, a \to e$

338

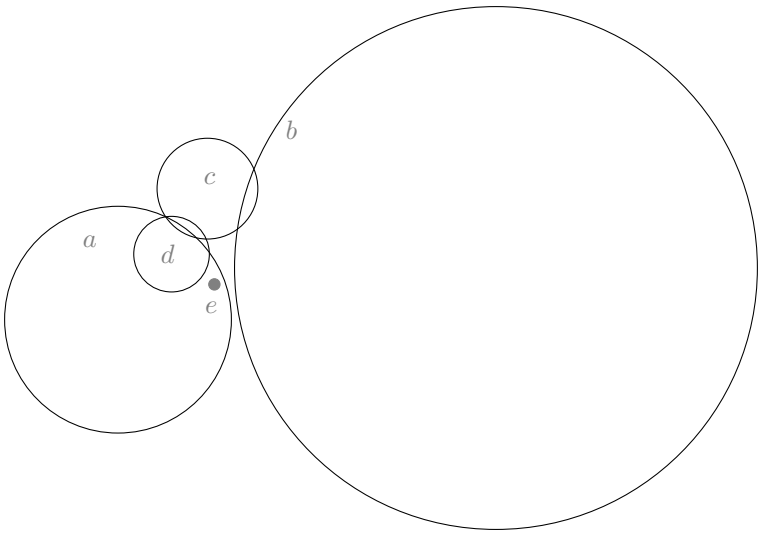


Figure 338: $bd \to e, bc \to e, ac \to de, ab \to cde, a \to e$

339

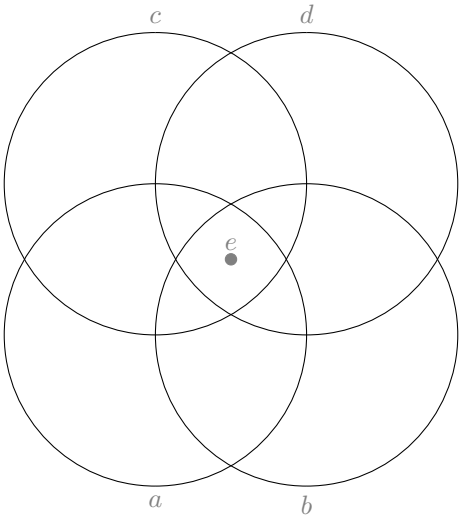


Figure 339: $a \to e$, $b \to e$, $c \to e$, $d \to e$

340

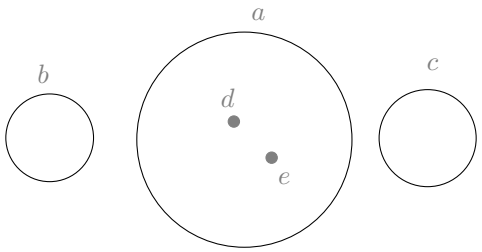


Figure 340: $a \to de, bc \to de$

341

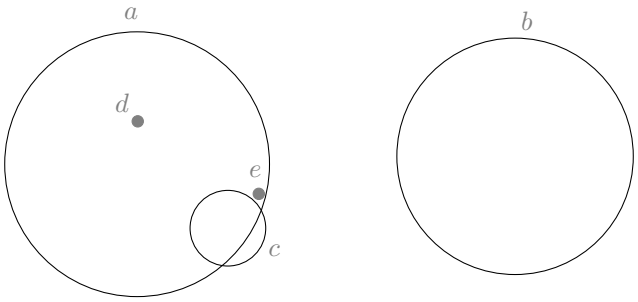


Figure 341: $a \to de, ab \to cde, bc \to e$

342

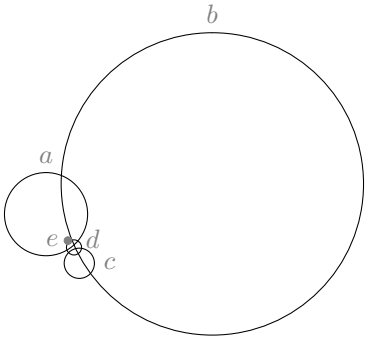


Figure 342: $a \to e$, $ac \to de$, $bc \to de$, $ab \to cde$

343

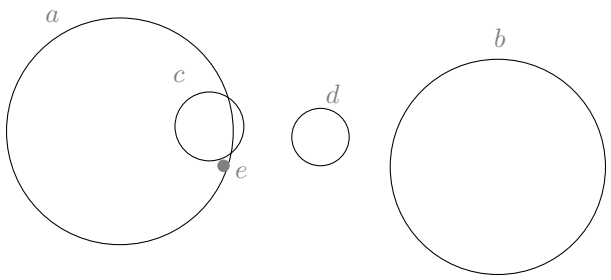


Figure 343: $a \to e, ab \to cde, ad \to ce, bc \to de$

344

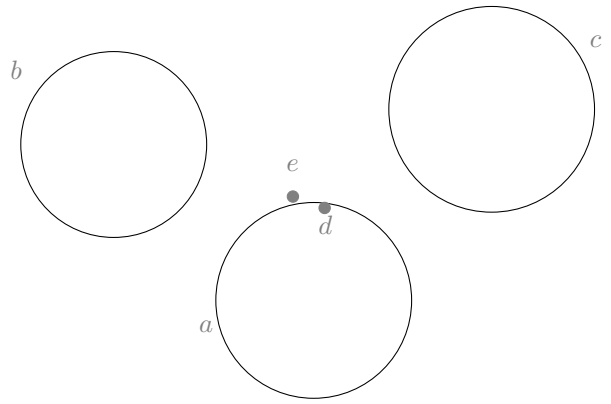


Figure 344: $a \to d, ab \to de, ac \to de, bc \to de, bd \to e$

345

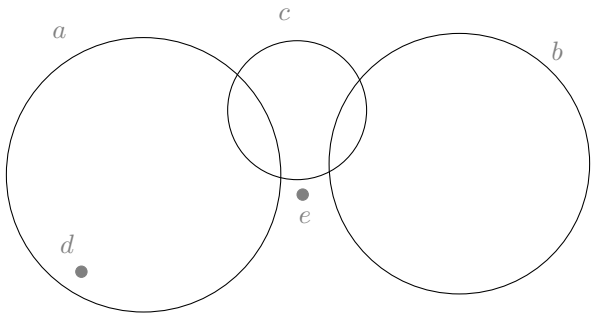


Figure 345: $a \to d, ab \to cde, ac \to de, bc \to e, bd \to e$

346

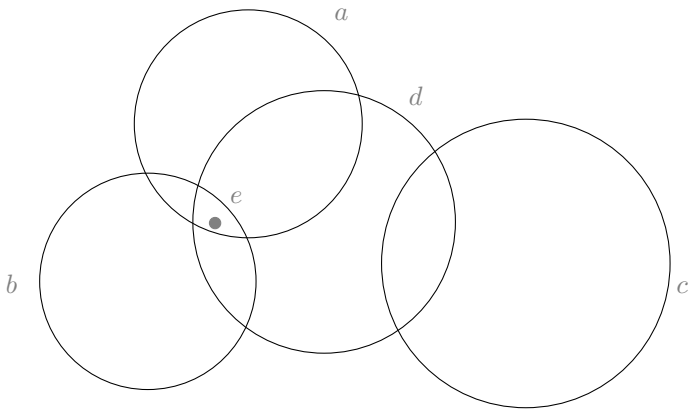


Figure 346: $a \to e, b \to e, d \to e, abc \to de$

347

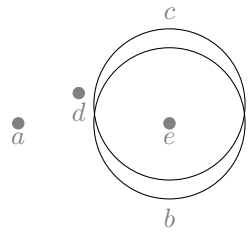


Figure 347: $b \to e, c \to e, ab \to de, ac \to de$

348

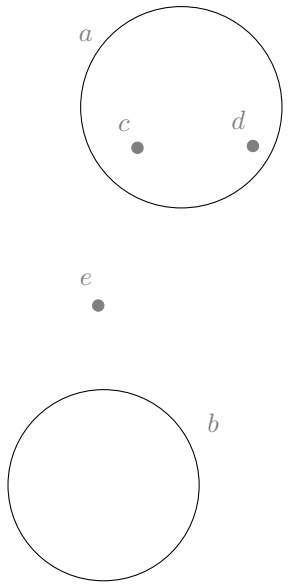


Figure 348: $a \to cd, ab \to cde, bc \to e$

349

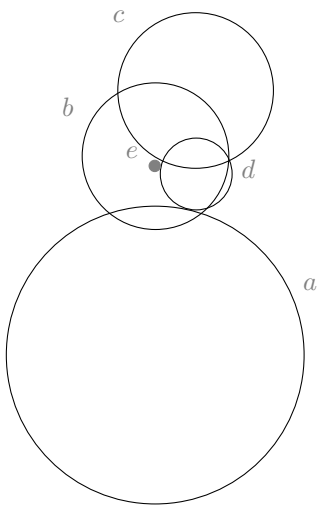


Figure 349: $b \to e, ab \to de, ac \to de, ad \to e, cd \to e$

350

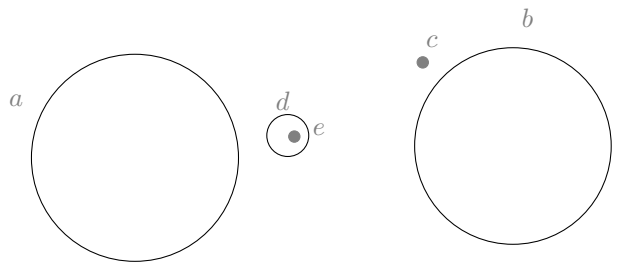


Figure 350: $d \to e, ab \to cde, ac \to de$

352

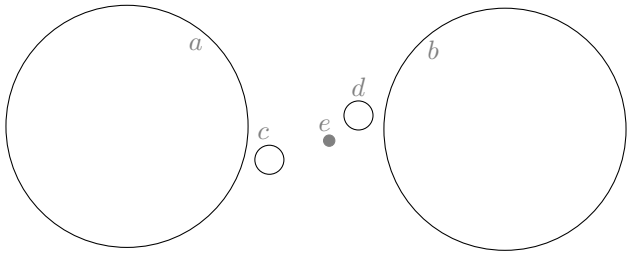


Figure 352: $ae \to c, cd \to e, ad \to ce, bc \to de, ab \to cde$

353

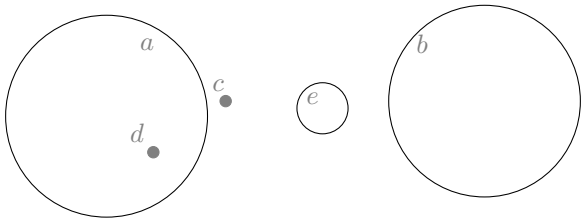


Figure 353: $ae \to cd, bd \to e, bc \to e, ab \to cde, a \to d$

354

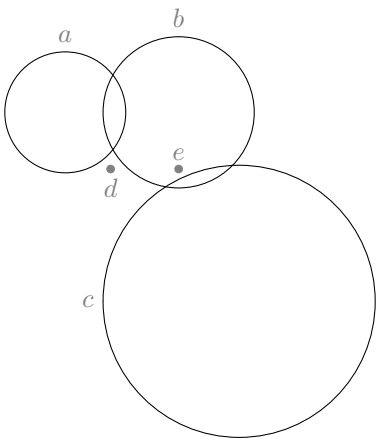


Figure 354: $b \to e$, $ae \to d$, $cd \to e$, $bc \to de$, $ac \to de$, $ab \to de$

355

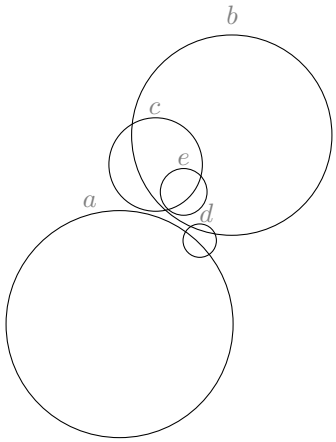


Figure 355: $ae \to d, cd \to e, ac \to de, ab \to cde, b \to e$

356

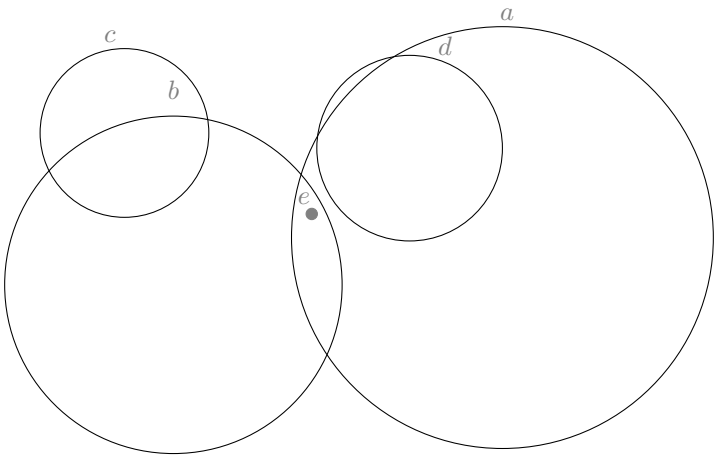


Figure 356: $cd \to e, ac \to de, ab \to de, b \to e, a \to e$

357

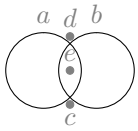


Figure 357: $a \to e$, $b \to e$, $cd \to e$, $ab \to cde$

358

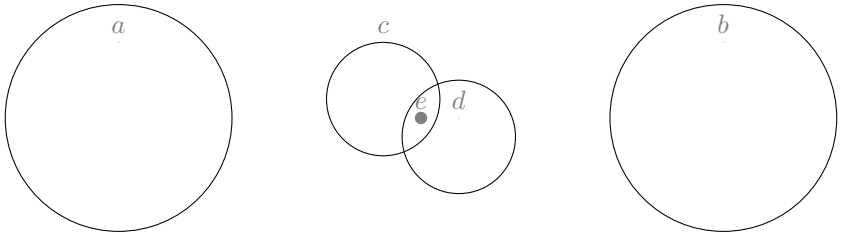


Figure 358: $d \to e$, $c \to e$, $ab \to cde$

359

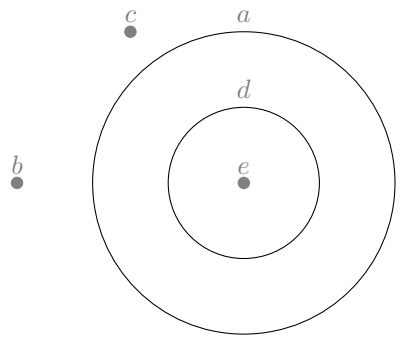


Figure 359: $a \to de$, $d \to e$

360

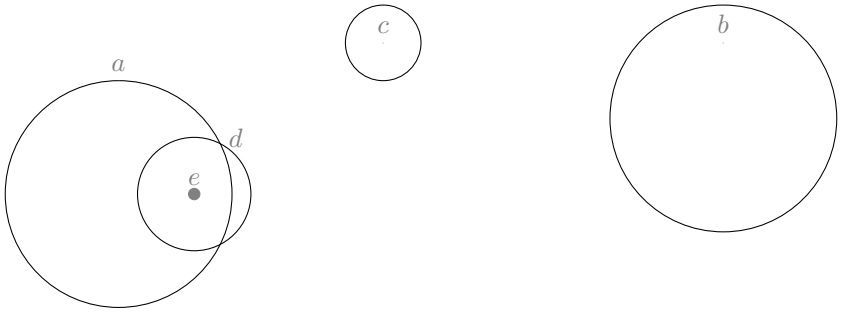


Figure 360: $d \to e$, $a \to e$, $ab \to cde$, $ac \to de$

361

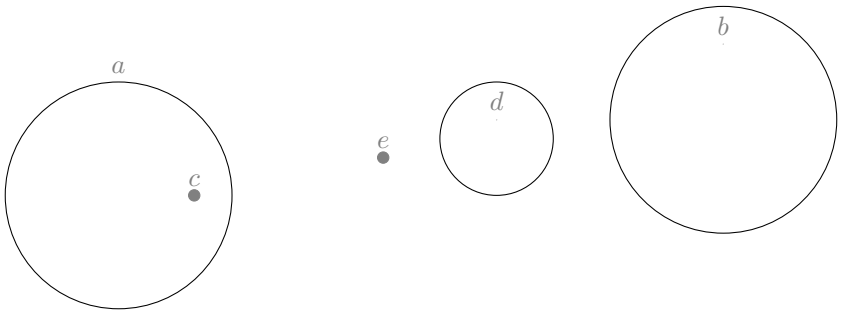


Figure 361: $cd \to e$, $ad \to ce$, $bc \to de$, $ab \to cde$, $a \to c$

362

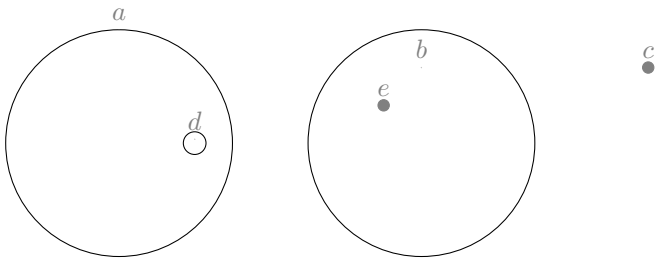


Figure 362: $cd \to e$, $ac \to de$, $ab \to de$, $b \to e$, $a \to d$

363

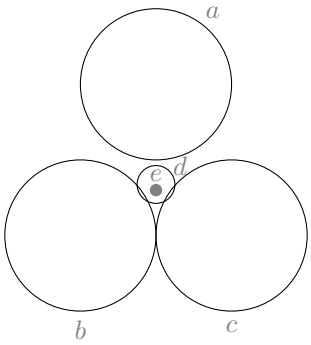


Figure 363: $ab \to d$, $ac \to d$, $bc \to d$, $d \to e$

364

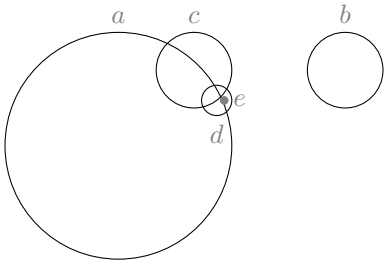


Figure 364: $d \to e$, $bc \to e$, $ac \to de$, $ab \to cde$

365

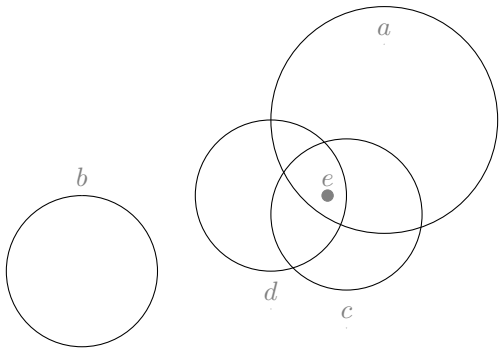


Figure 365: $d \to e$, $c \to e$, $ab \to de$, $a \to e$

366

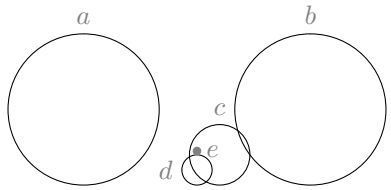


Figure 366: $c \to e$, $ad \to e$, $bd \to e$, $ac \to de$, $ab \to cde$

367

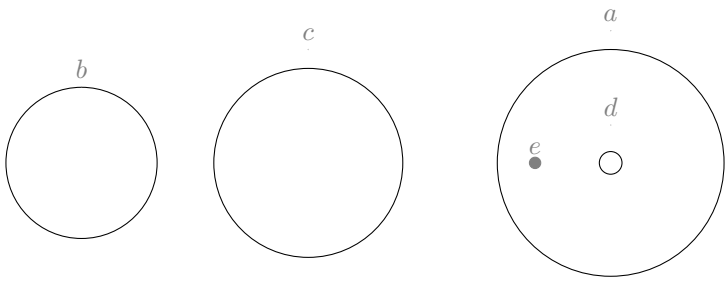


Figure 367: $cd \to e$, $bd \to e$, $ab \to cde$, $a \to de$

368

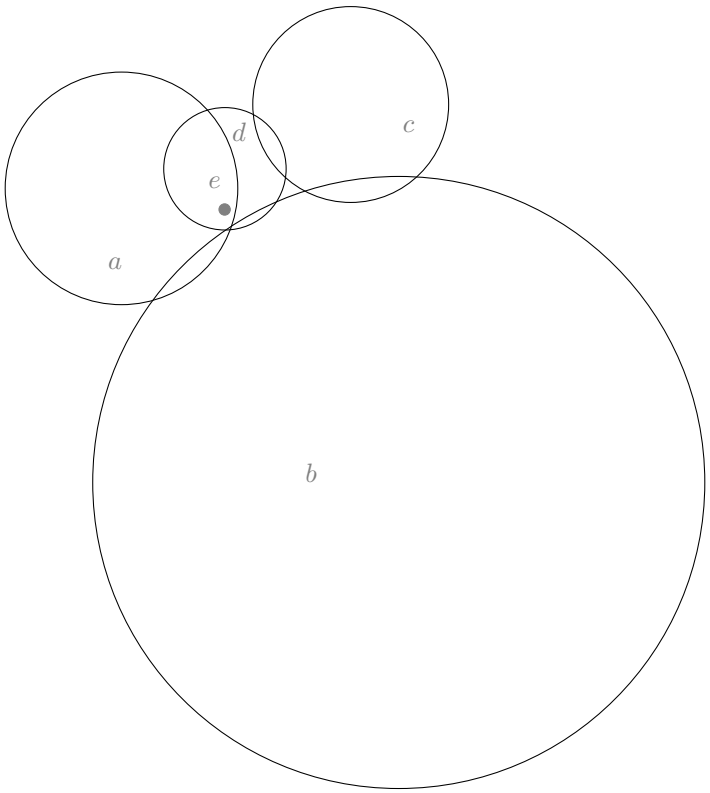


Figure 368: $d \to e, bc \to e, ac \to de, ab \to de, a \to e$

369

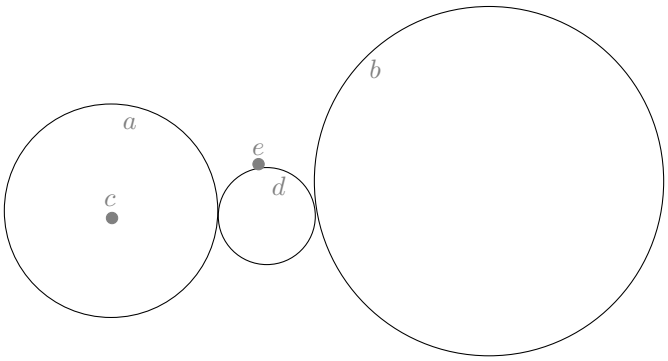


Figure 369: $bd \to e, ad \to ce, bc \to de, ab \to cde, a \to c$

370

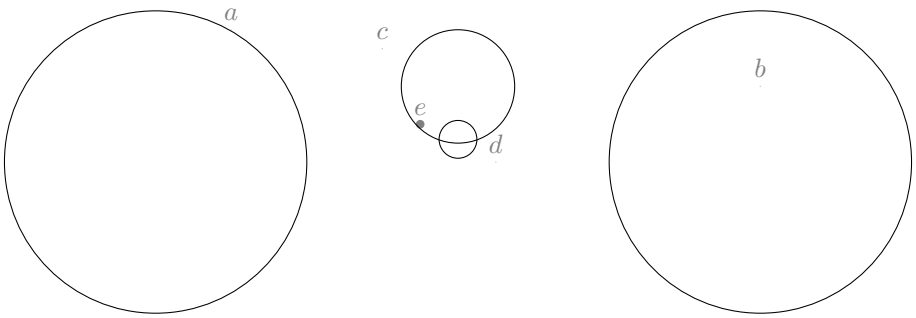


Figure 370: $ad \to e$, $bc \to de$, $ac \to de$, $c \to e$, $ab \to cde$

371

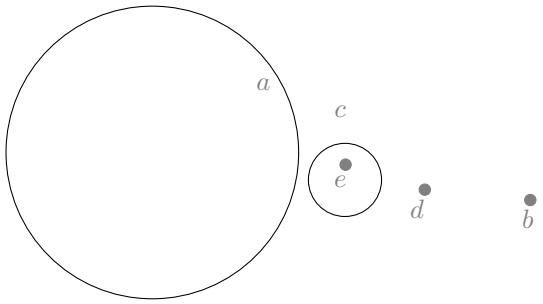


Figure 371: $ad \to ce, bc \to de, c \to e, ab \to cde$

372

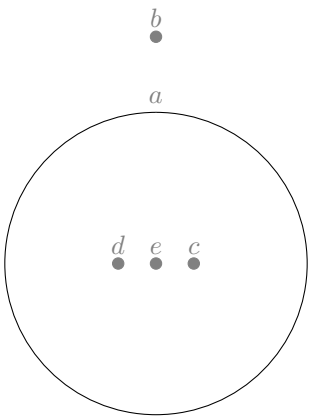


Figure 372: $a \to cde$, $cd \to e$

373

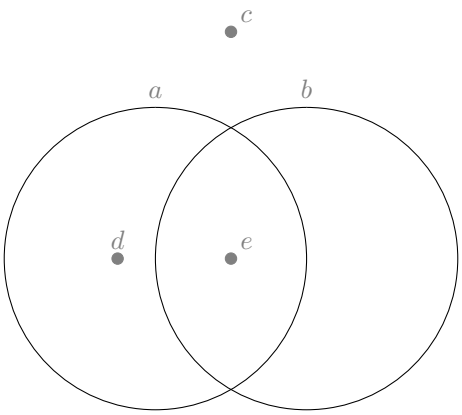


Figure 373: $a \to de$, $b \to e$

374

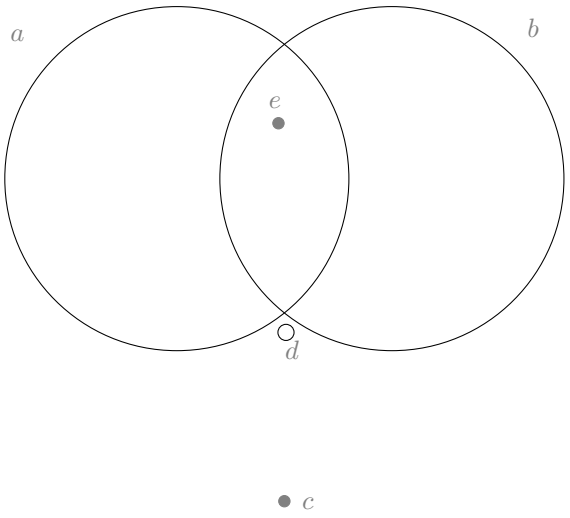


Figure 374: $bc \to de, ac \to de, ab \to de, b \to e, a \to e$

375

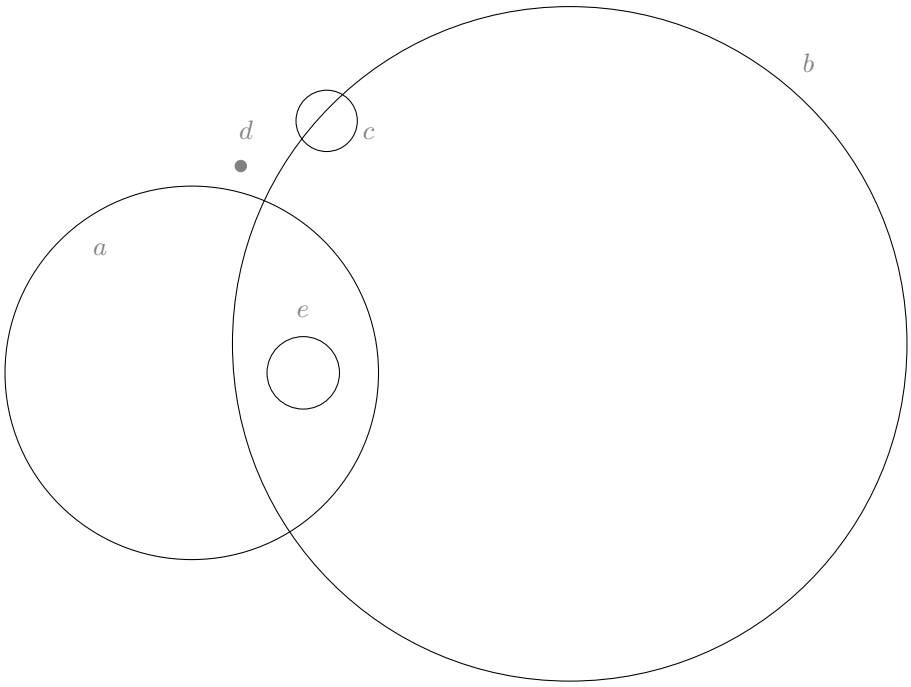


Figure 375: $ac \to de, ab \to cde, b \to e, a \to e$

376

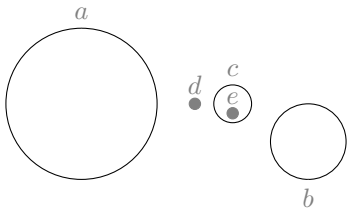


Figure 376: $ab \to cd$, $ac \to d$, $ae \to d$, $c \to e$, $bd \to e$

377

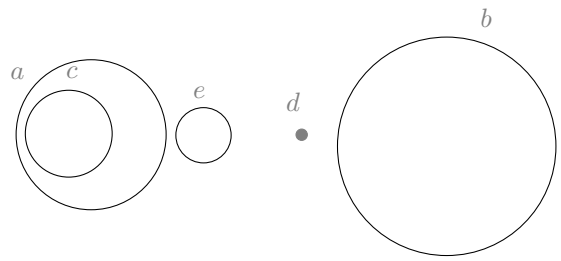


Figure 377: $be \to d, ad \to ce, bc \to de, ab \to cde, a \to c$

378

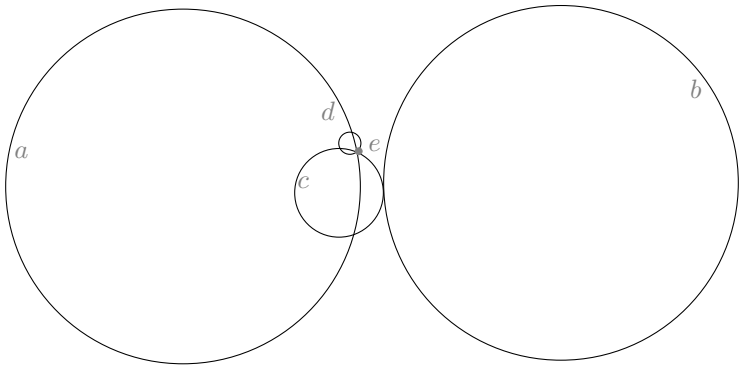


Figure 378: $ab \to cde, ac \to de, bc \to de, cd \to e, bd \to e, ad \to e$

379

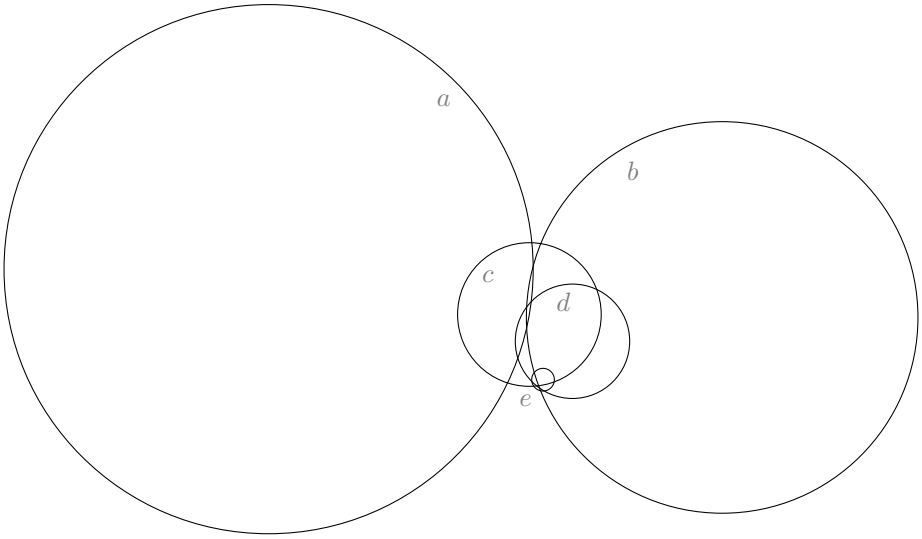


Figure 379: $cd \to e$, $bd \to e$, $ad \to ce$, $bc \to de$, $ac \to e$, $ab \to cde$

380

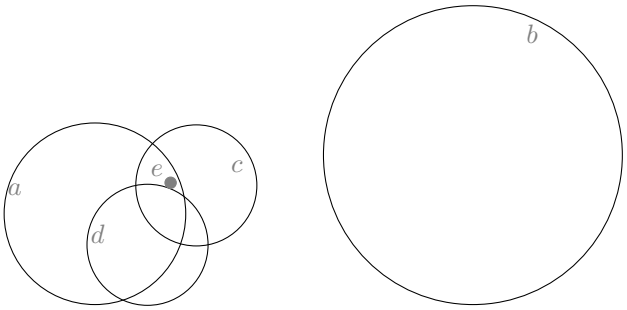


Figure 380: $ab \to cd, bd \to e, a \to e, c \to e$

381

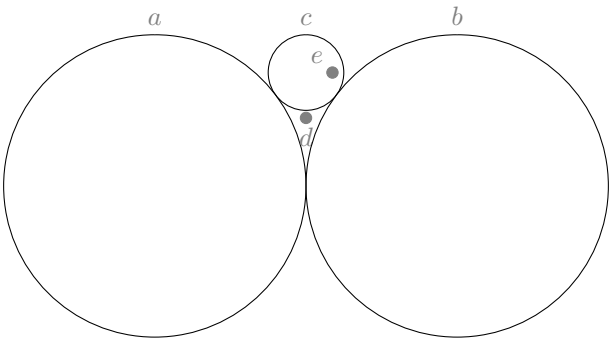


Figure 381: $ac \to d$, $bc \to d$, $ab \to cd$, $c \to e$, $ae \to d$

382

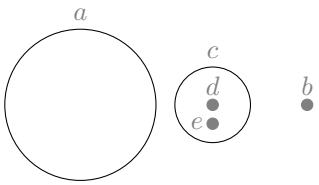


Figure 382: $c \to de, ab \to cde$

383

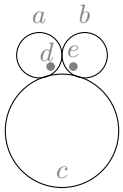


Figure 383: $a \to d$, $b \to e$, $ac \to de$, $bc \to de$

384

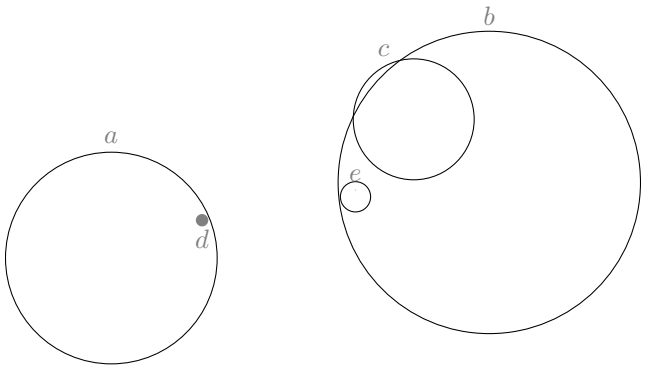


Figure 384: $ac \to de, ab \to cde, b \to e, a \to d$

385

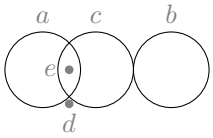


Figure 385: $a \to e$, $c \to e$, $ac \to de$, $ab \to cde$

386

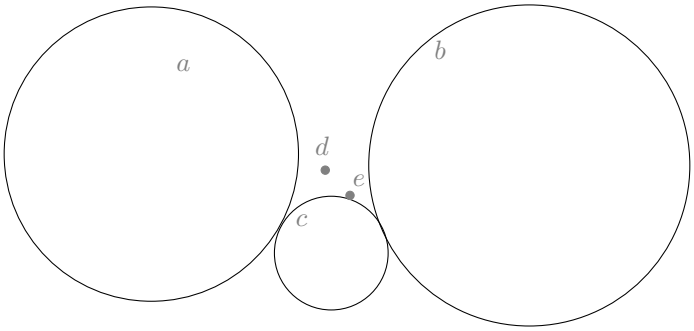


Figure 386: $ae \to d, cd \to e, bd \to e, bc \to de, ac \to de, ab \to cde$

387

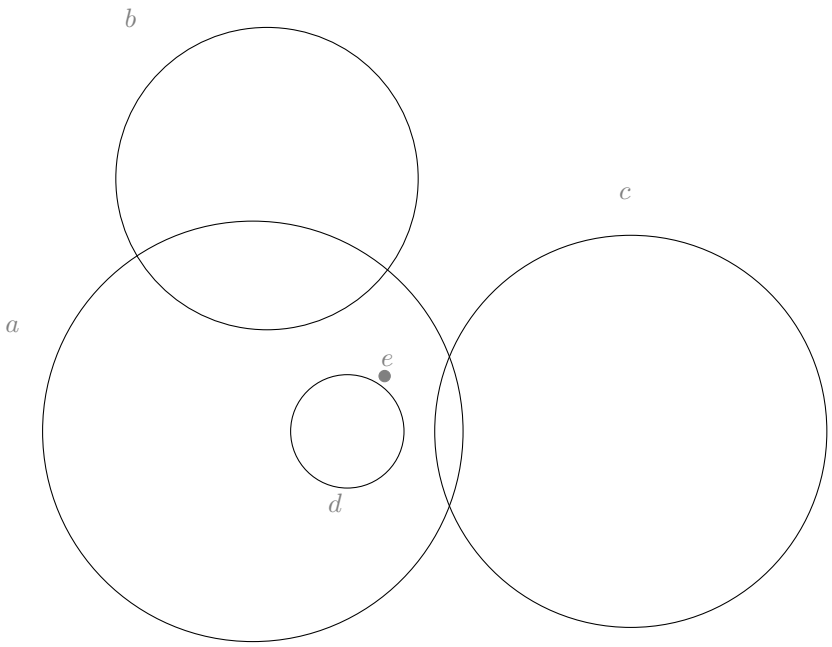


Figure 387: $cd \to e, bd \to e, bc \to e, a \to de$

388

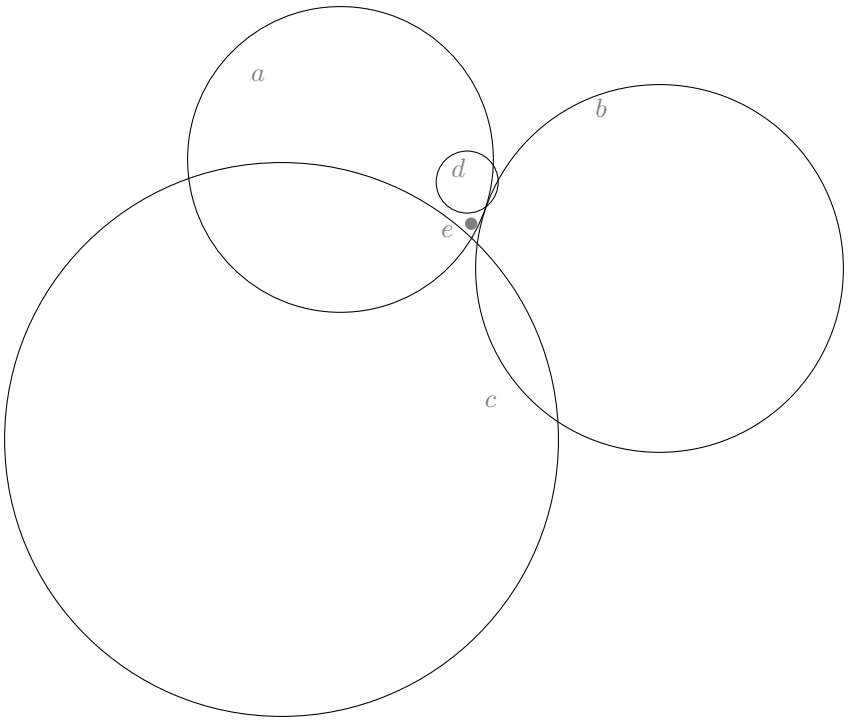


Figure 388: $cd \to e, bd \to e, bc \to de, ac \to de, ab \to de, a \to e$

389

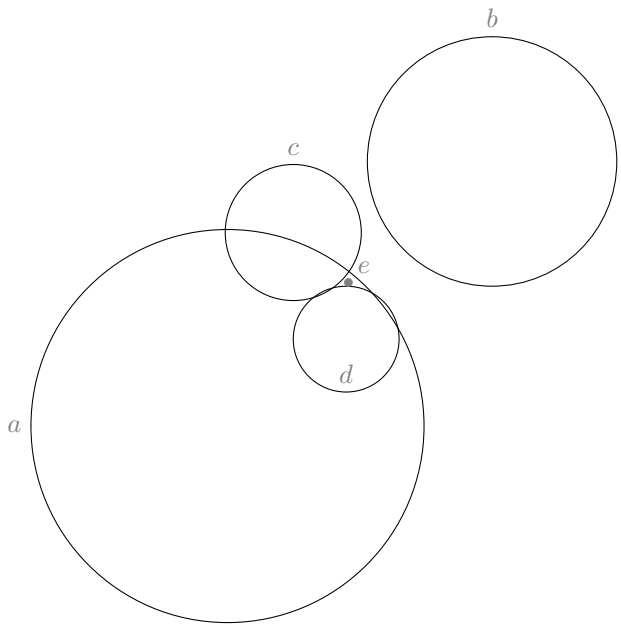


Figure 389: $a \to e$, $bc \to e$, $bd \to e$, $cd \to e$, $ac \to de$, $ab \to cde$

390

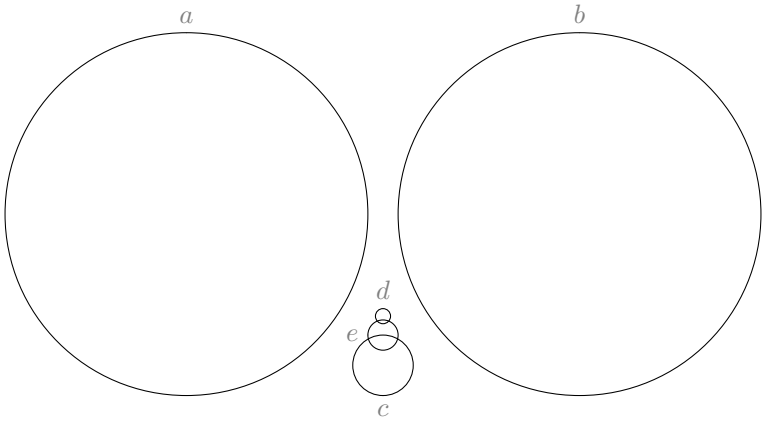


Figure 390: $be \to d$, $ae \to d$, $cd \to e$, $bc \to de$, $ac \to de$, $ab \to cde$

391

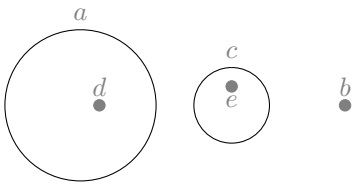


Figure 391: $a \to d$, $ab \to c$, $c \to e$

392

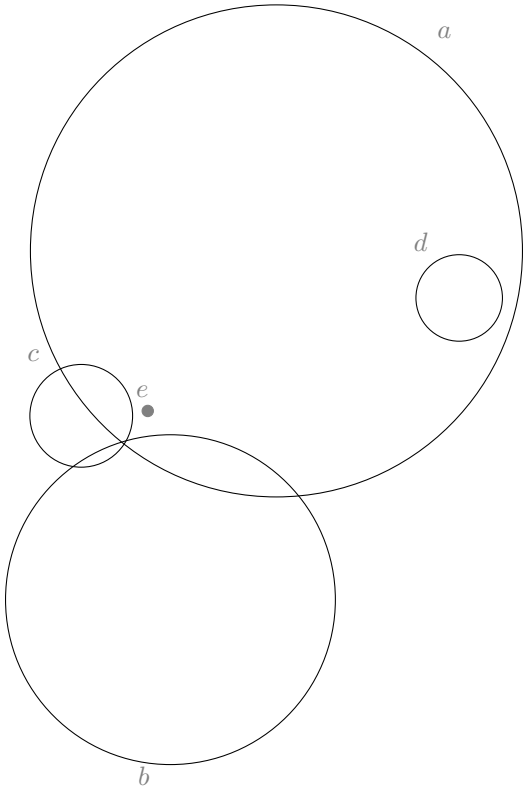


Figure 392: $cd \to e, bc \to e, ab \to cde, a \to de$

393

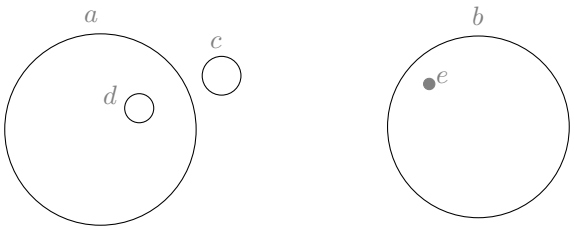


Figure 393: $ae \to cd, ab \to cde, b \to e, a \to d$

394

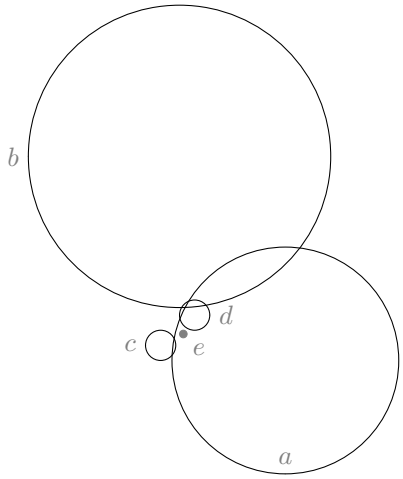


Figure 394: $a \to e$, $cd \to e$, $bc \to de$, $ac \to de$, $ab \to cde$

395

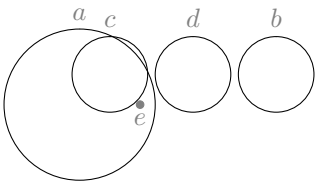


Figure 395: $a \to e$, $cd \to e$, $ad \to ce$, $bc \to de$, $ab \to cde$

396

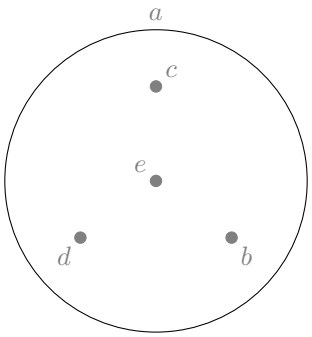


Figure 396: $bcd \to e, a \to bcde$

397

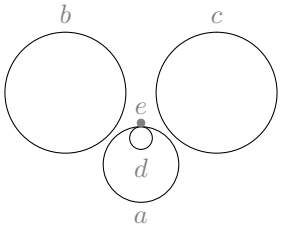


Figure 397: $a \to d$, $cd \to e$, $bd \to e$, $bc \to de$, $ac \to de$, $ab \to de$

398

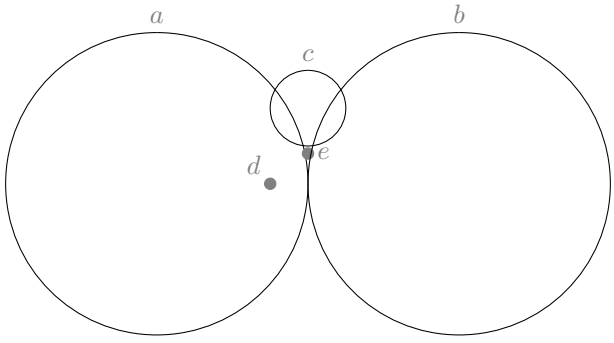


Figure 398: $ac \to e$, $bc \to e$, $ab \to ce$, $cd \to e$, $bd \to e$, $a \to d$

399

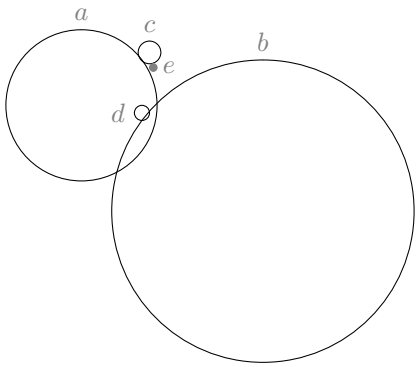


Figure 399: $a \to d$, $cd \to e$, $bc \to de$, $ac \to de$, $ab \to cde$

400

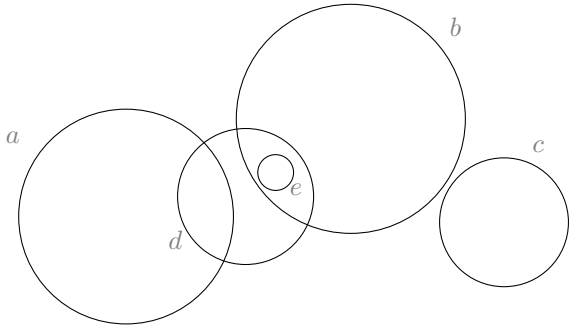


Figure 400: $ab \to de, ac \to de, d \to e, b \to e$

401

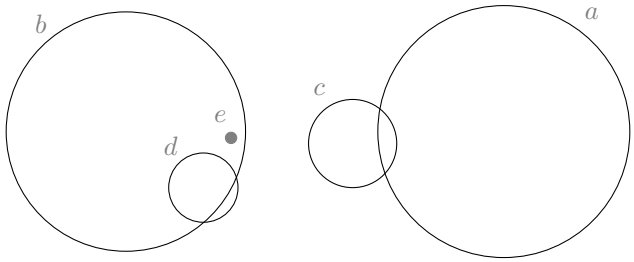


Figure 401: $ae \to c, ad \to ce, bc \to de, ab \to cde, b \to e$

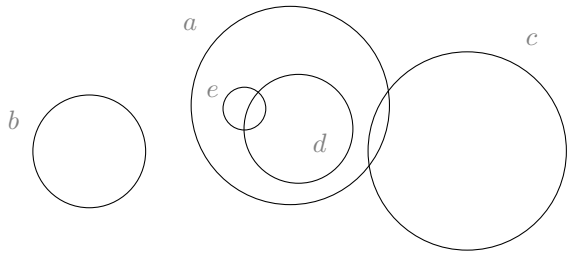


Figure 402: $bc \to de, bd \to e, a \to de$

403

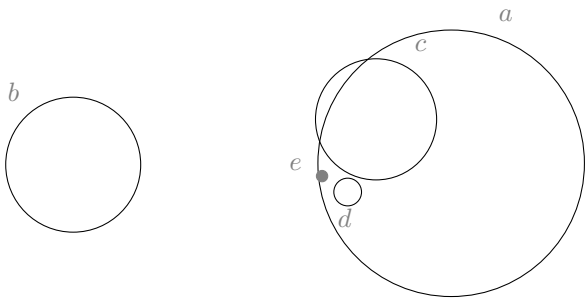


Figure 403: $ab \to cde, bd \to e, bc \to e, a \to de$

404

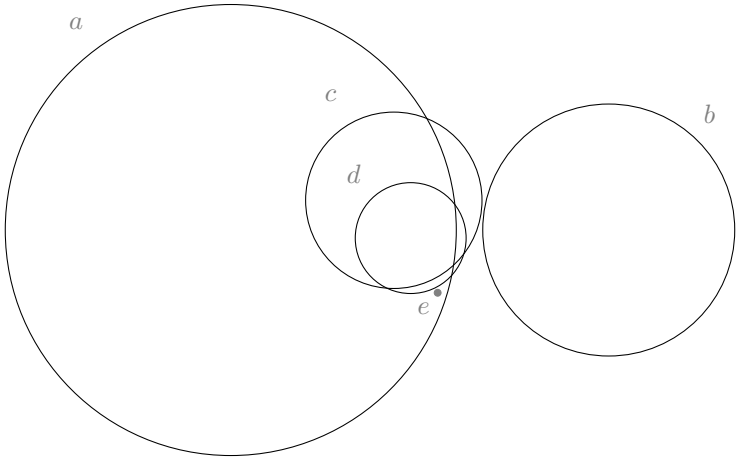


Figure 404: $bd \to e, bc \to de, ac \to de, ab \to cde, a \to e$

405

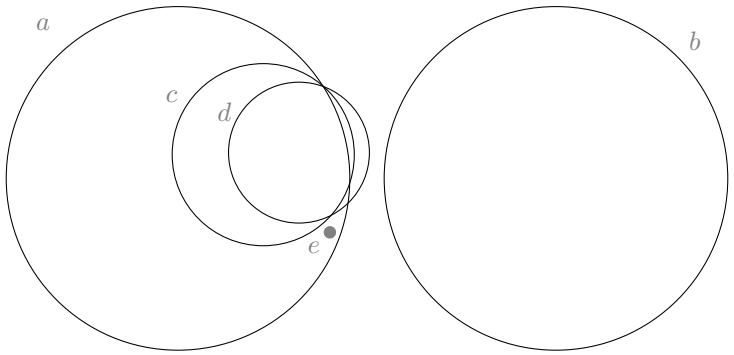


Figure 405: $bd \to e, ad \to c, bc \to de, ab \to cd, a \to e$

406

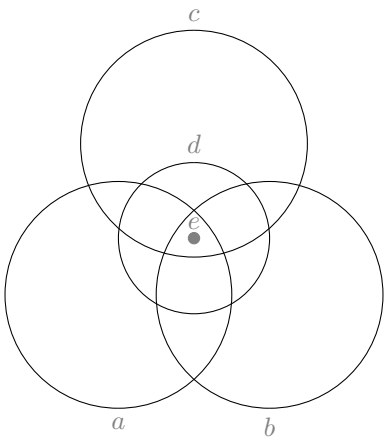


Figure 406: $a \to e$, $b \to e$, $c \to e$, $d \to e$, $abc \to de$

407

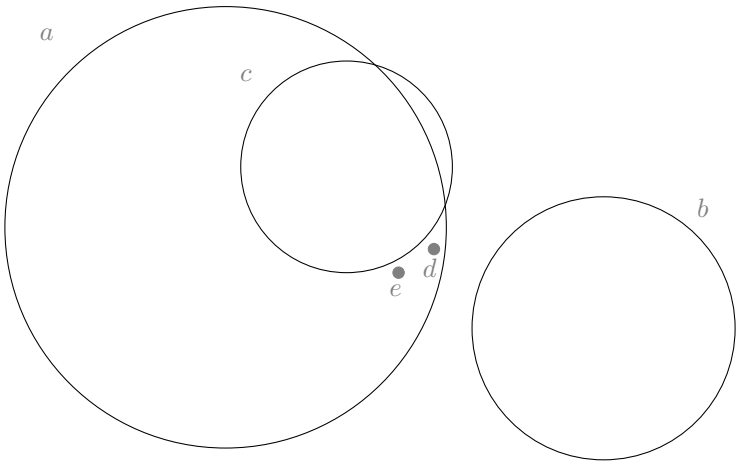


Figure 407: $bc \to de, ab \to cde, a \to de$

408

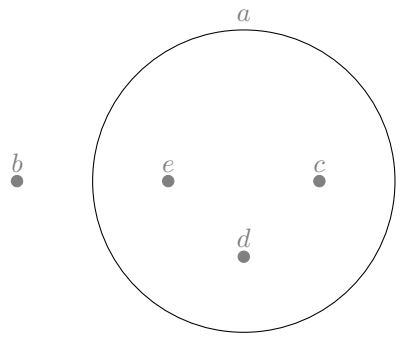


Figure 408: $a \to cde$, $bc \to e$

409

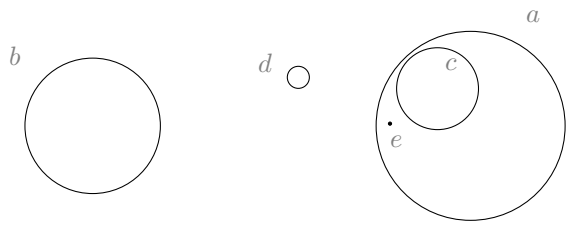


Figure 409: $ab \to cde, bc \to de, a \to ce$

410

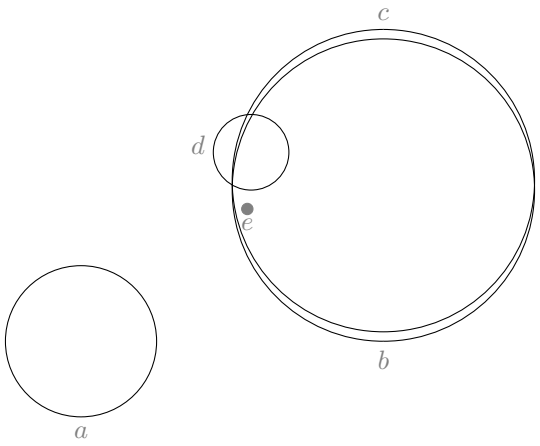


Figure 410: $b \to e, c \to e, ab \to de, ac \to de, ad \to e$

411

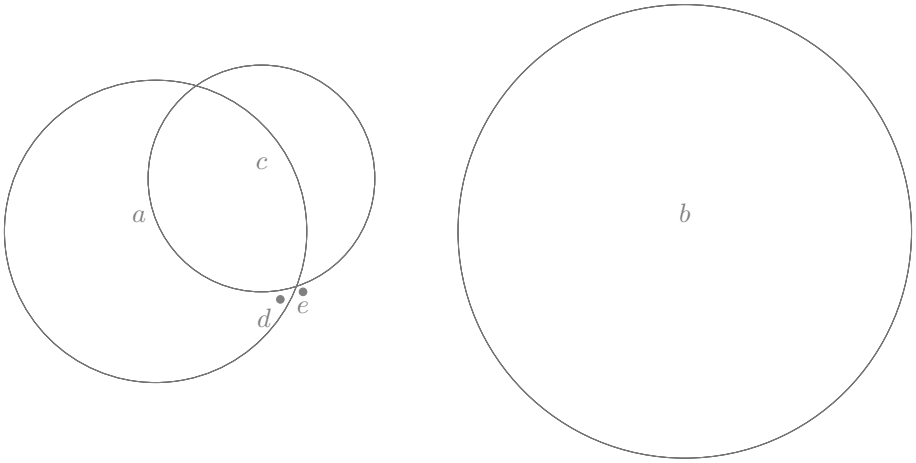


Figure 411: $bd \to e$, $bc \to de$, $ac \to de$, $ab \to cde$, $a \to d$

412

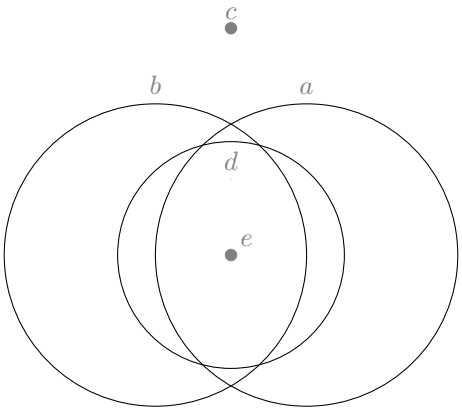


Figure 412: $a \to e$, $b \to e$, $d \to e$, $ab \to de$

413

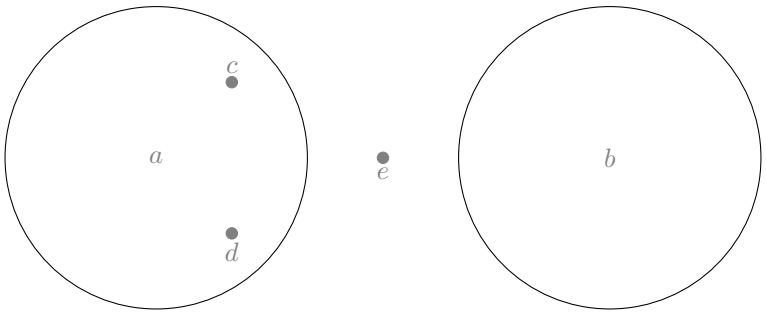


Figure 413: $bd \to e$, $bc \to e$, $ab \to cde$, $a \to cd$

414

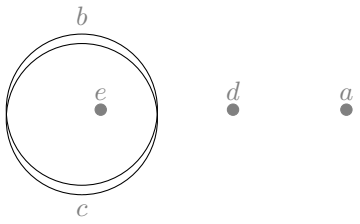


Figure 414: $b \to e, c \to e, ab \to de, ac \to de, ae \to d$

415

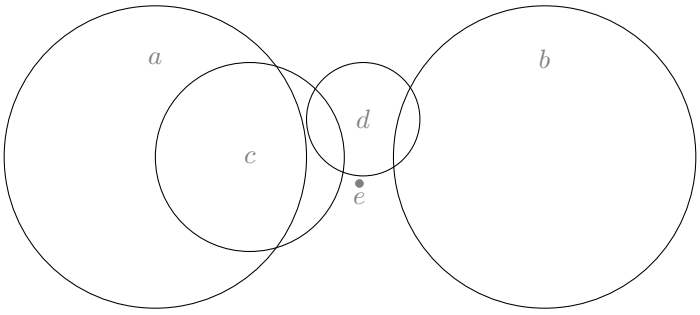


Figure 415: $ae \to c$, $cd \to e$, $bd \to e$, $ad \to ce$, $bc \to de$, $ab \to cde$

416

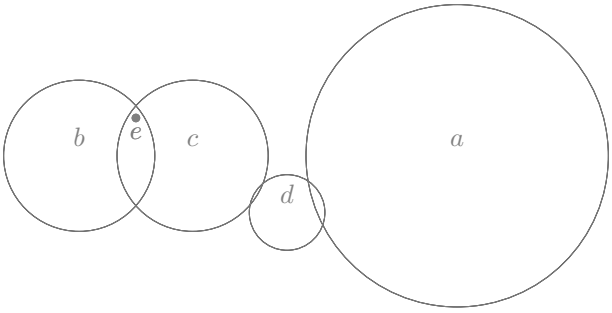


Figure 416: $ac \to de$, $c \to e$, $ab \to cde$, $b \to e$

417

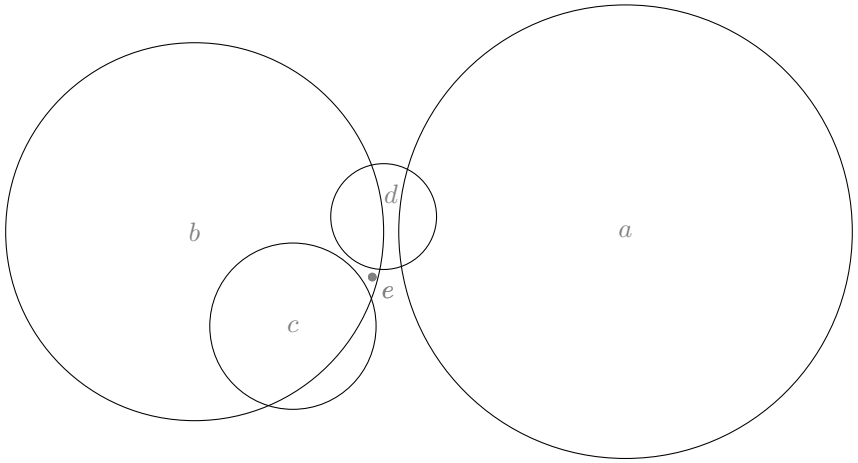


Figure 417: $cd \to e$, $ad \to e$, $ac \to de$, $ab \to cde$, $b \to e$

418

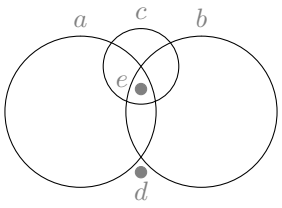


Figure 418: $ab \to de$, $a \to e$, $b \to e$, $c \to e$

419

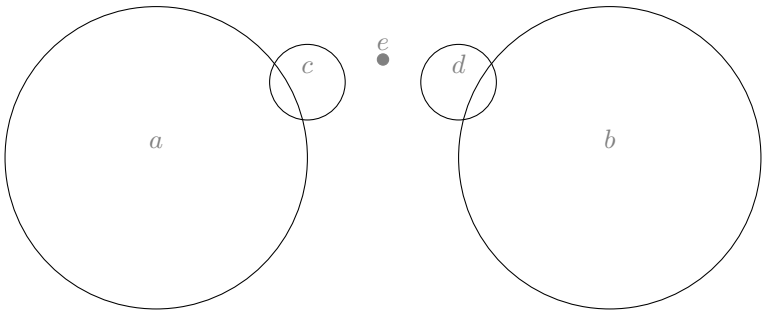


Figure 419: $be \to d$, $ae \to c$, $cd \to e$, $ad \to ce$, $bc \to de$, $ab \to cde$

420

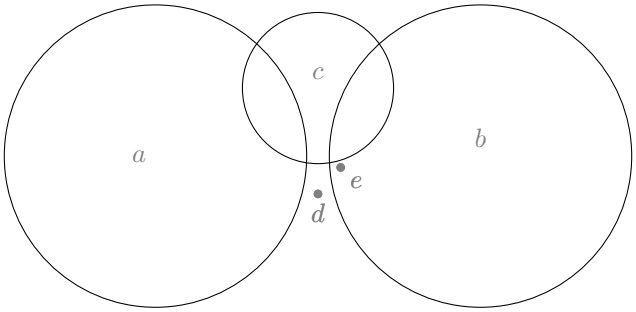


Figure 420: $ae \to d$, $cd \to e$, $bc \to de$, $ac \to de$, $ab \to cde$, $b \to e$

421

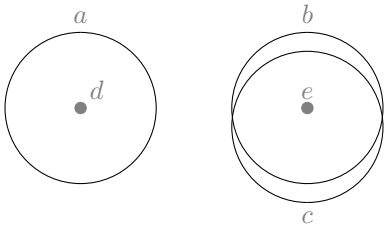


Figure 421: $a \to d$, $b \to e$, $c \to e$

422

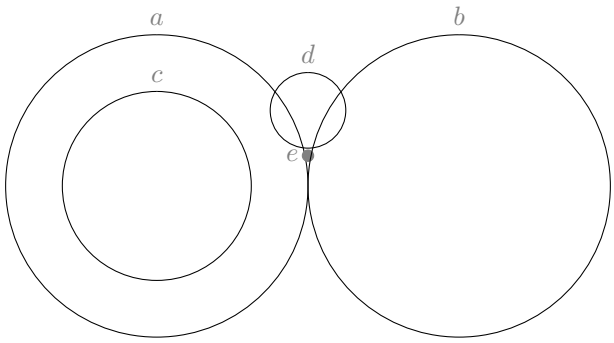


Figure 422: $ad \to e$, $bd \to e$, $ab \to de$, $bc \to de$, $cd \to e$, $a \to c$

423

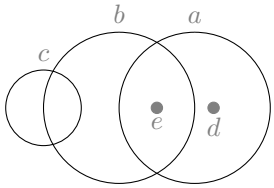


Figure 423: $b \to e$, $a \to de$, $cd \to e$

424

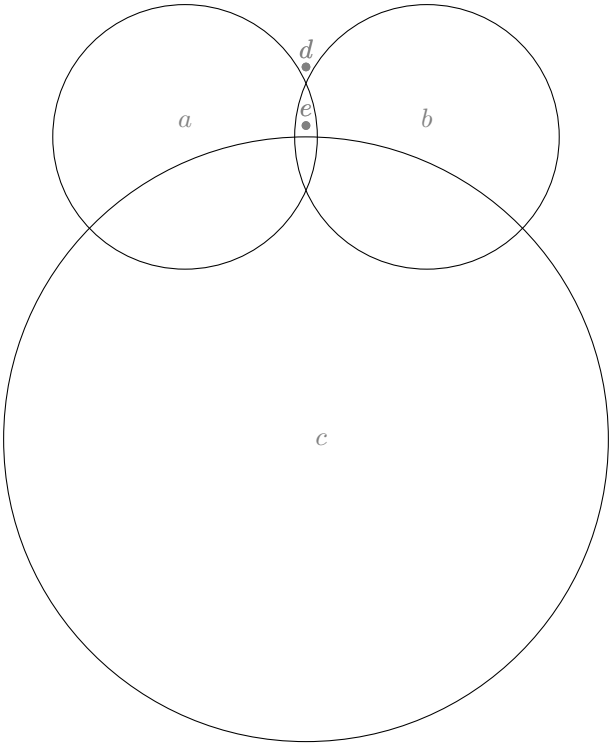


Figure 424: $cd \to e$, $bc \to de$, $ac \to de$, $ab \to de$, $b \to e$, $a \to e$

425

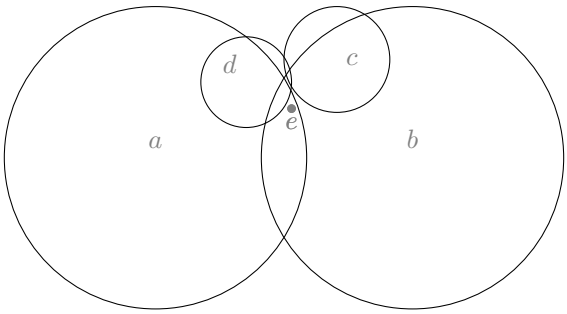


Figure 425: $cd \to e$, $ac \to de$, $ab \to cde$, $b \to e$, $a \to e$

426

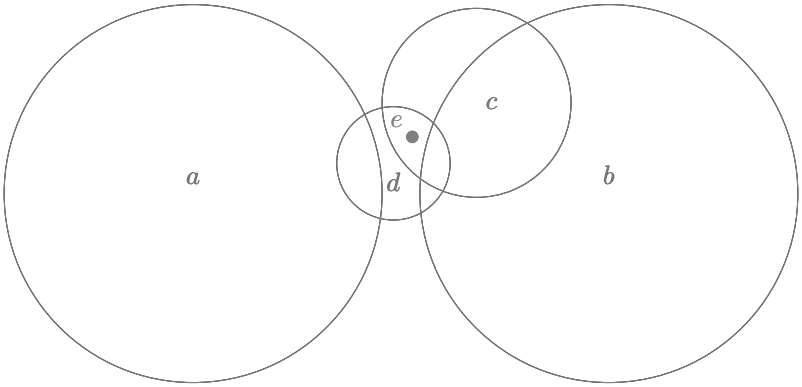


Figure 426: $d \to e$, $ac \to de$, $c \to e$, $ab \to cde$

427

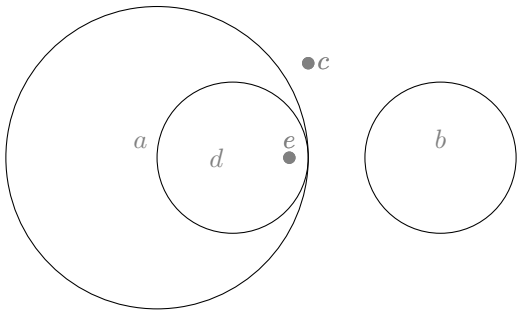


Figure 427: $d \to e$, $ab \to cde$, $a \to de$

428

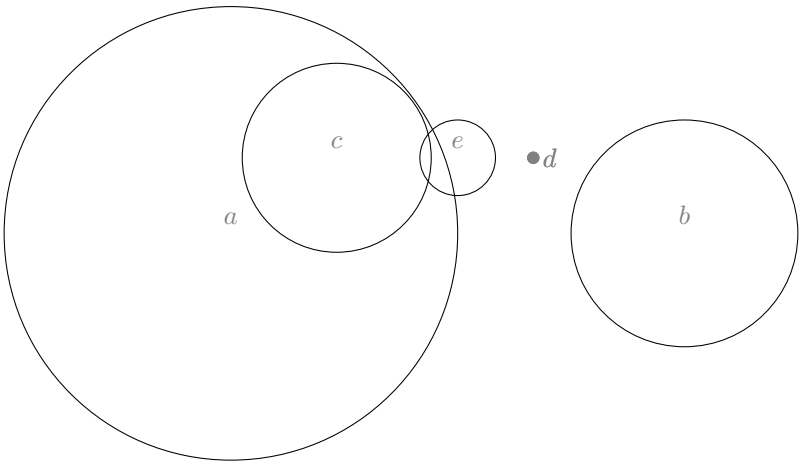


Figure 428: $be \to d$, $cd \to e$, $ad \to ce$, $bc \to de$, $ab \to cde$, $a \to c$

429

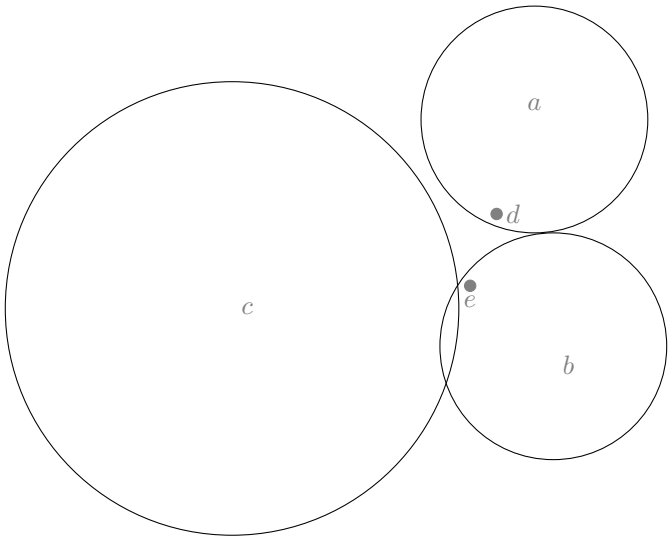


Figure 429: $cd \to e$, $bc \to de$, $ac \to de$, $ab \to de$, $b \to e$, $a \to d$

430

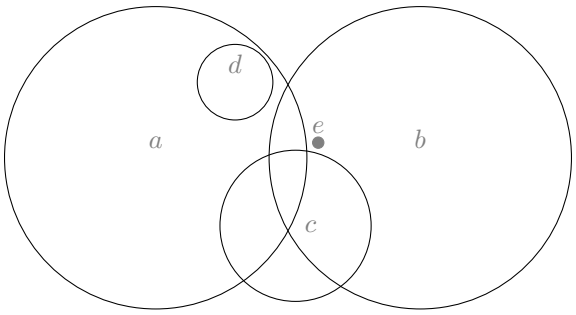


Figure 430: $cd \to e$, $ac \to de$, $ab \to cde$, $b \to e$, $a \to d$

431

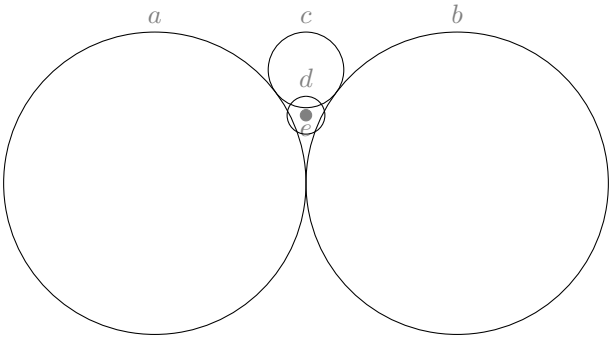


Figure 431: $ac \to d$, $bc \to d$, $ab \to cd$, $d \to e$

432

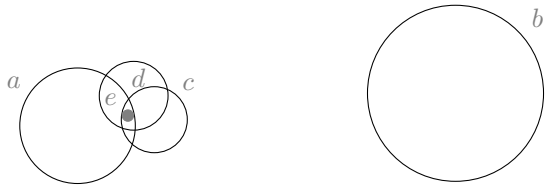


Figure 432: $d \to e, c \to e, ab \to cde, a \to e$

433

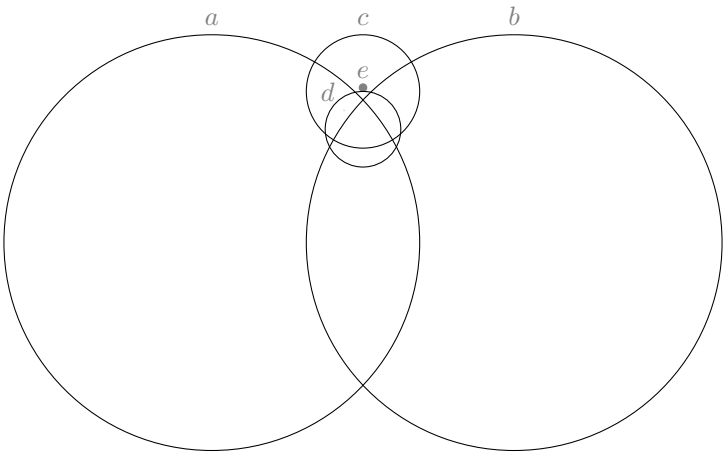


Figure 433: $ac \to d$, $bc \to d$, $ab \to cd$, $ad \to e$, $bd \to e$, $c \to e$

434

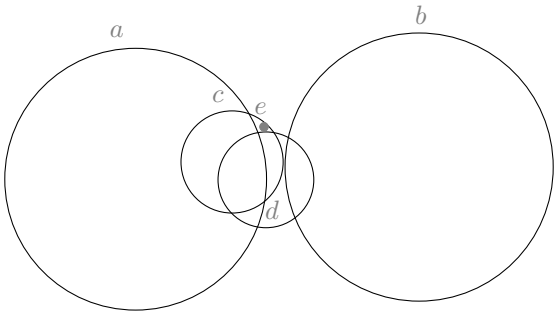


Figure 434: $ab \to cde, c \to e, bc \to de, ad \to ce, bd \to e$

435

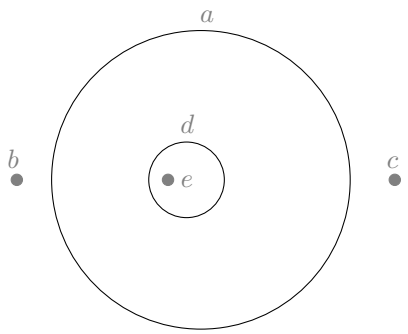


Figure 435: $a \to de, d \to e, bc \to e$

436

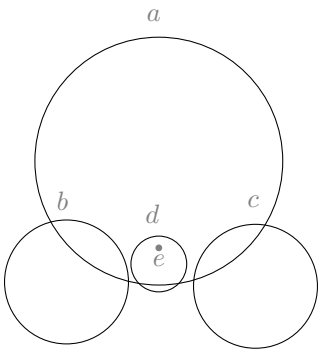


Figure 436: $a \to e, bc \to de, ab \to de, ac \to de, d \to e$

437

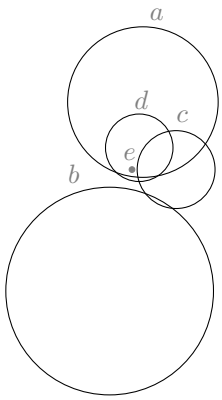


Figure 437: $a \to e, bc \to e, ac \to de, ab \to cde, d \to e$

438

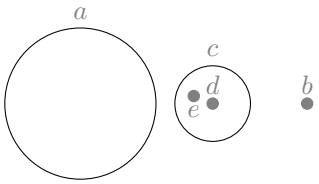


Figure 438: $c \to de, ab \to cde, ad \to e$

439

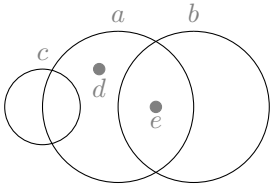


Figure 439: $a \to de$, $b \to e$, $bc \to d$

440

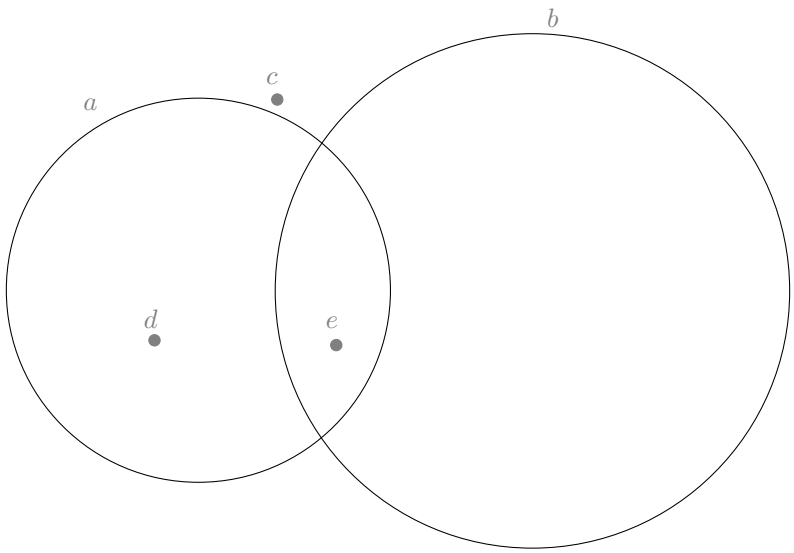


Figure 440: $ab \to cde, b \to e, a \to de$

441

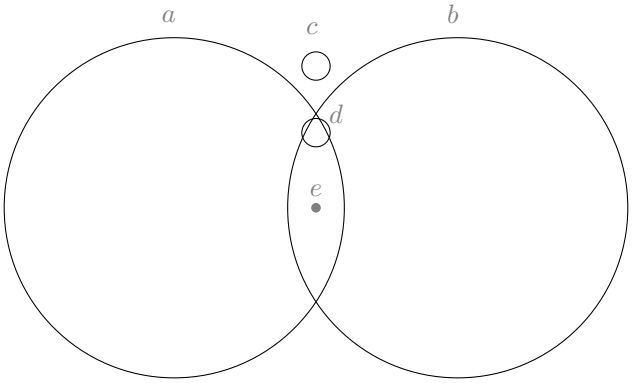


Figure 441: $bc \to de, ac \to de, ab \to cde, b \to e, a \to e$

442

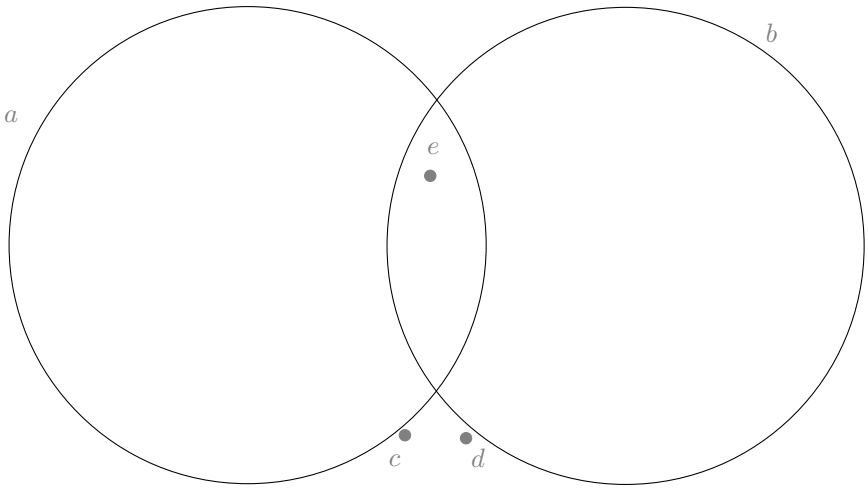


Figure 442: $a \to e, b \to e, ab \to cde, ad \to ce, bc \to de$

443

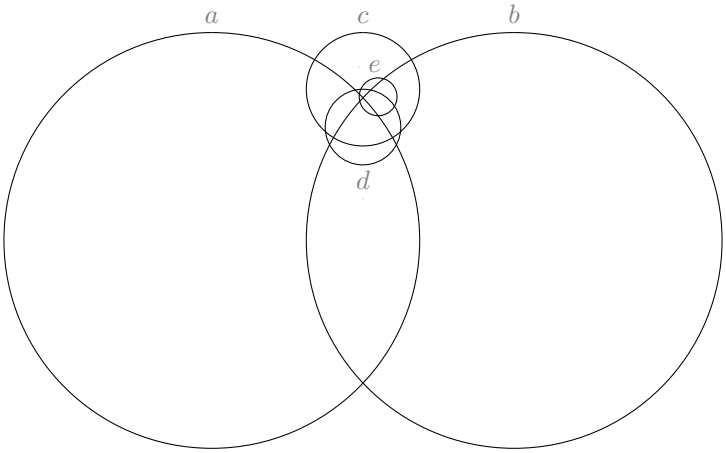


Figure 443: $ac \to d$, $ae \to d$, $bc \to d$, $ab \to cd$, $bd \to e$, $c \to e$

444

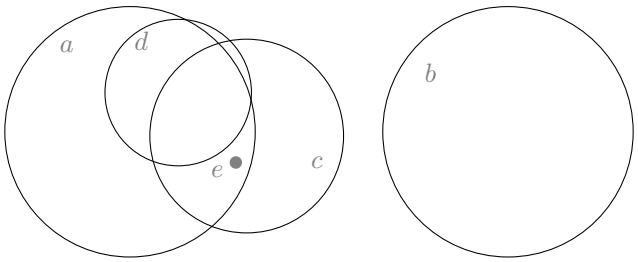


Figure 444: $bd \to e$, $ac \to de$, $c \to e$, $ab \to cde$, $a \to e$

445

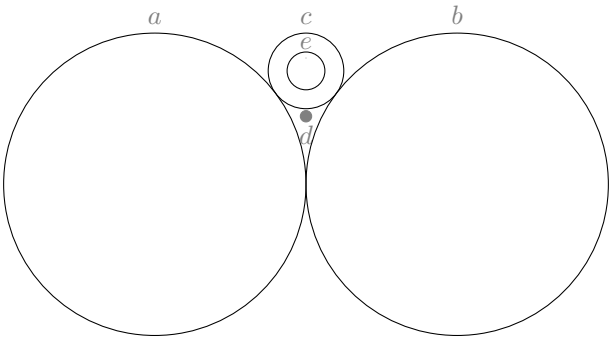


Figure 445: $ac \to d$, $bc \to d$, $ab \to cd$, $c \to e$, $ae \to d$, $be \to d$

446

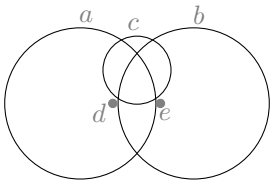


Figure 446: $bc \to de$, $ac \to de$, $ab \to cde$, $b \to e$, $a \to d$

447

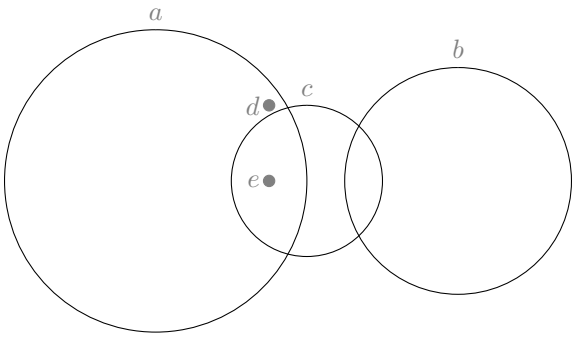


Figure 447: $a \to de$, $c \to e$, $ab \to ce$

448

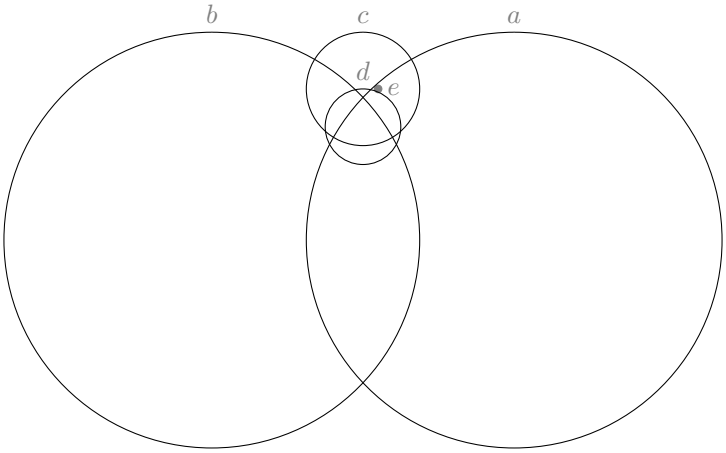


Figure 448: $ac \to d$, $bc \to d$, $ab \to cd$, $a \to e$, $c \to e$

449

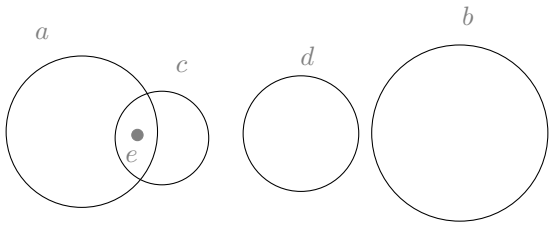


Figure 449: $a \to e, c \to e, ab \to cde, ad \to ce, bc \to de$

450

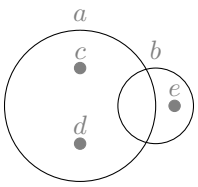


Figure 450: $ab \to cde, b \to e, a \to cd$

451

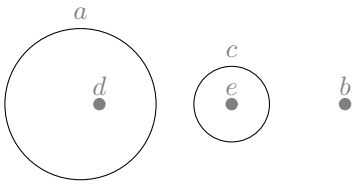


Figure 451: $a \to d$, $bd \to e$, $ab \to c$, $c \to e$

452

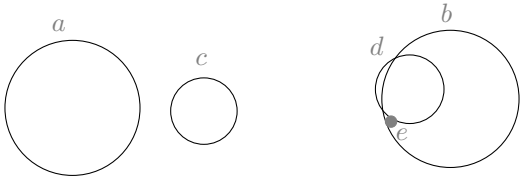


Figure 452: $ae \to c, cd \to e, ad \to ce, bc \to de, ab \to cde, b \to e$

453

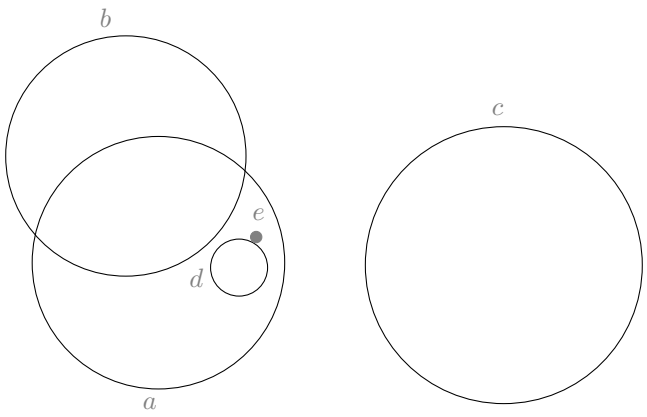


Figure 453: $cd \to e, bd \to e, bc \to de, a \to de$

454

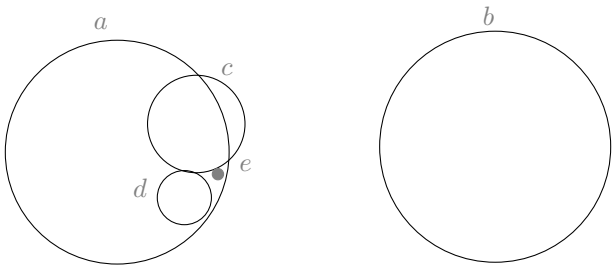


Figure 454: $cd \to e, bd \to e, bc \to e, ab \to cde, a \to de$

455

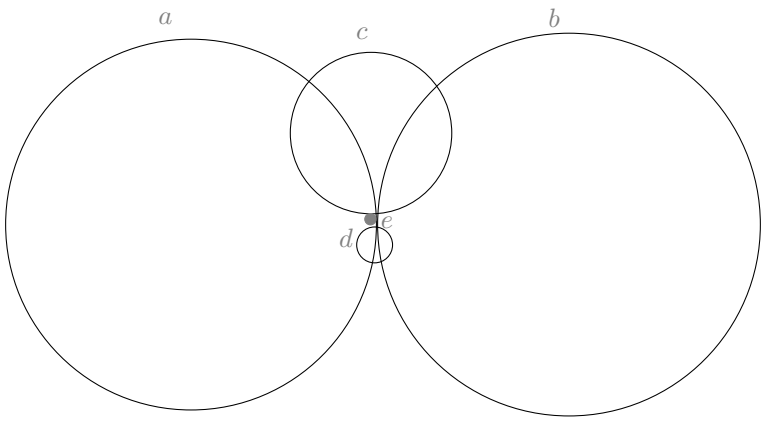


Figure 455: $cd \to e, bd \to e, bc \to de, ac \to de, ab \to cde, a \to e$

456

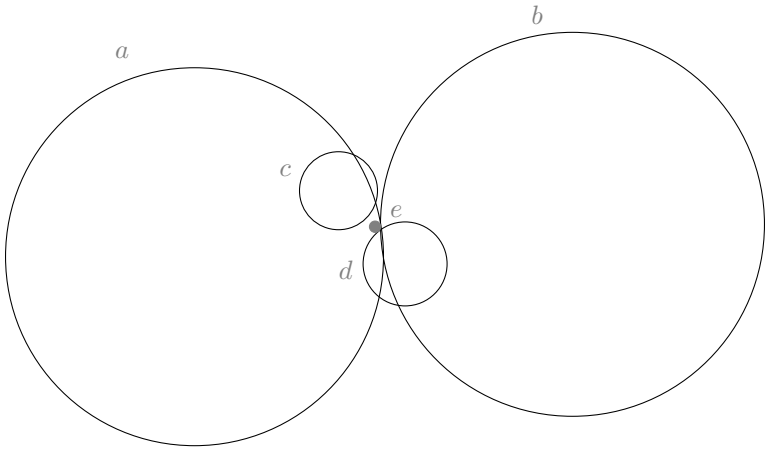


Figure 456: $cd \to e, bd \to e, ad \to ce, bc \to de, ab \to cde, a \to e$

457

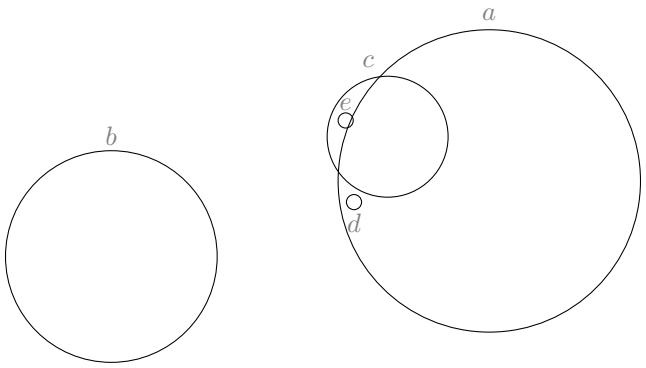


Figure 457: $bc \to de, ac \to de, c \to e, ab \to cde, a \to d$

458

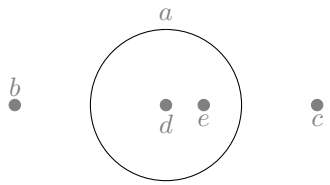


Figure 458: $be \to d, cd \to e, bc \to de, a \to de$

459

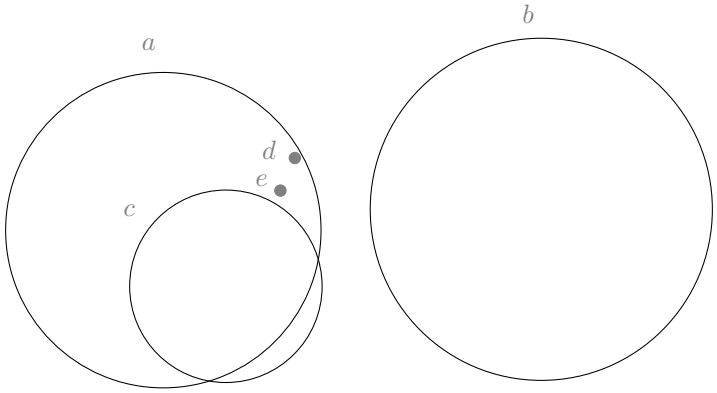


Figure 459: $cd \to e, bc \to de, ab \to cde, a \to de$

460

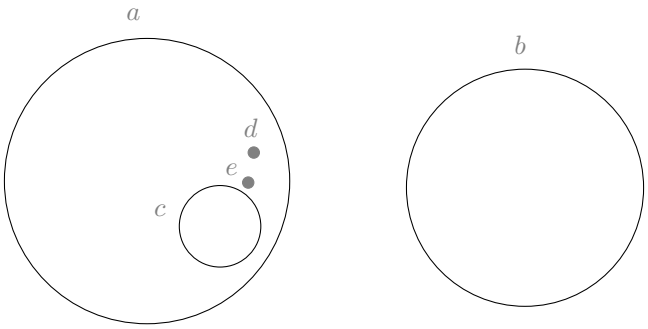


Figure 460: $cd \to e, bc \to e, a \to cde$

461

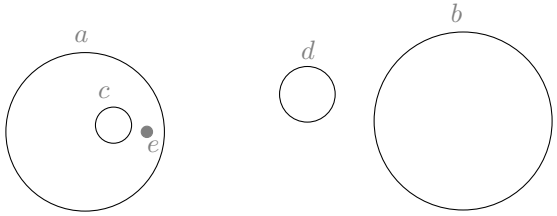


Figure 461: $cd \to e, bc \to de, ab \to cde, a \to ce$

462

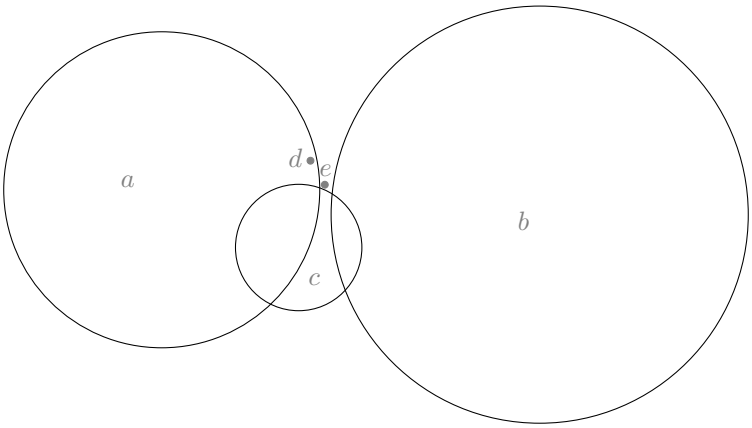


Figure 462: $cd \to e, bd \to e, bc \to de, ac \to de, ab \to cde, a \to d$

463

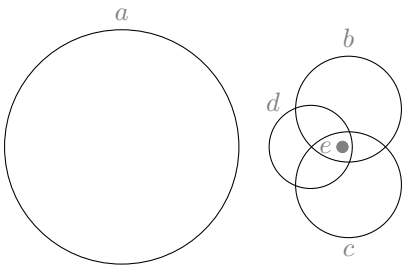


Figure 463: $d \to e$, $ac \to de$, $c \to e$, $ab \to de$, $b \to e$

464

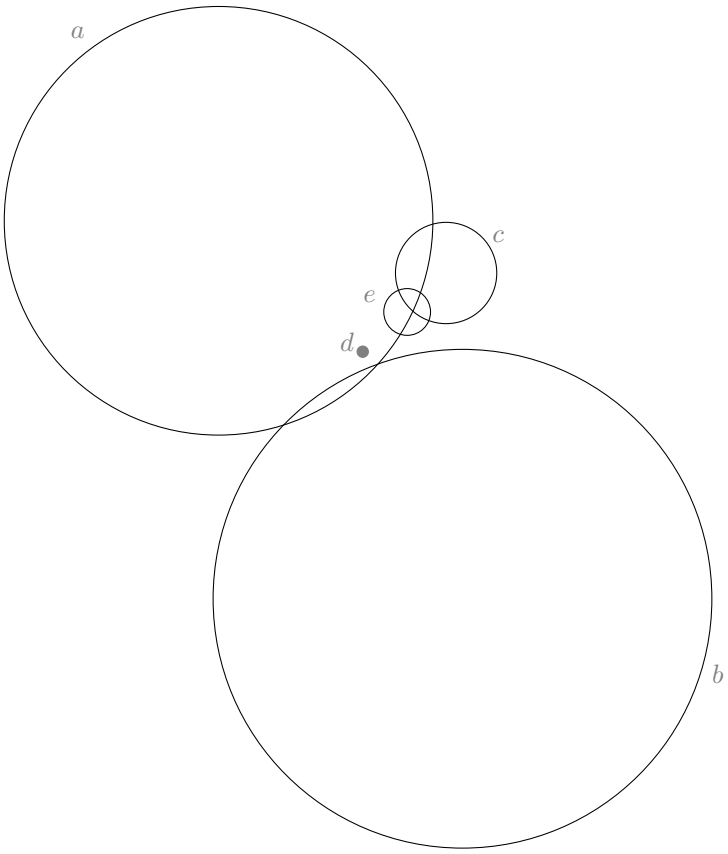


Figure 464: $be \to d, cd \to e, bc \to de, ac \to de, ab \to cde, a \to d$

465

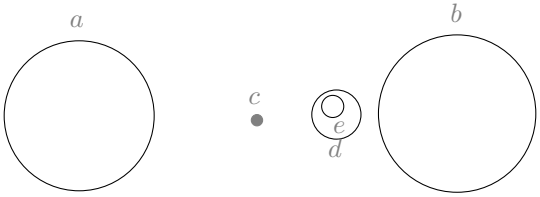


Figure 465: $ae \to c, ad \to ce, d \to e, bc \to de, ab \to cde$

466

Figure 466: $d \to e, ac \to de, ab \to cde, b \to e$

467

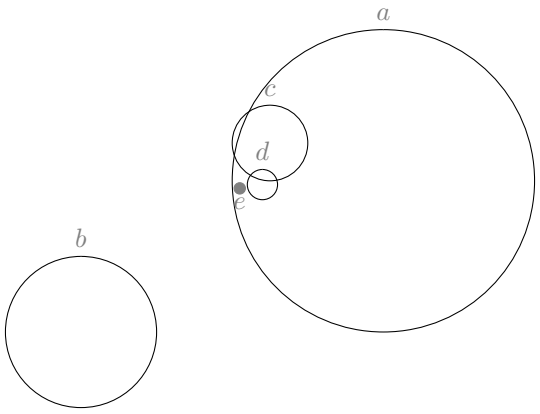


Figure 467: $bd \to e, bc \to de, ab \to cde, a \to de$

468

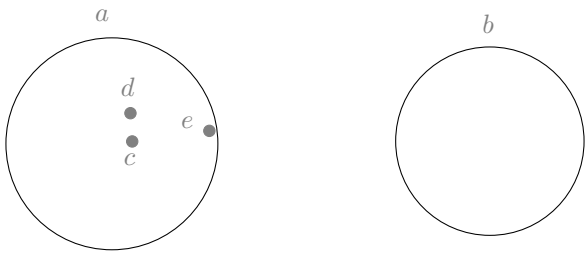


Figure 468: $bd \to e, bc \to e, a \to cde$

469

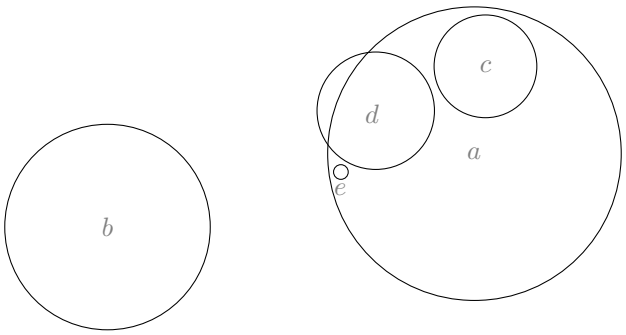


Figure 469: $bd \to e, bc \to de, ab \to cde, a \to ce$

470

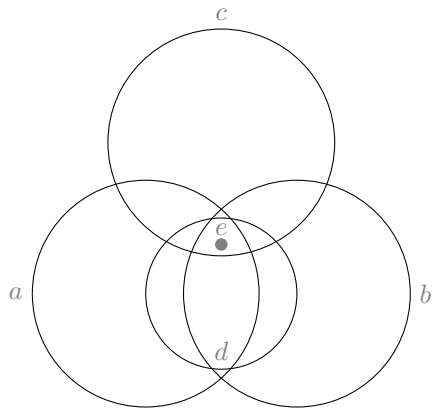


Figure 470: $a \to e$, $b \to e$, $c \to e$, $d \to e$, $ab \to de$

471

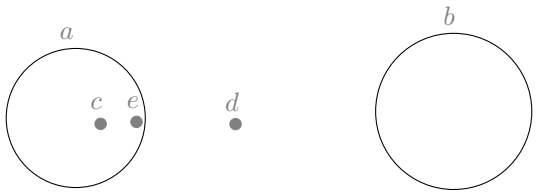


Figure 471: $be \to d, bc \to de, ab \to cde, a \to ce$

472

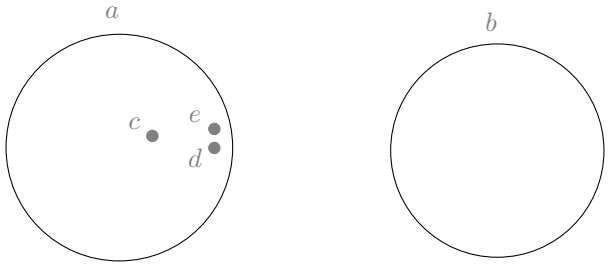


Figure 472: $bc \to de, a \to cde$

473

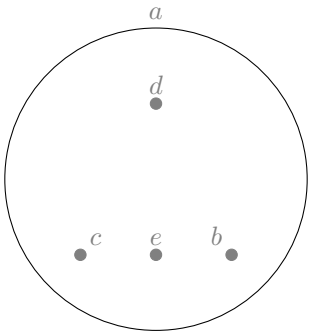


Figure 473: $bc \to e, a \to bcde$

474

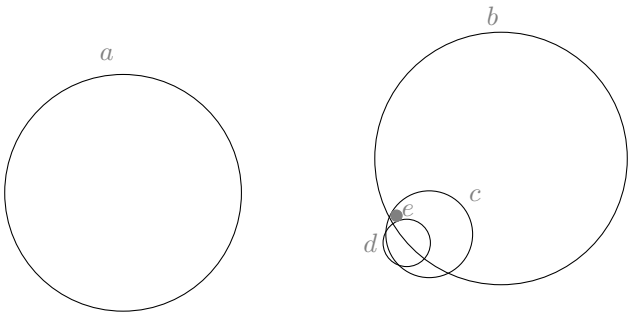


Figure 474: $ad \to e, ac \to de, c \to e, ab \to cde, b \to e$

475

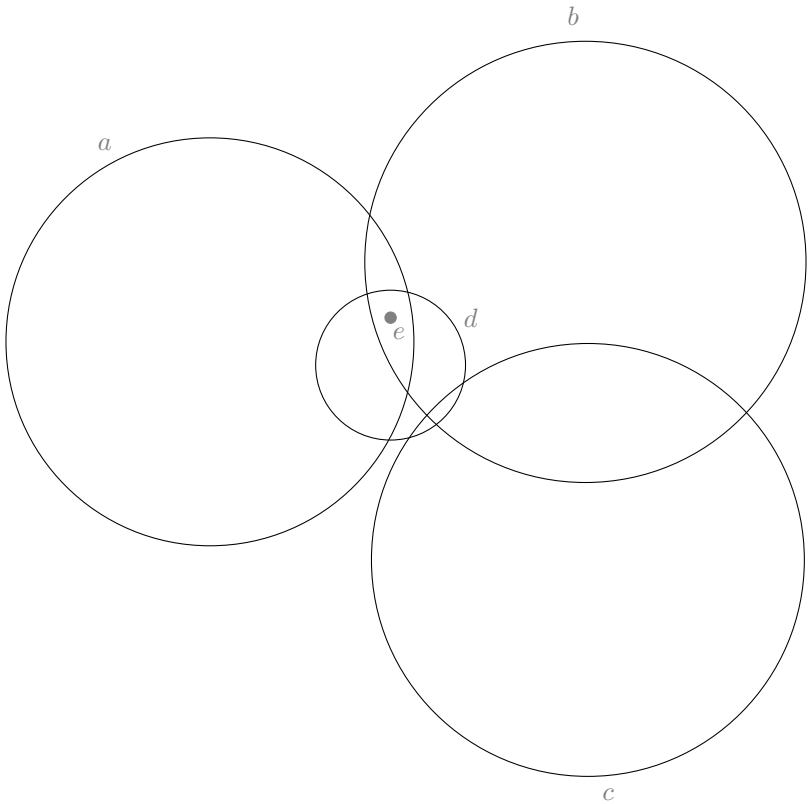


Figure 475: $d \to e, ac \to de, ab \to de, b \to e, a \to e$

476

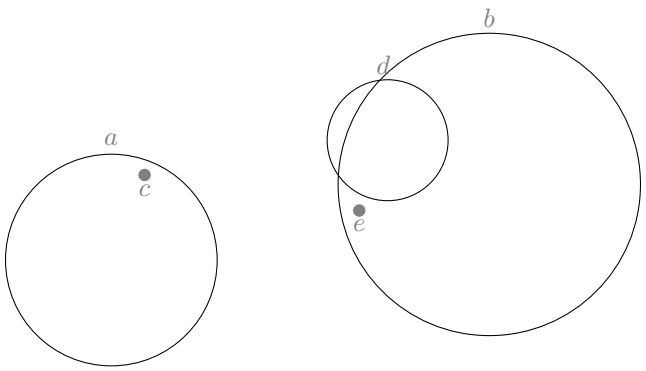


Figure 476: $ad \to ce, bc \to de, ab \to cde, b \to e, a \to c$

477

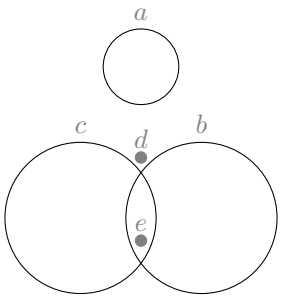


Figure 477: $b \to e$, $c \to e$, $ab \to de$, $ac \to de$, $bc \to de$, $ae \to d$

478

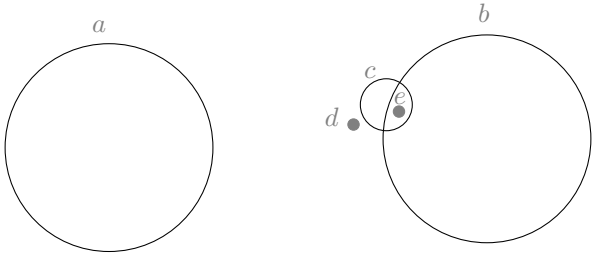


Figure 478: $ae \to d, ac \to de, c \to e, ab \to cde, b \to e$

479

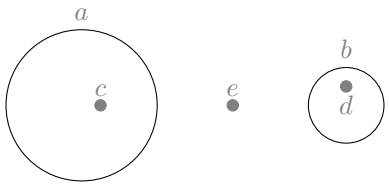


Figure 479: $a \to c$, $bc \to e$, $ab \to e$, $b \to d$, $ad \to e$

480

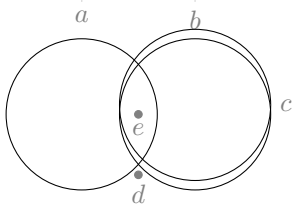


Figure 480: $a \to e, b \to e, c \to e, ab \to de, ac \to de$

481

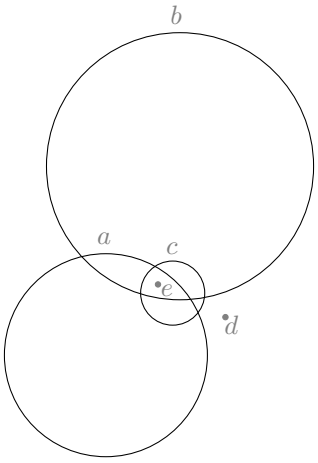


Figure 481: $ab \to cd,\ a \to e, b \to e, c \to e$

482

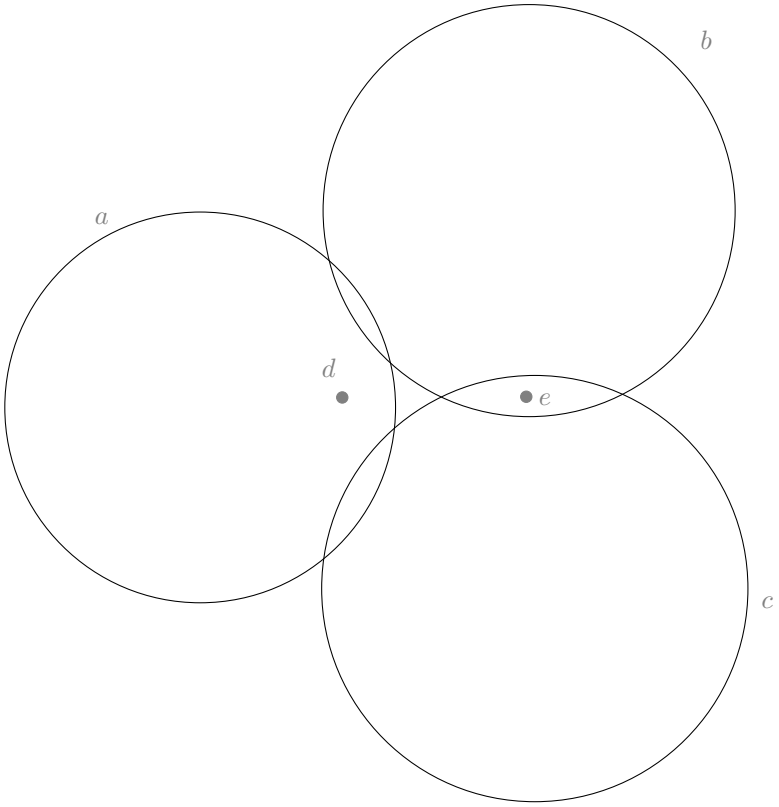


Figure 482: $bc \to de, ac \to de, c \to e, ab \to de, b \to e, a \to d$

483

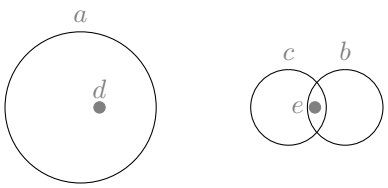


Figure 483: $a \to d$, $b \to e$, $c \to e$, $ab \to c$

484

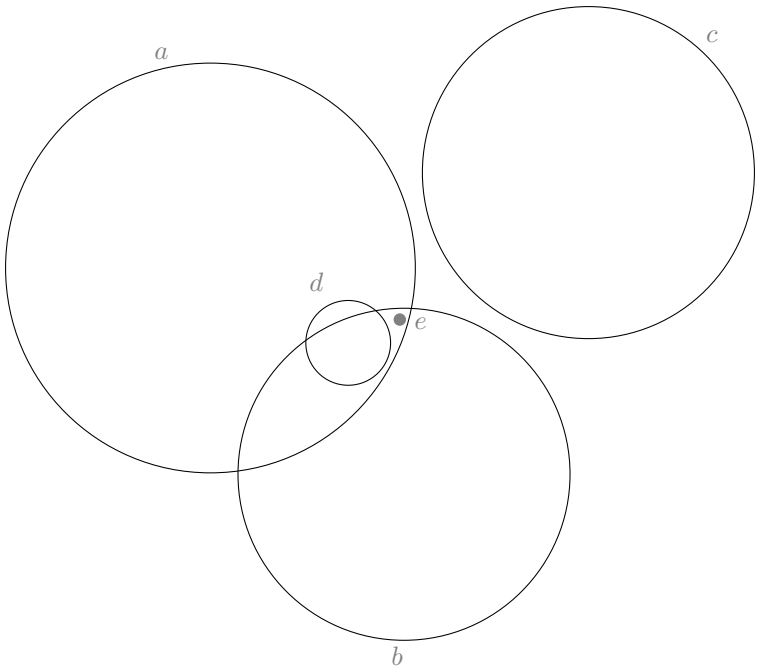


Figure 484: $cd \to e, bc \to de, b \to e, a \to de$

485

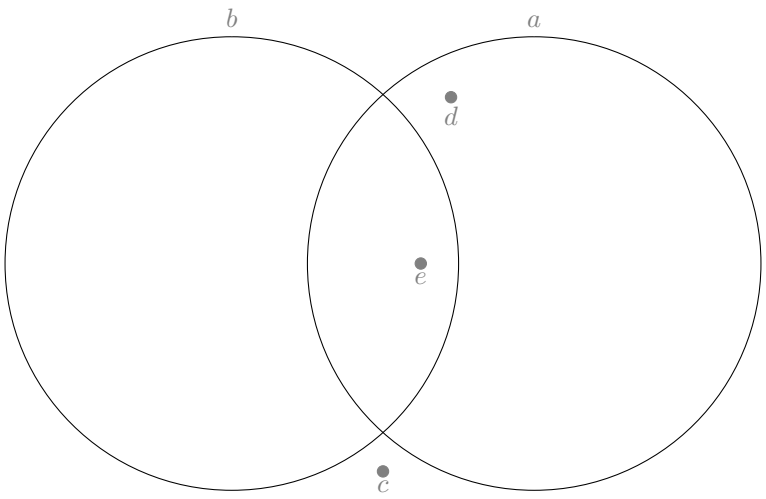


Figure 485: $a \to e, ab \to cde, b \to e, cd \to e$.

486

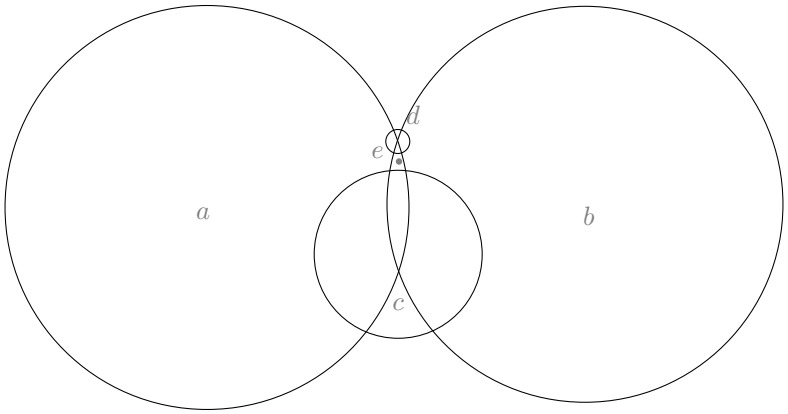


Figure 486: $cd \to e, b \to e, bc \to de, ac \to de, ab \to cde, a \to e$

487

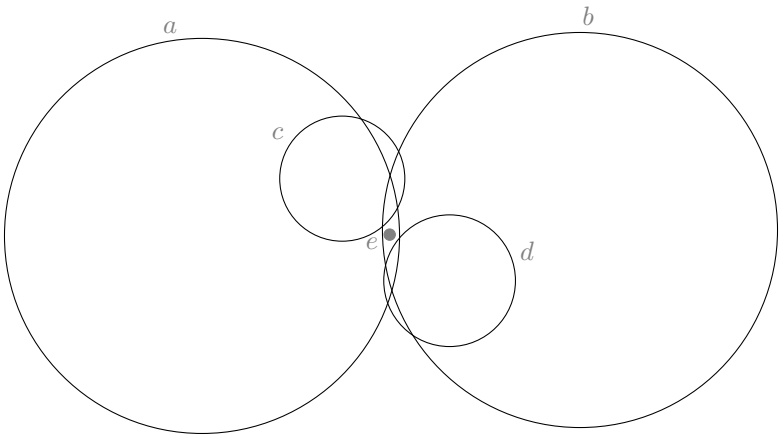


Figure 487: $cd \to e, ad \to ce, bc \to de, ab \to cde, b \to e, a \to e$

488

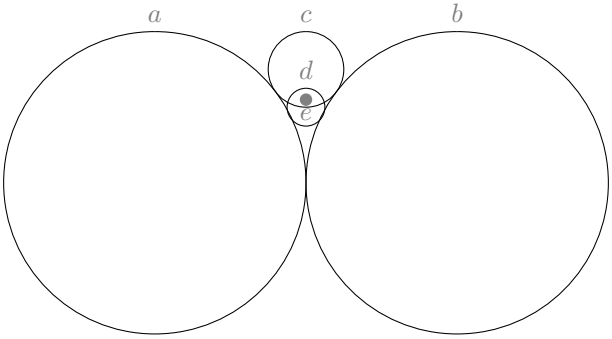


Figure 488: $ac \to d$, $bc \to d$, $ab \to cd$, $c \to e$, $d \to e$

489

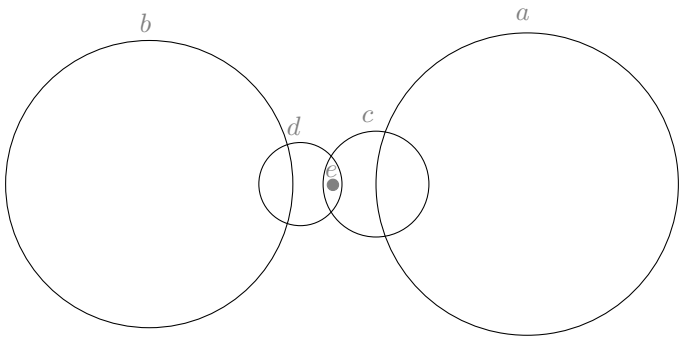


Figure 489: $ad \to ce, d \to e, bc \to de, c \to e, ab \to cde$

490

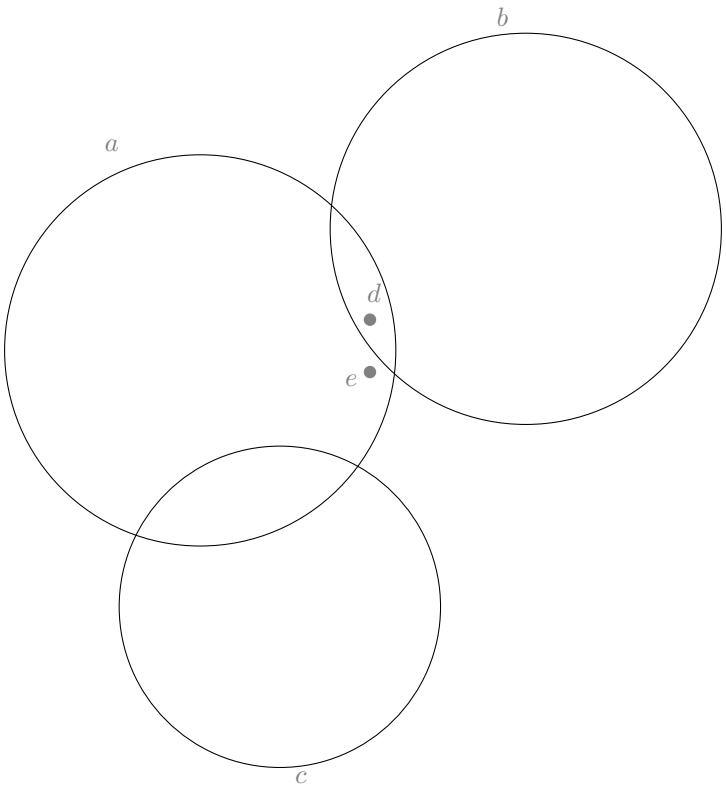


Figure 490: $cd \to e, bc \to de, b \to d, a \to de$

491

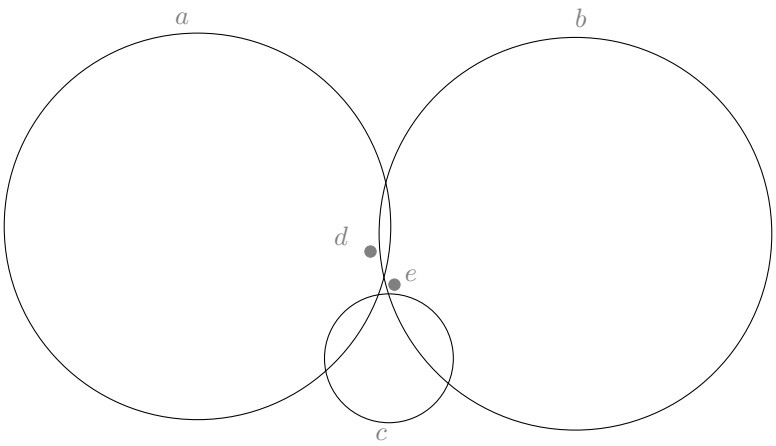


Figure 491: $cd \to e, bc \to de, ac \to de, ab \to cde, b \to e, a \to d$

492

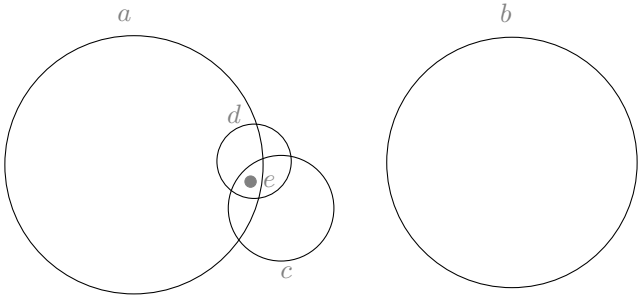


Figure 492: $d \to e, ac \to de, c \to e, ab \to cde, a \to e$

493

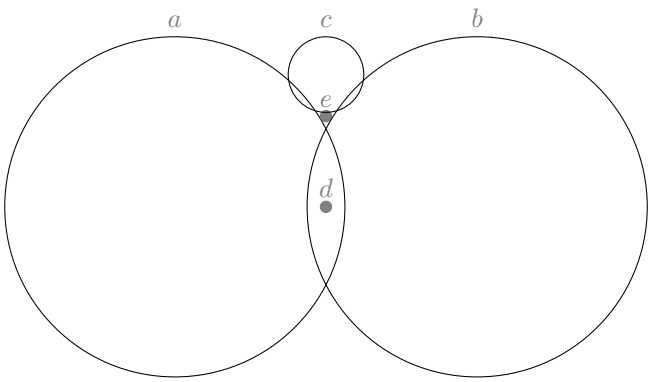


Figure 493: $ac \to e$, $bc \to e$, $ab \to ce$, $a \to d$, $b \to d$, $cd \to e$

494

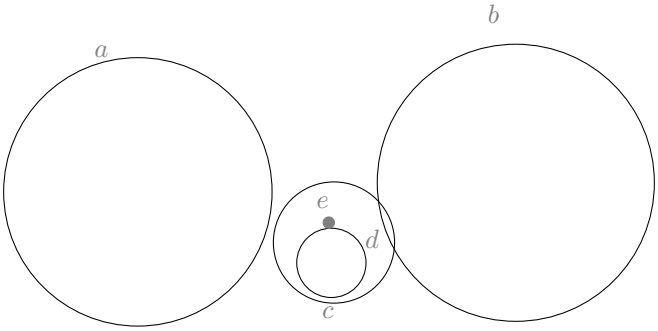


Figure 494: $bd \to e, ad \to e, c \to de, ab \to cde$

495

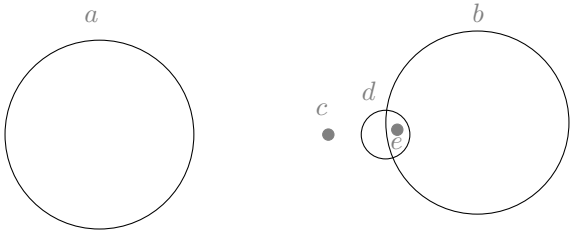


Figure 495: $ae \to c, ad \to ce, d \to e, bc \to de, ab \to cde, b \to e$

496

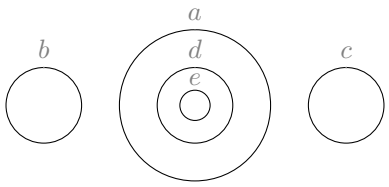


Figure 496: $a \to de, bc \to de, d \to e$

497

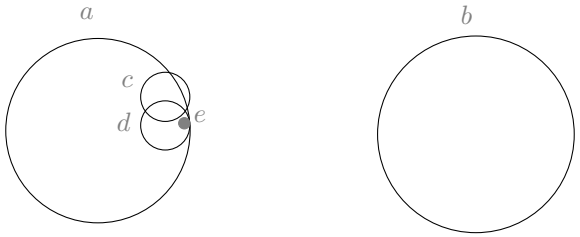


Figure 497: $d \to e, bc \to e, ab \to cde, a \to de$

498

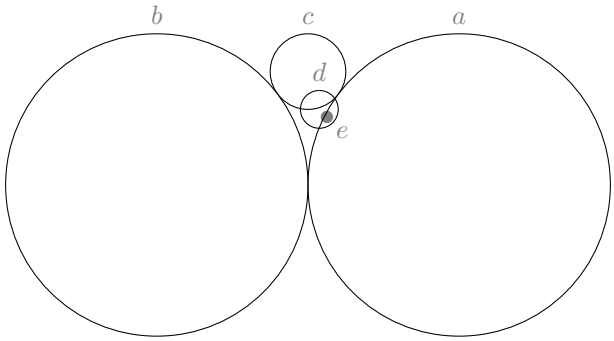


Figure 498: $ac \to d$, $bc \to d$, $ab \to cd$, $a \to e$, $d \to e$

499

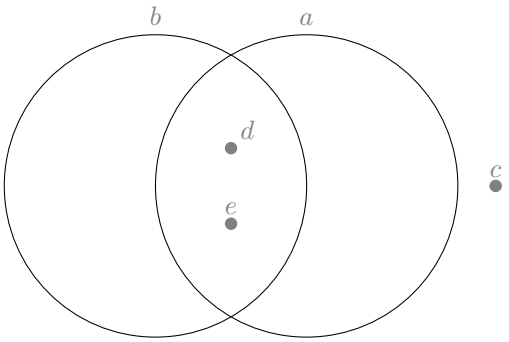


Figure 499: $a \to de$, $b \to de$

500

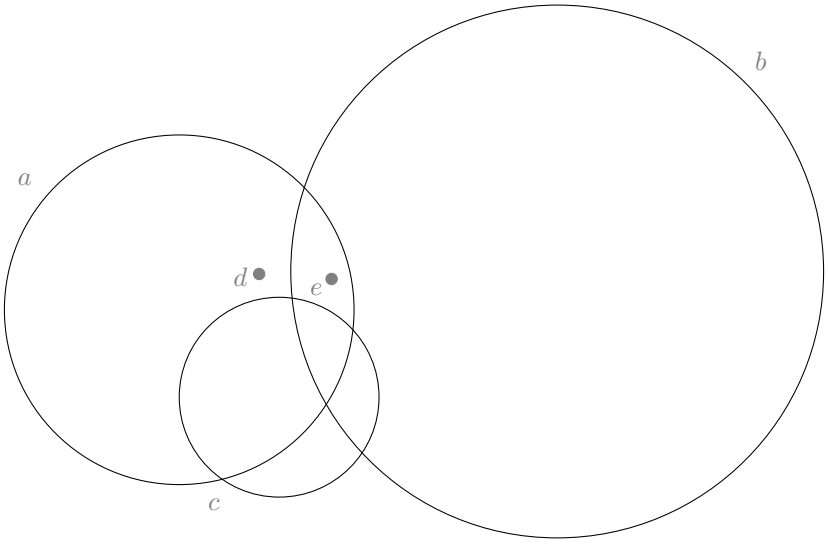


Figure 500: $bc \to de, ab \to cde, b \to e, a \to de$

501

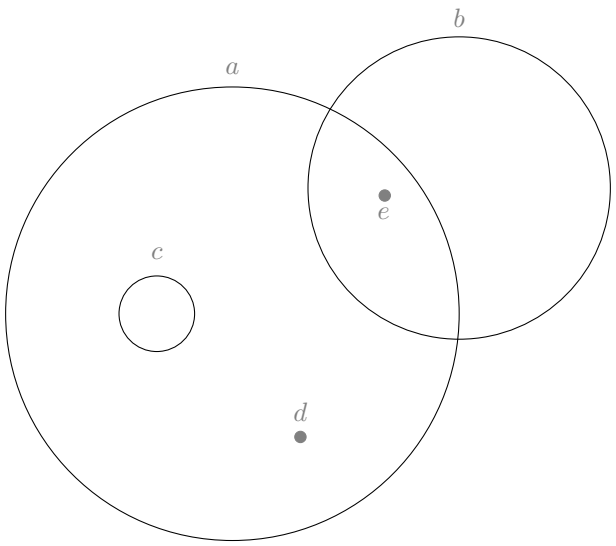


Figure 501: $b \to e, a \to cde$

502

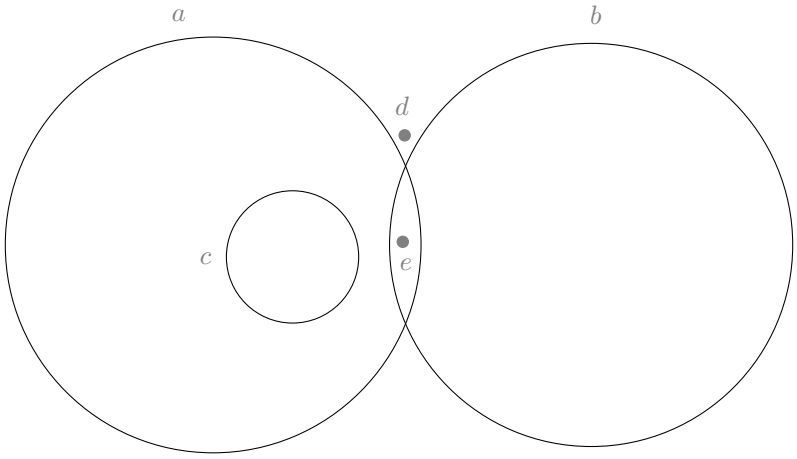


Figure 502: $bc \to de, ab \to cde, b \to e, a \to ce$

503

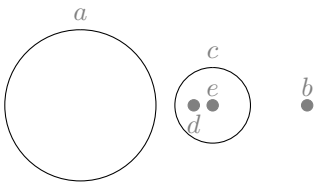


Figure 503: $c \to de, ab \to cde, ae \to d, bd \to e$

504

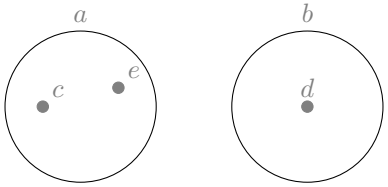


Figure 504: $a \to ce$, $b \to d$, $bc \to e$

505

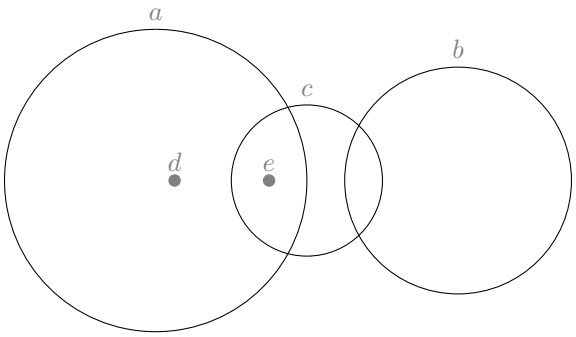


Figure 505: $a \to de$, $c \to e$, $ab \to c$, $bd \to e$

506

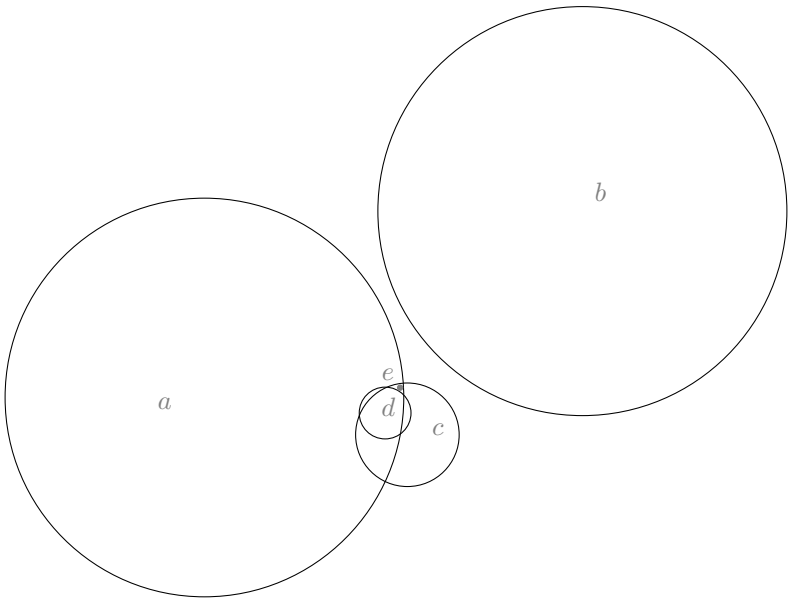


Figure 506: $bd \to e, bc \to de, ac \to de, c \to e, ab \to cde, a \to e$

507

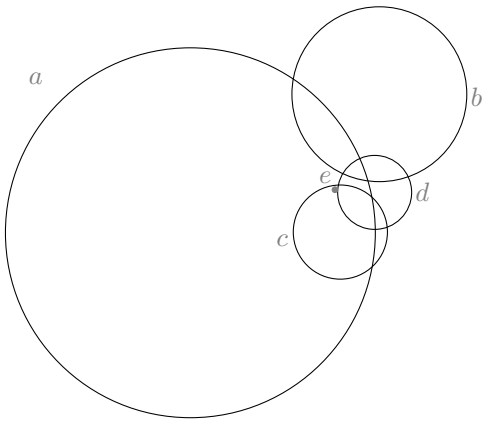


Figure 507: $a \to e, c \to e, ab \to cde, ad \to ce, bc \to de, bd \to e$

508

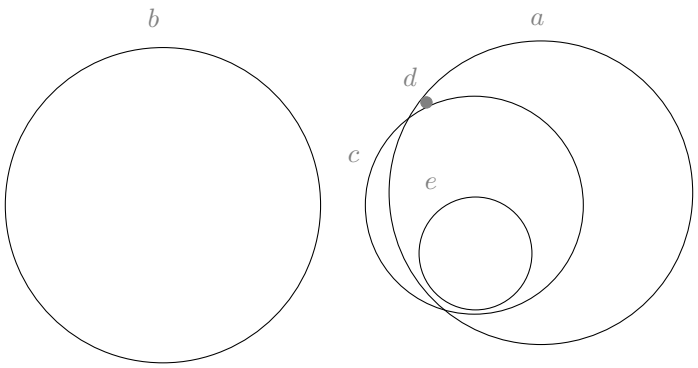


Figure 508: $bc \to de, c \to e, ab \to cde, a \to de$

509

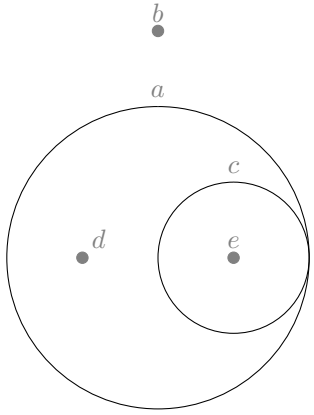


Figure 509: $c \to e$, $a \to cde$

510

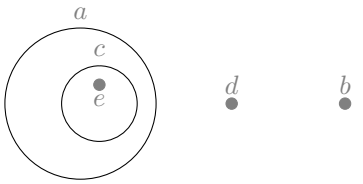


Figure 510: $a \to ce$, $bc \to de$, $ab \to cde$, $c \to e$

511

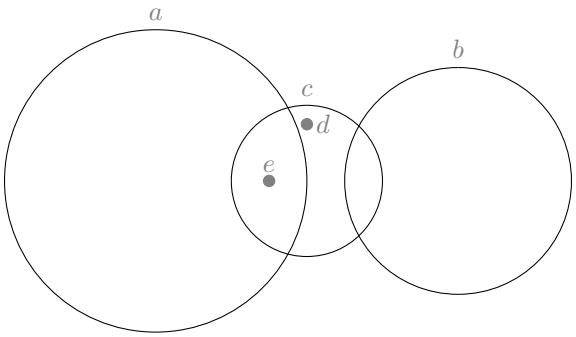


Figure 511: $a \to e$, $c \to de$, $ab \to cde$

512

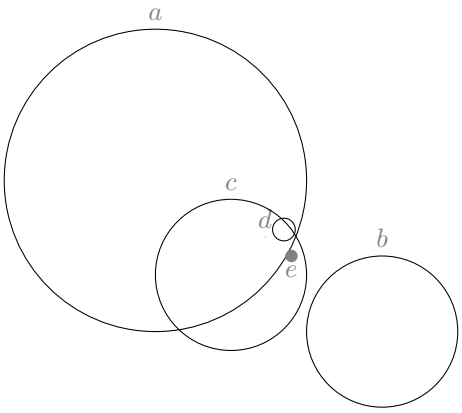


Figure 512: $bd \to e$, $bc \to de$, $ac \to de$, $c \to e$, $ab \to cde$, $a \to d$

513

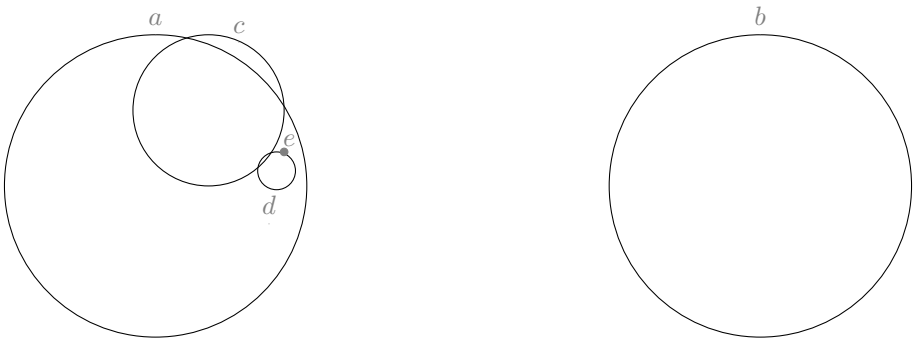


Figure 513: $cd \to e$, $bd \to e$, $bc \to de$, $ab \to cde$, $a \to de$

514

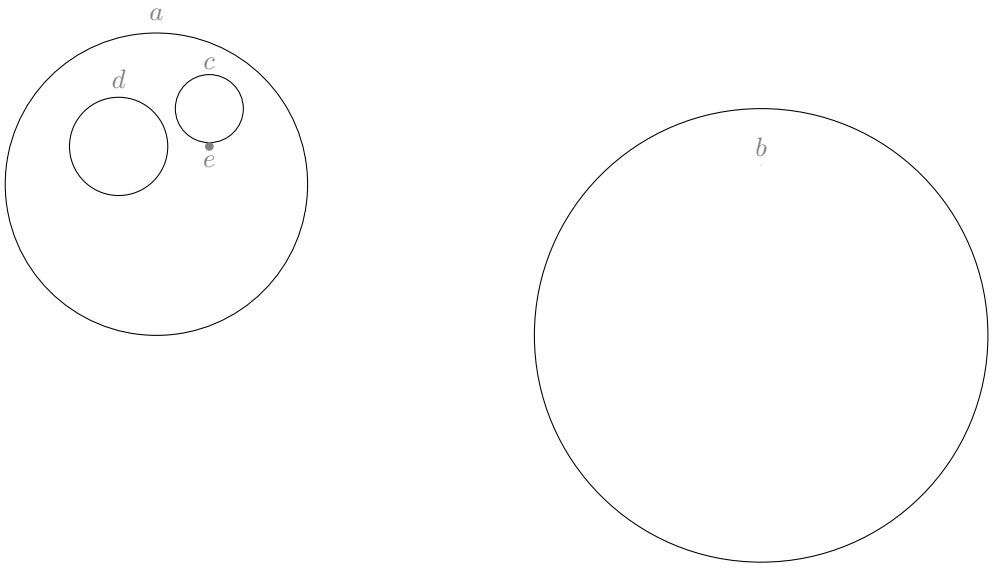


Figure 514: $cd \to e$, $bd \to e$, $bc \to e$, $a \to cde$

515

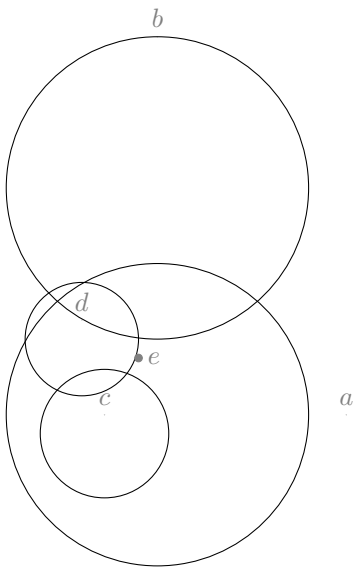


Figure 515: $cd \to e$, $bd \to e$, $bc \to de$, $ab \to cde$, $a \to ce$

516

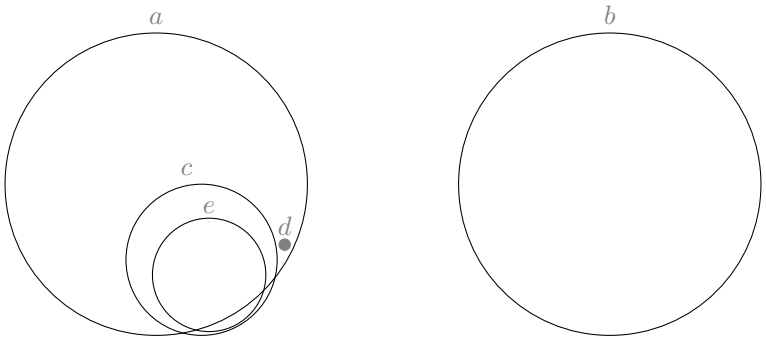


Figure 516: $be \to d$, $bc \to de$, $c \to e$, $ab \to cde$, $a \to d$

517

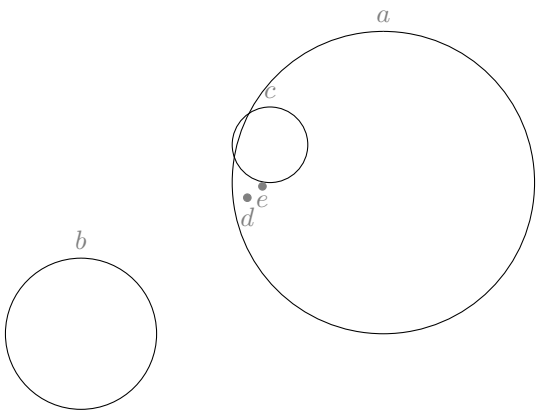


Figure 517: $be \to d, cd \to e, bc \to de, ab \to cde, a \to de$

518

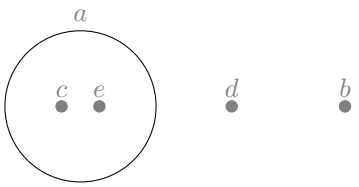


Figure 518: $a \to ce$, $bc \to de$, $ab \to cde$, $be \to d$, $cd \to e$

519

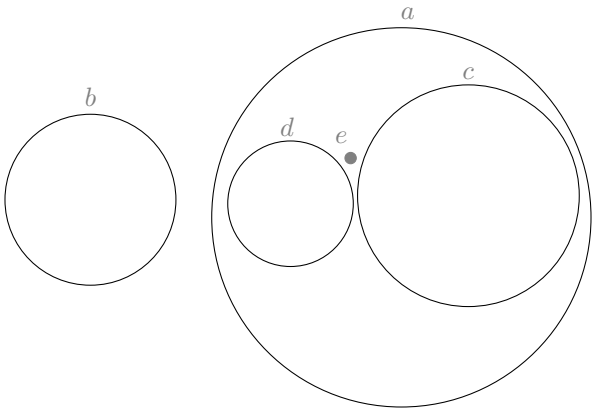


Figure 519: $cd \to e, bc \to de, a \to cde$

520

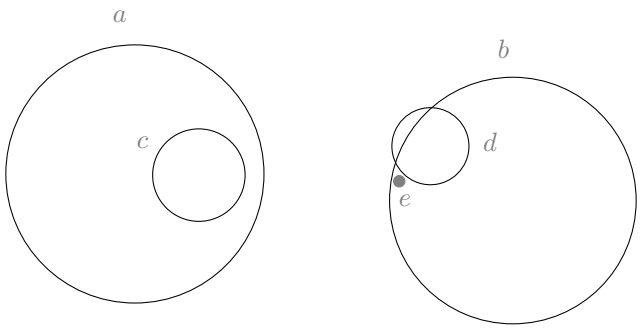


Figure 520: $cd \to e, ad \to ce, bc \to de, ab \to cde, b \to e, a \to c$

521

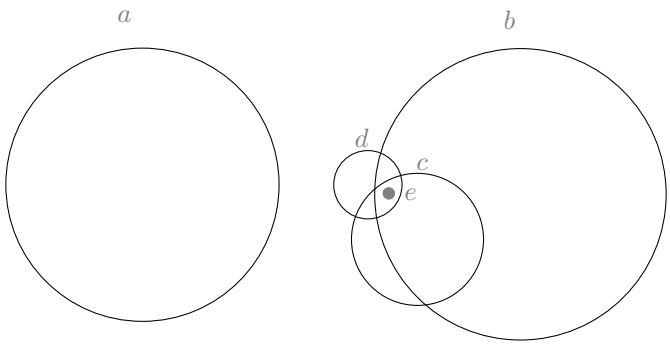


Figure 521: $d \to e, ac \to de, c \to e, ab \to cde, b \to e$

522

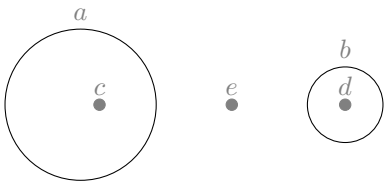


Figure 522: $a \to c$, $bc \to de$, $ab \to cde$, $b \to d$, $cd \to e$, $ad \to ce$

523

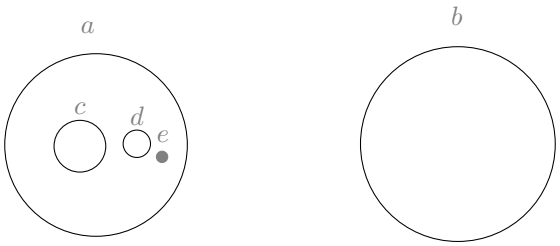


Figure 523: $bd \to e, bc \to de, a \to cde$

524

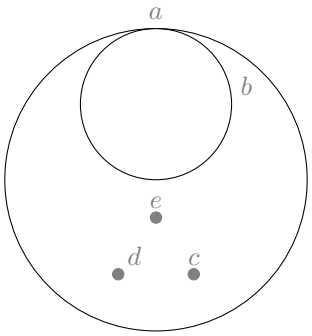


Figure 524: $bc \to e, bd \to e, a \to bcde$

525

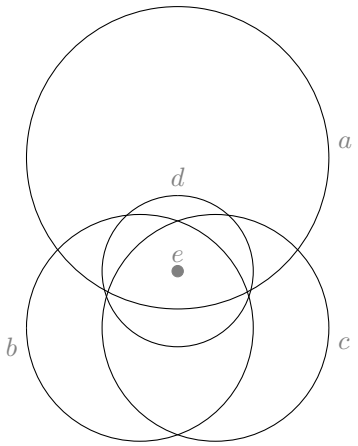


Figure 525: $a \to e$, $b \to e$, $c \to e$, $d \to e$, $ab \to de$, $ac \to de$

526

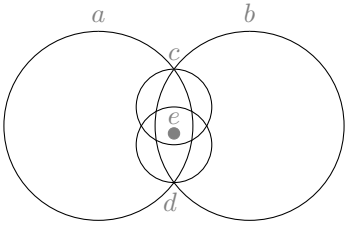


Figure 526: $a \to e$, $b \to e$, $c \to e$, $d \to e$, $ab \to cde$

527

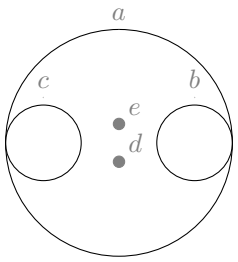


Figure 527: $bc \to de, a \to bcde$

528

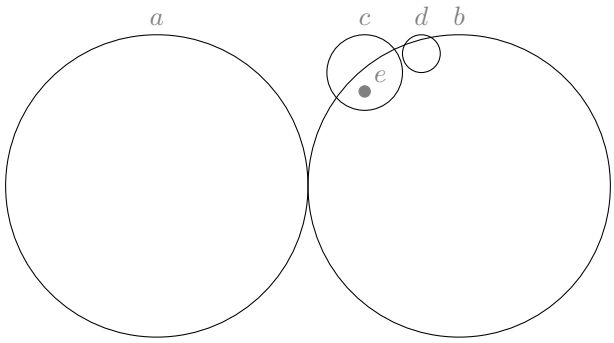


Figure 528: $b \to e$, $c \to e$, $ad \to ce$, $bc \to de$, $ab \to cde$

529

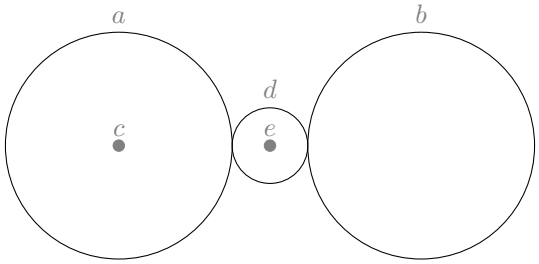


Figure 529: $a \to c$, $d \to e$, $bc \to de$, $ab \to cde$

530

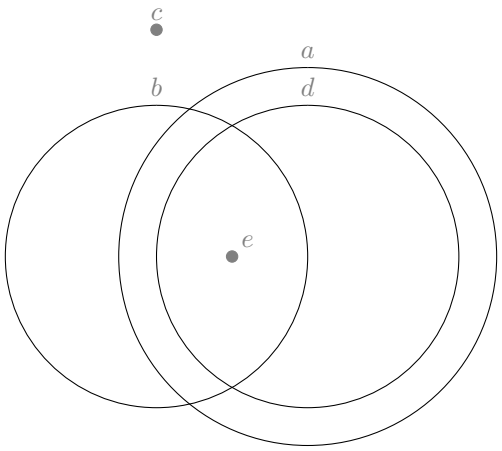


Figure 530: $a \to de$, $b \to e$, $d \to e$

531

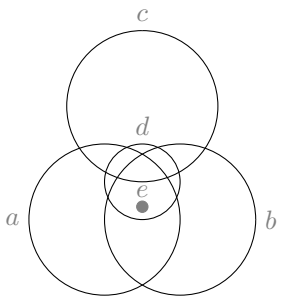


Figure 531: $a \to e$, $b \to e$, $d \to e$, $ab \to de$, $ac \to de$, $bc \to de$

532

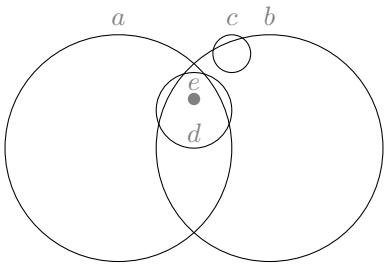


Figure 532: $a \to e$, $b \to e$, $d \to e$, $ac \to de$, $ab \to cde$

533

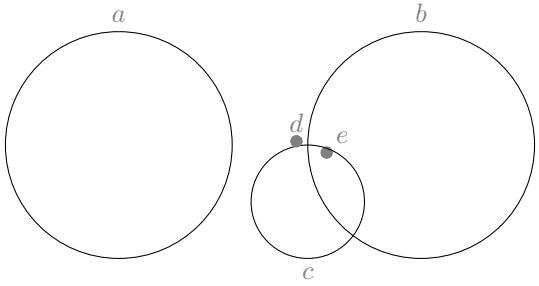


Figure 533: $b \to e$, $c \to e$, $ac \to de$, $bc \to de$, $ae \to d$, $ab \to cde$

534

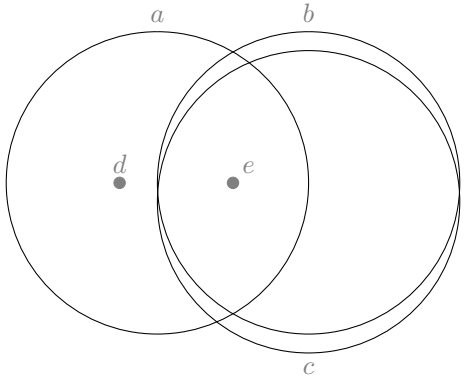


Figure 534: $a \to de$, $b \to e$, $c \to e$

535

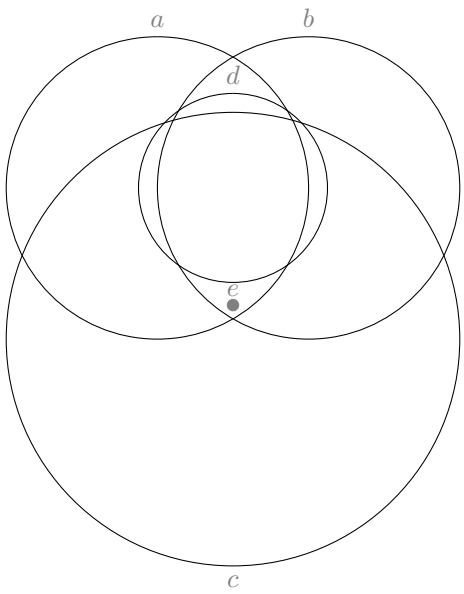


Figure 535: $ac \to de$, $bc \to de$, $c \to e$, $b \to e$, $a \to e$, $ab \to de$

536

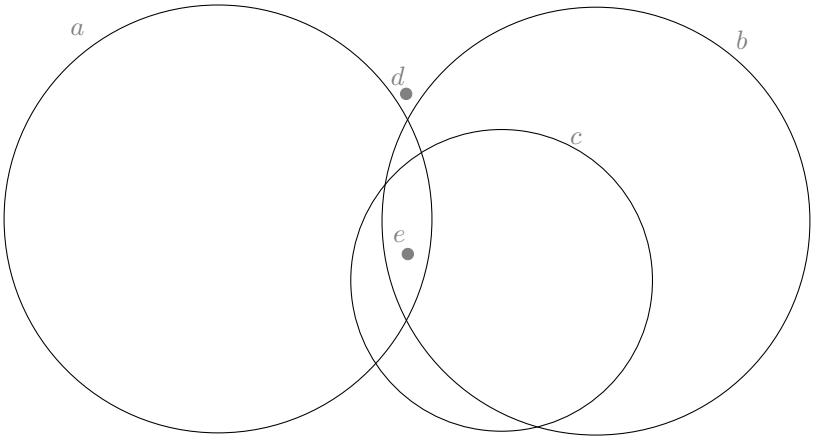


Figure 536: $ac \to de, c \to e, ab \to cde, b \to e, a \to e$

537

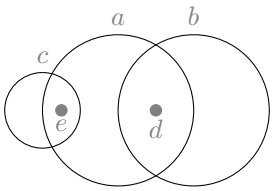


Figure 537: $a \to de$, $b \to d$, $c \to e$

538

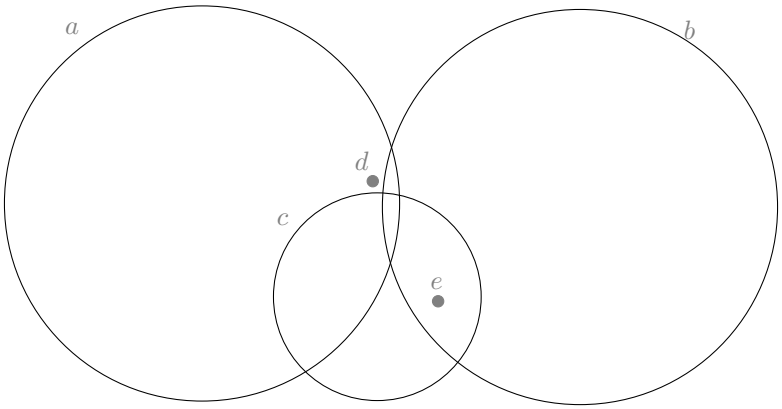


Figure 538: $bc \to de, c \to e, ab \to cde, b \to e, a \to d$

539

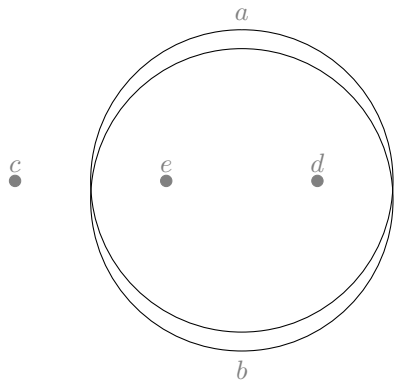


Figure 539: $a \to de$, $b \to de$, $cd \to e$

540

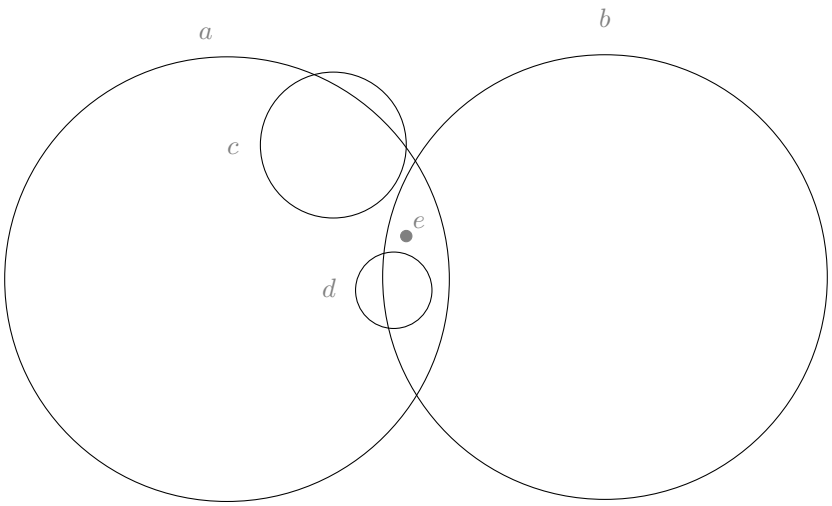


Figure 540: $cd \to e, bc \to de, ab \to cde, b \to e, a \to de$

541

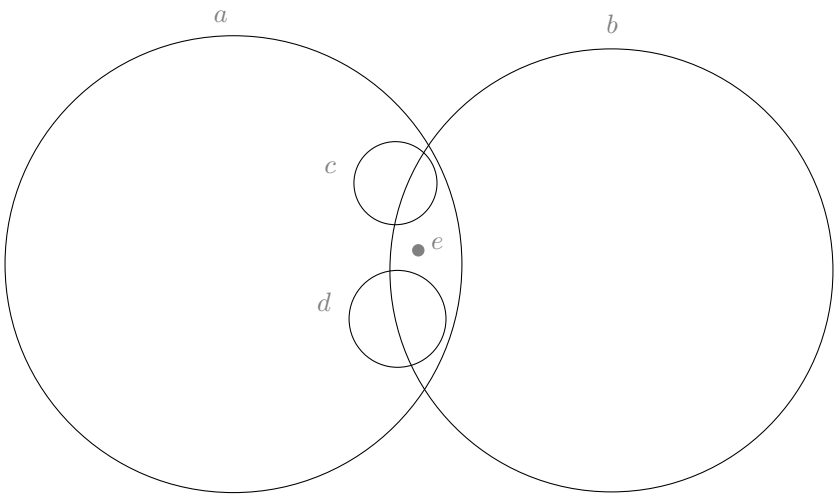


Figure 541: $cd \to e, b \to e, a \to cde$

542

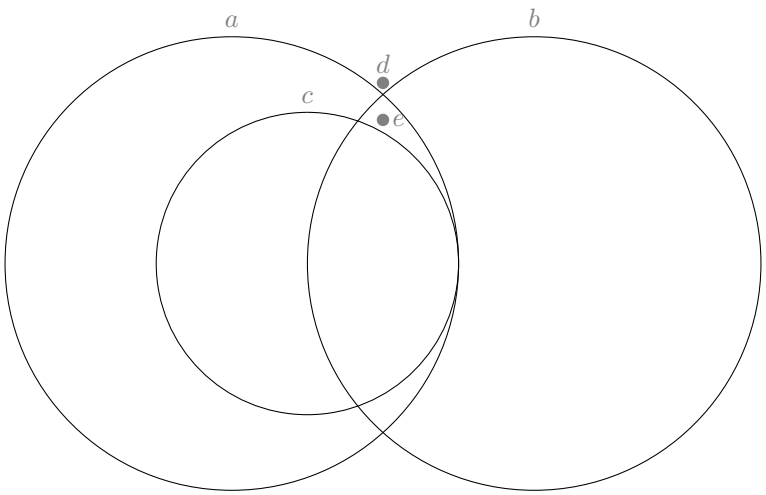


Figure 542: $cd \to e, bc \to de, ab \to cde, b \to e, a \to ce$.

543

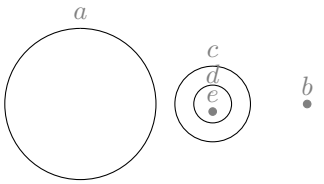


Figure 543: $c \to de, ab \to cde, d \to e$

544

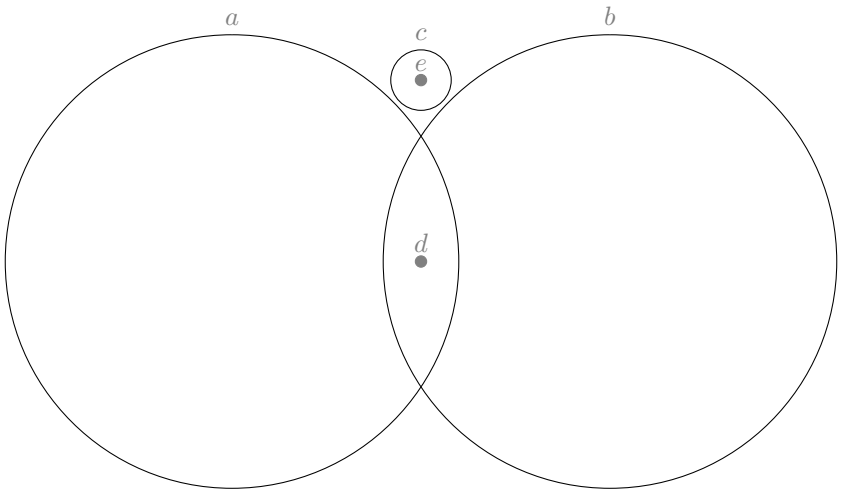


Figure 544: $c \to e, ab \to cde, b \to d, a \to d$

545

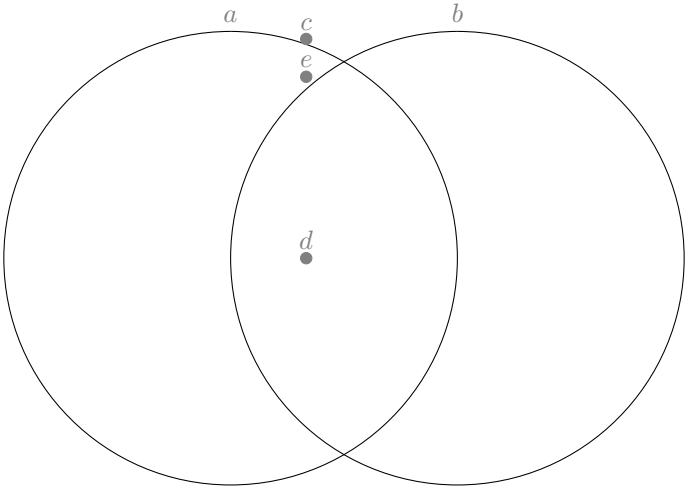


Figure 545: $cd \to e, bc \to de, ab \to cde, b \to d, a \to de$

546

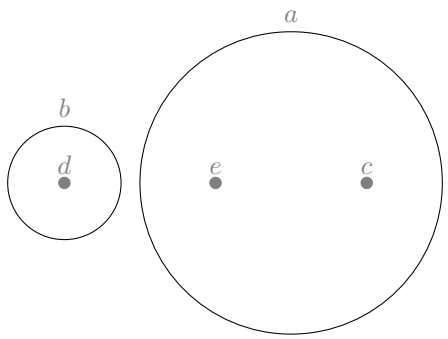


Figure 546: $a \to ce$, $bc \to de$, $cd \to e$, $b \to d$

547

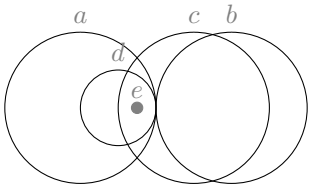


Figure 547: $a \to de$, $ab \to cde$, $c \to e$, $d \to e$

548

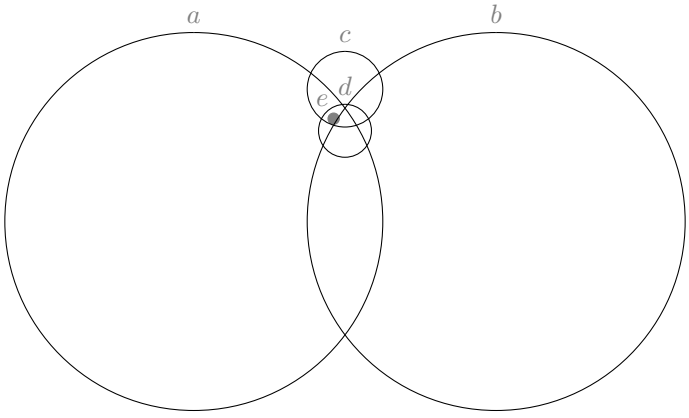


Figure 548: $ac \to de$, $bc \to de$, $ab \to cde$, $a \to e$, $c \to e$, $d \to e$

549

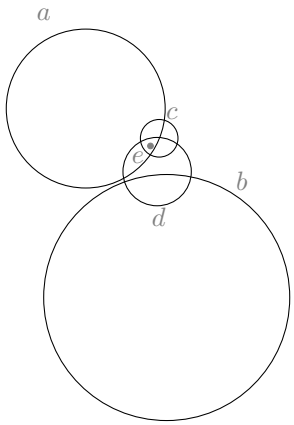


Figure 549: $a \to e, c \to e, d \to e, ab \to cde, ad \to ce, bc \to de$

550

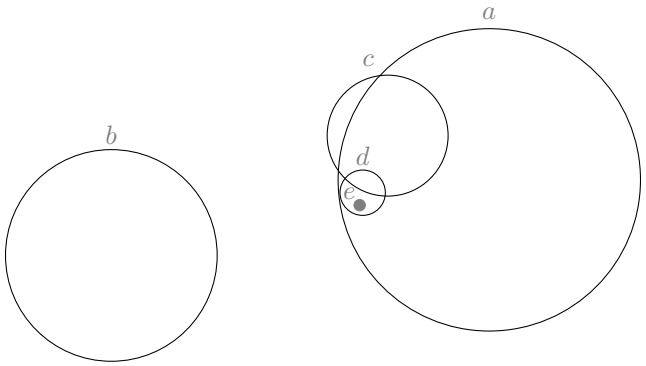


Figure 550: $bc \to de, d \to e, ab \to cde, a \to de$

551

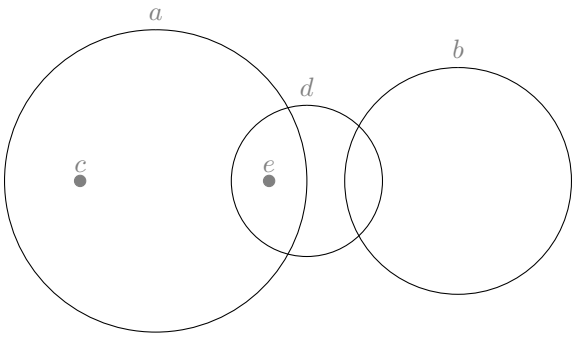


Figure 551: $a \to ce$, $d \to e$, $ab \to de$, $bc \to de$

552

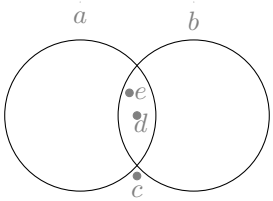


Figure 552: $a \to de, b \to de, ab \to cde$

553

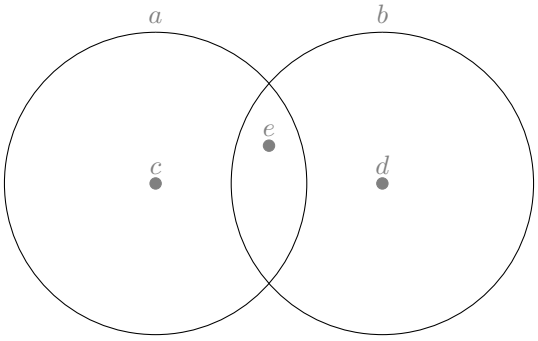


Figure 553: $a \to ce$, $b \to de$

554

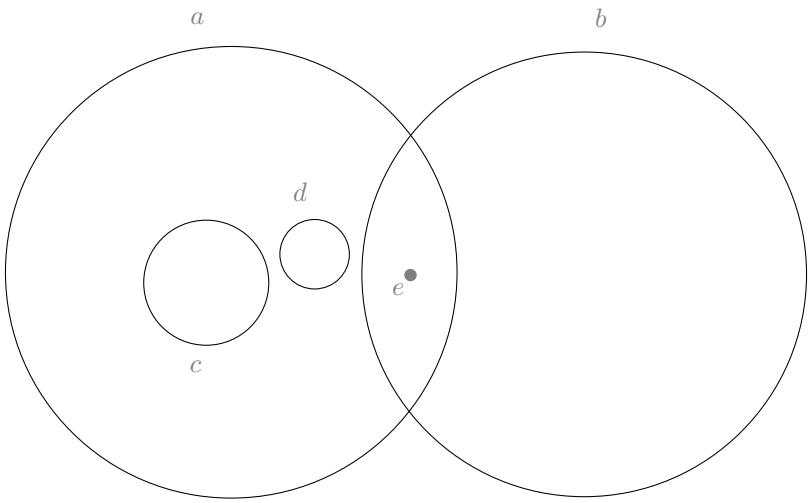


Figure 554: $bc \to de, b \to e, a \to cde$

555

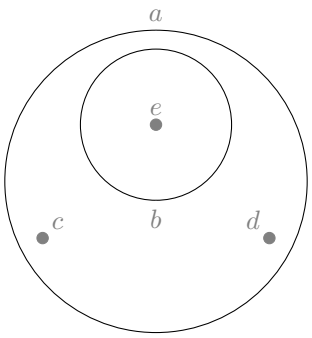


Figure 555: $b \to e, a \to bcde$

556

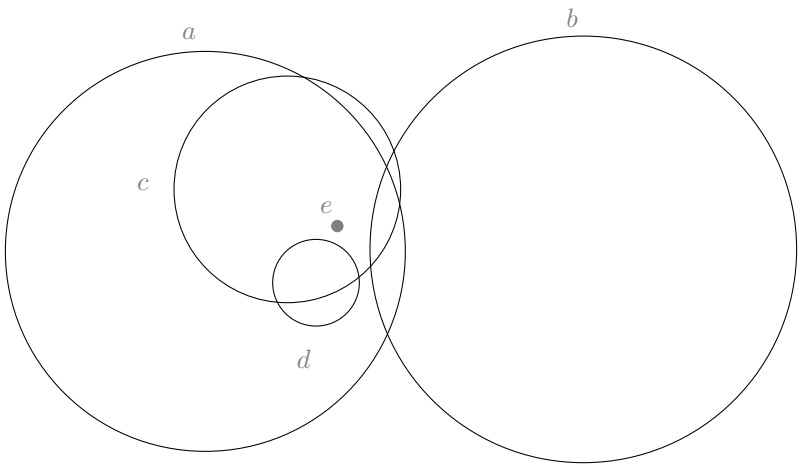


Figure 556: $bd \to e, bc \to de, c \to e, ab \to cde, a \to de$

557

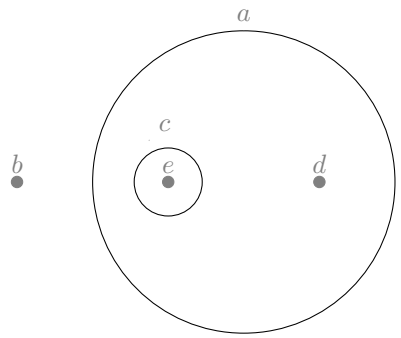


Figure 557: $a \to cde$, $bd \to e, c \to e$

558

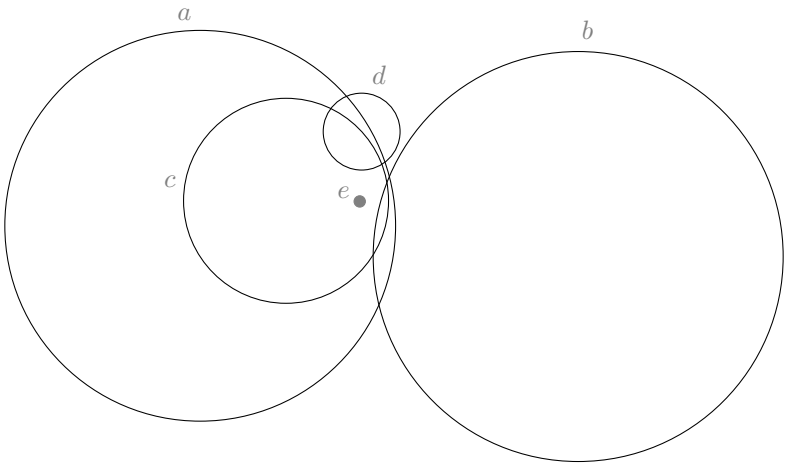


Figure 558: $bd \to e, bc \to de, c \to e, ab \to cde, a \to ce$

559

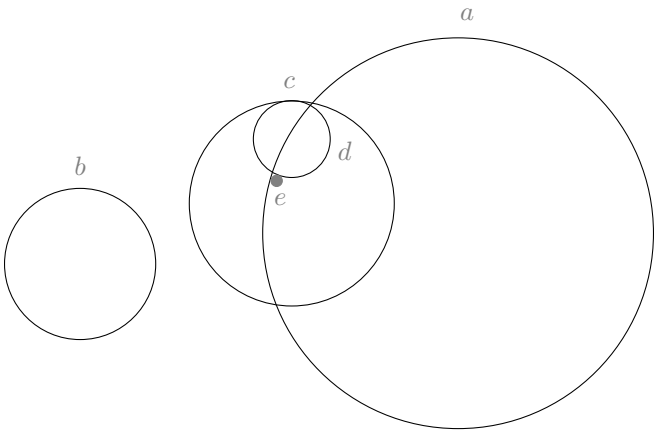


Figure 559: $bd \to e, c \to de, ab \to cde, a \to e$

560

Figure 560: $a \to c$, $b \to e$, $d \to e$, $bc \to d$, $ab \to d$

561

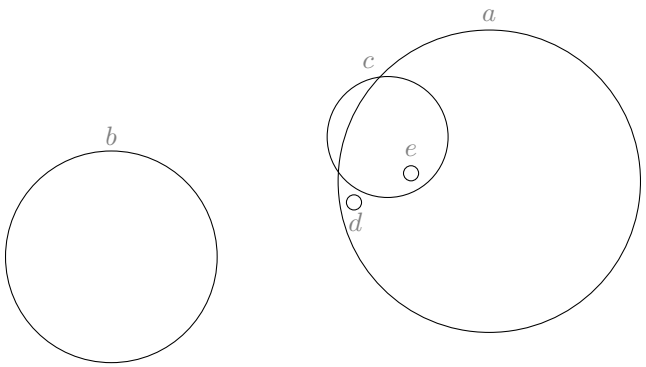


Figure 561: $be \to d, bc \to de, c \to e, ab \to cde, a \to de$

562

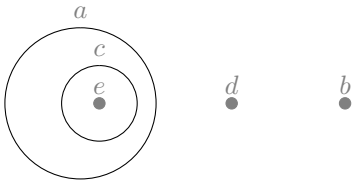


Figure 562: $a \to c$, $bc \to d$, $ab \to d$, $c \to e$, $be \to d$

563

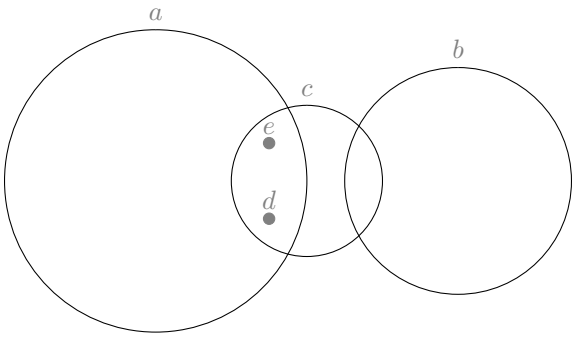


Figure 563: $a \to de$, $c \to de$, $ab \to cd$

564

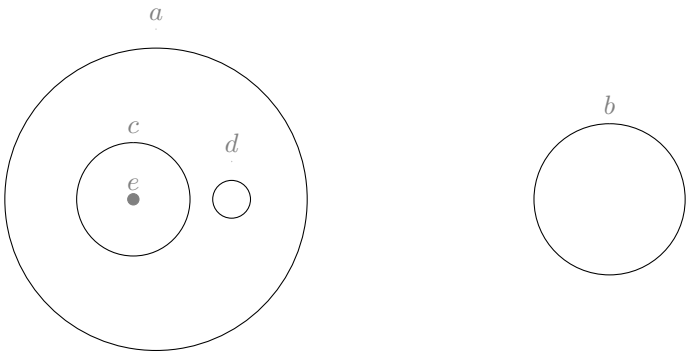


Figure 564: $bc \to de, c \to e, a \to cde$

565

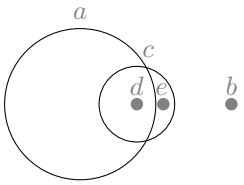


Figure 565: $c \to de, ab \to cde, a \to d, bd \to e$

566

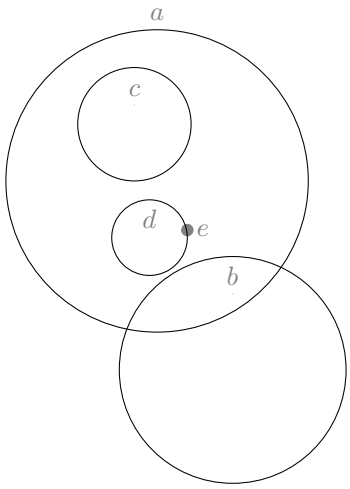


Figure 566: $cd \to e, bd \to e, bc \to de, a \to cde$

567

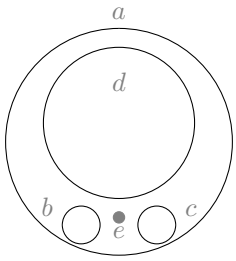


Figure 567: $bd \to e, cd \to e, bc \to e, a \to bcde$

568

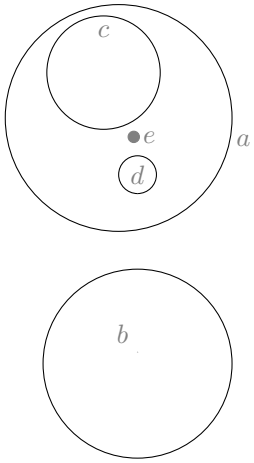


Figure 568: $be \to d, cd \to e, bc \to de, a \to cde$

569

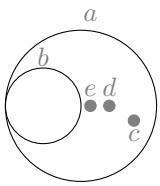


Figure 569: $bd \to e, bc \to de, a \to bcde$

570

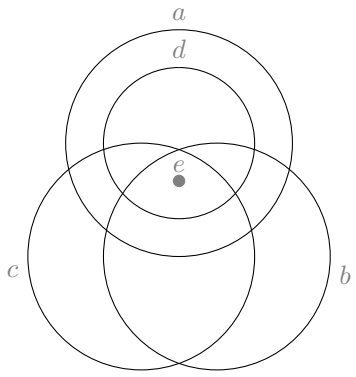


Figure 570: $a \to de$, $b \to e$, $c \to e$, $d \to e$

571

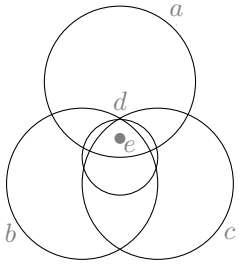


Figure 571: $a \to e$, $b \to e$, $c \to e$, $d \to e$, $ab \to de$, $ac \to de$, $bc \to de$

572

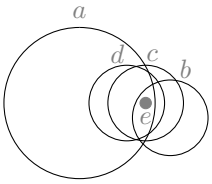


Figure 572: $a \to e$, $b \to e$, $c \to e$, $d \to e$, $ab \to cde$, $ac \to de$

573

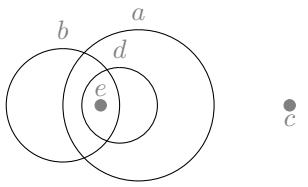


Figure 573: $d \to e, bc \to de, b \to e, a \to de$

574

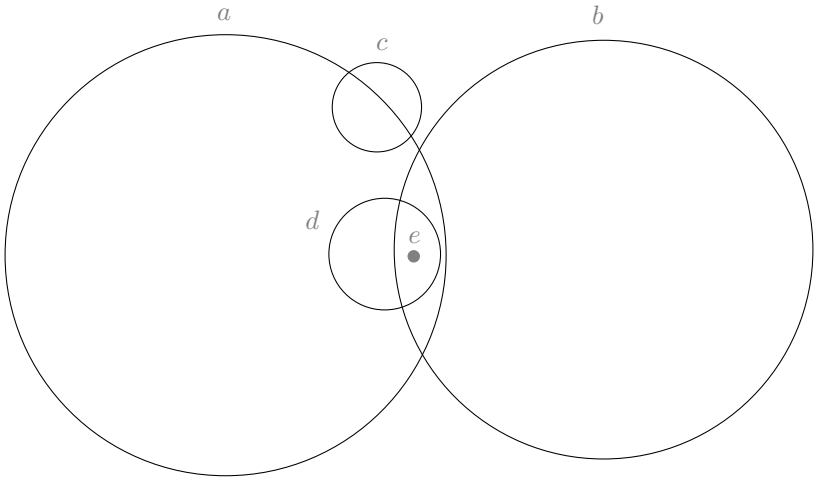


Figure 574: $d \to e, ab \to cde, b \to e, a \to de$

575

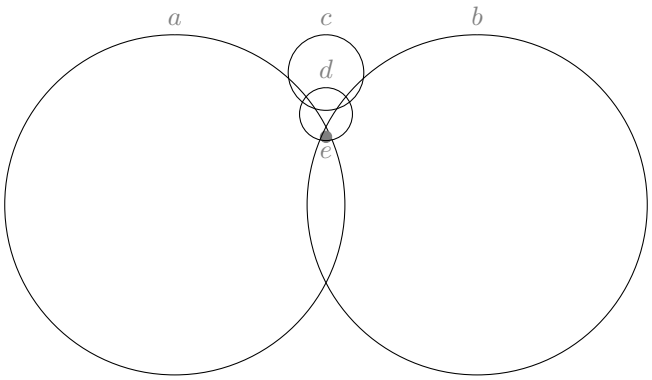


Figure 575: $ac \to d$, $bc \to d$, $ab \to cd$, $a \to e$, $b \to e$, $d \to e$

576

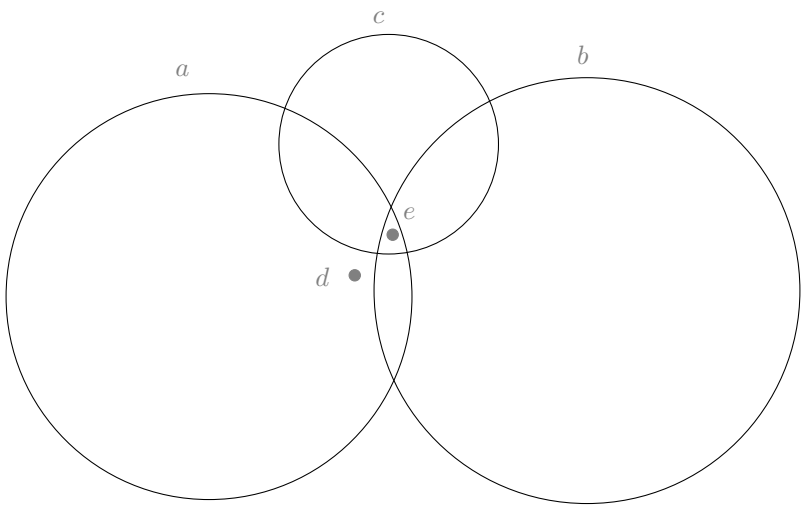


Figure 576: $bc \to de, c \to e, b \to e, a \to de$

577

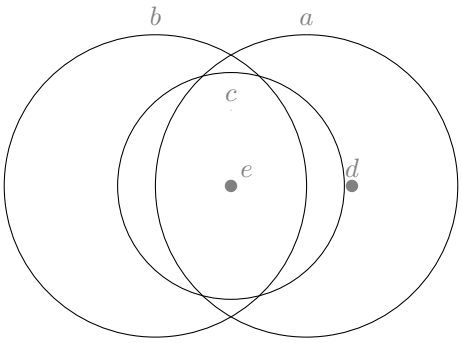


Figure 577: $a \to de$, $b \to e$, $c \to e$, $ab \to c$

578

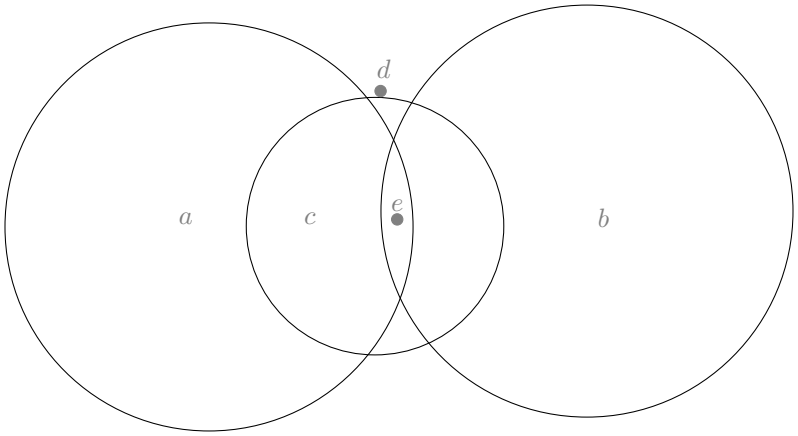


Figure 578: $bc \to de, ac \to de, c \to e, ab \to cde, b \to e, a \to e$

579

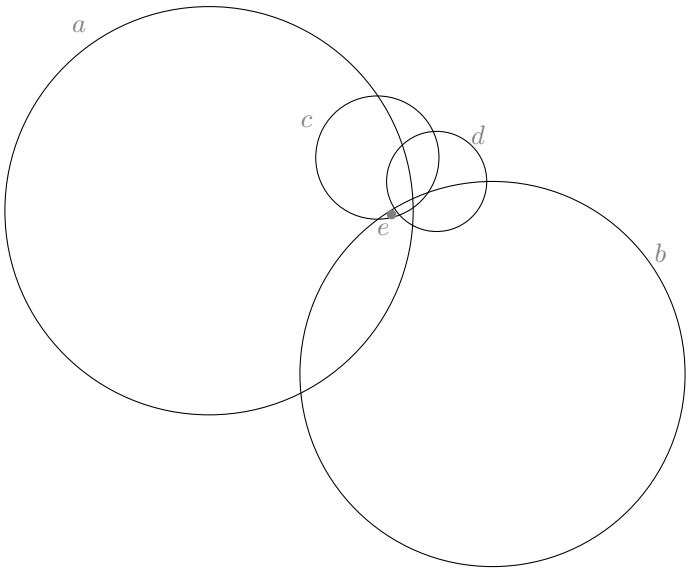


Figure 579: $a \to e, b \to e, c \to e, ab \to cde, ad \to ce, bc \to de$

580

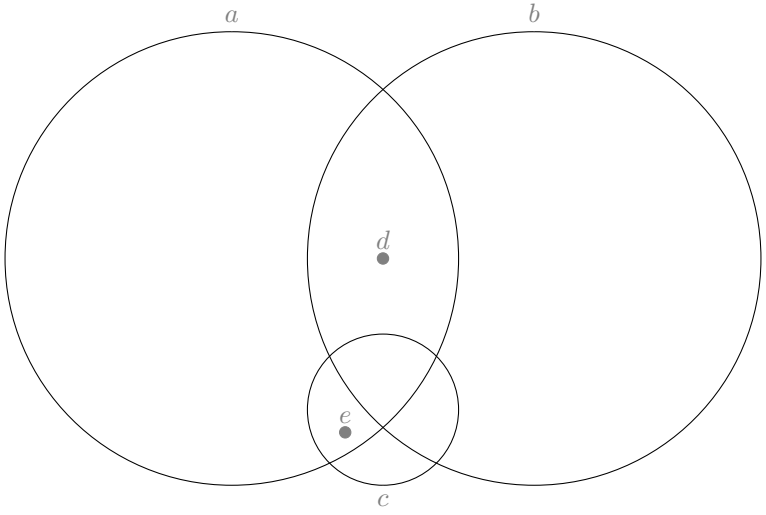


Figure 580: $bc \to de, c \to e, ab \to cde, b \to d, a \to de$.

581

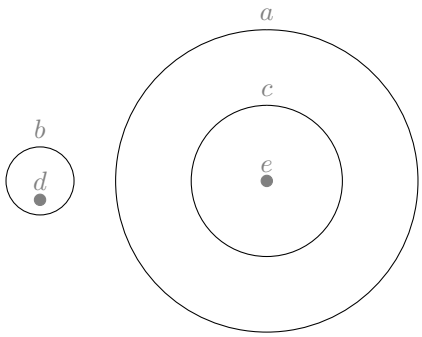


Figure 581: $a \to ce$, $c \to e$, $b \to d$

582

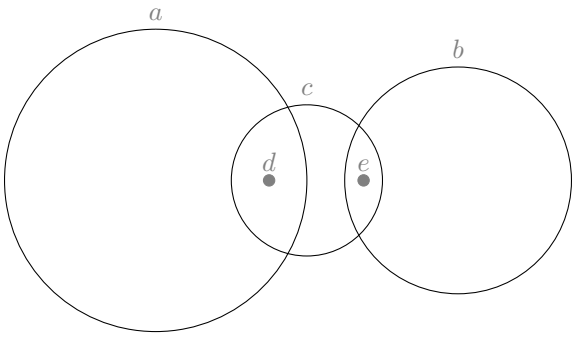


Figure 582: $a \to d$, $c \to de$, $ab \to cd$, $b \to e$

583

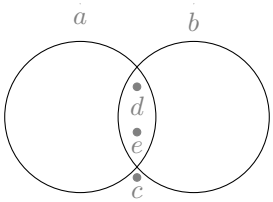


Figure 583: $a \to de, b \to de, ab \to cde, cd \to e$

584

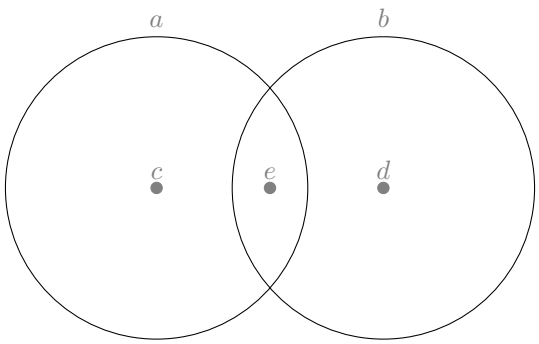


Figure 584: $a \to ce$, $b \to de$, $cd \to e$

585

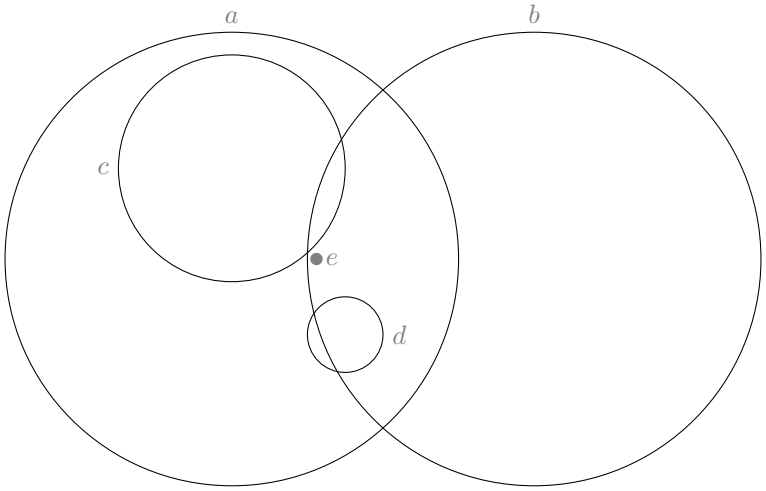


Figure 585: $cd \to e, bc \to de, b \to e, a \to cde$.

586

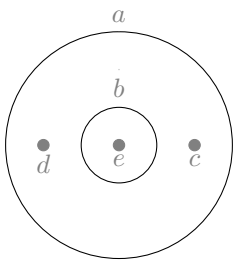


Figure 586: $b \to e, cd \to e, a \to bcde$

587

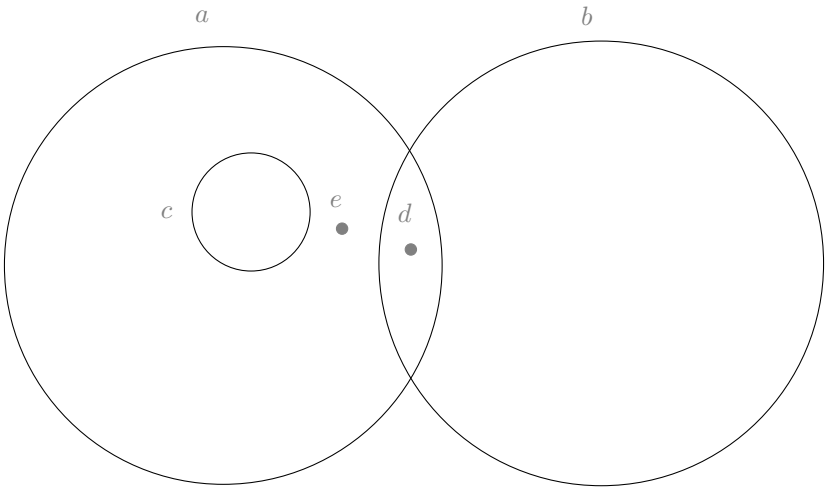


Figure 587: $cd \to e, bc \to de, b \to d, a \to cde$

588

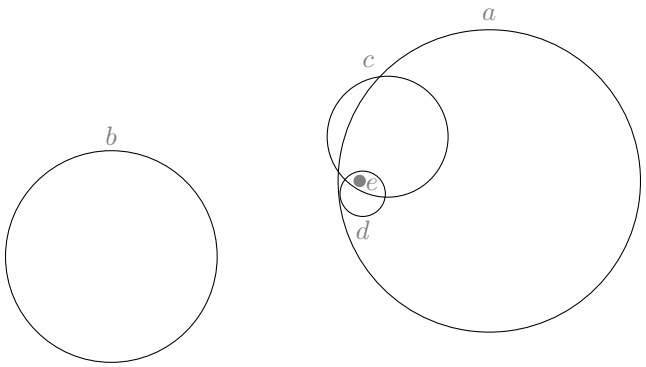


Figure 588: $d \to e, bc \to de, c \to e, ab \to cde, a \to de$

589

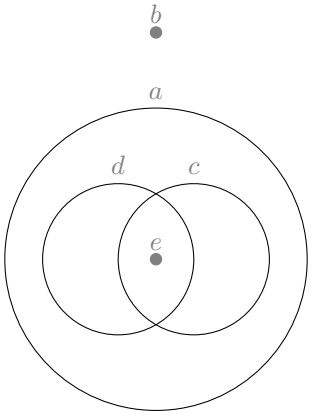


Figure 589: $a \to cde$, $c \to e$, $d \to e$

590

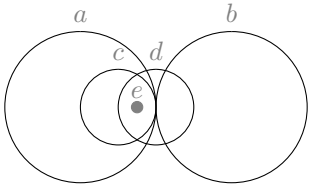


Figure 590: $a \to c$, $bc \to d$, $ab \to d$, $c \to e$, $d \to e$

591

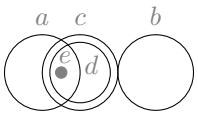


Figure 591: $a \to e$, $d \to e$, $c \to de$, $ab \to cde$

592

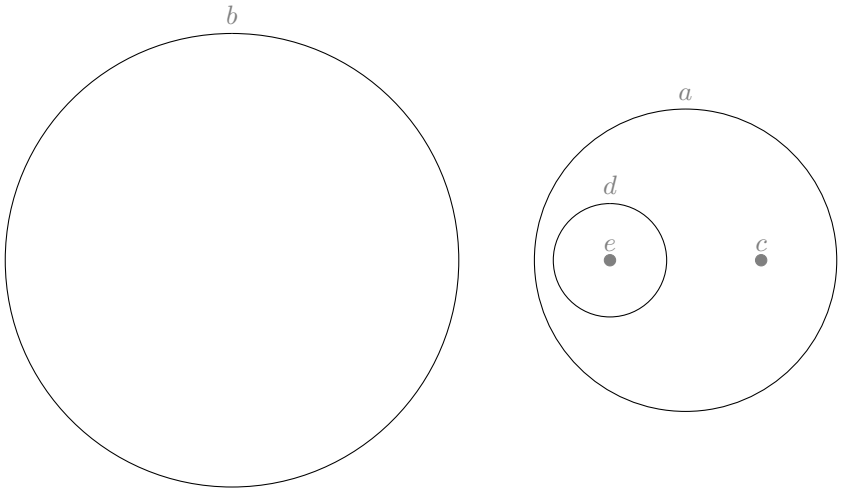


Figure 592: $d \to e, bc \to de, a \to cde$.

593

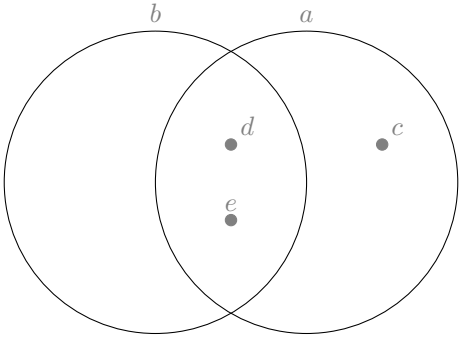


Figure 593: $b \to de$, $a \to cde$

594

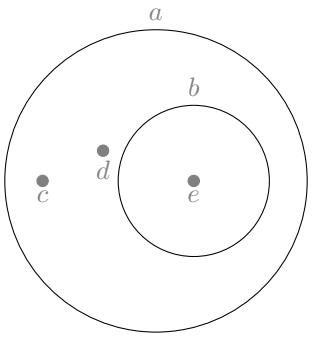


Figure 594: $b \to e, bc \to de, a \to bcde$

595

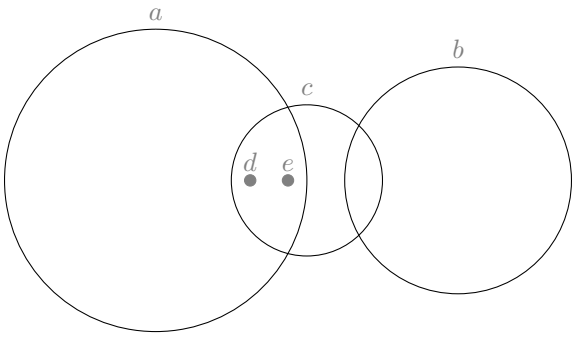


Figure 595: $a \to de$, $c \to de$, $ab \to c$, $bd \to e$

596

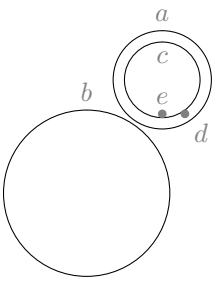


Figure 596: $a \to cde$, $c \to e$, $bd \to e$, $bc \to de$

597

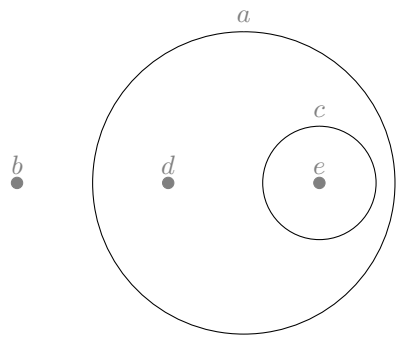


Figure 597: $a \to cde$, $bc \to de$, $be \to d$, $c \to e$

598

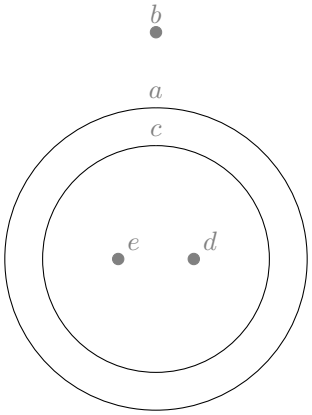


Figure 598: $c \to de$, $a \to cde$

599

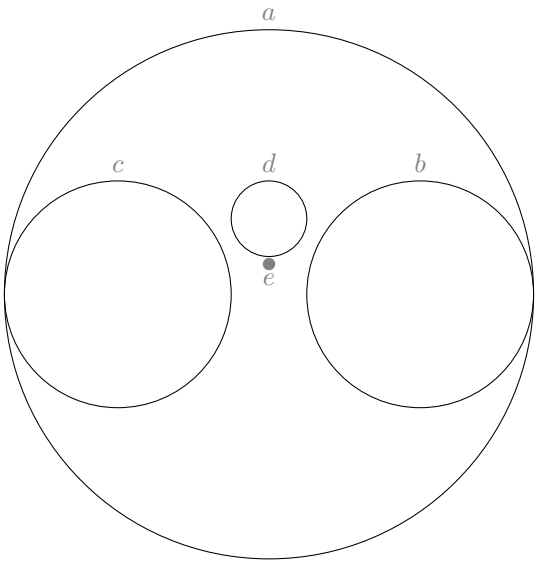


Figure 599: $bd \to e, cd \to e, bc \to de, a \to bcde$

600

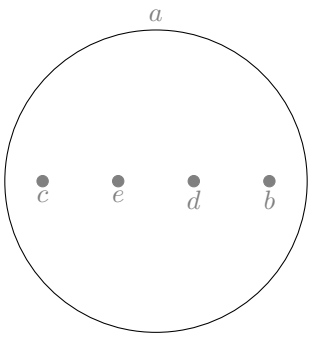


Figure 600: $be \to d, cd \to e, bc \to de, a \to bcde$

601

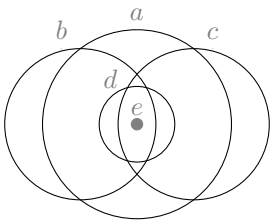


Figure 601: $a \to de$, $b \to e$, $c \to e$, $d \to e$, $bc \to de$

602

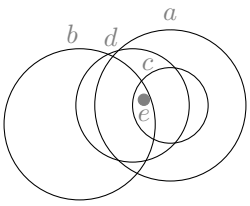


Figure 602: $a \to de$, $b \to e$, $c \to e$, $d \to e$, $ab \to cde$

603

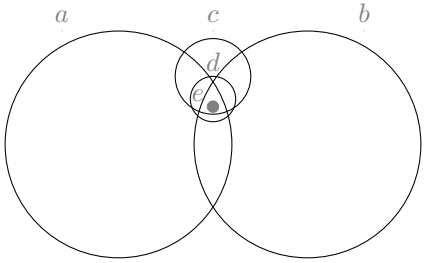


Figure 603: $a \to e$, $b \to e$, $c \to e$, $d \to e$, $ac \to de$, $bc \to de$, $ab \to cde$

604

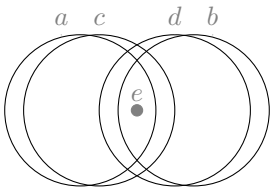


Figure 604: $a \to e$, $b \to e$, $c \to e$, $d \to e$, $ad \to ce$, $bc \to de$, $ab \to cde$

605

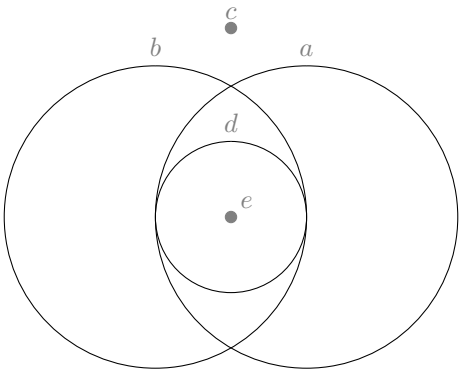


Figure 605: $a \to de$, $b \to de$, $d \to e$

606

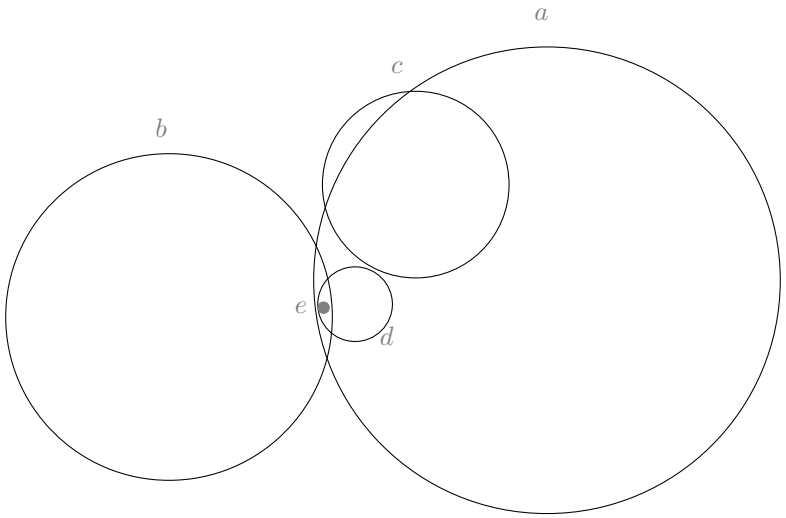


Figure 606: $d \to e, bc \to de, ab \to cde, b \to e, a \to de$

607

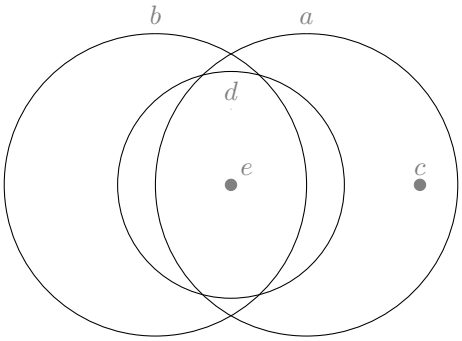


Figure 607: $a \to ce$, $b \to e$, $d \to e$, $ab \to cde$, $bc \to de$

608

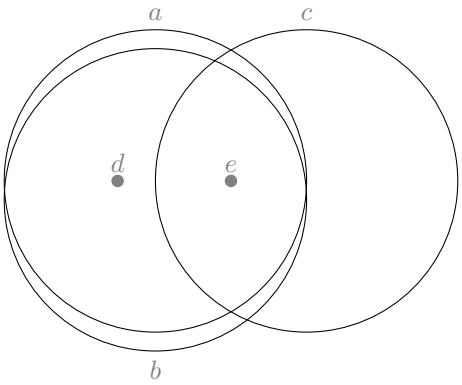


Figure 608: $a \to de$, $b \to de$, $c \to e$

609

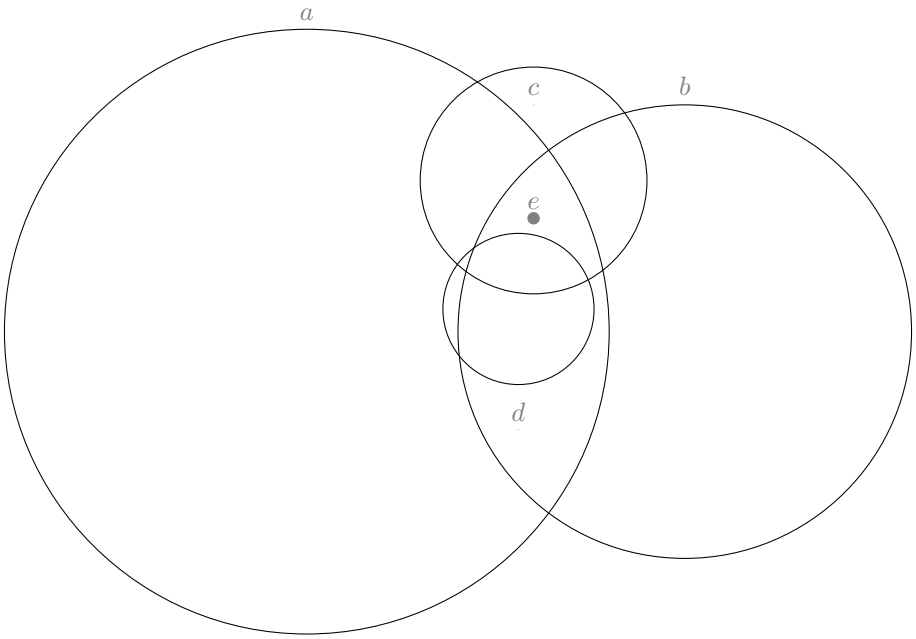


Figure 609: $bc \to de, c \to e, ab \to cde, b \to e, a \to de$.

610

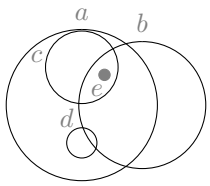


Figure 610: $a \to cde, b \to e, c \to e$

611

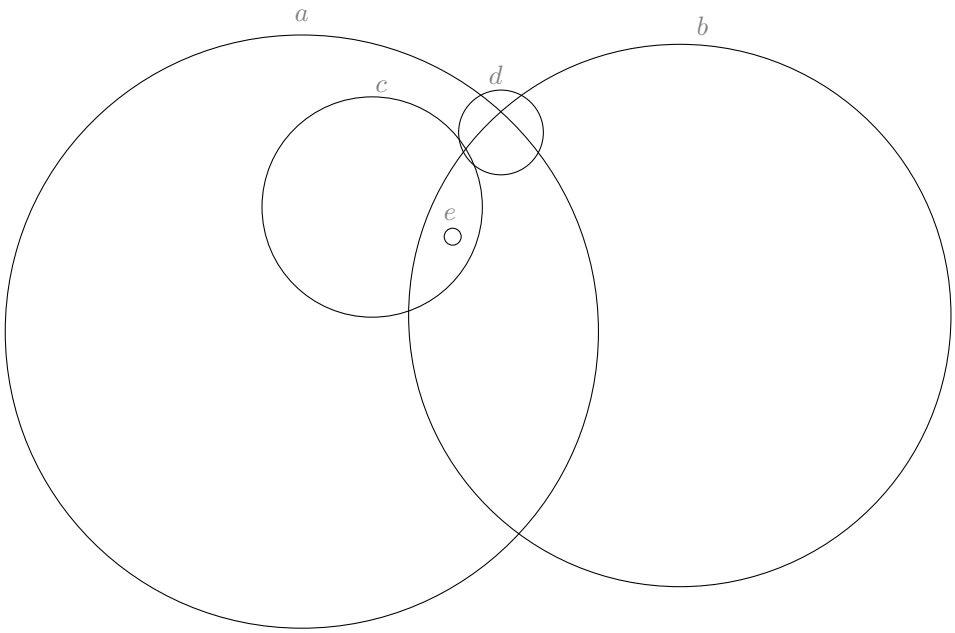


Figure 611: $bc \to de, ab \to cde, a \to ce, b \to e, c \to e$

612

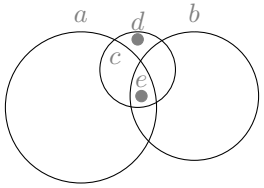


Figure 612: $c \to de, ab \to cde, a \to e, b \to e$

613

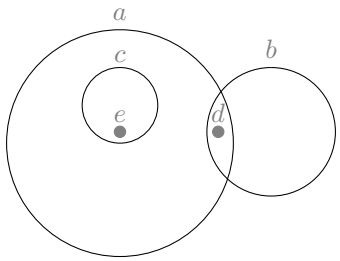


Figure 613: $bc \to de, b \to d, c \to e, a \to cde$

614

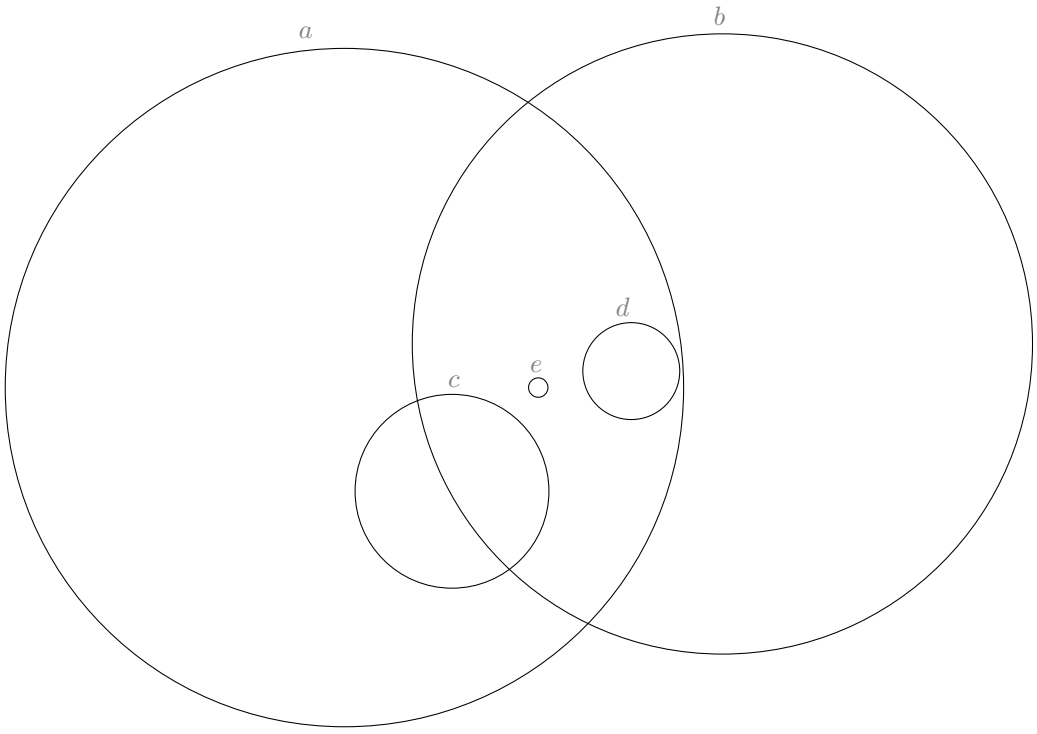


Figure 614: $cd \to e, a \to cde, b \to de$

615

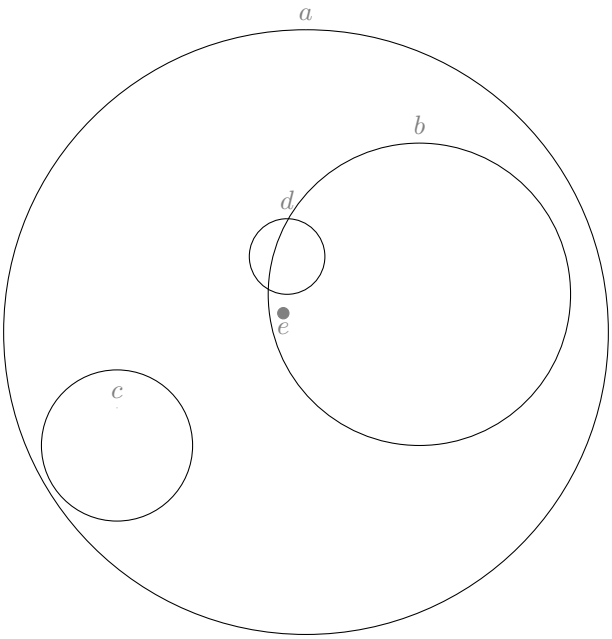


Figure 615: $b \to e, bc \to de, cd \to e, a \to bcde$

616

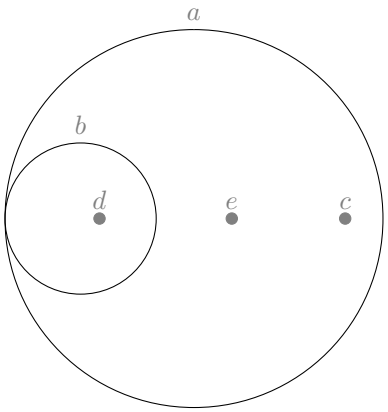


Figure 616: $b \to d, bc \to de, cd \to e, a \to bcde$

617

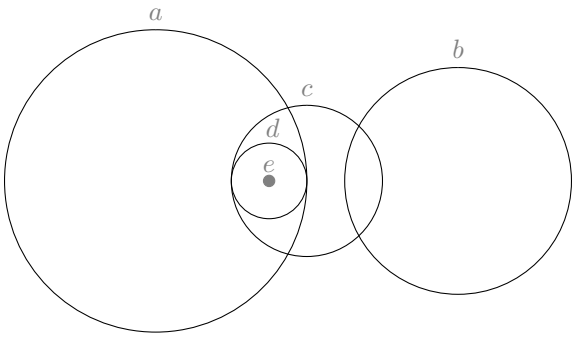


Figure 617: $a \to de$, $c \to de$, $ab \to cde$, $d \to e$

618

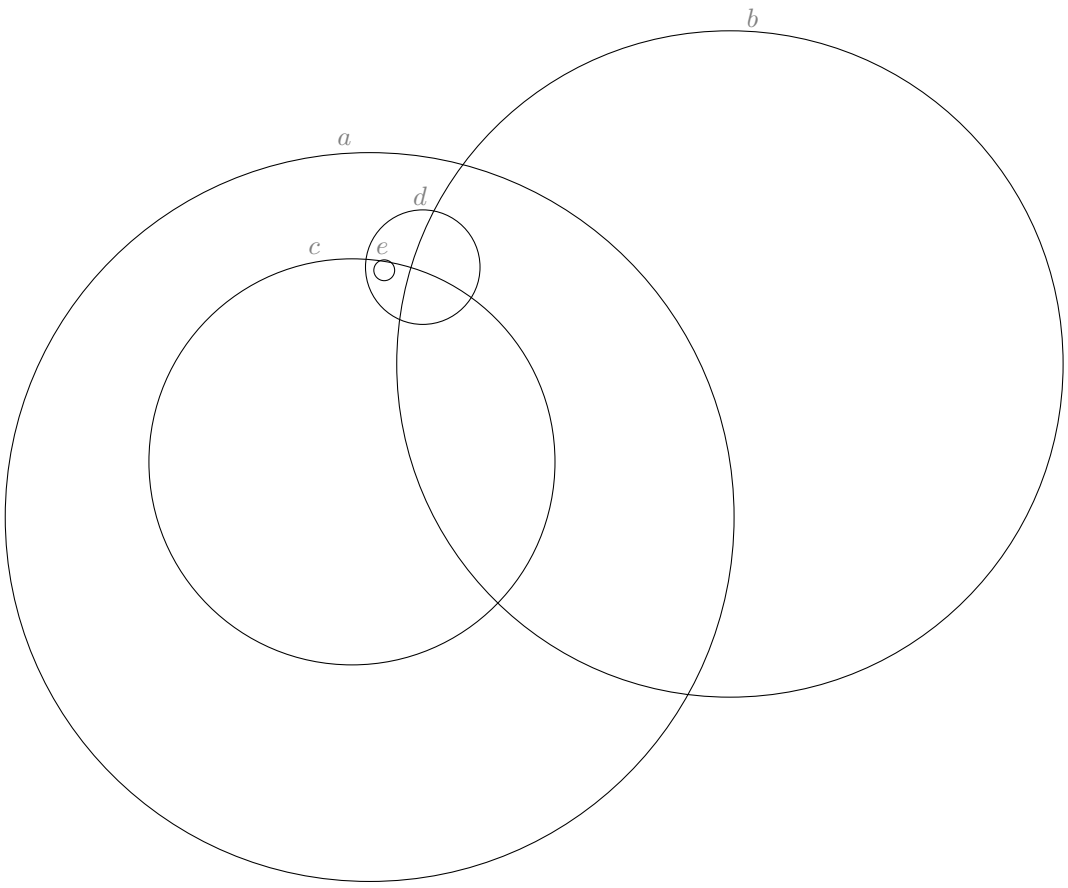


Figure 618: $c \to e, d \to e, a \to cde, bc \to de$

619

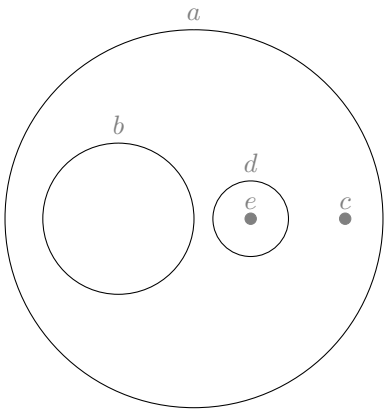


Figure 619: $d \to e, bc \to de, a \to bcde$

620

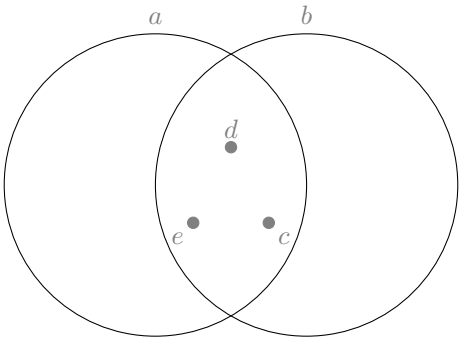


Figure 620: $a \to cde, b \to cde$

621

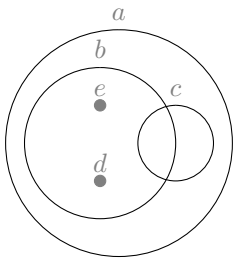


Figure 621: $b \to de, a \to bcde$

622

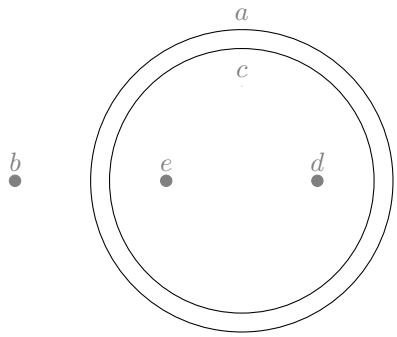


Figure 622: $a \to cde$, $bd \to e$, $c \to de$

623

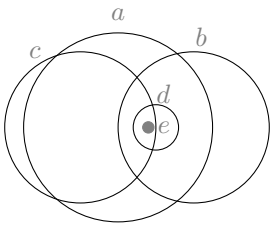


Figure 623: $a \to de$, $b \to de$, $c \to e$, $d \to e$,

624

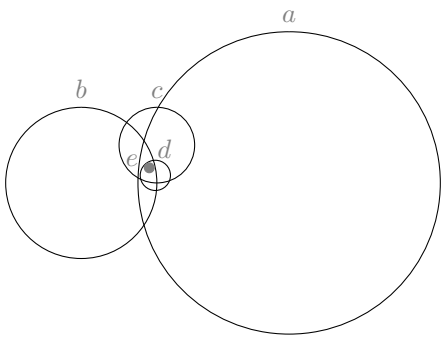


Figure 624: $a \to de$, $b \to e$, $c \to e$, $d \to e$, $bc \to de$, $ab \to cde$

625

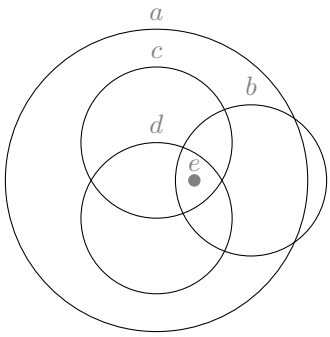


Figure 625: $a \to cde$, $b \to e$, $c \to e$, $d \to e$,

626

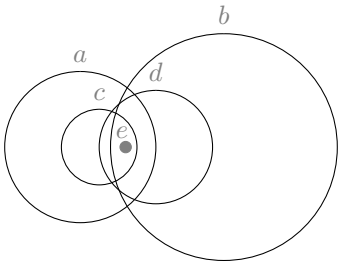


Figure 626: $a \to ce$, $b \to e$, $c \to e$, $d \to e$, $bc \to de$, $ab \to cde$

627

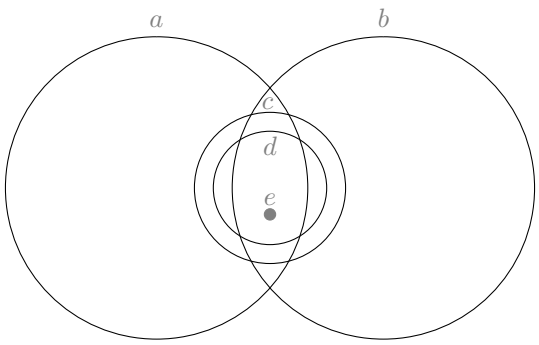


Figure 627: $a \to e$, $b \to e$, $c \to de$, $d \to e$, $ab \to cde$

628

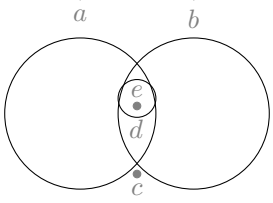


Figure 628: $a \to de, b \to de, ab \to cde, d \to e$

629

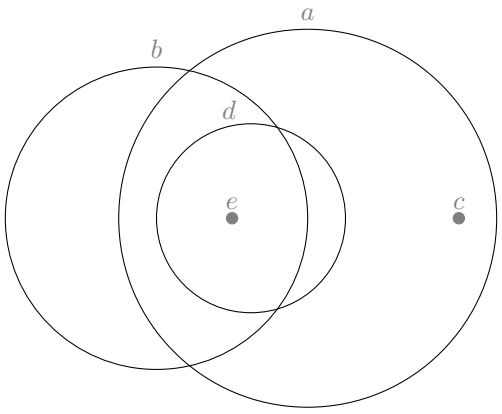


Figure 629: $a \to cde$, $d \to e$, $b \to e$, $bc \to de$

630

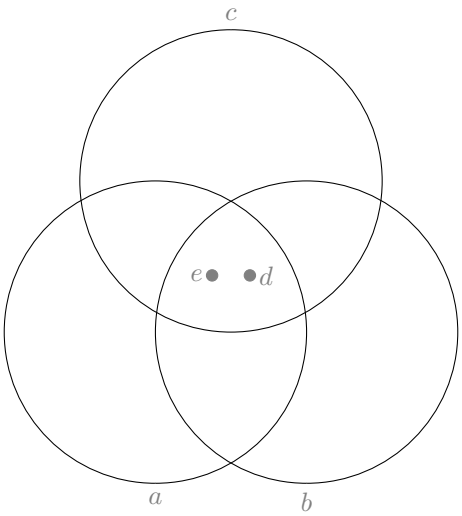


Figure 630: $a \to de, b \to de, c \to de$

631

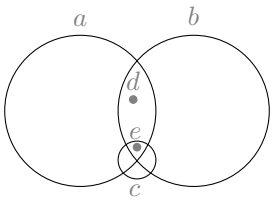


Figure 631: $a \to de, b \to de, ab \to cde, c \to e$

632

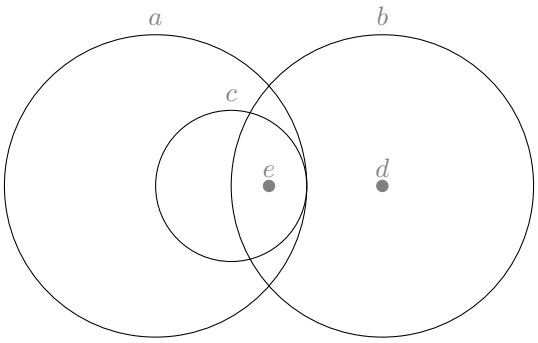


Figure 632: $a \to ce$, $b \to de$, $c \to e$, $ab \to cde$

633

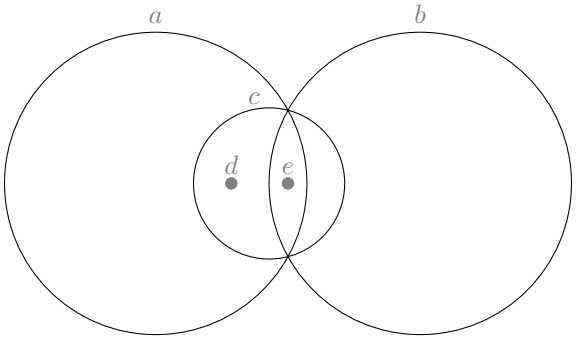


Figure 633: $a \to de$, $c \to de$, $ab \to cde$, $b \to e$

634

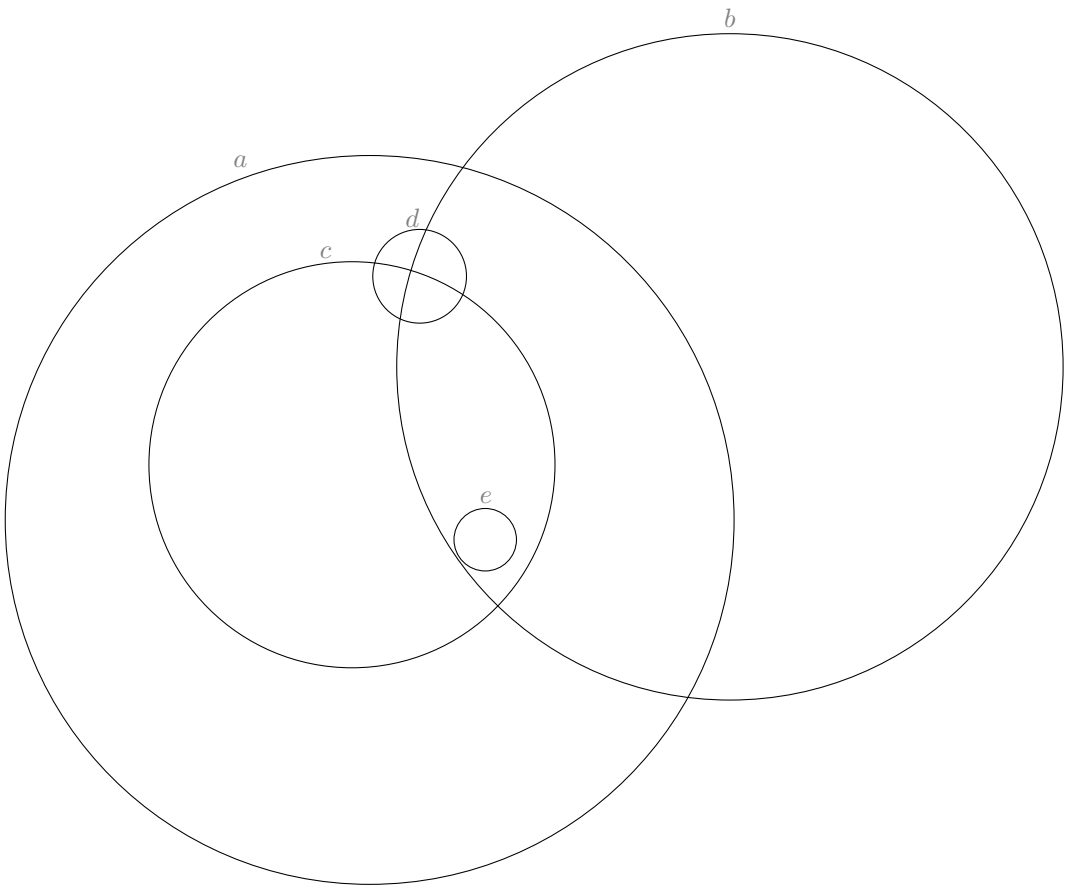


Figure 634: $c \to e, a \to cde, b \to e, bc \to de$

635

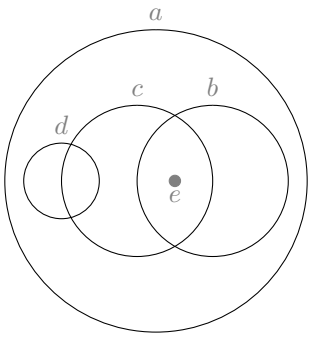


Figure 635: $c \to e, b \to e, a \to bcde$

636

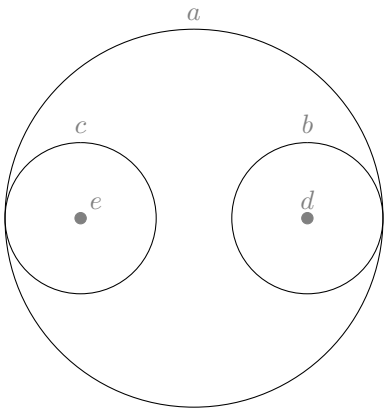


Figure 636: $c \to e, b \to d, a \to bcde, bc \to de$

637

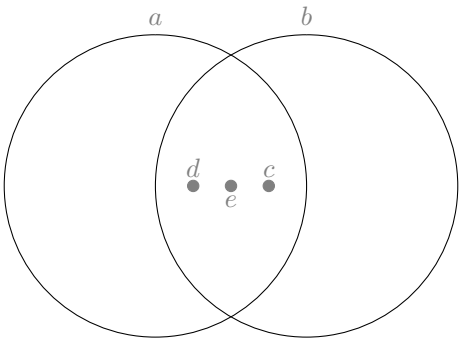


Figure 637: $a \to cde, b \to cde, cd \to e$

638

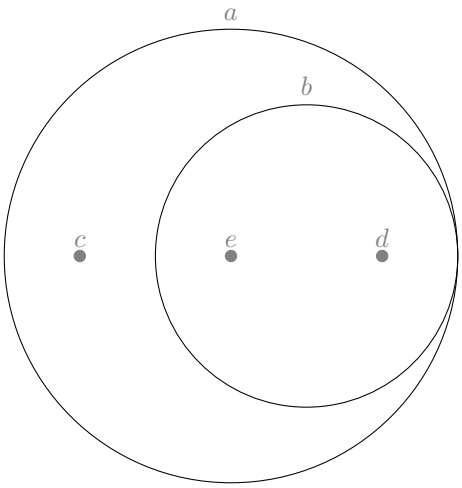


Figure 638: $b \to de, cd \to e, a \to bcde$

639

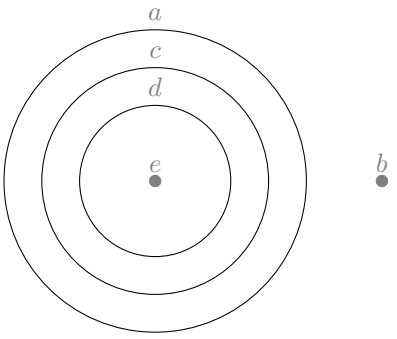


Figure 639: $a \to cde, c \to de, d \to e$

640

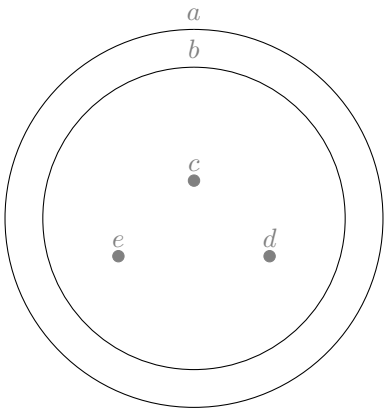


Figure 640: $b \to cde, a \to bcde$

641

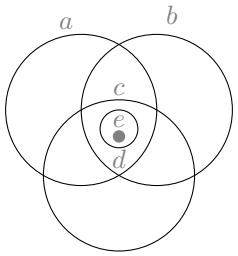


Figure 641: $a \to de$, $b \to de$, $c \to de$, $d \to e$

642

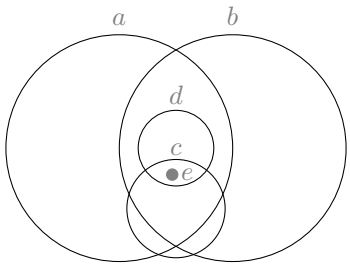


Figure 642: $a \to de$, $b \to de$, $c \to e$, $d \to e$, $ab \to cde$

643

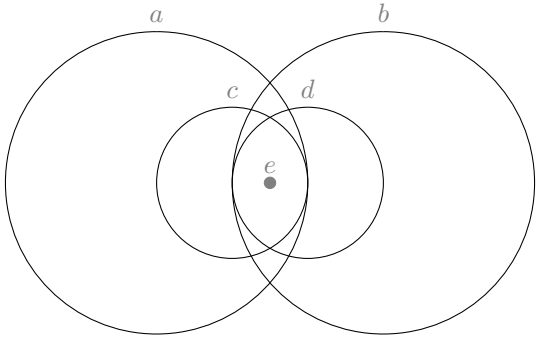


Figure 643: $a \to ce$, $b \to de$, $c \to e$, $d \to e$, $ab \to cde$

644

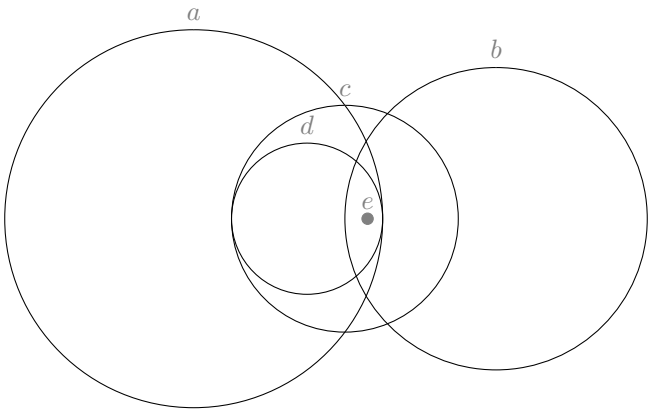


Figure 644: $a \to de$, $b \to e$, $c \to de$, $d \to e$, $ab \to cde$

645

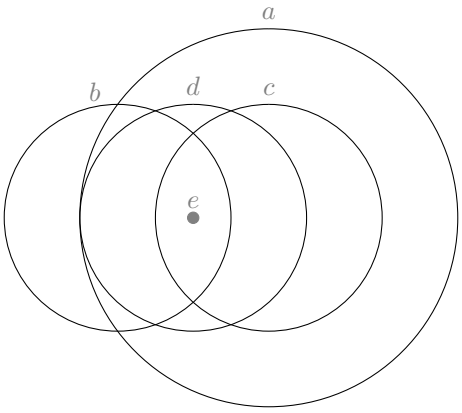


Figure 645: $a \to cde$, $b \to e$, $c \to e$, $d \to e$, $bc \to de$

646

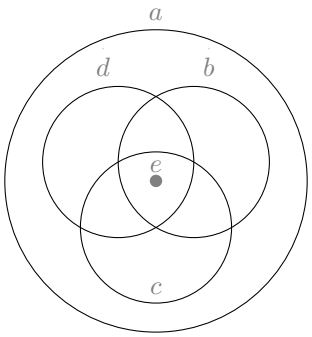


Figure 646: $d \to e, b \to e, c \to e, a \to bcde$

647

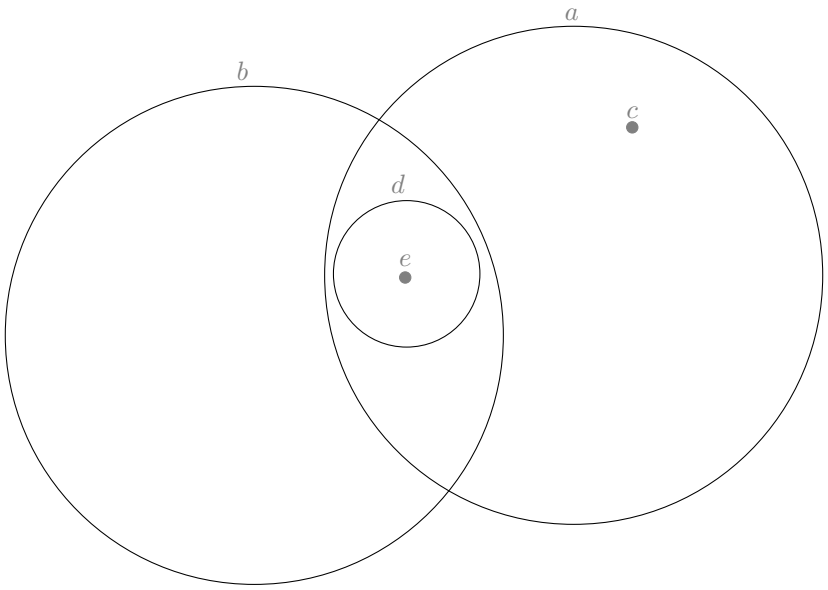


Figure 647: $d \to e, b \to de, a \to cde$

648

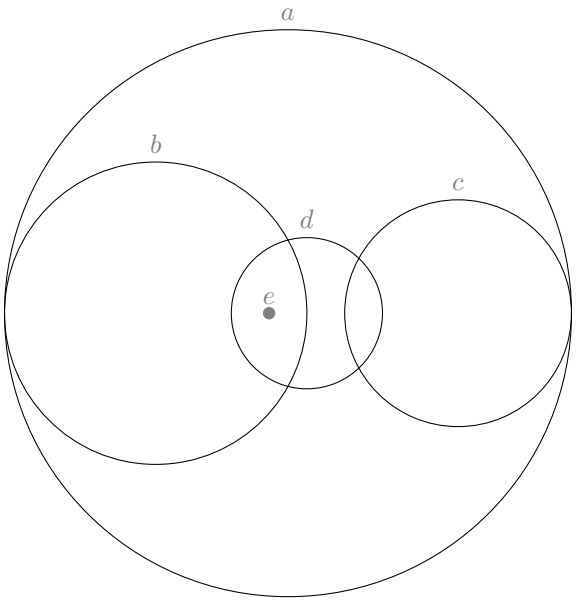


Figure 648: $b \to e, d \to e, bc \to de, a \to bcde$

649

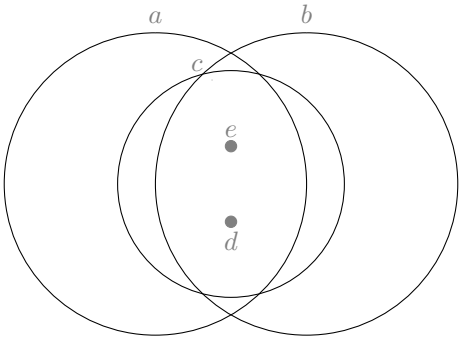


Figure 649: $a \to de, b \to de, c \to de, ab \to cde$

650

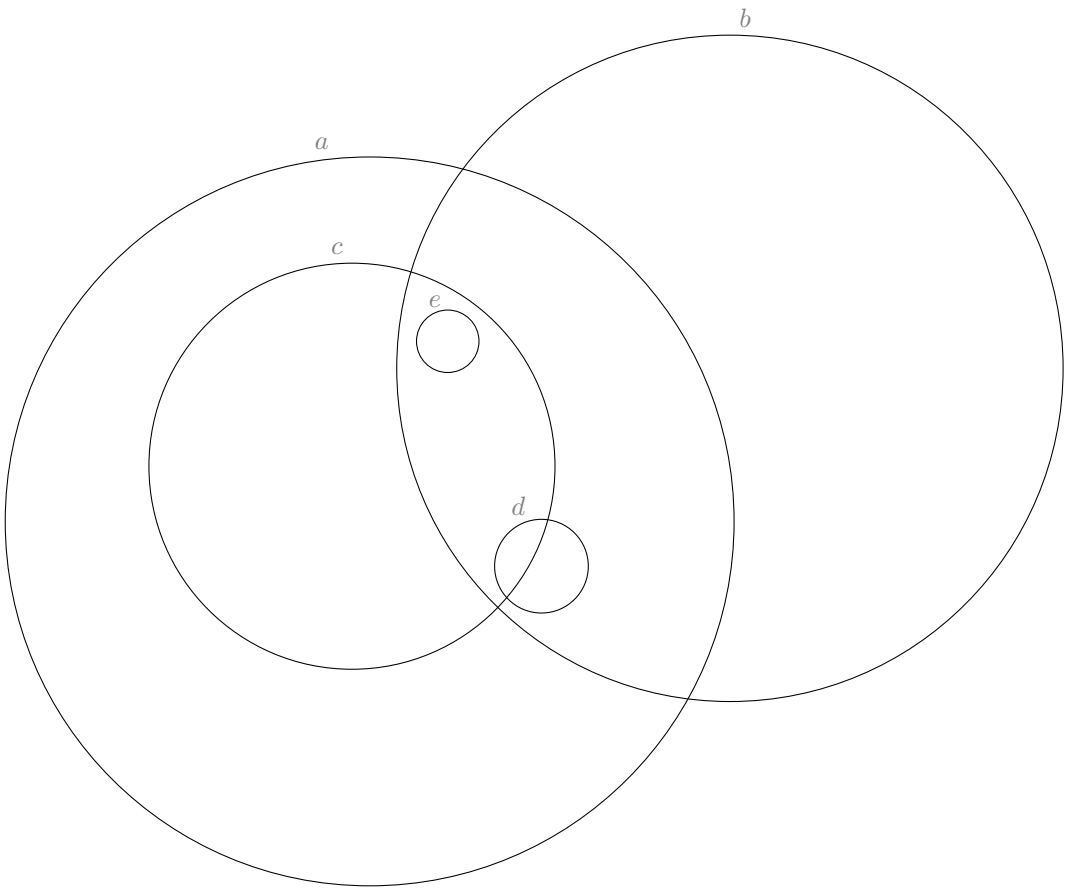


Figure 650: $c \to e, a \to cde, b \to de$

651

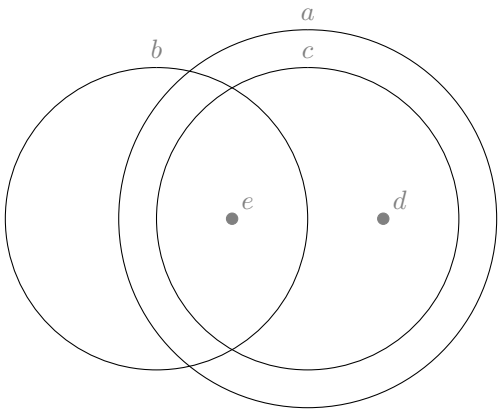


Figure 651: $a \to cde$, $c \to de$, $b \to e$

652

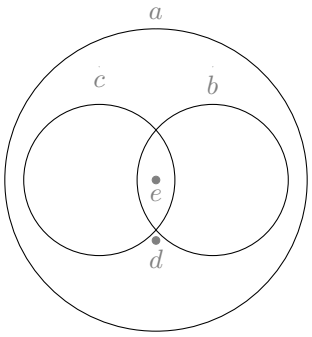


Figure 652: $c \to e, b \to e, bc \to de, a \to bcde$

653

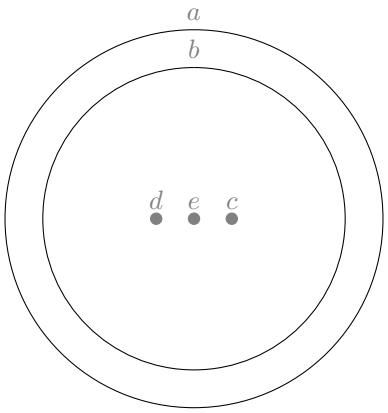


Figure 653: $b \to cde, cd \to e, a \to bcde$

654

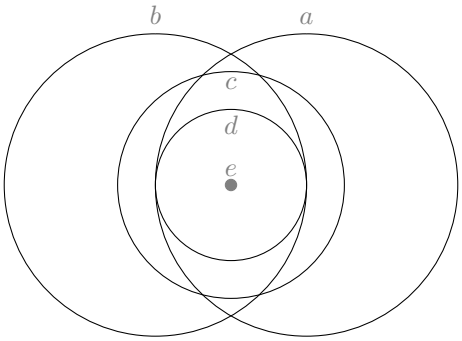


Figure 654: $a \to de$, $b \to de$, $c \to de$, $d \to e$, $ab \to cde$

655

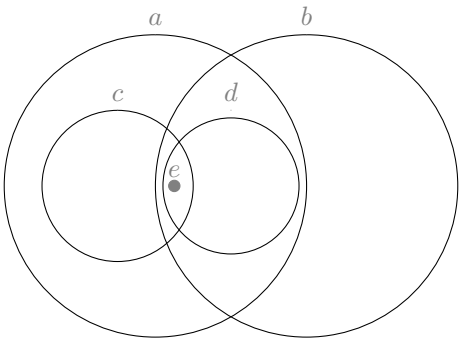


Figure 655: $a \to cde$, $b \to de$, $c \to e$, $d \to e$

656

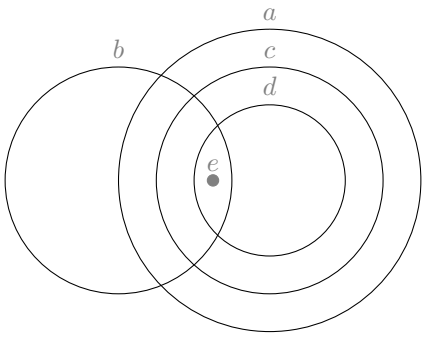


Figure 656: $a \to cde$, $b \to e$, $c \to de$, $d \to e$

657

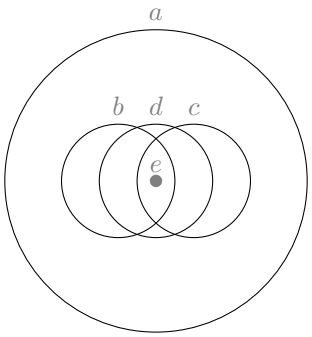


Figure 657: $a \to bcde$, $b \to e$, $c \to e$, $d \to e$, $bc \to de$

658

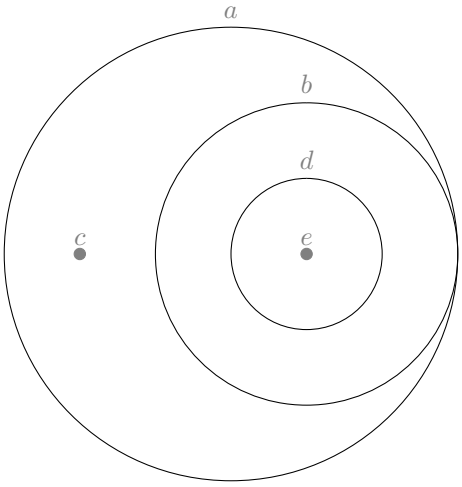


Figure 658: $b \to de, d \to e, a \to bcde$

659

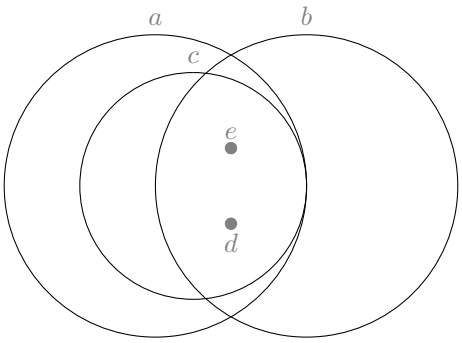


Figure 659: $a \to cde, b \to de, c \to de$

660

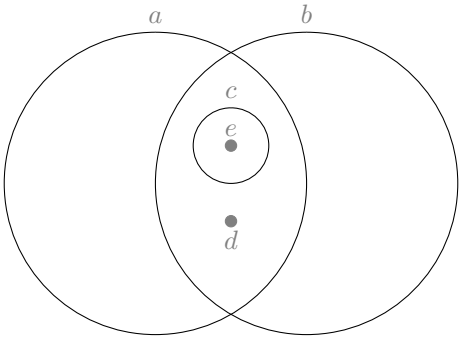


Figure 660: $a \to cde, b \to cde, c \to e$

661

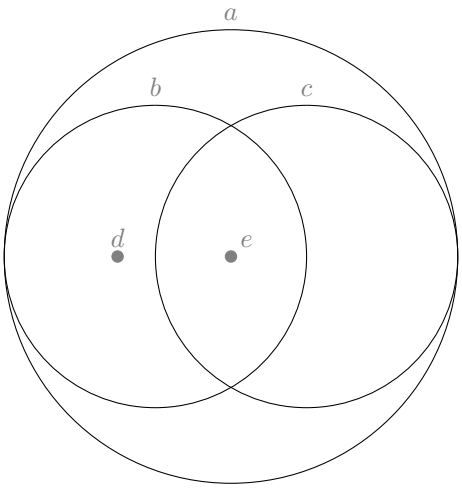


Figure 661: $b \to de, c \to e, a \to bcde$

662

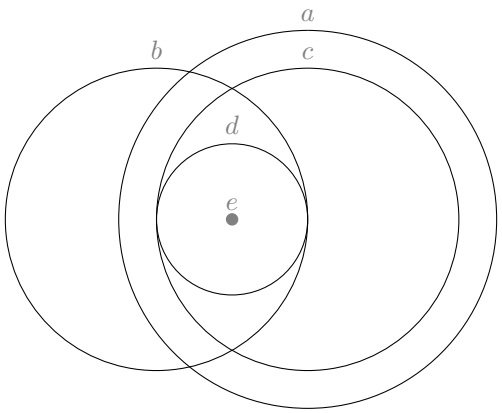


Figure 662: $a \to cde$, $b \to de$, $c \to de$, $d \to e$

663

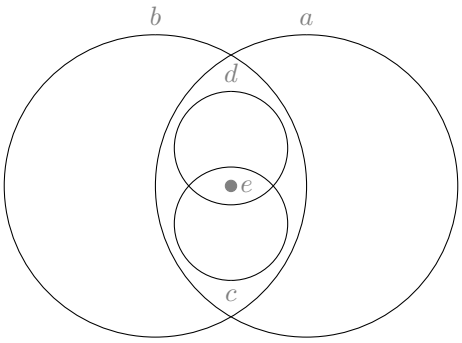


Figure 663: $a \to cde$, $b \to cde$, $c \to e$, $d \to e$

664

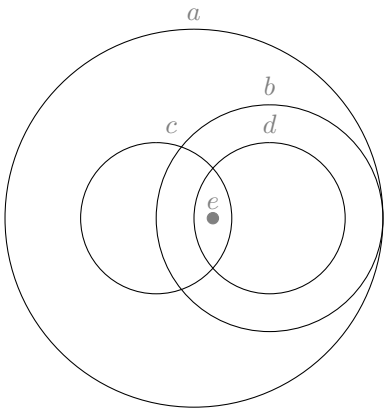


Figure 664: $a \to bcde$, $b \to de$, $c \to e$, $d \to e$

665

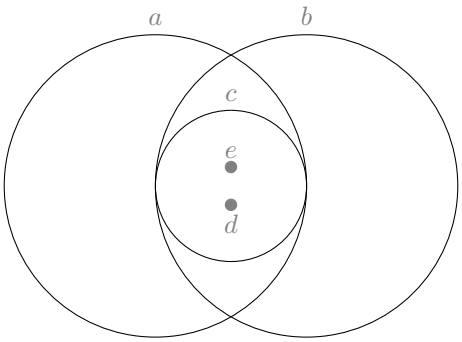


Figure 665: $a \to cde, b \to cde, c \to de$

666

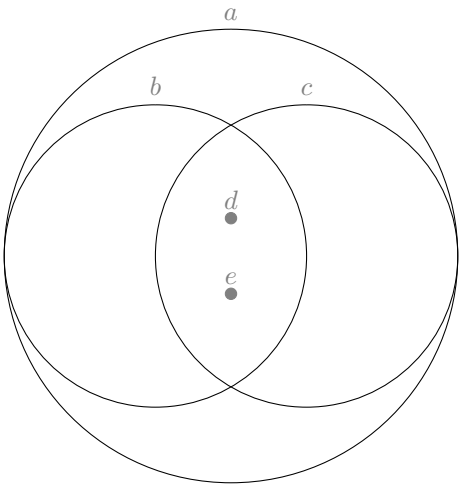


Figure 666: $b \to de, c \to de, a \to bcde$

667

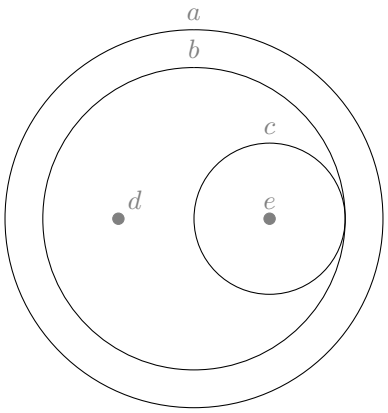


Figure 667: $c \to e, b \to cde, a \to bcde$

668

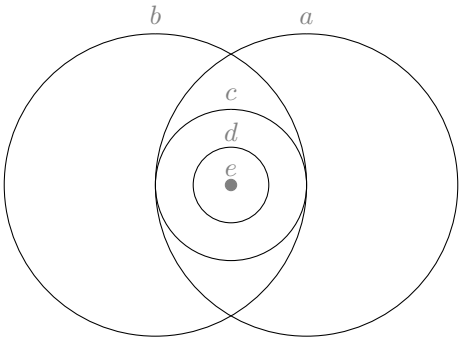


Figure 668: $a \to cde$, $b \to cde$, $c \to de$, $d \to e$

669

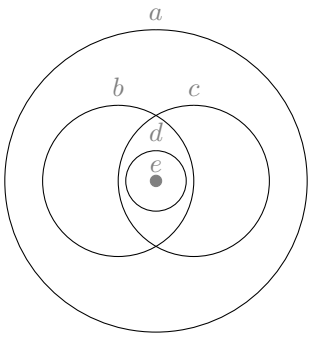


Figure 669: $a \to bcde$, $b \to de$, $c \to de$, $d \to e$

670

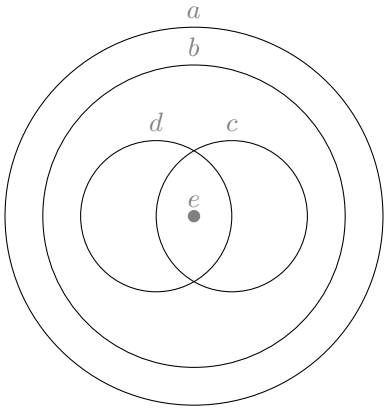


Figure 670: $a \to bcde, b \to cde, c \to e, d \to e$

671

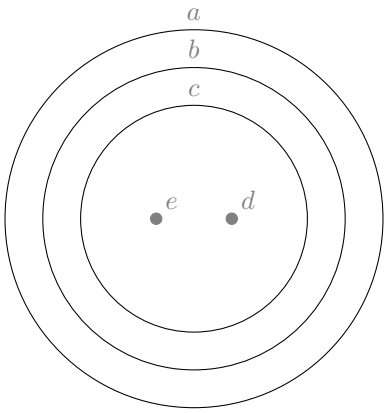


Figure 671: $b \to cde, c \to de, a \to bcde$

672

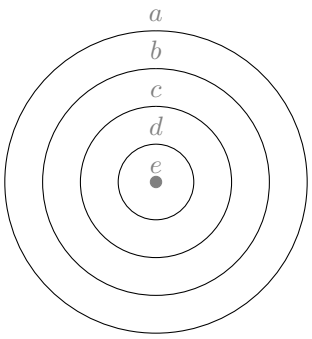


Figure 672: $a \to bcde$, $b \to cde$, $c \to de$, $d \to e$

B.3 Impossible Geometries

```
4. Weak Carousel Rule
abc -> de
dimension: 6

7. Triangle Property
acd -> e
abd -> e
abc -> e
dimension: 4

11. Triangle Property
bcd -> e
acd -> e
abd -> e
abc -> e
dimension: 6

12. Triangle Property
acd -> e
abd -> e
abc -> de
dimension: 5

14.
bcd -> e
acd -> e
ab -> e
dimension: 5

15.
acd -> e
abc -> de
ab -> e
dimension: 5

18. Triangle Property
bcd -> e
acd -> e
abd -> e
abc -> de
dimension: 6

21. Triangle Property
abe -> d
```

```
bcd -> e
acd -> e
abc -> de
dimension: 4

23. Triangle Property Corollary
bcd -> e
acd -> e
abc -> de
ab -> e
dimension: 6

26.
bcd -> e
ad -> e
abc -> de
dimension: 5

27.
abe -> d
bcd -> e
abc -> de
ac -> e
dimension: 4

33.
abc -> de
ac -> e
ab -> e
dimension: 5

34. Triangle Property
bcd -> e
acd -> e
abc -> de
ab -> d
dimension: 4

35.
bcd -> e
ad -> e
abc -> de
ab -> e
dimension: 5

39.
```

```
bcd -> e
acd -> e
ab -> de
dimension: 5

43.
ace -> d
bcd -> e
ab -> de
dimension: 4

45.
ad -> e
abc -> de
bc -> e
dimension: 5

46.
bcd -> e
ad -> e
ac -> e
ab -> e
dimension: 4

47.
abe -> d
abc -> de
bc -> e
ac -> e
dimension: 4

49.
bcd -> e
abc -> de
ac -> e
ab -> e
dimension: 6

54.
bcd -> e
acd -> e
ab -> cde
dimension: 5

56.
ad -> e
```

```
abc -> de
bc -> e
ab -> e
dimension: 5

57.
ace -> d
bcd -> e
ab -> cde
dimension: 4

60.
bcd -> e
ad -> e
abc -> de
ac -> e
ab -> e
dimension: 5

62.
bcd -> e
ac -> e
ab -> de
dimension: 5

69.
bcd -> e
ac -> d
ab -> de
dimension: 4

70.
abc -> de
bc -> e
ac -> e
ab -> e
dimension: 6

74.
cd -> e
bd -> e
ad -> e
abc -> de
dimension: 4

84.
```

```
bcd -> e
ad -> e
ac -> e
ab -> de
dimension: 4

85.
ad -> e
ac -> de
ab -> de
dimension: 4

86.
ad -> e
ac -> e
ab -> cde
dimension: 5

87.
bcd -> e
ac -> de
ab -> de
dimension: 5

88.
bcd -> e
ac -> e
ab -> cde
dimension: 5

89.
bcd -> e
abc -> de
a -> e
dimension: 5

94.
ad -> e
abc -> de
bc -> e
ac -> e
ab -> e
dimension: 5

95.
bcd -> e
```

```
ac -> d
ab -> cde
dimension: 4

96.
cd -> e
bd -> e
ad -> e
abc -> de
ab -> e
dimension: 4

105.
cd -> e
bd -> e
ad -> e
abc -> de
ac -> e
ab -> e
dimension: 4

114.
bcd -> e
ad -> e
ac -> de
ab -> de
dimension: 4

115.
bcd -> e
ad -> e
ac -> e
ab -> cde
dimension: 5

122.
bcd -> e
ac -> de
ab -> cde
dimension: 5

129.
ad -> e
bc -> e
ac -> e
ab -> de
```

```
dimension: 4

132.
bc -> e
ac -> de
ab -> de
dimension: 5

134.
abc -> de
bc -> e
a -> e
dimension: 5

143.
cd -> e
bd -> e
abc -> de
a -> e
dimension: 4

147.
cd -> e
bd -> e
ad -> e
abc -> de
bc -> e
ac -> e
ab -> e
dimension: 4

153.
bcd -> e
ad -> e
ac -> de
ab -> cde
dimension: 4

161.
bcd -> e
ac -> de
ab -> de
a -> e
dimension: 4

175.
```

```
bc -> de
ac -> de
ab -> de
dimension: 6

206.
cd -> e
bd -> e
abc -> de
bc -> e
a -> e
dimension: 4

211.
bcd -> e
ac -> de
ab -> cde
a -> e
dimension: 4

235.
cd -> e
abc -> de
b -> e
a -> e
dimension: 4

351.
abc -> de
c -> e
b -> e
a -> e
dimension: 4
```