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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

14[65-01, 65N30, 65N55, 73Cxx]—*The mathematical theory of finite element methods*, by Susanne C. Brenner and L. Ridgway Scott, Texts in Applied Mathematics, Vol. 15, Springer, New York, 1994, xx+294 pp., 24 cm, $39.00

The book is very well done, and it provides a nice basis for an undergraduate course on Finite Element Methods. In particular, Chapters 0 to 5 constitute an easy and effective approach to the mathematical foundations of the method, as they recall in a simple and convincing way the basic ideas and the main results of the whole theory. In Chapter 0 a first presentation of the Finite Element Method is given on very simple one-dimensional problems. The reader can immediately understand the power of the method, its interest and, at the same time, the need for more sophisticated mathematical tools which are to be used for the analysis of more general cases. Such tools are then introduced in the following chapters. In Chapter 1 the basic information on Sobolev spaces is recalled in an easy and elegant way. In Chapter 2 the variational formulation of elliptic boundary value problems is introduced at a rather abstract level, using mainly one-dimensional problems as examples to help the reader. In Chapter 3 the basic features of Finite Element spaces are presented, wisely skipping, for the moment, isoparametric elements. The general theory of polynomial approximation in Sobolev spaces comes then naturally in Chapter 4 with a major emphasis on affine elements and some hints on isoparametric ones. Finally, in Chapter 5 the variational formulation of elliptic boundary value problems is revisited, this time in the n-dimensional case, and the previous results on the approximation theory are used to get the basic error estimates. Up to this point, the book is a very appealing intermediate work between the two other masterpieces in the field, namely the book by P. G. Ciarlet (more detailed) and the book by C. Johnson (more accessible to beginners), although closer to Ciarlet than to Johnson.

In the subsequent chapters various excursions in different streams of development are made. Specifically, in Chapter 6 the basic concepts of multigrid methods are presented, in Chapter 7 the maximum-norm estimates are derived, and in Chapter 8 the case of nonconforming elements is considered. Chapter 9 introduces the reader to plane elasticity problems and to the related *locking* phenomenon, which is dealt with in more detail in Chapter 10 (Mixed methods). Chapter 11 gives a hint on resolution techniques for algebraic problems arising from mixed formulations, and finally Chapter 12 deals with various applications of operator-interpolation theory, essentially showing how to get error estimates in less traditional norms or in classical norms but with intermediate regularity for the solution.

This second part of the book is also very nice, but does not match the expectations. In some sense, I would have expected glimpses into hotter research areas, such as advection-dominated flows, domain decomposition techniques, hierarchical bases. The relationships of finite element methods with other recent techniques, such as spectral elements or p-methods, finite volumes and wavelets, would also have been welcome. Surely, multigrid methods or mixed methods are the object of very active research, but one cannot use this book as a starting point for working in these areas, as the material presented here is too meager and, sometimes, almost misleading. For instance, the use of linear nonconforming methods to avoid *locking* in linear elasticity is recommended in Chapter 9 of the book for Dirichlet boundary conditions, which is a case of no practical interest, but the information that the method does not work for more general cases is hidden in Exercise 10.x.5 at the end of the subsequent chapter. A simple-minded reader might think that he got a sound way for escaping locking, but in truth he got only a mathematical exercise.

More generally speaking, I think that the recent book by A. Quarteroni and A. Valli is a much more powerful instrument as a starting point for somebody who wants to start doing research. But even Johnson's book, in its simplicity, opens much wider perspectives for a beginning applied mathematician. Still the final part of the present book can be useful for a reader with intellectual curiosity, willing to have some ideas on selected topics that go beyond the basic results of the first six chapters, and I strongly recommend it for that.

A final unhappy remark has to deal with citations. As I said, the book is intended (or it should be) as a didactic one more than one oriented toward research. As such, a basic lack of citations can be tolerated. It is also normal for an author to refer explicitly to his own work more often than to the work of others. But the present authors are overdoing it a bit. Finally, I am curious to know what the basic tools contained in the book are that (quoting from the preface) "are commonly used by researchers in the field but never published". I hope they do not refer to all the results obtained by others (and regularly published), presented here without citations.

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15[00A69, 34-01, 35-01, 65-01]—*Industrial mathematics: A course in solving real-world problems*, by Avner Friedman and Walter Littman, SIAM, Philadelphia, PA, 1994, xiv+136 pp., $25\frac{1}{2}$ cm, softcover, \$22.50

This brief, ground-breaking, scholarly text is written for persons who have a two-year basic Calculus background, that includes functions of several variables, along with rudimentary knowledge of ordinary differential equations, linear algebra, infinite series, vector analysis, and who have elementary computer experience (with at least one of these—Fortran, Pascal, C, Maple, Matlab, etc.). In seven concise chapters, mathematical models for the following topics are covered:

(1) Black and white photography, silver crystal growth.
{Ordinary differential equations, theory and methods for numerical solution.}
(2) Air quality.
{Partial differential equations for advection and for advection-diffusion; numerical methods for solution, stability of difference schemes.}

(3) Electron beam lithography (computer chips).
{Integral transforms, heat equation, scattering, Fourier series.}
(4) Color film negative development.
{Diffusion, maximum principles.}
(5) Automobile catalytic converter.
{Optimal control, calculus of variations.}
(6) Photocopy machine (electric image).
{Poisson equation, finite differences, direct and iterative methods for solution.}
(7) Photocopy machine (visible image).
{Free boundary problems.}

This wide range of topics is succinctly and carefully presented. Hence the volume is a valuable teaching resource for a variety of upper level undergraduate and beginning graduate courses. Each chapter begins with a description of the physical phenomena, followed by a mathematical formulation of a model and simplification(s) thereof. The simplest models permit complete mathematical analysis and numerical solution. All of the models have been thoroughly studied in the previously published six volumes by the first author and in other, also given, references. Each chapter concludes with a summary of the topics that were covered. Problems are presented within each chapter. These exercises are generally nontrivial. The authors suggest that some of the computing projects may serve as final examinations, that could well take considerable time and effort to complete!

Here is a challenging noncomputational problem that is given in Chapter 5. The classical brachistochrone problem is stated and provides the motivation for a discussion of simple variational problems. The derivation of the Euler-Lagrange equation is then produced. This Euler equation is a necessary condition that is satisfied by the solution of any simple variational problem. The exercise presented at this point states: "Use the Euler equation to find the solution of the brachistochrone problem."

The authors have performed a valuable service by choosing interesting examples of real-world problems and formulating them as mathematical problems. They achieve their aim to show that Calculus and computers serve as ubiquitous tools for today and tomorrow. The material was used by the second author and a colleague in a one-year upper-level course during 1992–3. The students came "from mathematics, physics, computer science, and various engineering departments." The authors recommend, "One attitude to encourage is that the class is one research organization attacking some problems. Thus, it makes sense to divide the problems among various students or groups of students."

Eugene Isaacson

16[65-06, 65Y05]—*Environments and tools for parallel scientific computing*, Jack J. Dongarra and Bernard Tourancheau (Editors), SIAM Proceedings Series, SIAM, Philadelphia, PA, 1994, xii+292 pp., $25\frac{1}{2}$ cm, softcover, \$38.50

This book is based on the proceedings of *The Second Workshop on Environments and Tools for Parallel Scientific Computing* which took place at Townsend, Tennessee, on May 25–27, 1994. The book is organized in four parts. The first part addresses issues related to data mapping in HPF, run-time support libraries, and editors to support programming in FORTRAN D and HPF. The second part deals with libraries and languages to support various existing parallel programming models and some activities in parallel numerical software. It includes papers on message passing interface (MPI), varied communication models, migratable, exportable and multi-threaded versions of PVM for homogeneous and heterogeneous network-based computing, the C^{++} version of HPF pC^{++}, and a new C^{++} based language that allows parallel programming models to be implemented as libraries. The third and fourth parts of the book present environments that attempt to support various parallel programming paradigms and integrate compiling, debugging and tracing, performance evaluation, and visualization tools at various levels. It includes papers on various parallel environments such as CODE 2.0 (task/dependence), LHPC (distributed shared memory), τ (TAU:pC^{++}), PPPE (MPI and HPF), PARADYN (PVM), ParaGraph and CAPSE (message passing), EPPP (HPC), TOPSYS (Munich multitasking kernel), and IMPOV. The book will be useful to students and researchers working in the field of high performance computing.

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17[68-01, 65-01]—*Solving Problems in Scientific Computing Using MAPLE and MATLAB*, by Walter Gander and Jiří Hřebíček, Springer, Berlin, 1993, xiv+268 pp., $23\frac{1}{2}$ cm, softcover, $39.00

This text presents the solution to several interesting scientific computation problems via the use of either one of the computer languages of MAPLE or MATLAB. The solution to these problems would be difficult and time-consuming without the use of MAPLE or MATLAB. On the other hand, the authors make effective use of these powerful languages, enabling their solutions to these nontrivial problems presentable in a classroom setting.

The authors intend the book as a text for students in scientific computing. It is not a text for learning MAPLE or MATLAB; rather, the authors assume that the reader is familiar with these languages.

The text had several contributors, from three university sources:

The Department of Theoretical Physics and Astrophysics of Masaryk University, Brno, Chech Republic;

The Institute of Physics of the University of Agriculture and Forestry, Brno, Chech Republic; and

The Institute of Scientific Computing, ETH, Zürich, Switzerland.

The following people, listed with reference to the Universities as cited above, contributed solutions to problems in this text:

Stanislav Barton (Brno), Joroslav Buchar (Brno), Ivan Daler (Brno), Walter Gander (ETH), Dominik Gruntz (ETH), Jürgen Halin (ETH), Jirí Hřebíček (Brno), František Klvaňa (Brno), Urs von Matt (ETH), and Jörg Waldvogel (ETH).

The text consists of 19 chapters, with each chapter presenting a different problem, and with the following titles providing reasonably informative descriptions of the contents:

1. The Tractrix and Similar Curves;
2. Trajectory of a Spinning Tennis Ball;

3. The Illumination Problem;
4. Orbits in the Planar Three-Body Problem;
5. The Internal Field in Semiconductors;
6. Some Least Squares Problems;
7. The Generalized Billiard Problem;
8. Mirror Curves;
9. Smoothing Filters;
10. The Radar Problem;
11. Conformal Mapping of a Circle;
12. The Spinning Top;
13. The Calibration Problem;
14. Heat Flow Problems;
15. The Penetration of a Long Rod into a Semi-infinite Target;
16. Heat Capacity of a System of Bose Particles;
17. Compression of a Metal Disc;
18. Gauss Quadrature; and
19. Symbolic Computation of Explicit Runge-Kutta Formulas.

The presentation is unique, and extremely interesting. I was thrilled to read this text, and to learn the powerful problem-solving skills presented by these authors. I recommend the text highly, as a learning experience, not only to engineering students, but also to anyone interested in computation.

Frank Stenger

18[68-01, 68Q40]—*Maple* V *by example*, by Martha L. Abell and James P. Braselton, AP Professional, Boston, MA, 1994, xii+500 pp., 23 cm, softcover, $39.95

This book appears to be aimed at first-year undergraduate students who are not specializing in either mathematics or computer science but are using mathematics and computers purely as tools, and are trying to use Maple for the first time. It presents computational models for a range of standard elementary mathematical tasks to which Maple can be applied. It is analogous to a book of recipes rather than a book about cookery, and whilst it presents the mathematical background to some topics, it discusses hardly any of the programming background.

After a general introduction, the book has chapters discussing the following topics: basic arithmetic and algebra; calculus; sets, lists and tables; matrices and vectors; more on linear algebra; a long and rather laboured chapter on differential equations; and finally a chapter showing the obligatory plots of weird shapes in three dimensions. The book has a detailed index—so detailed that it indexes two occurrences of ellipsis, neither of which refers to its Maple significance. At the back of the book is a gratuitous tear-out "Quick Reference" card that reminds the reader that, among other things, "+" is a "frequently used abbreviation" for addition, and briefly illustrates 23 elementary Maple commands in the general areas of calculus, linear algebra and graphics.

Students, especially the less academic, seem to like copious explicit examples, and that requirement is certainly met by this book. However, students tend to believe what is printed in textbooks in preference to what their lecturers write or say in class. Hence, authors of student textbooks have a considerable responsibility to get it right. Unfortunately, the authors of this book have not got it right, because it contains a prodigious number of errors, misguided examples and confusing assertions.

For example, the following all occur on page 20 alone. The Maple trigonometric function `arccos` is misspelled as `arcos`. It is asserted that "`e ^ (−5)` could have been entered instead of exp(−5)", whereas in fact `e` has no special significance in Maple and it is `E` that represents the exponential number (the base of natural logarithms)—Maple is case-sensitive. The authors write "Notice that Maple returns an exact value unless otherwise specified with `evalf` or `evalc`." In fact, the value returned by `evalc` is as exact as its argument, and it is only `evalf` that forces approximate numerical evaluation of exact expressions. Such errors would be less disastrous in a monograph aimed at experienced Maple users, but unfortunately there are no pearls of wisdom for such readers in this book.

The book describes Maple V Release 2, which is the release before the current one. However, it is almost impossible to describe the latest version of software that is developing as rapidly as computer algebra systems unless one is a member of the inner circle of developers. A less excusable aspect of this book is that all descriptions of the user interface are specific to the Macintosh implementation. Hence, the illustrations of the graphical user interface and the discussion of details of its operation will be either inappropriate or wrong for users of Maple on other systems. For example, the comments about the distinction between the *Return* and *Enter* keys, and some comments about the Help system, are wrong in the context of Microsoft Windows.

There are a number of subtleties to Maple that seem to cause users fairly widespread confusion. Some are conceptual difficulties common to many computer algebra systems. One of these is the need to use free (i.e., unassigned) variables in some contexts, which the book confuses totally. It is *never* necessary to clear or unassign a variable immediately before assigning to it, but this is done (inconsistently) in perhaps half the examples in this book.

I will discuss a few of the more pervasive of the many other errors, inconsistencies and sources of confusion in the book.

Maple supports functions or mappings as objects independent of any arguments. This is an elegant and useful feature, which is not provided by all computer algebra systems. However, it is important not to confuse *functions* with their values, which are *expressions* obtained by applying the functions to arguments. This book does nothing to reduce this confusion. For example, on page 42, the authors write "The following example illustrates how to graph several functions using operator notation". In fact, nothing at all that could be regarded either as a Maple function or as Maple operator notation appears in the example. Three variables are (unnecessarily) unassigned and then assigned *expressions* containing the free variable x (which could usefully have been unassigned to ensure that it is free, but has not been). These three expressions are then plotted together using the normal Maple syntax for plotting expressions (which requires the free variable to be specified). This example illustrates a lack of understanding of the significance of free variables and the distinctions among functions, expressions and operators.

Maple supports sequences, lists and sets as distinct (but closely related) primitive data types, and it supports vectors as special cases of arrays. At various points the book confuses these data types. On page 33, within the space of four lines, the book uses both notations (x, y) and $\langle x, y\rangle$ to represent vectors in the context of writing *vector*-valued functions, but in fact the examples define functions that return *lists*.

Whilst it is true that many Maple functions accept lists and vectors interchangeably, they are fundamentally different data types. For example, it is possible to assign to the elements of a Maple vector but not to the elements of a Maple list. Similarly, on page 225 the book asserts that "A matrix is simply a list of lists...", which is not true, although the two structures have a natural one-to-one correspondence that Maple uses in some contexts.

On page 177 the authors assert that "the command `seq(f(i), i = 0..n)` creates the list `[f(1), f(2), ..., f(n)]`", which is not true: the list must be explicitly constructed from the sequence by using square brackets. On page 186 the authors present an example that completely confuses sets and sequences. The output shown would not be produced by the input. To produce the output shown would require the addition of explicit set braces around the sequences, in which case it would be preferable to use the conventional infix syntax "`setone union settwo`" rather than the more obscure prefix syntax "`‘union’(setone, settwo)`". On page 334, the authors create a *set* of expressions (which the authors call functions) and then map a function over the set to create a new set. It is meaningless to use sets in this context: they should be lists, because the ordering of the elements is *essential* in order to retain the correspondence between sources and images under the map. On page 347, the authors construct a list of expressions (but the output contains a spurious comma, thereby creating nonsense), which they then call a set. There is a lamentable lack of precision in the authors' use of sequences, lists and sets.

On page 35, the authors assert that "... in general, $''''''\ldots''$ (k times) refers to the kth most recent output." This is true only for $1 \le k \le 3$, which is not made clear. There are subtle dangers attached to the use of the $''$ (ditto) operator in a worksheet environment because ditto refers to the value computed most recently in time rather than in space. Anyone who teaches Maple must have observed that this confuses inexperienced users, but I found no mention of this danger in the book.

When illustrating the process of taking a limit, the authors evaluate an expression on a set of points in the neighbourhood of the limit point. They take great trouble to randomize this set of points, but I see no reason to believe that randomness plays any role in taking limits, and its role is not explained. I found this discussion particularly misleading.

On page 260 the book advocates the use of the cut-and-paste facilities (I assume) provided by the (Macintosh) graphical user interface as a way of outputting data to a file for later re-input to Maple. This is generally a tortuous process because it is necessary both to ensure that linear character-based (`lprint`) output is used *and* to proceed via another application such as an editor. Maple provides several *much better* and more elegant mechanisms. The `save` command can be used to save data in either ASCII or binary format, and `writeto`, `appendto` and `printf` can be used to output arbitrary data to a file. Several analogous functions are provided to read data back into Maple. These functions all existed in Release 2, but I could not find any of them mentioned in the book—they certainly do not appear in the index.

On page 300 the authors illustrate the use of rotation matrices to rotate a square. Apart from two syntax errors, they omit to specify the plot option "`scaling = CONSTRAINED`", with the consequence that their square is plotted as a rectangle which is then distorted into a parallelogram as it is rotated. This is much more likely to obscure the underlying geometry than to illuminate it!

This book represents a good idea that is unfortunately marred by what I assume was haste and carelessness, and perhaps some lack of understanding of the more subtle aspects of Maple. The publishers must surely also take some blame for not ensuring adequate proofreading.

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19[49-02, 49K40, 65K05]—*Nonlinear programming*, by Olvi L. Mangasarian, Classics in Applied Mathematics, Vol. 10, SIAM, Philadelphia, PA, 1994, xvi+220 pp., 23 cm, softcover, $28.50

This influential book on the theory of nonlinear optimization was first published in 1969, and went out of print a few years ago. SIAM has now republished it as part of a series "Classics in Applied Mathematics".

This beautiful book contains a lucid exposition of the mathematical foundations of optimization. The presentation is always simple and concise, and contains complete and rigorous proofs of most of the results. This is a formative book, which can be read by a student with a good background of calculus of several variables, in that it exposes the reader to most of the basic mathematical ideas of nonlinear optimization. The book is self-contained: the excellent appendices and the introduction review all the mathematical concepts needed to understand the material of the book.

There is a very good treatment of convexity and its generalizations—quasiconvexity and pseudoconvexity. Optimality and duality are covered in great generality, and the exposition of theorems of the alternative is a pleasure to read. There is also an extensive discussion of constraint qualification. All of these results make the book a valuable reference.

Much work has been done in convexity and duality in the 25 years since this book was published, and our views of the foundations of mathematical programming have changed. Nevertheless, the theory developed in this book remains central to the foundations of nonlinear optimization, and the style remains as effective and delightful today as when the book was first published. Every person interested in nonlinear optimization should own this book.

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20[49-02, 49M35, 49M45, 65K05]—*Interior-point polynomial algorithms in convex programming*, by Yurii Nesterov and Arkadii Nemirovskii, SIAM Studies in Applied Mathematics, Vol. 13, SIAM, Philadelphia, PA, 1994, x+405 pp., 26 cm, $68.50[1]

The appearance of Karmarkar's method ten years ago opened a new chapter in the study of complexity in mathematical programming, which has since resulted

[1] This is an abridged version of a review that appeared in OPTIMA, a newsletter of the Mathematical Programming Society.

in the production of hundreds of papers. Karmarkar's method had a slight theoretical advantage and a very significant computational advantage over the only other polynomial-time algorithm for linear programming at the time—Khachiyan's modification of the ellipsoid method of Yudin and Nemirovsky and Shor, originally devised for nonsmooth convex optimization. The computational developments since Karmarkar's paper, both for interior-point and simplex methods, have been significant. These developments, however, are not the subject of the present book, which provides a truly comprehensive study of the foundations of interior-point methods for convex programming.

Karmarkar used projective transformations and an auxiliary potential function in his algorithm, which was presented for linear programming problems in a rather restrictive form. A large amount of effort went into understanding and extending these ideas and removing the restrictive assumptions over the next few years. Also, connections with classical barrier methods and methods of centers were established, and the first path-following methods, with a superior theoretical complexity bound, were developed by Renegar and soon thereafter Gonzaga. These led to primal-dual algorithms and the explosion of research referred to above.

At the same time as these developments, mainly concerned with complexity issues and practical computation for linear programming, Nesterov and Nemirovsky began their path-breaking research into what the key elements of interior-point methods were, what allowed polynomial complexity bounds to be established, and to what general classes of problems such analyses could be extended. This book is the result of five years of their investigations.

The key idea is that of a *self-concordant barrier* for the constraint set, a closed convex set or cone in a finite-dimensional space. The notion of self-concordance requires that the convex barrier function satisfy certain inequalities between its various derivatives; roughly, its third and first derivatives should be suitably bounded when measured in terms of its second derivative, which defines a seminorm at every point of the interior of the convex set or cone. These conditions ensure, for instance, that Newton's method behaves nicely in a reasonably global sense. From such a barrier one can construct path-following methods, of either barrier or method-of-centers type; if the barrier satisfies an additional property natural for a convex cone, one obtains potential-reduction methods. In all cases, the number of iterations necessary to obtain an ϵ-optimal solution depends polynomially on $\ln(1/\epsilon)$ and θ, a parameter associated with the barrier.

Chapter 1 of the book provides a very useful overview of the ideas underlying the work and the contents of each chapter. Then Chapter 2 contains the basic definitions and properties of self-concordant functions and barriers, including the beautiful result that every convex set in $\mathfrak{R}^n$ admits a self-concordant barrier (the *universal* barrier) with parameter θ of order n. Chapters 3 and 4 are concerned with path-following and potential-reduction algorithms, respectively, and demonstrate that the main requirement for efficiently solving a convex programming problem (without loss of generality, with a linear objective function) is the knowledge of a self-concordant barrier, together with its first two derivatives, for the constraint set, with a reasonably small value for its parameter. (The authors also show concern for the practical efficiency of variants of their methods.)

Chapter 5 provides tools for constructing such barriers and several examples. While the result quoted above assures the existence of a barrier with parameter of the order of the dimension, such a barrier may not be easily computable. For example, the usual barrier for a polyhedral set in $\mathfrak{R}^n$ defined by m inequalities is the standard logarithmic barrier, with parameter m not n. On the other hand, the cone of positive semidefinite matrices of order n, a set of dimension $n(n+1)/2$, admits a barrier of parameter n. (This cone arises frequently in important optimization problems.) Chapter 6 discusses applications of the tools developed previously to a wide range of nonlinear problems, and hence obtains efficient methods for their solution. Chapters 7 and 8 address extensions to variational inequalities and various acceleration techniques, respectively.

This is a book that every mathematical programmer should look at, and every serious student of complexity issues in optimization should own. For a brief idea of the approach, the first chapter, the introductory material in subsequent chapters, and the bibliographical notes at the end of the book can be read. For a more detailed study, a serious commitment is necessary; this is a technically demanding tour-de-force. The authors provide motivation and examples, but many of the beautiful ideas require long technical analyses. The reader is advised to skim forwards and backwards to help understand some of the definitions and results. For example, the standard logarithmic barrier function $-\sum_j \ln x_j$ for the nonnegative orthant is introduced on page 40 (with related barriers on pages 33 and 34), but it is helpful for motivation and illustration where self-concordant functions are first defined on page 12. Likewise, a hint of the barrier-generated family on page 66 would assist in understanding the definition of a self-concordant family on page 58. The authors' overview in Chapter 1 is also very helpful in showing the direction the argument will take.

There seem to be very few misprints. One possibly confusing one appears in (2.2.16): ω^2 should be ω in the numerator. Also, the material on representing problems using second-order cones (§6.2.3) states that the parameter of the barrier F is $2|\mu|$, whereas it is only $2k$; this error propagates throughout the section in the complexity bounds. And the excellent bibliographical notes were prepared for an earlier version of the book; the chapter-by-chapter remarks need the chapter numbers incremented by one. The bibliography itself is somewhat limited and often refers to reports that have since appeared in print.

In summary, this is an outstanding book, a landmark in the study of complexity in mathematical programming. It will be cited frequently for several years, and is likely to become a classic in the field.

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21[65-02, 65F05, 65F10]—*Iterative solution methods*, by Owe Axelsson, Cambridge University Press, Cambridge, 1994, xiv+654 pp., $23\frac{1}{2}$ cm, $59.95

The best place to start reading this book is in Chapter 5, where Gauss is quoted on the subject of iterative methods: "I recommend this modus operandi. You will hardly eliminate directly anymore, at least not when you have more than two unknowns. The indirect method can be pursued while half asleep or while thinking about other things." It would be nice if this assertion were valid for arbitrary problems, since direct methods are prohibitively expensive for large systems arising in many computational models. In fact, however, the development of robust and rapidly convergent techniques is the objective of considerable research today. Methods can be derived using purely algebraic considerations or by exploiting the relationship between the matrix problem and an underlying source such as a differential equation. In some domains (e.g., elliptic problems), the most powerful results derive from both points of view. The algebraic approach has the advantage of being broadly applicable to a wide variety of problems. The author of this volume has made fundamental contributions using both approaches. The book is concerned with solution methods derived from the algebraic point of view.

The book begins with a chapter on direct methods, and then it is divided into four general areas. Chapters 2 through 4 cover topics in matrix theory of use for analyzing the convergence properties of iterative methods. These include general properties of eigenvalues, the Perron-Frobenius theory for nonnegative matrices, Gerschgorin analysis, and Schur complements. Chapters 5 and 6 contain definitions and analysis of "classical" methods based on splitting operators, such as the successive over-relaxation method, and Chebyshev polynomials. Chapters 7 through 10 present preconditioners derived from algebraic considerations, such as incomplete factorizations, and tools for analyzing their performance. Chapters 11 through 13 discuss the conjugate gradient method and some variants for nonsymmetric systems. There are three appendices on additional matrix theory and Chebyshev polynomials.

The book should be valuable as a reference volume, updating the classic books of Varga [1] and Young [2], and it also has potential as a text for an advanced graduate course. The chapters on matrix theory give a good concise overview of material for analyzing iterative methods, and many of the topics, such as generalizations of regular splittings and uses of the field of values, are not seen in standard texts. The chapters on classical and conjugate gradient-like methods comprise an excellent summary of what is known about convergence rates of such methods, including important results for special cases of eigenvalue distributions and for nonsymmetric systems. The chapters on preconditioners are somewhat more difficult to read than the rest of the book, in part because there is little background on the discrete partial differential equations from which most of the ideas derive. For example, many results in Chapter 9 were designed for multilevel methods for elliptic problems, which is hard to appreciate from the text. However, the large amount of analysis in these chapters makes them of potential long-term use for reference. For use as a text, the first seven chapters contain a very good collection of exercises as well as the nice feature (through Chapter 6) of the presentation of definitions in the introduction to the chapters. As noted in the book's preface, the material is best suited for a matrix-theoretically oriented course. In addition, such a course would almost certainly have to be supplemented with material on discrete partial differential equations.

In general, the organization and presentation of the book is good, although there are a few technical concepts that are used before they are defined. The most notable weakness concerns the references. Citations appear at the end of individual chapters, but there is no index of references, and it is difficult to find individual references. This is coupled with other minor problems such as inconsistent placement of citations (sometimes near stated results, sometimes in remarks spread throughout chapters) as well as more significant ones such as omission of references (for example, to the concepts of probing and polynomial preconditioning in Chapter 8). As a consequence, I believe it will be somewhat difficult to follow up in the published literature.

Despite this flaw, I believe the book will serve as an excellent reference on the subject of iterative methods. It is a good introduction to the topic albeit at a fairly advanced level, and it is also a potential source of new ideas.

References

1. R. S. Varga, *Matrix iterative analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1962. MR **28:**1725
2. D. M. Young, *Iterative solution of large linear systems*, Academic Press, New York, 1970. MR **46:**4698

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22[15A06, 65F05, 68Q25]—*Polynomial and matrix computations: Fundamental algorithms,* Vol. 1, by Dario Bini and Victor Y. Pan, Progress in Theoretical Computer Science, Vol. 12, Birkhäuser, Boston, 1994, xvi+415 pp., 24 cm, $64.50

Bini and Pan describe their book as being "about algebraic and symbolic computation and numerical computing (with matrices and polynomials)", and they note that it extends the study of these topics found in the classic 1970s books by Aho, Hopcroft and Ullman [1] and Borodin and Munro [2]. A great deal of research has been done since the 1970s, and the authors state that most of the material they present has not previously appeared in textbooks. For a taster of the book's subject matter, see Pan's paper [3], which is mentioned in the preface as surveying a substantial part of the material of the book.

The book achieves its goals of presenting a "systematic treatment of algorithms and complexity in the areas of matrix and polynomial computations", as would be expected in view of the authors' eminence in the field. Both serial and parallel algorithms are described, the latter in the fourth and final chapter. There is very little treatment of computation in floating-point arithmetic, and the emphasis is on the computational complexity of algorithms rather than their actual cost in a computer implementation. The book can be described as being more theoretical than a numerical analysis textbook, but more practical than a textbook in computational complexity, and it makes contributions to both areas.

A strength is the treatment of structured matrices. Toeplitz, Hankel, Hilbert, Sylvester, Bézout, Vandermonde and Frobenius matrices are given a unified treatment and their connection with polynomial computations is explored.

The organization of the book is a little unusual in that important topics are often relegated to appendices and exercises. For example, the fast Fourier transform (FFT) is used and analyzed, but the statement and derivation of the algorithm appears in an exercise, and a discussion of floating-point numbers and error analysis appears in an appendix to Chapter 3, "Bit-Operation (Boolean) Cost of Arithmetic Computations". Unfortunately, solutions or references are rarely given to the many interesting exercises, so the reader struggling with a proof or wanting to know more will need to look elsewhere.

A full rounding error analysis is given for the FFT (Proposition 4.1 of Chapter 3) and claimed to be new, but several similar results have been published in the last 30 years and are referenced in [4, §1.4].

The presentation is good, the book having been prepared with TEX, but it could be improved. The numbering of equations, theorems and so on does not reflect the chapter number, which makes cross-referencing difficult. Some grammatical errors are present, and "Cholesky" is misspelled "Choleski" on page 100. Citations are given using an ugly alphanumeric scheme, under which the book under review would be cited as [BP94]; this makes finding a reference in the extensive bibliography more difficult than if numbers were used.

Volume 2 is referred to on page 94 in a reference to "Chapter 5", but the contents and publication date of the second volume are not mentioned. It appears that Volume 2 will include a treatment of fast matrix multiplication, which is only briefly considered in Volume 1.

Bini and Pan's book fills a gap in the literature and can be recommended as an accessible presentation of much recent research in computations with matrices and polynomials.

References

1. A. V. Aho, J. E. Hopcroft, and J. D. Ullman, *The design and analysis of computer algorithms*, Addison-Wesley, Reading, MA, 1974. MR **54:**1706
2. A. Borodin and I. Munro, *The computational complexity of algebraic and numeric problems*, American Elsevier, New York, 1975. MR **57:**8145
3. Victor Pan, *Complexity of computations with matrices and polynomials*, SIAM Rev. **34** (1992), 225–262. MR **93g:**65179
4. Charles F. Van Loan, *Computational frameworks for the Fast Fourier Transform*, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1992. ISBN 0-89871-285-8, xiii + 273 pp. MR **93a:**65186

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23[65-02, 65D17]—*Mathematical aspects of geometrical modeling*, by Charles A. Micchelli, CBMS-NSF Regional Conference Series in Applied Mathematics, Vol. 65, SIAM, Philadelphia, PA, 1995, x+256 pp., 25 cm, softcover, $37.50

This monograph grew out of the author's 1990 lectures at a Regional Conference sponsored by the Conference Board of the Mathematical Sciences. This was held at Kent State University, and had as its theme, "Curves and Surfaces: An Algorithmic Approach". The book, however, concentrates on the content of only a portion of the lectures, and enters into considerable detail on the chosen topics. These topics are quickly enumerated by giving the chapter headings: 1. Matrix Subdivision, 2. Stationary Subdivision, 3. Piecewise Polynomial Curves, 4. Geometric Methods for Piecewise Polynomial Surfaces, and 5. Recursive Algorithms for Polynomial Evaluation. Chapter 1 begins with the Casteljau subdivision algorithm for the Bernstein-Bézier representation of a polynomial curve in a Euclidean space of arbitrary dimension. This leads naturally into the subject of general matrix subdivision schemes. Chapter 2 discusses the de Rham-Chaikin algorithm and the Lane-Reisenfeld algorithm. Here we encounter the *refinement* equation that plays an important role in wavelets, and one section is devoted to applications in that subject. Chapter 3 concerns representation of curves parametrically by spline functions. Many subtopics are dealt with, such as knot insertion, variation-diminishing properties of the B-spline basis, and connection matrices (which relate adjacent parts of the piecewise polynomial). In Chapter 4 the emphasis is on multivariate splines and their use in surface representation. The geometric interpretation of higher-dimensional spline functions as volumes of slices of polyhedra is central. This chapter closes with a historical vignette in the form of letters by I. J. Schoenberg and H. B. Curry. Chapter 5 discusses, among other topics, blossoming, pyramid schemes, and subdivision for multivariate polynomials. This book of 256 pages offers a concentrated mathematical development of the representation of curves and surfaces by one of the most authoritative experts in the field. It certainly establishes the current status of this rapidly unfolding area.

E. W. Cheney

24[41-06, 41A30, 41A99, 65-06]—*Wavelets: theory, algorithms, and applications*, Charles K. Chui, Laura Montefusco, and Luigia Puccio (Editors), Wavelet Analysis and Its Applications, Vol. 5, AP Professional, San Diego, CA, 1994, xvi+627 pp., $23\frac{1}{2}$ cm, \$59.95

This is a formidable book to review. It contains twenty-eight papers, broadly sorted into seven categories. It records the transactions of a conference held in October 1993 at Taormina, Italy. Thirteen of the articles were invited specifically for the volume and fifteen were selected from a pool of contributions.

There are four papers in the first group on multiresolution analysis. Authors represented here are Cohen, Heller, Wells, Dahlke, Goodman, and Micchelli. In the second group, on wavelet transforms, there are three papers, by Terrésani, Kautsky, Turcajová, Plonka, and Tasche. The third group, on spline wavelets, contains four articles, by Steidl, Sakakibara, Lyche, Schumaker, Chui, Jetter, and Stöckler. The fourth group, on other mathematical tools for time-frequency analysis, has four papers, by Donoho, Davis, Mallat, Zhang, Suter, Oxley, Courbebaisse, Escudié, and Paul. In the fifth group there are two papers on wavelets and fractals, by Jaffard and Holschneider. The sixth group addresses numerical methods. Here there are five papers, with authors Dahmen, Prössdorf, Schneider, Bertoluzza, Naldi, Ravel, Karayannakis, Montefusco, Fischer, and Defranceschi. The seventh group is concerned with applications and contains six articles, by Wickerhauser, Farge, Goirand, Wesfreid, Cubillo, Mayer, Hudgins, Friehe, Druilhet, Attié, de Abreu Sá, Durand, Bénech, Olmo, Presti, Denjean, and Castanié.

This volume provides an excellent snapshot of the broad activity in wavelet theory. For readers of *Mathematics of Computation*, the papers in the sixth group may be of particular interest. Here one finds a detailed analysis of Galerkin schemes for pseudodifferential equations on smooth manifolds, a discussion of wavelet solutions of two-point boundary value problems on an interval, a paper on bounds for the

Franklin wavelet, an investigation of using orthonormal wavelets in parallel algorithms for numerical linear algebra, and a study of the Hartree-Fock equation by the use of wavelets.

E. W. Cheney

25[65-01, 65D07, 65Y25, 68U07]—*NURB curves and surfaces: from projective geometry to practical use*, by Gerald E. Farin, A K Peters, Wellesley, MA, 1995, xii+229 pp., $24\frac{1}{2}$ cm, $39.95

First, many readers, such as this reviewer, need to be told that "NURB" means "nonuniform rational B-spline". NURBS are basic objects that are the building blocks for representing curves and surfaces. Such representations, in turn, are essential in computerized design, drafting, modeling, and so on. NURBS are sufficiently versatile to fit several distinct systems for computerized design. In the 1950s, such systems grew up independently in different companies (mainly in the automobile and aircraft industries) and even in different branches of the same company. NURBS eventually made it possible to avoid the chaos in this field that the industry was apparently facing. The author gives a little of this history in his preface.

The book is intended as a textbook for a course in computer-aided-design at the beginning graduate level. Prerequisites are knowledge of linear algebra, calculus, and basic computer graphics. Since formal geometry is NOT a prerequisite, the author begins with a snappy account of projective geometry. I particularly like his definition of the projective plane, which requires just three simple sentences. By page 17 we have learned all about pencils, Pappus' theorem, duality, the affine plane, and various models of the projective plane.

Chapter 2 is devoted to projective maps, affine maps, Moebius transformations, perspectivities and collineations. In Chapter 3, conics are introduced in a manner going back to Steiner. The four-tangent theorem and Pascal's theorem are proved. In Chapter 4, more concrete representations of conics are considered, in parametric form. Here we meet the Bernstein form of a conic and the de Casteljau algorithm for computing points on it. The notion of a control polygon is introduced in this context. Interpolating conics, blossoms, and polars make their entrance. In Chapter 5, emphasis shifts from projective geometry to affine geometry, which is closer to the environment of most applications. Now the parametric form of a conic appears as a rational function containing "control points" and "weights". In Chapter 6, "conic splines" are introduced. These are curves made up piecewise from conics, with certain smoothness imposed at the junctions. Chapter 7, one of the longer chapters, discusses rational Bézier curves, which are basic to all piecewise rational curve strategies. We have a Bernstein representation, again with control points and weights, either of which can be manipulated to affect the shape of the curve. There is a projective form of the de Casteljau algorithm, due to the author (1983). Degree raising and reduction, reparametrization, blossoming, and hybrid Bézier curves are all treated. Rational cubics are the subject of Chapter 8. Rational cubic splines are treated from the projective viewpoint in Chapter 9, with second-order smoothness imposed by use of the osculants. Chapter 10 is devoted to the general NURBS. Thus, rational B-splines of arbitrary degree are permitted. The basic operation of knot insertion is described.

In the remaining chapters, attention shifts to surfaces. Rational Bézier patches play a central role. The bilinear and the bicubic cases are singled out. Surfaces of revolution and developable surfaces are considered specially. Triangular patches, quadric surfaces, and Gregory patches are topics considered in the later chapters. The fifteenth and last chapter gives some examples and a discussion of the IGES standards for NURBS. There is a good bibliography and a good index. Copious references to the literature are made throughout the book.

All-in-all, this is a very appealing book that should have a stimulating effect on the teaching of this important subject. It can certainly be recommended for solo study because of the gentle expository style of the writing.

E. W. Cheney

26[65-06, 65D05, 65D07, 65D17]—*Designing fair curves and surfaces*, Nickolas S. Sapidis (Editor), Geometric Design Publications, SIAM, Philadelphia, PA, 1994, xii+318 pp., $25\frac{1}{2}$ cm, softcover, \$61.50

This volume, the seventh in a series of geometric design publications from SIAM, focuses on the problem of "visually appealing" line/surface construction. Its twelve chapters explore various ways of (i) defining "fairness" or "shape quality" mathematically, (ii) developing new curve and surface schemes that guarantee fairness, (iii) enabling a user to identify and remove local shape aberrations without global disturbance.

A common thread is the use of differential geometry constructs to express fairness. Thus, we encounter *arc length s, curvature κ, radius of curvature $\rho = 1/\kappa$*, and *torsion τ* in the study of lines, *principal curvatures* k_1, k_2, *mean curvature* $H = (k_1 + k_2)/2$, and *Gaussian curvature* $K = k_1 k_2$ in the study of surfaces. Typically, not these quantities alone, but also their arcwise derivatives (or divided differences), are the determinants of shape quality. In the volume's broadest-gauged chapter, Roulier and Rando list eight different fairness metrics for lines, of which the first, $\mu = \int [\rho^2\tau^2 + (\rho')^2]^{1/2}\, ds$, is representative. They propose the minimization of μ over a preselected family of design curves as an answer to (i) and (ii) above. Surfaces are to be treated similarly, with at least five double-integral fairness metrics to choose from.

Other chapters present comparable schemes. Moreton and Séquin construct interpolatory quintic spline curves that minimize the functional $\int \|\overrightarrow{\kappa}'\|^2\, ds$, and biquintic surface patches wherein (loosely speaking) the total of such functionals over all lines of principal curvature is minimal. Eck and Jaspert work with point sets only. They interpolate data by a polygon, invoke *difference geometry* to obtain discrete curvature and torsion derivatives $\kappa_i, \kappa_i', \kappa_i'', \tau_i, \tau_i'$ at each inner vertex, and perturb these vertices iteratively, so as to minimize $\sum_i [(\kappa_i'')^2 + (\tau_i')^2]$. Feldman obtains discrete curvature in the same way for a planar polygon with vertices (L_i, X_i), and takes the length μ of the derived polygon (L_i, κ_i) as a fairness metric. His aim is to minimize μ by perturbing the ordinates X_i between prescribed tolerance limits.

Several authors prefer inequality constraints on $\kappa, \kappa', \ldots$ to the metric approach. Burchard et al. fit discrete points in the plane by a circular spline with curvature of uniform sign, monotone and log-convex as a function of s, between designated nodes. Ginnis et al. fit the same points by a polynomial spline of nonuniform degree. They allow small perturbation of the data, one point at a time, and local degree elevation as needed to satisfy certain shape-preservation and fairness criteria, holding κ and κ' to the fewest possible changes of sign. A. K. Jones proposes to fit planar data by a (parametric) polynomial spline whose curvature profile approximates, as closely as possible in a least squares sense, a user-supplied target profile and may also have to satisfy positivity and/or monotonicity and/or convexity constraints. Skillful handling leads to an optimization problem wherein both objective and constraint functions are polynomial in the unknown spline coefficients.

Rounding out the volume are various papers that speak to the issue of shape control without explicitly defining, or attempting to measure, fairness: Gallagher and Piper on convexity-preserving surface interpolation, Bloor and Wilson on interactive design using PDEs, J. Peters on surfaces of arbitrary topology using biquadratic and bicubic splines, Zhao and Rockwood on a convolution approach to N-sided patches, Beier and Chen on a simplified reflection model for interactive smoothness evaluation.

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27[12Y05, 65T10, 94B05]—*Computational number theory and digital signal processing: Fast algorithms and error control techniques*, by Hari Krishna, Bal Krishna, Kuo-Yu Lin, and Jenn-Dong Sun, CRC Press, Boca Raton, FL, 1994, xviii+330 pp., 24 cm, $59.95

Two main topics of this book are number-theoretic transforms and fast algorithms for the calculation of convolutions in digital signal processing. The relevant parts of the book are written in the same spirit as the classical monograph of McClellan and Rader [2]. The underlying algebraic structures are the residue class rings $Z(M)$ of the integers modulo M as well as polynomial rings over $Z(M)$. The third main topic of the book is error detection and correction by linear codes over $Z(M)$, with special reference to fault tolerance in modular arithmetic. It will be hard to find the material on this topic in any other book.

The authors appear to be comfortable with the algorithmic and signal processing aspects of their material, but the treatment of the algebraic background leaves a lot to be desired. The Chinese Remainder Theorem is proved several times over for various rings, when it would have been more efficient to establish once and for all the general Chinese Remainder Theorem for rings as in Lang [1, Chapter II]. Several basic definitions are wrong. For instance, on p. 32 it is said that two polynomials are relatively prime if they have no factors in common, and a similar error occurs on p. 34 in the definition of irreducible polynomials. In the definition of the order of an element on p. 46, replace "smallest non-zero value" by "smallest positive value". The book is replete with awkward formulations such as "A polynomial $A(u)$ is called *monic* if the coefficient of its highest degree is equal to 1" (Definition 3.2) and "Here the entire theorem is defined over the ring $Z(p^\alpha)$" (Theorem 4.6). On p. 80 we read that "It is necessary and sufficient that $P(u)$ be monic and of degree greater than $n-1$", but it is not said for which property this is necessary and sufficient. On p. 55 replace "Legrange" by "Lagrange". This is just a sample selection of deficiencies.

This book is useful as a reference for experts, but because of the weaknesses noted above it is not suitable as an introductory textbook.

References

1. S. Lang, *Algebra*, Addison-Wesley, Reading, MA, 1965. MR **33:**5416
2. J. H. McClellan and C. M. Rader, *Number theory in digital signal processing*, Prentice-Hall, Englewood Cliffs, NJ, 1979.

Harald Niederreiter

28[11-00, 11B83]—*The encyclopedia of integer sequences*, by N. J. A. Sloane and Simon Plouffe, Academic Press, San Diego, CA, 1995, xiv+587 pp., $23\frac{1}{2}$ cm, \$44.95

The title of this book is an accurate description, although some might argue that "A Dictionary of Integer Sequences" is a better title: it is literally a listing of some 5488 integer sequences, together with a brief description for each. The book is an updated printing, with more than twice as many entries, of an earlier book by Sloane [1].

The entries are listed in lexicographic order, except that for some reason the authors chose not to use zeros and ones in this ordering. The listing for an individual entry typically includes a sequence identification number for cross references, such as "M1234"; the leading elements of the sequence itself (typically two lines or so); a brief description, such as "orders of simple groups"; and, in many cases, an abbreviated reference, such as "MOC 21 246 67" (*Mathematics of Computation*, vol. 21, pg. 246, 1967).

For a few of the particularly interesting entries, the authors include a part- or full-page figure explaining the origin and significance of the sequence. Accompanying the entry M0692, which is a Fibonacci sequence $(1, 1, 2, 3, 5, 8, 13, 21, \ldots)$, the authors define the Fibonacci and Lucas numbers with the help of tree diagrams. Accompanying entry M1141, which is the sequence of generalized Catalan numbers $(1, 1, 1, 2, 4, 8, 17, 37, \ldots)$, is an inset explaining that these numbers arise in the enumeration of structures of RNA molecules. Accompanying entry M3987, the authors graphically list different ways in which an $n \times n$ chess board may be dissected into four congruent pieces. Accompanying entry M3218, the theta series of a lattice is defined and described in some detail.

In addition to the entries listed above, the book includes entries as diverse as the digits of π, the Euler numbers, the denominators of the Bernoulli numbers, successive values of the Euler totient function, the continued fraction elements of e, the elements of the recursion $a_n = a_{n-1} + a_{n-3}$, the numbers of planar maps with n edges, the numbers of irreducible positions of size n in Montreal solitaire and the Euler-Jacobi pseudoprimes.

In order to ascertain the completeness of the reference, this reviewer attempted to find a number of sequences that he has encountered in various research activities. In the majority of cases, the sequence was found. Here are some that were not: the continued fraction expansions for $\log 2$ and $\sqrt[3]{2}$, the denominators in the Taylor expansions of $\tan x$ and $e^x \cos x$, and the known Wieferich primes.

Such an exercise highlights both the value and the limitations of this type of reference. On one hand, it is very useful to be able to quickly identify an integer sequence that one encounters in research work. On the other hand, any reference book of this sort will be necessary incomplete. What is really needed are some effective algorithms and robust computational tools. The authors, in an introductory section, give an overview of known techniques of this sort, plus some tools that are available. For some readers, this will be the most valuable section of the book.

References

1. N. J. A. Sloane, *A handbook of integer sequences*, Academic Press, New York, 1973. MR **50:**9760

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