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What Comes Beyond the Standard Models

Bled, September 16–26, 2006

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Preface

The series of workshops on ”What Comes Beyond the Standard Model?” started in 1998 with the idea of organizing a real workshop, in which participants would spend most of the time in discussions, confronting different approaches and ideas. The picturesque town of Bled by the lake of the same name, surrounded by beautiful mountains and offering pleasant walks, was chosen to stimulate the discussions.
The idea was successful and has developed into an annual workshop, which is taking place every year since 1998. Very open-minded and fruitful discussions have become the trade-mark of our workshop, producing several published works. It takes place in the house of Plemelj, which belongs to the Society of Mathematicians, Physicists and Astronomers of Slovenia.
In this ninth workshop, which took place from 16 to 26 of September 2006 at Bled, Slovenia, there were some changes: the date for this year's workshop was moved due to participants' other obligations from a customary mid July to September, and several members of prof. Sannino's group were present thanks to bilateral Slovene-Danish collaboration project. They delivered talks, write-ups of some of which you can read in this volume and enriched our discussions. This ninth workshop differs from the previous ones in the fact that it is a very short period between the workshop and the deadline for sending the contributions for the proceedings. Because of this many a participant has not succeeded to send the contribution in time. We promise to include those, which were received too late to be included in this proceedings, in the next year proceedings. Also the discussion section, which usually is quite the rich one, is in this time missing several contributions, from the same reason - too short time.
We have tried to answer some of the open questions which the Standard models leave unanswered, like:

- Why has Nature made a choice of four (noticeable) dimensions? While all the others, if existing, are hidden? And what are the properties of space-time in the hidden dimensions?
- How could Nature make the decision about the breaking of symmetries down to the noticeable ones, if coming from some higher dimension d?
- Why is the metric of space-time Minkowskian and how is the choice of metric connected with the evolution of our universe(s)?
- Why do massless fields exist at all? Where does the weak scale come from?
- Why do only left-handed fermions carry the weak charge? Why does the weak charge break parity?

- What is the origin of Higgs fields? Where does the Higgs mass come from?
- Where does the small hierarchy come from? (Or why are some Yukawa couplings so small and where do they come from?)
- Where do the generations come from?
- Can all known elementary particles be understood as different states of only one particle, with a unique internal space of spins and charges?
- How can all gauge fields (including gravity) be unified and quantized?
- How can different geometries and boundary conditions influence conservation laws?
- Does noncommutativity of coordinate manifest in Nature?
- Can one make the Dirac see working for fermions and bosons?
- What is our universe made out of (besides the baryonic matter)?
- What is the role of symmetries in Nature?

We have discussed these and other questions for ten days. Some results of this efforts appear in these Proceedings. Some of the ideas are treated in a very preliminary way. Some ideas still wait to be discussed (maybe in the next workshop) and understood better before appearing in the next proceedings of the Bled workshops. The discussion will certainly continue next year, again at Bled, again in the house of Josip Plemelj.

Physics and mathematics are to our understanding both a part of Nature. To have ideas how to try to understand Nature, physicists need besides the knowledge also the intuition, inspiration, imagination and much more. These fundamental questions also receive quite an attention by the general public — see for example articles on these topics in several science magazines. Among them we should perhaps mention the article '*Create your own universe*' by Zeeya Merali in the New Scientist magazine (Issue 2559, 10 July 2006), mentioning the work of our contributor Eduardo Guendelman (see the article by Ansoldi and Guendelman on this very topic in this issue).

The organizers are grateful to all the participants for the lively discussions and the good working atmosphere. Support for the bilateral Slovene-Danish collaboration project by the Research Agency of Slovenia is gratefully acknowledged.

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1 Child Universes in the Laboratory

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Abstract. Although cosmology is usually considered an observational science, where there is little or no space for experimentation, other approaches can (and have been) also considered. In particular, we can change rather drastically the above, more passive, observational perspective and ask the following question: could it be possible, and how, to create a universe in a laboratory? As a matter of fact, this seems to be possible, according to at least two different paradigms; both of them help to evade the consequences of singularity theorems. In this contribution we will review some of these models and we will also discuss possible extensions and generalizations, by paying a critical attention to the still open issues as, for instance, the detectability of child universes and the properties of quantum tunnelling processes.

1.1 The studies so far . . .

The world Cosmology stems from the Greek word *cosmos*, which meant *beauty*, *harmony*, and is the name of that branch of science which studies the origin and evolution of the universe. Thus, considering its name and the object of its study, it is, perhaps, natural to take a "passive" point of view when dealing with cosmological problems, where we use the word *passive* to emphasize that our experience of cosmology is mainly observational in nature. This may undoubtedly be a condition that seems hard to change *in practice*: after all we are dealing with problems, as the birth of our universe and its evolution in the present state, which do not appear suitable for a direct experimental approach. On the other hand, we do not see any reason why this should prevent us from changing our *attitude* toward the problem, switching from a contemplative to a more active one. In our opinion, a stimulation in this sense is coming already from the theory which first gave us the opportunity to address cosmological problems quantitatively, i.e. General Relativity. General Relativity raises for the first time the concept of causality as a central one in physics. This means that, taking a very pragmatic point of view, we have to admit that only a subset of what exists in our universe can be experienced/observed by us. This is not because of our limited capabilities as humans,

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but, more fundamentally, because of the restrictions imposed by the spacetime structure on the causal relations among objects. At the same time causality also brings a challenge to cosmologists in connection with the *large scale* spacetime structure; this is because the simplest models of the universe which are built according to General Relativity and whose late time predictions have a reasonable degree of consistency with what we observe, seem doomed to have an initial singularity in their past so that field equations break down exactly where we would like to set up the initial conditions. This undesirable situation looks even more disappointing after the observation that many parameters describing the state of the early universe are quite far from the domain of "very large scales" which characterizes the present observable universe. Let us, for instance, consider a Grand Unified Theory scale of 10^{14} Gev: the universe could then emerge from a classical bubble which starts from a very small size and has a mass of of the order of about 10 Kg (by using quantum tunnelling the mass of the bubble could be arbitrarily small, but the probability of production of a new universe out of it would be reduced). The density of the universe would, admittedly, have been quite higher than what we could realize with present technologies, but the orders of magnitude of the other parameters make not unreasonable to ask the question: might we have the possibility of building a child universe in the laboratory?

As a matter of fact, a positive answer to this question was already envisaged some years ago (for a popular level discussion see [28]). In particular Farhi *et al.* suggested an interesting model able to describe universe creation starting from a non-singular configuration and involving semiclassical effects. This proposal, actually, leaves some open issues, for instance about the semiclassical part of the process and the global (Euclidean) structure of the solution. Although since then, a few more proposal have appeared, addressing in more detail qualitative issues, it is interesting to observe that most of the problems which emerged in the earliest formulation are, somehow, still open. It is our hope that the present review of the different approaches which have been developed along this interesting research line, will stimulate to study in more detail and with systematic rigor these problems as well as other realistic answers to the above question. In our opinion, this question is not a purely academic one, and might help not only to change our perspective (passing from an observational to an experimental one) in addressing cosmological problems, but also to shed some light on the importance of the interplay of gravitational and quantum phenomena.

We would also like to remark how, this complementary perspective can be considered much more promising nowadays than some years ago, thanks to the results of recent observations. These observations are helping us in focusing our field of view back in time, closer and closer to the earliest stages of life of our universe and are providing us with a large amount of data and information that will, hopefully, help us in sharpen our theoretical models. This has already allowed tighter constrains on the parameters of models of the early universe, giving us the chance for a more decisive attack of most of the still open problems. This will be a great help also for a "child universe formation in the laboratory" program; it can make easier to identify the fundamental elements (building blocks) required to model the *creation* of a universe that will evolve in something similar to the present one. At the same time, it will help us to narrow our selection of the fundamental principles that forged the earliest evolution of the universe, and, as we said already before, strengthen our hope to enlighten a crucial one, which is the interplay between General Relativity and Quantum Theory.

This said, in the rest of this section, keeping in mind the above preliminary discussion, we are going to give a concise review of the state of the art in the field and to to make a closer contact with some of the models for child universe formation; in particular we are going to review some of the existing works on the dynamics of vacuum bubbles and on topological inflation (both also considered in a semiclassical framework).

Callan and Coleman initiated the study of vacuum decay more than 30 years ago [10,9]; after their seminal papers the interest in the subject rapidly increased. The possible interplay of true vacuum bubbles with gravitation was then considered [11,25]. More or less at the same time and as opposed with the true vacuum bubbles of Coleman *et al.*, false vacuum bubbles were also considered. The classical behavior of regions of false vacuum coupled to gravity was studied by Sato et *al.* [33,23,31,27,32,22] and followed by the works of Blau *et al.* [7] and Berezin *et al.* [5,6]. The analysis in [7] clarified some aspects in the study of false vacuum dynamics coupled to gravity; in it, for the first time, the problem was formulated using geodesically complete coordinate systems: this made more clear the issue of wormhole formation, with all its rich sequel of stimulating properties and consequences.

The presence of wormholes makes possible a feature of false vacuum bubbles that is otherwise counterintuitive, which is that these objects can undergo an exponential inflation without displacing the space outside of the bubble itself; this could seem strange at a first look and is due to the fact that they have an energy density which is higher than that of the surrounding spacetime and which is responsible for keeping the required pressure difference. Because of this, *child universe* solutions appear as expanding bubbles of false vacuum which disconnect from the exterior region. Apart from the already mentioned wormhole, they are also characterized by the presence of a *white-hole like* initial singularity; the simplest example can be obtained by modelling the region inside the bubble with a domain of de Sitter spacetime and the region outside the bubble with a domain of Schwarzschild spacetime. These two regions are then joined across the bubble surface, using the well known Israel junction conditions [21,4]; Einstein equations, which hold independently in the two domains separated by the bubble, are also satisfied on the bubble surface if interpreted in a distributional sense; they determine the motion (embedding) of the bubble in the two domains of spacetime. Although there are various simple configurations of this system (as well as more elaborate generalizations) that are appropriate to describe the evolution of a newly formed universe (i.e. they are such that the expanding bubble can become very large), these *classical* models present also some undesirable features. In particular it turns out that only bubbles with masses above some critical value can expand from very small size to infinity. But then these solutions necessarily have a (white-hole) singularity in their past; in fact, for all of them the hypotheses of singularity theorems are satisfied.

In connection with the restriction on the values of the total mass, the situation could be improved in theories containing an appropriate multiplet of additional scalars[19][1]: then all bubbles that start evolving from zero radius can inflate to infinity if the scalars are in a "hedgehog" configuration, or global monopole of big enough strength. This effect also holds in the gauged case for magnetic monopoles with large enough magnetic charge: in this way the mass requirement is traded for requirements about the properties of magnetic monopoles.

A possible connection of this approach with the problem of the initial singularity appears, then, from the work of Borde *et al.* [8]: they proposed a mechanism which, by means of the coalescence of two regular magnetic monopoles (with *below critical* magnetic charge), is able to produce a *supercritical* one, which then inflates giving rise to a child universe. This idea might help addressing the singularity problem and in this context it is very interesting the work of Sakai *et al.* [30]: in it the interaction of a magnetic monopole with a collapsing surrounding membrane is considered; also in this case a new universe can be created and the presence of an initial singularity *in the causal past of the newly formed universe* can be avoided.

To solve the problem of initial singularity, there are also other approaches which make a good use of quantum effects. Needless to say, these ideas are very suggestive because they require a proper interplay of quantum and gravitational physics, for which a consistent general framework is still missing. This is the main reason why most of these investigations try to obtain a simplified description of the system by requiring a high degree of symmetry from the very beginning. In particular, if we describe the bubble separating the inflating spacetime domain from the surrounding spacetime in terms of Israel junction conditions [21,4], under the additional assumption of spherical symmetry, the dynamics of the system is determined by the dynamics of an effective system with only one degree of freedom: this is called the *minisuperspace approximation*; in this framework the problem of the semiclassical quantization of the system, even in the absence of an underlying quantum gravity theory, can be undertaken with less (but still formidable) technical problems using as a direct guideline the semiclassical procedure with which we are familiar in ordinary Quantum Mechanics. This has been the seminal idea of Farhi *et al.* [12] and of Fishler *et al.* [14,13]. One additional difficulty in these approaches was in connection with the stability of the classical initial state. Interestingly enough, this could be solved by the introduction of massless scalars or gauge fields that live on the shell and produce a classical stabilization effect of false vacuum bubbles. By quantum tunnelling, these bubbles can then become child universes [15] and, at least in a $2+1$-dimensional example [18], it has been shown that the tunnelling can be arbitrarily small.

1.2 ... and their future perspectives

From the above discussion, we think it is already clear that there are many interesting aspects in the study of models for child universe creation in the laboratory.

[1] The subject of inflation assisted by topological defects was also studied later in [35] and [26].

We would also like to remember how most of these models are based on a very well-known and studied classical system, usually known as a *general relativistic shell* [21,4]. The classical dynamics of this system is thus "under control", many analytical results can be found in the literature and numerical methods have also been employed (see the introduction of [1] for additional references). On the other hand there has been little progress in the development of the quantized theory, which still remains a non-systematized research field. We stress how a progress in this direction would be decisive for a more detailed analysis of the semiclassical process of universe creation.

Before coming back to the quantum side of the problem, let us first consider what could be done on the classical one. We will concentrate mainly on the works of Borde *et al.* [8] and of Sakai *et al.* [30], which suggest many interesting ideas for further developments. For instance, it is certainly important to extend the analysis in [8], which is mainly qualitative in nature, to take fully into account the highly non-linear details of the collision process by means of which a supercritical monopole is created (this is certainly instrumental for a quantitatively meaningful use of the idea of topological inflation). Also the study performed in [30] should be extended; to obtain some definitive conclusion about the stability of the initial configuration, it is, in fact, necessary to study the spacetime structure of the model for all possible values of the parameters; it could then be possible to determine if stability is a general feature of monopole models or an *accident* of some particular configurations. From the classical point of view, in both the above models another central point is the study of their causal structure; it can be obtained by well-known techniques, but, again, a full classification of all the possibilities that can arise is certainly required to gain support for the proposed mechanisms. Known subtleties which require closer scrutiny (as for example, the presence of singularities in the causal past of the created universe *but* not in the past of the experimenter creating the universe in the laboratory or, sometimes, the presence of timelike *naked* singularities) make a discussion of the problem of initial conditions not only interesting but necessary, especially in this context[2].

A suggestive complement to the classical aspects discussed above, is represented, of course, by the quantum (more precisely semiclassical) ones, where quantum effects are advocated to realize the tunnelling between classical solutions. If (i) the classical solution used to describe the initial state can be formed without an initial singularity and is stable, (ii) the classical solution which represents the final state can describe an inflating universe and (iii) we can master properly the tunnelling process, then we could use the quantum creation of an inflating universe *via* quantum tunnelling to evade the consequences of singularity theorems. The construction of proper initial and final states has already been successfully accomplished. The stability of the initial classical configuration has been, instead, only partly analyzed [15] and it would be certainly interesting to consider the tunnelling process in more general situations, where, for example, the stabilization can be still classical in origin. Although there is some evidence [30] of a general way to solve this issue in the context of monopole configurations,

[2] The proper analysis of the Cauchy problem will, in fact, involve resolution or proper handling of these singularities.

as we mentioned above, the analysis should be extended to the whole of the parameter space. At the same time a complementary possibility is that *semiclassical* effects might stabilize the initial configuration. In particular, closely related to the problem of instabilities present in many models, is the fact that the spacetime surrounding the vacuum bubble has itself an instability due to presence of a white hole region (see, for instance, [34]). Also in this context quantum effects might stabilize the system and help solving the issue. This approach could require the determination of the stationary states of the system in the WKB approximation, a problem for which a generalization of the procedure presented in [1] (where this analysis was performed for the first time in a simplified model) could be useful.

Another equally (if not more) important point for future investigations is certainly related with the still open issues in the semiclassical tunnelling procedure. We will shortly discuss this by following, for definiteness, the clear, but non-conclusive, analysis developed by Farhi *et al.* [12]: it is shown in their paper that, when considering the tunnelling process, it is not possible to devise a clear procedure to build the manifold interpolating between the initial and final classical configurations; this manifold would describe the instanton that is assumed to mediate the process. According to the discussion of Farhi *et al.* it seems possible to build only what they call a *pseudo-manifold*, i.e. a manifold in which various points have multiple covering. To make sense of this, they are forced to introduce a 'covering space' different from the standard spacetime manifold, in which they allow for a change of sign of the volume of integration required for the calculation of the tunnelling action and thus of tunnelling probabilities. It would be important to put on a more solid basis this interesting proposal, comparing it with other approaches which might help to give a more precise definition of this *pseudo-manifold*. In particular we would like to mention two possibilities. A first one uses the *two measures theory* [16]; considering four scalar fields it is possible to define an integration measure in the action from the determinant of the mapping between these scalar fields and the four spacetime coordinates; there can, of course, be configurations where this mapping is not of maximal rank and if we then interpret the scalar fields as coordinates in the *pseudo-manifold* of [12], then the non-Riemannian volume element of the two measures theory would be related to the non-Riemannian structure that could be associated to the *pseudo-manifold*. In this perspective, non-Riemannian volume elements could be essential to make sense of the quantum creation of a universe in the laboratory and it could be important to develop the theory of shell dynamics in the framework described by the two measures theory.

A second one, likely complementary, can come from a closer study of the Hamiltonian dynamics of the system. Let us preliminarily remember that the Hamiltonian for a general relativistic shell, which we are using as a model for the universe creation process, is a non quadratic function of the momentum (this comes from the non-linearities intrinsic to General Relativity); this makes the quantization procedure non-standard and quite subtle too. Moreover, although it is possible to determine an expression for the Euclidean momentum and use it to reproduce [2] standard results for the decay of vacuum bubbles (as for instance the results of Coleman *et al.* [11]) this momentum can have unusual properties along the tunnelling trajectory; some of these inconsistencies disappear if we consider the momentum as a function valued on the circle instead than on the real line [3] but further investigations in this direction are required; they will likely help us to obtain a better understanding of the semiclassical tunnelling creation of this general relativistic system and, perhaps, show us some interesting properties of the interplay between the quantum and the gravitational realms. In this context, it should be also explored how the Euclidean baby universes [29] could be matched continuously to the real time universes and in this way provide new ways to achieve spontaneous creation of real time baby universes

To complement the above discussion, we would now like to provide some additional contact points between theoretical ideas and experimental evidence. We start considering if all *creation efforts* might end in a child universe totally disconnected from its *creator* or not. Of course, there is not a definitive answer also to this problem yet, since this is tightly bound to the child universe creation model. Nevertheless, it it is certainly stimulating to address the question if, in some way, the new universe might be detectable. There is an indication in this direction from the analysis performed in [24]: here a junction with a Vaidya radiating metric is employed, so that the child universe could be detectable because of modifications to the Hawking radiation. Generalizations that apply to solitonic inspired universe creation[3] can be important, especially from the point of view of a quantum-gravitational scenario in which the exact and definite character of classical causal relations might be *waved* by quantum effects.

Other issues that could be tackled after having a more detailed model of child universe creation, are certainly phenomenological ones. They would also help to better understand the differences between purely classical and partly quantum processes, which is also a motivation to consider them explicitly and separately. Also the physical consequences of different values of the initial parameters characterizing the child universe formation process (initial conditions) should be analyzed[4] and in this context we would also like to recall the idea of Zee *et. al* [20], i.e. that a creator of a universe could pass a message to the future inhabitants of the created universe. From our point of view this is can be a suggestive way to represent the problem of both initial conditions and causal structure; this could be of relevance also for the problem of defining probabilities in the context of the multiverse theory and of eternal inflation.

A final point of phenomenological relevance would be in connection with observations that suggest the universe as super-accelerating. This seems to support the idea that some very unusual physics could be governing the universe, in the sense that standard energy conditions might not be satisfied. In the context of child universes creation in the laboratory in the absence of an initial singularity, it might very well be that a generalized behavior of the universe to try to raise its vacuum energy would manifest itself locally with the creation of bubbles of false vacuum (as seen by the surrounding spacetime), which would then led to

[3] It is, for instance, certainly possible to extend the metric describing the monopole, i.e. the Reißner-Nordström spacetime, to the Reißner-Nordström-Vaidya case.

[4] In particular different ways of creating a universe in the laboratory could lead to different coupling constants, gauge groups, etc..

child universes. In [17] a proposal, based on the two measures theory, to avoid initial singularities in a homogeneous cosmology has already been put forward. It would then be desirable to apply it to the non-singular child universe creation also.

To conclude we cannot miss to point out how all the above discussion about the possibility of producing child universes in the laboratory could take a completely new and concrete perspective in connection with the possible existence of new physics at the TeV scale in theories with large compact extra-dimensions, physics that might become available to our experimental testing at the colliders which will shortly start to operate.

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2 Relation between Finestructure Constants at the Planck Scale from Multiple Point Principle ⋆

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Abstract. We derive a relation between the three finestructure constants in the Standard Model from the assumptions of what we call "multiple point principle" (MPP) and "AntiGUT". By the first assumption we mean that we require coupling constants and mass parameters to be adjusted - by our multiple point principle - to be just so as to make several vacua have the same cosmological constants (from our point of view, basically zero). By AntiGUT we refer to our assumption of a more fundamental precursor to the usual Standard Model Group (SMG) consisting of the N_{gen}-fold cartesian product of the usual SMG such that each of the three families of quarks and leptons has its own set of gauge fields. The usual SMG comes about when SMG^3 breaks down to the diagonal subgroup at roughly a factor 10 below the Planck scale. Up to this scale we assume the absence of new physics. Relative to earlier work where the multiple point principle was used to get predictions for the gauge couplings independently of one another, the point here is to increase accuracy by considering a relation between all the gauge couplings (i.e., for U(1), and SU(N) with N=2 or 3)as a function of a N-dependent parameter d_N that is a characteristic of SU(N) groups. In doing this, the parameter d_N that initially only takes discrete values corresponding to the "N in SU(N) is promoted to being a continuous variable (corresponding to fantasy groups for $N \notin \mathbf{Z}$). By an approprite extrapolation in the variable d_N to a fantasy group for which the β-function for the magnetic coupling $\tilde{g}^2$ vanishes we avoid the problem of our ignorance of the ratio of the monopole mass scale to the fundamental scale. In addition to increasing the accuracy of our predictions for the gauge couplings by circumventing the uncertainty in our knowledge of this ratio, we interpret our results as being very supportive of the multiple point principle and AntiGut.

2.1 Introduction

In earlier work [1] we invented our Multiple Point Principle / AntiGUT (MPP / AntiGUT) gauge group model for the purpose of predicting the Planck scale values of the three Standard Model Group gauge couplings. These predictions were made independently for the three gauge couplings. In this work we test an

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alternative method of treatment of MPP/AntiGUT in which we seek a relation that would put a rather severe constraint on the values of the SMG couplings.

An important ingredient for the calculational technique in this paper is the Higgs monopole model description in which magnetic monopoles are thought of as particles described by a scalar field ϕ with an effective potential V_{eff} of the Weinberg-Coleman type [2],[3]. The MPP is implemented by requiring that the two minima of V_{eff} are degenerate. This requirement results in a relation between the square of the monopole charge $\tilde{g}^2$ and the self-coupling λ that defines a phase transition boundary between a Coulomb-like phase (with $<\phi>=0$) and a phase with a monopole condensate (with $<\phi>\neq 0$). For Abelian monopoles this phase boundary has been presented in earlier work [4],[5]. This phase boundary condition which has a term quadratic in $\tilde{g}^2$ (as well as terms linear and quadratic in λ) consists of tuples $(\lambda, \tilde{g}^2)$ of critical values of λ and $\tilde{g}^2$. A characteristic feature of this boundary is that it has negative curvature as a function of λ and hence a maximum value $\tilde{g}^2_{U(1)\ crit\, max}$ of $\tilde{g}^2$. It is important to keep in mind that this phase transition that we find by applying MPP to V_{eff} is assumed to be at the scale of the monopole mass. In the present work, the V_{eff} is generalized in such a way that it also embodies nonAbelian $SU(N)$ monopoles using the technique of Chan & Tsou in which a $SU(N)$ monopole is described as a $\underline{\mathbf{N}}$ under a dual Yang-Mills vector potential $\tilde{A}_\mu$ [6]. The corresponding phase boundary now has coefficients that depend on N (i.e., "N" as in $SU(N)$). But otherwise the phase boundary for a $SU(N)$ monopole is qualitatively the same as that for the Abelian monopole the important difference being that the maximum value of $\tilde{g}^2$ in the Abelian theory is different than in the nonAbelian $SU(N)$ theory: $\tilde{g}^2_{U(1)\ crit\ max} \neq \tilde{g}^2_{SU(N)\ crit\ max}$.

In fact a major trick in the present article is to consider a correspondance between Abelian and non-Abelian monopoles (the latter in the understanding of Chan-Tsou to be explained in section 2 below). This correspondence is defined using a N-dependent parameter $C = C(N)$ defined by

$$C \hat{=} \frac{\tilde{g}^2_{SU(N)\ crit\ max}}{\tilde{g}^2_{U(1)\ crit\ max}}. \tag{2.1}$$

We can think of this definition of parameter C as a definition of the Abelian theory with $\tilde{g}^2_{U(1)\ crit\ max}$ that corresponds to the nonAbelian $SU(N)$ theory with $\tilde{g}^2_{SU(N)\ crit\ max}$.

As shall be seen soon we need an unambiguous way to define the Abelian magnetic charge $\tilde{g}^2_{U(1)\ corresp.\ SU(N)}$ corresponding to *any* nonAbelian $SU(N)$ magnetic charge $\tilde{g}^2_{SU(N)}$. Our definition of such a correspondence is simply:

$$C \hat{=} \frac{\tilde{g}^2_{SU(N)}}{\tilde{g}^2_{U(1)\ corresp\ SU(N)}}, \tag{2.2}$$

where $\tilde{g}^2_{SU(N)}$ can have any value (not necessarily critical or critical maximum). We can thereby think of say the fundamental scale nonAbelian dual - i.e. monopole - coupling also as an Abelian one.

We now go to the Planck scale where the MPP/AntiGUT model was originally invented as a simple way of relating the experimental values of the SMG gauge couplings $g_{U(1)}$, $g_{SU(2)}$and $g_{SU(3)}$ (extrapolated to the Planck scale in the absence of new physics underway) to the critical values of these three coupling as determined using lattice gauge theory. This MPP/AntiGUT relation in terms of the (critical values of the) gauge couplings g_i $(i \in \{U(1), SU(2), SU(3)\})$ is in this work reformulated in terms of the dual (critical values of the) of the magnetic charges using the Dirac relations $g\tilde{g} = 2\pi$ and $g\tilde{g} = 4\pi$ for respectively Abelian and nonAbelian monopole theories. Recall that we already have a convention for calculating the (squared) Abelian magnetic charge that corresponds to a (squared) nonAbelian magnetic charge $\tilde{g}^2_{SU(N)}$ using the parameter C:

$$\tilde{g}^2_{U(1)\ \mathrm{corresp.}\ SU(N)} = \frac{\tilde{g}^2_{SU(N)}}{C}. \tag{2.3}$$

Now assuming for the moment that our MMP/AntiGUT model is in fact a law of Nature it would not be unreasonable to discuss whether the Abelian correspondent couplings $\tilde{g}^2_{U(1)\ \mathrm{corresp.}SU(2)}$ coupling is smooth or not as a function of a gauge group characteristica such as N for SU(N) groups. Also the U(1) gauge group can be taken into consideration using our nonAbelian to Abelian correspondence relation in the special case

$$\tilde{g}^2_{U(1)\ \mathrm{corresp.}U(1)} \equiv \frac{\tilde{g}^2_{U(1)}}{C=1} \tag{2.4}$$

which will be seen below to correspond to $d_N = 0$

Actually for later convenience we shall instead of N use an N-dependent parameter d_N as our independent variable and the quantitiy $\frac{3\tilde{g}^2_{U(1)\ \mathrm{corresp.}SU(2)}}{\pi}$ instead of $\tilde{g}^2_{U(1)\ \mathrm{corresp.}SU(2)}$) as our on d_N analytically dependent variable. We have now three points $(d_N, \frac{3\tilde{g}^2_{U(1)\ \mathrm{corresp.}\ SU(N)}}{\pi})$ belonging to our hypothesized in d_N analytic function namely the points

$$(0, \frac{3\tilde{g}^2_{U(1)}}{\pi}),\quad (d_2, \frac{3\tilde{g}^2_{U(1)\ \mathrm{corresp.}SU(2)}}{\pi}),\quad (d_3, \frac{3\tilde{g}^2_{U(1)\ \mathrm{corresp.}SU(3)}}{\pi}). \tag{2.5}$$

Now we make the guess that the analytic function in d_N that we seek is the parabolic fit obtained using these three points. It must be emphasized that our hypothesized function analytic in d_N lives at the Planck scale while the phase transition boundary discussed above lives at the (unknown) scale of the monopole mass and consists of critical λ and $\tilde{g}^2$ values that both run with scale. So the problem is how to connect hypothesized Planck scale physics in the form of our in d_N analytic function $\frac{3\tilde{g}^2_{U(1)\ \mathrm{corresp.}SU(2)}}{\pi}$ with say critical values of the function $\tilde{g}^2_{U(1)\ \mathrm{corresp.}\ SU(N)\ \mathrm{crit}}$ at the unknown scale of the monopole mass.

There is one value of $\tilde{g}^2$ for which this connection would be trivial namely the special point $(d_{N_{spec}}, \frac{3\tilde{g}^2_{spec}}{\pi})$ lying at the intersection of our in d_N analytically continued function with the function defined by requiring that the β-function for $\tilde{g}^2$ vanishes. I.e., $\beta_{\tilde{g}^2} = 0$. Just this value of $\frac{3\tilde{g}^2_{spec}}{\pi}$ is the same at the Planck scale and at the (unknown) scale of the monopole mass. We describe now briefly how we find the "fantasy" (and nonexistent!) gauge group "'$SU(N_{spec})$"' for which the corresponding $\tilde{g}^2$ does not run with scale. But first a little digression on how $\beta_{\tilde{g}^2}$ depends on d_N

Starting from the definition of the Abelian magnetic charge correspondent to a nonAbelian magnetic charge we use the nonAbelian Dirac relation to obtain

$$\tilde{g}^2_{U(1)\ corresp.SU(N)} \hat{=} \frac{\tilde{g}^2_{SU(N)}}{C} = \frac{1}{C}\frac{16\pi^2}{g^2_{SU(N)}} \tag{2.6}$$

We now require that the Dirac relation remain intact under scaling; i.e.,

$$\frac{d}{dt}\tilde{g}^2_{U(1)\ corresp.SU(N)} = \frac{16\pi^2}{C}\frac{d}{dt}\left(\frac{1}{g^2_{SU(N)}}\right) = \frac{16\pi^2}{C}\frac{1}{4\pi}\frac{d}{dt}(\alpha^{-1}_{SU(N)}). \tag{2.7}$$

Using that $\frac{d}{dt}(\alpha^{-1}_{SU(N)}) = \frac{11N}{12\pi}$ (which is just the usual Yang-Mills contribution to the β-function for $\alpha^{-1}_{SU(N)}$ using t=lnμ^2) we get the Yang-Mills contribution to the running of $\tilde{g}^2_{U(1)\ corresp.\ SU(N)}$:

$$\beta_{\tilde{g}^2_{U(1)\ corresp.\ SU(N)}}|_{Y.-M.\ contrib} = \frac{16\pi^2}{C}\frac{1}{4\pi}\frac{11N}{12\pi} = \frac{11N}{3C}\hat{=}d_N. \tag{2.8}$$

So even though there is of course no Yang-Mills contribution to $\beta_{U(1)}$ we see that $\beta_{U(1)\ corresp.\ SU(N)}$ inherits a dependence on $\beta_{g^2_{SU(N)}}$ through the requirement that the Dirac relation remain intact under scale changes. Using the known β-function for $\tilde{g}^2_{U(1)}$ (to 2-loops):

$$\beta_{\tilde{g}^2_{U(1)}} = \frac{\tilde{g}^4}{48\pi^2} + \frac{\tilde{g}^6}{(16\pi^2)^2} \tag{2.9}$$

which leads to the β-function for $\tilde{g}^2_{U(1)\ corresp,\ SU(N)}$:

$$\beta_{\tilde{g}^2_{U(1)\ corresp,\ SU(N)}} = \beta_{\tilde{g}^2_{U(1)}} + d_N = \frac{\tilde{g}^4}{48\pi^2} + \frac{\tilde{g}^6}{(16\pi)^2} + d_N. \tag{2.10}$$

The intersection point (with no running of $\tilde{g}^2$)is readily found as the intersection of

$$\beta_{\tilde{g}^2_{U(1)\ corresp,\ SU(N)}} = 0 \tag{2.11}$$

and our in d_N analytically extrapolated function $\frac{3\tilde{g}^2_{U(1)\ corresp.\ SU(N)}}{\pi}$. At the intersection point we have

$$(d_N, \frac{3\tilde{g}^2}{\pi}) = (-0.57, 14.54). \tag{2.12}$$

So any intersection point would have a $\tilde{g}^2$ that necessarily has a RG trajectory parallel to the λ axis in the space spanned by $(\lambda, \tilde{g}^2)$ where the phase transition boundary lives. But only one of the horizontal (i.e., parallel to the λ axis) RG trajectories can be tangent to the phase transition boundary and the point of tangency must neccessarily be at the top point of the phase transition curve. The interesting result of this work is that our intersection point (which depends of course on the experimental values of the SMG finestructure constants) to high accuracy is the value of $\tilde{g}^2$ with the RG trajectory that is tangent to the phase boundary at its top point where $\tilde{g}^2 = \tilde{g}^2_{U(1)\ crit\ max} = 15.11$. Had our intersection point singled out any other RG trajectory our MPP would have been falsified (see Figures 2.1 and 2.2).

2.2 The Chan-Tsou duality and monopole critical coupling calculation

Investigating nonAbelian theories, we have used the quantum Yang-Mills theory by Chan-Tsou [6] for a system of fields with chromo-electric charge g and chromo-magnetic charge $\tilde{g}$ (monopoles). This theory describes symmetrically the non-dual and the dual sectors of theory with nonAbelian vector potentials A_μ and $\tilde{A}_\mu$ covariantly interacting with chromo-electric j_μ and chromo-magnetic $\tilde{j}_\mu$ currents respectively. As a result, the Chan-Tsou nonAbelian theory has a doubling of symmetry from $SU(N)$ to

$$SU(N) \times \widetilde{SU(N)}$$

and reveals the generalized dual symmetry which reduces to the well-known electromagnetic (Hodge star) duality in the Abelian case.

We want in principle to consider three phase transitions connected with a single nonAbelian monopole which in the philosophy of the Chan-Tsou-theory to be described below is an $\underline{N}$-plet under the by Chan-Tsou introduced dual Yang Mills four vector field $\tilde{A}_\mu$, namely 1) a confining phase, 2) a Coulomb phase, and 3) a phase with monopole condensate. According to the Multiple Point Principle the coupling constants and mass parameters should then be adjusted in Nature to just make these phase degenerate (i.e. same cosmological constant). Earlier we have used two loop calculations to obtain the Abelian gauge group a phase transition between the monopole-condensate phase and the Coulomb phase using the Coleman-Weinberg effective potential technique, which led to a relation between the self coupling λ for the monopole Higgs field and the monopole charge $\tilde{g}$. To determine the monopole mass we should, however, in principle involve one more phase, i.e. the monopole confining one, but that would need a string description in the language used for the two other phases and doing that sufficiently accurately for the fit towards which we aim in this article is not undertaken here. Hence we shall assume only that the ratio of the mass scale of the monopole condensate or approximately equivalently the monopole mass to fundamental scale, taken here to be the Planck scale, is an analytical function of some group characteristic, which we shall take to be the quantity d_N that we shall return to shortly. The basic point is that we derive by the Coleman-Weinberg effective potential an a priori scale independent phase transition curve in as far as the monopole mass drops out of the relation describing the phase border between the Coulomb phase and the monopole condensate one, so that the only scale dependence of this relation comes in via the renormalization group. The lack of a good technology for calculating the mass scale of the monopole therefore means that we have troubles in calculating the renormalization group correction of the by the Coleman-Weinberg-technique calculated relation between λ and $\tilde{g}^2$ to run it from the monopole mass scale to the fundamental scale. The major idea of the present article now is that this would be no problem if the beta-function for the monopole coupling $\tilde{g}$ had been zero. The trick now is to effectively achieve that zero beta function by extrapolating in the gauge group so to speak to a "fantasy" group having zero beta-function.

A priori magnetic monopole couplings for different gauge groups cannot be compared, and so to make the statement that the phase transition coupling is analytic as a function of some group characteristic, call it d_N say, is a priori not meaningfull. This is so because in principle one could vary notation from group to group, and such a choice of notation would not a priori be analytical. We shall primarily be interested in the phase transitions from a Coulomb phase to the monopole condensed phase as obtained from studying the effective potential as a function of the norm of the vacuum expectation value of the monopole scalar field in the manner of Coleman-Weinberg. We decide to take the a priori arbitrary ratio between the ratio of a gauge coupling for an $SU(N)$ gauge group and the coupling for the corresponding Abelian $U(1)$ theory to be the same as that for the critical values of these couplings. We take the Lagrangian densities for a $U(1)$ theory and an $SU(N)$ Higgs Yang Mills theory respectively as

$$\mathcal{L} = -\frac{1}{4\tilde{g}^2}\tilde{F}^2_{\mu\nu} + |\tilde{D}_\mu \phi|^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}|\phi|^4 \tag{2.13}$$

and

$$\mathcal{L} = -\frac{1}{4\tilde{g}^2}\tilde{F}^{j2}_{\mu\nu} + |\tilde{D}_\mu \phi^a|^2 - \frac{1}{2}\mu^2|\phi^a|^2 - \frac{\lambda}{4}(|\phi^a|^2)^2 \tag{2.14}$$

where ϕ^a is a monopole $\underline{N}$-plet, $\tilde{D}_\mu$ is the covariant derivative for dual gauge field $\tilde{A}_\mu$ and $\tilde{g}$ is magnetic charge, such that the meaning of the mass μ and the self coupling λ becomes related in as far as we decide to identify as having corresponding meaning of the length squares of the fields; i.e. we identify

$$|\phi|^2 = \sum_{a=1}^{N} |\phi^a|^2 \tag{2.15}$$

as is natural, since from the derivative part in the kinetic term respectively $\frac{1}{2}|\partial_\mu\phi|^2$ and $\frac{1}{2}|\partial_\mu\phi^a|^2$ we can claim that a given size of $|\phi|^2$ and $|\phi^a|^2$ corresponds to a given density of Higgs particles, a number of particles per unit volume being the same in both theories. Accepting (2.15) as a physically meaningfull identification we can also claim that the λ and the μ in (2.13) and (2.14) are naturally identified i an N-independent way (i.e., N as in SU(N)).

Denoting (2.15) by just $|\phi|^2$ one can write - as is seen by a significant amount of calculation or by using Coleman-Weinberg [2] and Sher [3] - the one-loop effective potential for $U(1)$ and $SU(N)$ gauge groups as

$$V_{eff} = -\frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4 + \frac{|\phi|^4}{64\pi^2}$$

$$[3B\tilde{g}^4 \ln\frac{|\phi|^2}{M^2} + (-\mu^2 + 3\lambda|\phi|^2)^2 \ln\frac{-\mu^2 + 3\lambda|\phi|^2}{M^2}$$

$$+ A(-\mu^2 + \lambda|\phi|^2)^2 \ln\frac{-\mu^2 + \lambda|\phi|^2}{M^2}], \tag{2.16}$$

where

$$A = B = 1 \qquad \text{for Abelian case,} \tag{2.17}$$

and

$$A = 2N - 1, \tag{2.18}$$

$$B = \frac{(N-1)(N^2 + 2N - 2)}{8N^2} \qquad \text{for SU(N) gauge group.} \tag{2.19}$$

The $SU(N)$ formula we used here were derived using an N-plet monopole and with a convention for the covariant derivative

$$\tilde{D}_\mu = \partial_\mu - i\tilde{A}^j_\mu \lambda^j/2 \tag{2.20}$$

in the convention with absorbed coupling where the generators $\lambda^j/2$ were normalized to

$$Tr(\frac{\lambda^j}{2}\frac{\lambda^k}{2}) = \frac{1}{2}\delta_{jk}, \tag{2.21}$$

while for the Abelian theory we used the convention

$$\tilde{D}_\mu = \partial_\mu - i\tilde{A}_\mu. \tag{2.22}$$

We have stable or meta-stable vacua when we have minima in the effective potential (2.16) which of course then means that the derivatives of it are zero there:

$$\frac{\partial V_{eff}(|\phi|^2)}{\partial|\phi|^2}|_{min\ i} = 0 \tag{2.23}$$

where i enumerates the various minima.

Now our multiple point principle asserts that there should be as many degenerate vacua as possible - i.e., the more degenerate vacua the more intensive parameters that become finetuned by the requirement of being at the multiple point (in parameter space). In the case we consider here there are just two degenerate vacua at say $\phi = \phi_{min1}$ and $\phi = \phi_{min2}$. This means that if we take the degenerate minima to have zero energy density (cosmological constant) that

$$V_{eff}(|\phi|^2_{min1}) = V_{eff}(|\phi|^2_{min2}) = 0. \tag{2.24}$$

The joint solution of equations (2.24) and (2.23) for the effective potential (2.16) gives the phase transition border curve between a Coulomb phase and monopole condesced phase:

$$3B\tilde{g}^4_{p.t.} + (5 + A)\lambda^2_{p.t.} + 16\pi^2\lambda_{p.t.} = 0. \tag{2.25}$$

All of the combinations $(\lambda, \tilde{g}^2)$ satisfying (2.25) are critical in the sense of separating phases. The maximum value of $\tilde{g}^2_{U(1)\ crit}$ - we have called it $\tilde{g}^2_{U(1)\ crit\ max}$ turns out to be interesting for us. Let us now find this top point of the phase boundary curve (2.25) (see also [4] and [5]).

$$\frac{d\tilde{g}^4}{d\lambda}|_{crit} = 0, \tag{2.26}$$

which gives

$$\tilde{g}^4_{crit} = \frac{(16\pi^2)^2}{12(5+A)B}, \tag{2.27}$$

and

$$\lambda_{crit} = -\frac{16\pi^2}{2(5+A)}. \tag{2.28}$$

From Eq. (2.27) we obtain:

for U(1) group:

A=1, B=1,

$$\tilde{g}^2_{crit,U(1)} = \frac{8\pi^2}{3\sqrt{2}} \approx 18.61, \tag{2.29}$$

for N=2:

A=3, $B = \frac{3}{16}$,

$$\tilde{g}^2_{crit,SU(2)} = \frac{8\pi^2}{3\sqrt{2}} \approx 37.22, \tag{2.30}$$

for N=3:

A=5, $B = \frac{13}{36}$,

$$\tilde{g}^2_{crit,SU(3)} = \sqrt{\frac{108}{65}} \cdot \frac{8\pi^2}{3\sqrt{2}} \approx 11.99. \tag{2.31}$$

In general we define the parameter C such that

$$\tilde{g}^2_{crit,SU(N)} = C\tilde{g}^2_{crit,U(1)}, \quad \text{where} \quad C = \sqrt{\frac{6}{(5+A)B}}. \tag{2.32}$$

We shall assume that this relationship between the Abelian and nonAbelian couplings is also valid when the couplings are not critical. These results are given at the scale of monopole mass or VEV.

2.3 Compilation and correction of finestructure constants in AntiGut model

Recall that MPP is to be applied to the N_{gen}-fold replication of SMG. For $N_{gen} = 3$ we have

$$(SMG)^3 = U(1)^3 \times SU(2)^3 \times SU(3)^3 \tag{2.33}$$

that breaks down to the diagonal subgroup at roughly the Planck scale. It is the couplings for the diagonal subgroup that are predicted to coincide with experimental gauge group couplings at the Planck scale [1]:

$$\begin{aligned}\alpha^{-1}_{1,exp} &= 6\alpha^{-1}_{1,crit},\\ \alpha^{-1}_{2,exp} &= 3\alpha^{-1}_{2,crit},\\ \alpha^{-1}_{3,exp} &= 3\alpha^{-1}_{3,crit}.\end{aligned} \tag{2.34}$$

According to the Particle Data Group results [7], we have:

$$\alpha^{-1}_{1,exp}(\mu_{Pl}) \approx 55.4; \qquad \alpha^{-1}_{2,exp}(\mu_{Pl}) \approx 49.0_3; \qquad \alpha^{-1}_{3,exp}(\mu_{Pl}) \approx 53.00. \tag{2.35}$$

In the Abelian theory the Dirac relation is

$$g\tilde{g} = 2\pi n \text{ where } n \in \mathbf{Z} \tag{2.36}$$

which leads to

$$\alpha\tilde{\alpha} = \frac{1}{4} \tag{2.37}$$

or

$$\alpha^{-1} = 4\tilde{\alpha} = \frac{\tilde{g}^2}{\pi}. \tag{2.38}$$

In the nonAbelian case the Dirac relation is

$$g\tilde{g} = 4\pi n \text{ where } n \in \mathbf{Z}. \tag{2.39}$$

This leads to

$$\alpha\tilde{\alpha} = 1 \tag{2.40}$$

or

$$\alpha^{-1} = \tilde{\alpha} = \frac{\tilde{g}^2}{4\pi} \tag{2.41}$$

From Eqs. (2.34) and (2.35) we have at the Planck scale:

$$\begin{aligned}\frac{3\tilde{g}^2_{U(1)}}{\pi} &\approx 27.7,\\ \frac{3\tilde{g}^2_{SU(2)/Z_2}}{\pi} &\approx 196,\\ \frac{3\tilde{g}^2_{SU(3)/Z_3}}{\pi} &\approx 212.0.\end{aligned} \tag{2.42}$$

But the correction of the running of finesructure constants from AntiGUT group (2.33) in the interval $\Delta t = \sqrt{40}$ gives:

$$\alpha^{-1}_{2,exp}(\mu_{Pl}) \approx 53.3; \qquad \alpha^{-1}_{3,exp}(\mu_{Pl}) \approx 59.4. \tag{2.43}$$

The corresponding Abelian values of $\frac{3\tilde{g}^2}{\pi}$ are:

for U(1):

$$\frac{3\tilde{g}^2_{U(1)}}{\pi} \approx 27.7, \tag{2.44}$$

for N=2:

$$\frac{3\tilde{g}^2_{U(1)}}{\pi} \approx 106.6, \tag{2.45}$$

for N=3:

$$\frac{3\tilde{g}^2_{U(1)}}{\pi} \approx 59.4 \cdot \sqrt{\frac{65}{27}} \approx 184.32. \tag{2.46}$$

2.4 The d_N-parameter.

As we suggested in the introduction we shall consider a correspondance between the nonAbelian and Abelian Chan Tsou monopoles and in this connection have a correspondance of couplings so that the ratio of corresponding monopole couplings is like that of the critical couplings for the same two theories; i.e., as in equation (2.32). Since we want to avoid having to try to use our bad knowledge of the ratio of the monopole mass scale at which the phase transition couplings are rather easily estimated and the Planck or fundamental scale, we are interested in beta-functions. So the most important feature of the gauge group for the purpose here is how the monopole coupling will run as a function of the scale. For this purpose we use $\beta_{\tilde{g}^2}$. Now if we consider - as is the simplest - an Abelian monopole we strictly speaking would have no group dependence of the beta function $\beta_{\tilde{g}^2}$, but now we imagine there there actually *is* an effect of the selfcouplings of the Yang Mills fields in the "electric" sector and include that. We shall do that by the assumption that there will be a term corresponding to it in such a way as to make the Dirac relation (or its replacement for a non Abelian monopole) (2.37) be valid for the running couplings at all scales. In (2.32) we have the relation between the Chan-Tsou $\widetilde{SU(N)}$ coupling $\tilde{g}^2_{SU(N)}$ for the critical coupling, which we here by definition of the relation between corresponding theories extend also to non-critical couplings:

$$\tilde{g}^2_{SU(N)} = C\tilde{g}^2_{U(1)\ \ corresp\ \ SU(N)}, \quad \text{where} \quad C = \sqrt{\frac{6}{(5+A)B}}. \tag{2.47}$$

In the Chan-Tsou formalism the replacement for the Dirac relation is that

$$\tilde{g}g = 4\pi n, \quad n \in \mathbf{Z}. \tag{2.48}$$

Combining (2.47) and (2.48) and taking n to be unity we get for the Abelian magnetic charge corresponding to that of a nonAbelian $SU(N)$

$$\tilde{g}^2_{U(1)\ \mathrm{corr}\ SU(N)} = \frac{(16\pi^2)}{Cg^2_{SU(N)}}. \tag{2.49}$$

With the postulate - but that is really true - that the running of the couplings shall be consistent with the Dirac relation we can take the scale dependence of this equation (2.49) on both sides to obtain

$$\beta_{\tilde{g}^2_{U(1)\ \mathrm{corresp}\ SU(N)}} = -(16\pi^2/C)\cdot\beta_{g^2_{SU(N)}}/g^4_{SU(N)}. \tag{2.50}$$

Now there is the group dependent contribution to the well known beta function

$$\beta_{g^2_{SU(N)}}|_{Y.M.contribution} = -g^4_{SU(N)}\cdot\frac{11N}{48\pi^2} \tag{2.51}$$

which to keep the Dirac relation valid at all scales must be tranfered to also exist in the beta function for the square of the monopole charge

$$\beta_{\tilde{g}^2}|_{Y.M.contribution} = (16\pi^2/C)\cdot\frac{11N}{48\pi^2} = \frac{11N}{3C} \triangleq d_N \tag{2.52}$$

2.5 Our monopole coupling versus d curve and successful agreement

Let us make it quite clear what we mean by our *intersection point* and our so called *fourth point*.

Our intersection point is the point at which two functions that live in the space spanned by the variables $(d_N, \tilde{g}^2_{U(1)})$ intersect one another. One of these functions is our in d_N extrapolated curve of "experimental" values of function $\tilde{g}^2_{U(1)\ \mathrm{corresp}\ SU(N)}$. The other function consists of values $(d_N, \tilde{g}^2_{U(1)\ \mathrm{corresp}\ SU(N)})$ that satisfy the condition

$$\beta_{\tilde{g}^2_{U(1)\,\mathrm{corresp},\,SU(N)}} = \beta_{\tilde{g}^2_{U(1)}} + d_N = \frac{\tilde{g}^4}{48\pi^2} + \frac{\tilde{g}^6}{(16\pi)^2} + d_N = 0. \tag{2.53}$$

The value of $\tilde{g}^2_{U(1)\ \mathrm{corresp}\ SU(N)}$ at the intersection point remains the same under scale changes of course since its β-function vanishes.

Our fourth point - we could denote it as $(d_{N_{4th}}, \tilde{g}^2_{U(1)\ \mathrm{crit}\ \max})$ -lives in the space of variables $(d_N, \tilde{g}^2)$. By definition the second coordinate is the maximum value value of $\tilde{g}^2_{U(1)\ \mathrm{crit}}$ on the phase transition bounday which lives in the space of the variables $(\lambda, \tilde{g}^2_{U(1)})$. We have above denoted this maximum alias top point by the symbol $\tilde{g}^2_{U(1)\ \mathrm{crit}\ \max}$. The first coordinate of the fourth point - i.e., $d_{N_{4th}}$ - is the value of d_N obtained when $\tilde{g}^2_{U(1)\ \mathrm{crit}\ \max}$ is substituted into Eqn (2.53) above. We get for the fourth point

$$(d_N, \tilde{g}^2_{U(1)\ \mathrm{crit}\ \max}) = (-0.62, 15.11). \tag{2.54}$$

The success that we have in this paper is that the intersection point coincides with the fourth point to very high accuracy.

Maybe it's instructive to think of choosing a bunch of $\tilde{g}^2_{U(1)}$ values satisfying Eqn (2.53) (corresponding of course to a bunch of different d_N values). With this bunch of $\tilde{g}^2_{U(1)}$ values we know how to RG run them back and forth between Planck scale and monopole mass scale - also in the space spanned by $(\lambda, \tilde{g}^2)$ where the phase transition curve is located with its top point

$$(\lambda_{crit}, \tilde{g}^2_{U(1)\,crit\;max}) = (-7.13, 15.11).$$

This bunch of $\tilde{g}^2_{U(1)}$ values don't RG run at all by definition so they must be parallel to the λ axis in the space in which the phase transition boundary lives. What happens to these parallel to λ RG tracks of the bunch of $\tilde{g}^2_{U(1)}$ values depends only on the height (i.e., $\tilde{g}^2_{U(1)\;\;crit\;\;max}$) and not on the details (including the unknown scale) of the phase transition curve. Our intersection point follows the one horizontal RG track that can become a tangent to the phase transition boundary and the point of tangency is necessarily the top point. RG tracks below the one corresponding to our (fortuitous for MPP) intersection point would hit the phase boundary below the top point and therefore corresspond to having only a monopole condensate phase. And being within the condensate phase and removed from the phase transition boundary so that the Coulomb phase is energetically inaccessible would violate MPP. The (horizontal) RG tracks of $\tilde{g}^2_{U(1)}$ values above the top point would miss hitting the phase transition boundary and hence correspond to being in the Coulomb phase more or less energetically prohibited from being in the monopole condensate phase depending on how far above the top point, that the RG track is. This is also in violation of MPP. The only RG trajectory allowed by MPP is the one that goes through the fourth point. And our intersection point singles out just this RG trajectory.

Actually, MPP is put to a very stringent test here. Had our intersection point picked out any other RG trajectory then the one that hits the fourth point our MPP would have been falsified.

2.6 Conclusion and outlook

In the figures we see the plot of the values of $3\tilde{g}^2/\pi$ – for the "corresponding " $U(1)$ monopole coupling – versus our group characterising quantity d_N. The extrapolation to the point where it crosses the curve of $(d_N, 3\tilde{g}^2/\pi)$ combinations for which the β-function for $\tilde{g}$ is zero (the crossing point is drawn both for the one loop and two loop calculation and lies of course exactly on the extrapolated curve but the intersecting curve $\beta_{\tilde{g}^2} = 0$ is not drawn) it is remarkably close to being critical in the sense that its ordinate is very close to the critical value $3\tilde{g}^2/\pi = 3 \cdot 15.11/\pi = 14.46$ (the two loop critical value for the case of zero beta function of $\tilde{g}^2$ is 15.11 corresponding to the top point of the phase transition curve). We have drawn on the plot the point with this ordinate calculated using the relation $\beta_{\tilde{g}^2} = 0$.

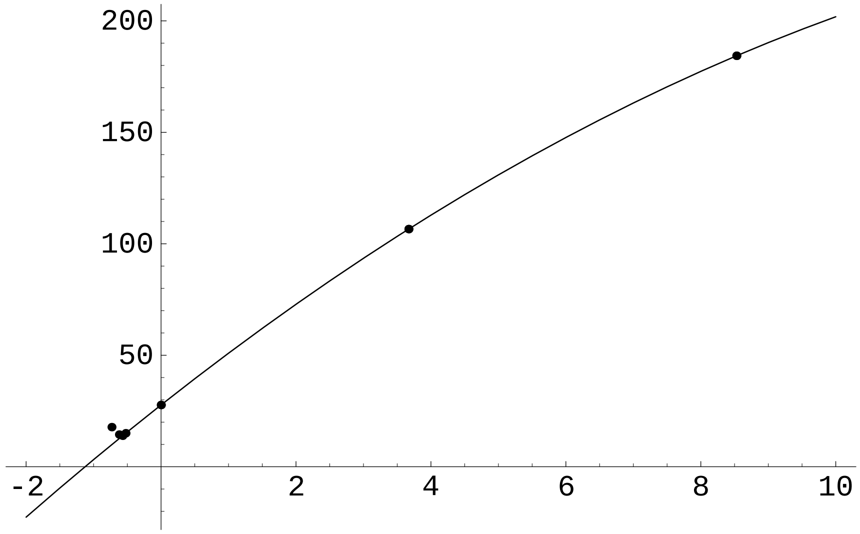

Fig. 2.1. Magnetic coupling $\frac{3\tilde{g}^2}{\pi}$ (ordinate) as a function of d_N (abcissa). The curve is determined to be the parabola that passes through the values of $\frac{3\tilde{g}^2}{\pi}$ corresponding to at "experimental values of gauge couplings (at positive d_N-values). The points at negative value of d_N lying off the parabolic fit (solid line) correspond to maximum values of $3\tilde{g}^2/\pi$ on phase transition curves calculated to one and two loops (see closeup in Fig. 2.2).

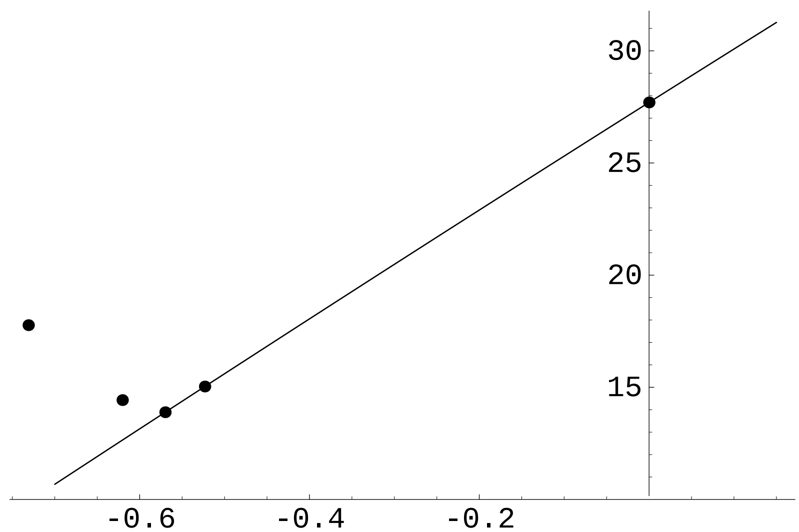

Fig. 2.2. Closeup of $\frac{3\tilde{g}^2}{\pi}$ vs. d_N for slightly negative values of d_N. The two points (at negative d) lying on the parabolic fit (solid line) correspond to to the intersection of the parabolic fit with the curves $\beta_{g^2} = 0$ calculated for one- and two loops the latter corresponding to the point at $d_N \approx -0.57$. The two points (also at negative values of d_N) not on the parabolic fit correspond to the maximum values of $3\tilde{g}^2/\pi$ on the phase transition curves calculated to one and two loop. The latter corresponds to the point at $d_N =\approx -0.62$. The points for highest order (i.e., at $d_N \approx -0.57$ on curve and at $d_N \approx -0.62$ off the curve) are seen to converge towards one another relative to the remaining two points calculated to one loop.

It is this coincidence which is either accidental or because our theory of Anti-GUT conbined with MPP is working so well that it only deviates in say the inverse $U(1)$ fine structure constant at the Planck scale by about 2·1.5 = 3 units.

If the Planck scale had not been taken to be $\mu_{Pl} = 1.2 \cdot 10^{19}$ GeV, as was the case for the figure but rather to the value obtained if we had used $8\pi G$ instead of G as the quantity from which to determine by dimensional arguments the Planck scale, the latter would have been a factor $\sqrt{8\pi}$ smaller. This would correspond to subtracting 1.61 from the $\ln \mu_{Pl}$ or 3.22 from the $\ln \mu_{Pl}^2$. With this lower value for the fundamental energy scale the $U(1)$ running coupling would be shifted up in value from the 55.4 of equation (23) by 1.84 to 57.2 which in turn would shift the 55.4/2=27.7 value plotted on the curve up by 0.92. This is seen by eye to shift the curve even closer to going through the fourth point than was the case for the Planck scale calculated simply using G. Have in mind that the $SU(2)$ point with almost compensating β-function contributions from the Yang Mills and the fermion contributions would only be moved very little by a slight change in the Planck scale so that this point would be almost unchanged by the replacement of G with $8\pi G$, while the $SU(3)$ point would go the opposite way, meaning down in $3\tilde{g}^2/\pi$-value, so that the curve would essentially be tilted so as to rise more slowly with increasing d_N.

Assuming that the correct fundamental scale should be obtained from $8\pi G$ rather than from G, the deviation would in terms of the inverse $U(1)$ fine structure constant be only about one out of 27.7 meaning about 1 in 55.4, which is only about 2%. We can not meaningfully expect better coincidence unless we were to calculate a three loop critical value since going from one to two loop in the critical monopole coupling squared gave a 20% correction so that 4% would be expected from three loops But 4% in the critical monopole coupling square would mean about 2% in the $U(1)$ finestructure constant (inverted) at the Planck scale.

We may also be concerned that we have not yet made two loop calculations of our correction factors C giving the correspondance between the nonAbelian couplings and the corresponding $U(1)$ coupling. But it is our experience numerically that the remarkable result of the intersection point having critical coupling is rather insensitive to the exact C-system used. We therefore believe that even with the C's only computed to one loop the accuracy of the calculation is already effectively a two loop calculation.

The question of the $8\pi G$ versus the G in determining the Planck scale or fundamental scale must be considered basically an uncertainty at least until after a very detailed discussion of this point.

Our result brings to mind the proverbial story of trying to find a needle in a large haystack. Actually we could even tell a better story. Start by standing at a distance from the haystack with an electric torch with a very narrow beam. Initially the torch is turned off. Now imagine that our task is to aim the torch in the dark it before we turn the torch on - we are not allowed to move the torch after turning it on. Now ask about the likelyhood of capturing the needle in our fixed narrow beam of light. We could improve the story by claiming that we stand with our torch at the Planck scale and must aim it at a haystack at the scale of the monopole mass before turning it on. There is even the added complication that we a priori don't know exatly what we're looking for. But we miraculously find the needle in our narrow beam of light. But maybe with the guiding light of our model this is not such a miracle after all. Nature may be telling us that we're on the right track with our MPP/AntiGUT model.

In this work we have only concerned ourselves with two of the three intersting phases for monopoles, namely the Coulomb phase and the phase with a monopole condensate. These phases have been treatable using the Higgs monopole description of monopoles as particles. We would also like to have the monopole confining phase at our multiple point. For every new phase we bring to the multiple point there is one more intensive parameter that becomes finetunned by MPP. This could be ratio of the monopole mass scale to that of the Planck mass if we had the confining phase. However taking the confining phase into account would require appending a string scenario to our approach here and doing this with the the high accuracy we otherwise have in this work is not yet possible.

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3 On the Origin of Families of Fermions and Their Mass Matrices — Approximate Analyses of Properties of Four Families Within Approach Unifying Spins and Charges

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Abstract. The approach unifying all the internal degrees of freedom - proposed by one of us[4,2,3,4,5,6,7,8,9,10] - is offering a new way of understanding families of quarks and leptons. Spinors, namely, living in $d\ (= 1 + 13)-$dimensional space, manifest in the observed $d(= 1 + 3)$-dimensional space (at ”physical energies”) all the known charges of quarks and leptons (with the mass protection property of the Standard model that only the left handed quarks and leptons carry the weak charge while the right handed ones are weak chargeless included), while a part of the starting Lagrange density in $d\ (= 1 + 13)$ transforms the right handed quarks and leptons into the left handed ones, manifesting a mass term in $d = 1 + 3$. Since a spinor carries two kinds of spins and interacts accordingly with two kinds of the spin connection fields, the approach predicts families and the corresponding Yukawa couplings. In the paper[11] the appearance of families of quarks and leptons within this approach was investigated and the explicit expressions for the corresponding Yukawa couplings, following from the approach after some approximations and simplifications, presented. In this paper we continue investigations of this new way of presenting families of quarks and leptons by further analyzing properties of mass matrices, treating quarks and leptons in an equivalent way. Since it is a long way from the starting simple action for a Weyl spinor in $d = 1 + 13$ to the observable phenomena at low energies, and we yet want to make (at least rough) predictions of the approach, we connect free parameters of the approach with the known experimental data and investigate a possibility that the fourth family of quarks and leptons appears at low enough energies to be observable with the new generation of accelerators.

3.1 Introduction

The Standard model of the electroweak and strong interactions (extended by assuming nonzero masses of the neutrinos) fits with around 25 parameters and constraints all the existing experimental data. However, it leaves unanswered many open questions, among which are also the questions about the origin of the families, the Yukawa couplings of quarks and leptons and the corresponding Higgs mechanism. Understanding the mechanism for generating families, their masses and mixing matrices might be one of the most promising ways to physics beyond the Standard model.

The approach, unifying spins and charges[4,2,3,4,5,6,7,8,9,10,11], might by offering a new way of describing families, give an explanation about the origin of the Yukawa couplings.

It was demonstrated in the references[6,8,9,10] that a left handed $SO(1,13)$ Weyl spinor multiplet includes, if the representation is analyzed in terms of the subgroups $SO(1,3)$, $SU(2)$, $SU(3)$ and the sum of the two $U(1)$'s, all the spinors of the Standard model - that is the left handed $SU(2)$ doublets and the right handed $SU(2)$ singlets of (with the group $SU(3)$ charged) quarks and (chargeless) leptons. There are the (two kinds of) spin connection fields and the vielbein fields in $d = (1+13)-$dimensional space, which might manifest - after some appropriate compactifications (or some other kind of making the rest of $d-4$ space unobservable at low energies) - in the four dimensional space as all the gauge fields of the known charges, as well as the Yukawa couplings.

The paper[11] analyzes, how do terms, which lead to masses of quarks and leptons, appear in the approach unifying spins and charges as a part of the spin connection and vielbein fields. No Higgs is needed in this approach to "dress" right handed spinors with the weak charge, since the terms of the starting Lagrangean, which include $\gamma^0\gamma^s$, with $s = 7, 8$, do the job of a Higgs field.

Since we have done no analyses (yet) about the way of breaking symmetries of the starting group $SO(1,13)$ to $SO(1,7) \times U(1) \times SU(3)$ and further within our approach (except some very rough estimations in ref.[12]), we do not know how might symmetry breaking in the ordinary space influence the fields (spin connections and vielbeins), which in the starting action determine the Yukawa couplings. We also do not know how do nonperturbative and other effects (like boundary conditions) influence after the break of symmetries the couplings of spinors to the part of the starting gauge fields which in $d = 1+3$ manifest at low energy as the Yukawa couplings. It is namely a long way from a simple starting action for a spinor with only one parameter and only the gravity as a gauge field to the observable quarks and leptons interacting with all the known gauge fields.

We can accordingly in this investigation, by connecting Yukawa couplings with the experimental data, only discuss about the appearance of the "vacuum expectation values" of the spin connection fields which enter into the Yukawa couplings, trying to guess how all the complicated breaks of the symmetries have lead from the starting action to the observable massess and mixing angles. We also have no explanation yet why the second kind of the Clifford algebra objects do not manifest in $d = 1+3$ any charges, which could appear in addition to the known ones.

Since the generators of the Lorentz transformations and the generators of families commute, and since only the generators of families contribute to non-diagonal elements of mass matrices (which means, that the off diagonal matrix elements of quarks and leptons are strongly correlated), the question arises, what makes leptons so different from quarks in the proposed approach. Can it be that at some energy level they are very alike and that there are some kinds of boundary conditions together with the nonperturbative effects which lead to observable properties, or might it be that Majorana like objects, not taken into account in these investigations up to now, are responsible for the observed differences?

To evaluate whether this way of going beyond the Standard model of the electroweak and colour interaction is in agreement with the nature, we must first make a rough estimate of what the approach predicts, before going to more sophisticated and therefore also more trustable predictions. Starting with only the gauge gravity in $d = 1 + 13$ (or in any d) and then coming down to the "the physical world", is a huge project, which needs to be made in several successive steps.

What turns out in our approach to be exciting is that one Weyl spinor in d=1+13 offers all the quarks and the leptons postulated by the Standard model and with just right quantum numbers answering the open question of the Standard model, how it is at all possible that the weak charge "knows" for the handedness (left handed weak charged quarks and leptons and right handed weak chargeless quarks and leptons), since in the Standard model the handedness and the weak charge belong to totally separated degrees of freedom.

Next exciting thing in the approach is, that it is a part of the starting Lagrangean which does, what the Standard model requires from the Higgs: makes the nonzero matrix elements between the weak chargeless right handed quarks and leptons and weak charged quarks and leptons, correspondingly, which then manifest as the mass matrices.

The third exciting thing of our approach is that it offers a mechanism for the appearance of families and accordingly the possibility to calculate the Yukawa couplings "from the first principle".

There are also severe problems, which the approach is confronting in the way down to the $d = 1 + 3$ world.

First severe problem is, how can at all exist at low energies any "non Planck scale" mass, if one starts from a very high scale? We are working hard[13] to overcome this "Witten's no go" theorem[14], which concerns all the Kaluza-Klein-like theories. We found for a toy model ($d = 1 + 5$) a particular boundary condition, which by requiring that spinors of only one handedness exist on the boundary makes that no Dirac mass can occur and therefore also the corresponding Yukawas do not appear after a particular break of a symmetry. We also found [15] that a Weyl spinor with no charge has no Majorana mass only in some dimensions, and d=1+13 is one of those. In this paper we assume that the extension of the toy model can work also in our more generalized case. The justification is under consideration.

One further severe problem is how to treat the "history" of spinors after the breaks of symmetries with all the perturbative but mostly nonperturbative effects, which are "dressing" spinors (quarks and leptons), since even in the hadron physics a similar problem is not yet solved. In this paper we assume that the "dressing" manifests, after going beyond "the tree level" in different "vacuum expectation values" of the omega fields.

Even proceeding in this way, this paper (making a first step towards more justified results by allowing several approximations and assumptions in order to find out, whether something very essential and unexpected can go wrong with our approach) was quite a work. We indeed come in this paper and in the previous one, which this one is following to promising results, that the approach might have a chance to go successfully beyond the Standard model.

In this paper we try to understand properties of quarks and leptons within the approach unifying spins and charges treating quarks and leptons equivalently. Within this approach we discuss also a possibility, that the fourth family of quarks and leptons appears at low enough energies to be observable with new accelerators.

In Sect.3.2 of this paper we present the action for a Weyl spinor in $(1 + 13)$-dimensional space and the part of the Lagrangean, which manifests at "physical energies" as an effective Lagrangean, with the Yukawa mass term included. This section is a brief repetition of the derivations presented in the ref.[11].

Also Sect.3.3 is a short summary of the paper[11] in the part in which the explicit expression for the four mass matrices of the four families of quarks and leptons is presented, derived under several assumptions and simplifications from the starting action of the approach unifying spins and charges. In Subsect.3.3.1 we study properties of the mass matrices in the approximation, that the "vacuum expectation values" of the gauge fields of the second kind of the Clifford algebra objects entering into mass matrices are the same for all the quarks and the leptons. In Subsect.3.3.2 we study properties of the mass matrices relaxing this requirement.

In Subsect.3.3.3 we discuss the problem of the appearance of negative masses in connection with the internal parity, defined within the presented approach.

In Sect.3.4 we present the numerical results after fitting the free parameters of the mass matrices with the experimental data, predicting masses and mixing matrices of the four families of quarks and leptons.

In Sect.6.7 we comment on the success of our approximate prediction of our approach.

3.2 Weyl spinors in $d = (1 + 13)$ manifesting at "physical energies" families of quarks and leptons

We assume a left handed Weyl spinor in $(1 + 13)$-dimensional space. A spinor carries only the spin (no charges) and interacts accordingly with only the gauge gravitational fields - with spin connections and vielbeins. We assume two kinds of the Clifford algebra objects and allow accordingly two kinds of gauge fields[1,2,3,4,5,6,7,8,9,10,11]. One kind is the ordinary gauge field (gauging the Poincaré symmetry in $d = 1 + 13$). The corresponding spin connection field appears for spinors as a gauge field of $S^{ab} = \frac{1}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)$, where γ^a are the ordinary Dirac operators. The contribution of these fields to the mass matrices manifests in only the diagonal terms - connecting the right handed weak chargeless quarks or leptons to the left handed weak charged partners within one family of spinors.

The second kind of gauge fields is in our approach responsible for families of spinors and couplings among families of spinors - contributing to diagonal matrix elements as well - and might explain the appearance of families of quarks and leptons and the Yukawa couplings of the Standard model of the electroweak and colour interactions. The corresponding spin connection fields appear for spinors as gauge fields of $\tilde{S}^{ab}$ ($\tilde{S}^{ab} = \frac{1}{2}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$) with $\tilde{\gamma}^a$, which are the Clifford algebra objects[2,16], like γ^a, but anticommute with γ^a.

Following the ref.[11] we write the action for a Weyl (massless) spinor in $d(= 1 + 13)$ - dimensional space as follows[1]

$$
\begin{aligned}
S &= \int d^d x \,\mathcal{L},\\
\mathcal{L} &= \frac{1}{2}(E\bar{\psi}\gamma^a p_{0a}\psi) + h.c. = \frac{1}{2}(E\bar{\psi}\gamma^a f^{\alpha}{}_a p_{0\alpha}\psi) + h.c.,\\
p_{0\alpha} &= p_\alpha - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}.
\end{aligned} \tag{3.1}
$$

Here $f^{\alpha}{}_a$ are vielbeins (inverted to the gauge field of the generators of translations $e^a{}_\alpha$, $e^a{}_\alpha f^\alpha{}_b = \delta^a_b$, $e^a{}_\alpha f^\beta{}_a = \delta_\alpha{}^\beta$), with $E = \det(e^a{}_\alpha)$, while $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ are the two kinds of the spin connection fields, the gauge fields of S^{ab} and $\tilde{S}^{ab}$, respectively, corresponding to the two kinds of the Clifford algebra objects[17,10], namely γ^a and $\tilde{\gamma}^a$, with the properties

$$
\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0, \tag{3.2}
$$

leading to $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$. We kindly ask the reader to learn about the properties of these two kinds of the Clifford algebra objects - γ^a and $\tilde{\gamma}^a$ and of the corresponding S^{ab} and $\tilde{S}^{ab}$ - and about our technique in the ref.[11] or the refs.[17,16].

One Weyl spinor representation in $d = (1+13)$ with the spin as the only internal degree of freedom, manifests, if analyzed in terms of the subgroups $SO(1,3) \times U(1) \times SU(2) \times SU(3)$ in four-dimensional physical space as the ordinary ($SO(1,3)$) spinor with all the known charges of one family of the left handed weak charged and the right handed weak chargeless quarks and leptons of the Standard model. The reader can see this analyses in the paper[11] (as well as in several references, like the one in the ref.[10]).

We may rewrite the Lagrangean of Eq.(3.1) so that it manifests the usual $(1+3)-$dimensional spinor Lagrangean part and the term manifesting as a mass term[11]

$$
\begin{aligned}
\mathcal{L} = \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_m)\psi + \\
\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\,\psi + \text{the rest}.
\end{aligned} \tag{3.3}
$$

Index A determines the charge groups ($SU(3), SU(2)$ and the two $U(1)$'s), index i determines the generators within one charge group. τ^{Ai} denote the generators of

[1] Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

the charge groups

$$\tau^{Ai} = \sum_{s,t} c^{Ai}{}_{st}\, S^{st},$$
$$\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk}\tau^{Ak}, \tag{3.4}$$

with $s,t \in 5,6,..,14$, while $A^{Ai}_m, m = 0,1,2,3$, denote the corresponding gauge fields (expressible in terms of ω_{stm}).

We have: $Y = \tau^{41} + \tau^{21}$, $\quad Y' = \tau^{41} - \tau^{21}$, with $\tau^{11} := \frac{1}{2}(\mathcal{S}^{58} - \mathcal{S}^{67})$, $\tau^{12} := \frac{1}{2}(\mathcal{S}^{57} + \mathcal{S}^{68})$, $\tau^{13} := \frac{1}{2}(\mathcal{S}^{56} - \mathcal{S}^{78})$, $\tau^{21} := \frac{1}{2}(\mathcal{S}^{56} + \mathcal{S}^{78})$, $\tau^{31} := \frac{1}{2}(\mathcal{S}^{9\;12} - \mathcal{S}^{10\;11})$, $\tau^{32} := \frac{1}{2}(\mathcal{S}^{9\;11} + \mathcal{S}^{10\;12})$, $\tau^{33} := \frac{1}{2}(\mathcal{S}^{9\;10} - \mathcal{S}^{11\;12})$, $\tau^{34} := \frac{1}{2}(\mathcal{S}^{9\;14} - \mathcal{S}^{10\;13})$, $\tau^{35} := \frac{1}{2}(\mathcal{S}^{9\;13} + \mathcal{S}^{10\;14})$, $\tau^{36} := \frac{1}{2}(\mathcal{S}^{11\;14} - \mathcal{S}^{12\;13})$, $\tau^{37} := \frac{1}{2}(\mathcal{S}^{11\;13} + \mathcal{S}^{12\;14})$, $\tau^{38} := \frac{1}{2\sqrt{3}}(\mathcal{S}^{9\;10} + \mathcal{S}^{11\;12} - 2\mathcal{S}^{13\;14})$, $\tau^{41} := -\frac{1}{3}(\mathcal{S}^{9\;10} + \mathcal{S}^{11\;12} + \mathcal{S}^{13\;14})$.

The subgroups are chosen so that the gauge fields in the physical region agree with the known gauge fields. If the break of symmetries in the $\tilde{S}^{ab}$ sector demonstrates the same symmetry after the break as in the S^{ab} sector, then also the corresponding operators with $\tilde{\tau}^{Ai}$ should be defined.

Making several assumptions, explained in details in the ref.[11] - we shall repeat them bellow - needed to manifest the observable phenomena (and can not yet be derived, since we do not yet know how the break of symmetries influences the starting Lagrangean), we are able to rewrite the mass term of spinors (fermions) from Eq.(3.3) ($\sum_{s=7,8}\ \bar{\psi}\gamma^s p_{0s}\ \psi$, neglecting the rest) by assuming that they are small in comparison with what we keep at "physical energies") as L_Y, demonstrating the Yukawa couplings of the Standard model

$$\begin{aligned}
\mathcal{L}_Y = \psi^+\gamma^0 \{ \; & \overset{78}{(+)} \Big(\sum_{y=Y,Y'} y A^y_+ + \sum_{\tilde{y}=\tilde{N}^+_3,\tilde{N}^-_3,\tilde{\tau}^{13},\tilde{Y},\tilde{Y}'} \tilde{y}\tilde{A}^{\tilde{y}}_+ \Big) + \\
& \overset{78}{(-)} \Big(\sum_{y=Y,Y'} y A^y_- + \sum_{\tilde{y}=\tilde{N}^+_3,\tilde{N}^-_3,\tilde{\tau}^{13},\tilde{Y},\tilde{Y}'} \tilde{y}\tilde{A}^{\tilde{y}}_- \Big) + \\
& \overset{78}{(+)} \sum_{\{(ac)(bd)\},k,l} \overset{ac}{(\tilde{k})}\overset{bd}{(\tilde{l})}\ \tilde{A}^{kl}_+((ac),(bd)) \; + \\
& \overset{78}{(-)} \sum_{\{(ac)(bd)\},k,l} \overset{ac}{(\tilde{k})}\overset{bd}{(\tilde{l})}\ \tilde{A}^{kl}_-((ac),(bd))\}\psi,
\end{aligned} \tag{3.5}$$

with

$$\overset{ab}{(k)} := \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^8), \quad \overset{ab}{(\tilde{k})} = \frac{1}{2}(\tilde{\gamma}^a + \frac{\eta^{aa}}{ik}\tilde{\gamma}^b) \tag{3.6}$$

and with $k = \pm 1$, if $\eta^{aa}\eta^{bb} = 1$ and $\pm i$, if $\eta^{aa}\eta^{bb} = -1$. While $\overset{ab}{(k)}$ are expressible in terms of ordinary γ^a and γ^b, $\overset{ab}{(\tilde{k})}$ are expressible in terms of the second kind of the Clifford algebra objects, namely in terms of $\tilde{\gamma}^a$ and $\tilde{\gamma}^b$.

The Yukawa part of the starting Lagrangean (Eq.(3.5)) has the diagonal terms, that is the terms manifesting the Yukawa couplings within each family, and the off diagonal terms, determining the Yukawa couplings among families.

The operators, which contribute to the non diagonal terms in mass matrices, are superposition of $\tilde{S}^{ab}$ (times the corresponding fields $\tilde{\omega}_{abc}$) and can be represented as factors of nilpotents

$$\stackrel{ab}{(\tilde{k})}\stackrel{cd}{(\tilde{l})}, \tag{3.7}$$

with indices (ab) and (cd) which belong to the Cartan subalgebra indices and the superposition of the fields $\tilde{\omega}_{abc}$. We may write accordingly

$$\sum_{(a,b)} -\frac{1}{2} \stackrel{78}{(\pm)} \tilde{S}^{ab}\tilde{\omega}_{ab\pm} = - \sum_{(ac),(bd),\, k,l} \stackrel{78}{(\pm)}\stackrel{ac}{(\tilde{k})}\stackrel{bd}{(\tilde{l})}\ \tilde{A}^{kl}_{\pm}((ac),(bd)), \tag{3.8}$$

where the pair (a,b) in the first sum runs over all the indices, which do not characterize the Cartan subalgebra, with $a,b = 0,\dots,8$, while the two pairs (ac) and (bd) denote only the Cartan subalgebra pairs (for SO(1,7) we only have the pairs $(03),(12)$; $(03),(56)$;$(03),(78)$; $(12),(56)$; $(12),(78)$; $(56),(78)$) ; k and l run over four possible values so that $k=\pm i$, if $(ac)=(03)$ and $k=\pm 1$ in all other cases, while $l=\pm 1$. The fields $\tilde{A}^{kl}_{\pm}((ac),(bd))$ can then be expressed by $\tilde{\omega}_{ab\pm}$ as follows

$$\begin{aligned}
\tilde{A}^{++}_{\pm}((ab),(cd)) &= -\frac{i}{2}(\tilde{\omega}_{ac\pm} - \frac{i}{r}\tilde{\omega}_{bc\pm} - i\tilde{\omega}_{ad\pm} - \frac{1}{r}\tilde{\omega}_{bd\pm}),\\
\tilde{A}^{--}_{\pm}((ab),(cd)) &= -\frac{i}{2}(\tilde{\omega}_{ac\pm} + \frac{i}{r}\tilde{\omega}_{bc\pm} + i\tilde{\omega}_{ad\pm} - \frac{1}{r}\tilde{\omega}_{bd\pm}),\\
\tilde{A}^{-+}_{\pm}((ab),(cd)) &= -\frac{i}{2}(\tilde{\omega}_{ac\pm} + \frac{i}{r}\tilde{\omega}_{bc\pm} - i\tilde{\omega}_{ad\pm} + \frac{1}{r}\tilde{\omega}_{bd\pm}),\\
\tilde{A}^{+-}_{\pm}((ab),(cd)) &= -\frac{i}{2}(\tilde{\omega}_{ac\pm} - \frac{i}{r}\tilde{\omega}_{bc\pm} + i\tilde{\omega}_{ad\pm} + \frac{1}{r}\tilde{\omega}_{bd\pm}),
\end{aligned} \tag{3.9}$$

with $r=i$, if $(ab)=(03)$ and $r=1$ otherwise. We simplify the index kl in the exponent of the fields $\tilde{A}^{kl}{}_{\pm}((ac),(bd))$ to $\pm$, omitting i.

We must point out that a way of breaking any of the two symmetries - the Poincaré one and the symmetry determined by the generators $\tilde{S}^{ab}$ in $d = 1+13$ - strongly influences the Yukawa couplings of Eq.(3.5), relating the parameters $\tilde{\omega}_{abc}$. Not necessarily any break of the Poincaré symmetry influences the break of the other symmetry and opposite. Although we expect that it does. Accordingly the coefficients $c^{Ai}{}_{ab}$ determining the operators τ^{Ai} in Eq.(3.4) and the coefficients $\tilde{c}^{\tilde{A}i}{}_{ab}$ determining the operators $\tilde{\tau}^{\tilde{A}i}$ in the relations

$$\begin{aligned}
\tilde{\tau}^{\tilde{A}i} &= \sum_{a,b} \tilde{c}^{\tilde{A}i}{}_{ab}\ \tilde{S}^{ab},\\
\{\tilde{\tau}^{\tilde{A}i}, \tilde{\tau}^{\tilde{B}j}\}_{-} &= i\delta^{\tilde{A}\tilde{B}}\tilde{f}^{\tilde{A}ijk}\tilde{\tau}^{\tilde{A}k}
\end{aligned} \tag{3.10}$$

might even not be correlated. If correlated (through boundary conditions, for example) the break of symmetries might cause that off diagonal matrix elements of Yukawa couplings distinguish between quarks and leptons.

We made, when deriving the mass matrices of quarks and leptons from the approach unifying spins and charges, several assumptions, approximations and simplifications in order to be able to make at the end some rough predictions about the properties of the families of quarks and leptons:

i. The break of symmetries of the group $SO(1,13)$ (the Poincaré group in $d = 1 + 13$) into $SO(1,7) \times SU(3) \times U(1)$ occurs in a way that only massless spinors in $d = 1 + 7$ with the charge $SU(3) \times U(1)$ survive, and yet the two $U(1)$ charges, following from $SO(6)$ and $SO(1,7)$, respectively, are related. (Our work on the compactification of a massless spinor in $d = 1 + 5$ into $d = 1 + 3$ and a finite disk gives us some hope that such an assumption might be justified[13].) The requirement that the terms with $S^{5a}\omega_{5ab}$ and $S^{6a}\omega_{6ab}$ do not contribute to the mass term, assures that the charge $Q = \tau^{41} + S^{56}$ is conserved at low energies.

ii. The break of symmetries influences both, the (Poincaré) symmetry described by S^{ab} and the symmetries described by $\tilde{S}^{ab}$, and in a way that there are no terms, which would transform $\overset{56}{(\tilde{+})}$ into $\overset{56}{[\tilde{+}]}$. This assumption was made that at "low energies" only four families have to be treated and can be explained by a break of the symmetry $SO(1,7)$ into $SO(1,5) \times U(1)$ in the $\tilde{S}^{ab}$ sector so that all the contributions of the type $\tilde{S}^{5a}\tilde{\omega}_{5ab}$ and $\tilde{S}^{6a}\tilde{\omega}_{6ab}$ are equal to zero. We also assume that the terms which include components $p_s, s = 5,..,14$, of the momentum p^a do not contribute to the mass matrices. We keep in mind that any further break of symmetries strongly influences the relations among $\tilde{\omega}_{abc}$, appearing in the paper [11] as "vacuum expectation values" in mass matrices, so that predictions in Sect.3.4 strongly depend on the way of breaking.

iii. We make estimations on a "tree level".

iv. We assume the mass matrices to be real and symmetric (expecting that complexity and nonsymmetric properties will not influence considerably masses and mixing matrices of quarks and leptons).

3.3 Four families of quarks and leptons

Taking into account the assumptions, presented in Sect.3.2, we end up with four families of quarks and leptons

$$
\begin{array}{rl}
\mathrm{I.} & \overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{(+)} \parallel ... \\
\mathrm{II.} & \overset{03}{[+i]}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{(+)} \parallel ... \\
\mathrm{III.} & \overset{03}{[+i]}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{[+]} \parallel ... \\
\mathrm{IV.} & \overset{03}{(+i)}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{[+]} \parallel ....
\end{array}
\tag{3.11}
$$

The Yukawa couplings for these four families are for u-quarks and neutrinos presented on Table 3.1, where α stays for u-quarks and neutrinos.

The corresponding mass matrix for the d-quarks and the electrons is presented on Table 3.2, where β stays for d-quarks and electrons.

The explicit form of the diagonal matrix elements for the above choice of assumptions in terms of $\omega_{abc\delta}$'s, $\delta = \alpha, \beta$ and $A^{y}_{\pm}, y = Y$ and Y', $\tilde{\omega}_{abc\delta}$ and $\tilde{A}^{41}_{\pm}$

α	I_R	II_R	III_R	IV_R
I_L	A^{I}_{α}	$\tilde{A}^{++}_{\alpha}((03),(12)) = \frac{\tilde{\omega}_{327\alpha}+\tilde{\omega}_{018\alpha}}{2}$	$\tilde{A}^{++}_{\alpha}((03),(78)) = \frac{\tilde{\omega}_{387\alpha}+\tilde{\omega}_{078\alpha}}{2}$	$-\tilde{A}^{++}_{\alpha}((12),(78)) = \frac{\tilde{\omega}_{277\alpha}+\tilde{\omega}_{187\alpha}}{2}$
II_L	$\tilde{A}^{--}_{\alpha}((03),(12)) = \frac{\tilde{\omega}_{327\alpha}+\tilde{\omega}_{018\alpha}}{2}$	$A^{\mathrm{II}}_{\alpha} = A^{\mathrm{I}}_{\alpha} + (\tilde{\omega}_{127\alpha} - \tilde{\omega}_{038\alpha})$	$\tilde{A}^{-+}_{\alpha}((12),(78)) = -\frac{\tilde{\omega}_{277\alpha}-\tilde{\omega}_{187\alpha}}{2}$	$-\tilde{A}^{-+}_{\alpha}((03),(78)) = \frac{\tilde{\omega}_{387\alpha}-\tilde{\omega}_{078\alpha}}{2}$
III_L	$\tilde{A}^{--}_{\alpha}((03),(78)) = \frac{\tilde{\omega}_{387\alpha}+\tilde{\omega}_{078\alpha}}{2}$	$-\tilde{A}^{+-}_{\alpha}((12),(78)) = -\frac{\tilde{\omega}_{277\alpha}-\tilde{\omega}_{187\alpha}}{2}$	$A^{\mathrm{III}}_{\alpha} = A^{\mathrm{I}}_{\alpha} + (\tilde{\omega}_{787\alpha} - \tilde{\omega}_{038\alpha})$	$\tilde{A}^{-+}_{\alpha}((03),(12)) = -\frac{\tilde{\omega}_{327\alpha}-\tilde{\omega}_{018\alpha}}{2}$
IV_L	$\tilde{A}^{--}_{\alpha}((12),(78)) = \frac{\tilde{\omega}_{277\alpha}+\tilde{\omega}_{187\alpha}}{2}$	$-\tilde{A}^{+-}_{\alpha}((03),(78)) = \frac{\tilde{\omega}_{387\alpha}-\tilde{\omega}_{078\alpha}}{2}$	$\tilde{A}^{+-}_{\alpha}((03),(12))$ $-\frac{\tilde{\omega}_{327\alpha}-\tilde{\omega}_{018\alpha}}{2}$	$A^{\mathrm{IV}}_{\alpha} = A^{\mathrm{I}}_{\alpha} + (\tilde{\omega}_{127\alpha} + \tilde{\omega}_{787\alpha})$

Table 3.1. The mass matrix of four families of u-quarks and neutrinos, obtained within the approach unifying spins and charges under the assumptions i.-iv. (see also the ref.[11]). According to Eq.(3.12) and Table 3.1 and 3.2 there are 13 free parameters, expressed in terms of the fields A^{I}_{α} and $\tilde{\omega}_{\alpha abc}$, if $\tilde{\omega}_{\alpha abc}$ are the same for different α. They then accordingly determine (due to assumptions i.-iv.) all the properties of the four families of the two types of quarks and the two types of leptons.

β	I_R	II_R	III_R	IV_R
I_L	A^{I}_{β}	$\tilde{A}^{++}_{\beta}((03),(12)) = \frac{\tilde{\omega}_{327\beta}-\tilde{\omega}_{018\beta}}{2}$	$-\tilde{A}^{++}_{\beta}((03),(78)) = -\frac{\tilde{\omega}_{387\beta}-\tilde{\omega}_{078\beta}}{2}$	$\tilde{A}^{++}_{\beta}((12),(78)) = -\frac{\tilde{\omega}_{277\beta}+\tilde{\omega}_{187\beta}}{2}$
II_L	$\tilde{A}^{--}_{\beta}((03),(12)) = \frac{\tilde{\omega}_{327\beta}-\tilde{\omega}_{018\beta}}{2}$	$A^{\mathrm{II}}_{\beta} = A^{\mathrm{I}}_{\beta} + (\tilde{\omega}_{127\beta} + \tilde{\omega}_{038\beta})$	$-\tilde{A}^{-+}_{\beta}((12),(78)) = \frac{\tilde{\omega}_{277\beta}-\tilde{\omega}_{187\beta}}{2}$	$\tilde{A}^{-+}_{\beta}((03),(78)) = -\frac{\tilde{\omega}_{387\beta}+\tilde{\omega}_{078\beta}}{2}$
III_L	$-\tilde{A}^{--}_{\beta}((03),(78)) = -\frac{\tilde{\omega}_{387\beta}-\tilde{\omega}_{078\beta}}{2}$	$\tilde{A}^{+-}_{\beta}((12),(78)) = \frac{\tilde{\omega}_{277\beta}-\tilde{\omega}_{187\beta}}{2}$	$A^{\mathrm{III}}_{\beta} = A^{\mathrm{I}}_{\beta} + (\tilde{\omega}_{787\beta} + \tilde{\omega}_{038\beta})$	$\tilde{A}^{-+}_{\beta}((03),(12)) = -\frac{\tilde{\omega}_{018\beta}+\tilde{\omega}_{327\beta}}{2}$
IV_L	$-\tilde{A}^{--}_{\beta}((12),(78)) = -\frac{\tilde{\omega}_{277\beta}+\tilde{\omega}_{187\beta}}{2}$	$\tilde{A}^{+-}_{\beta}((03),(78)) = -\frac{\tilde{\omega}_{387\beta}+\tilde{\omega}_{078\beta}}{2}$	$\tilde{A}^{+-}_{\beta}((03),(12))$ $-\frac{\tilde{\omega}_{018\beta}+\tilde{\omega}_{327\beta}}{2}$	$A^{\mathrm{IV}}_{\beta} A^{\mathrm{I}}_{\beta} + (\tilde{\omega}_{127\beta} + \tilde{\omega}_{787\beta})$

Table 3.2. The mass matrix of four families of the d-quarks and electrons, β stays for the d-quarks and the electrons. Comments are the same as on Table 3.1.

is as follows

$$
\begin{aligned}
A^{I}_{u} &= \frac{2}{3}A^{Y}_{-u} - \frac{1}{3}A^{Y'}_{-u} + \tilde{\omega}^{I}_{-u}, & A^{I}_{\nu} &= -A^{Y'}_{-\nu} + \tilde{\omega}^{I}_{-\nu},\\
A^{I}_{d} &= -\frac{1}{3}A^{Y}_{+d} + \frac{2}{3}A^{Y'}_{+d} + \tilde{\omega}^{I}_{+d}, & A^{I}_{e} &= -A^{Y}_{+e} + \tilde{\omega}^{I}_{+e},\\
A^{II}_{\alpha} &= A^{I}_{\alpha} + (i\tilde{\omega}_{03-\alpha} + \tilde{\omega}_{12-\alpha}), & A^{II}_{\beta} &= A^{I}_{\beta} + (i\tilde{\omega}_{03+\beta} + \tilde{\omega}_{12+\beta}),\\
A^{III}_{\alpha} &= A^{I}_{\alpha} + (i\tilde{\omega}_{03-\alpha} + \tilde{\omega}_{78-\alpha}), & A^{III}_{\beta} &= A^{I}_{\beta} + (i\tilde{\omega}_{03+\beta} + \tilde{\omega}_{78+\beta}),\\
A^{IV}_{\alpha} &= A^{I}_{\alpha} + (\tilde{\omega}_{12-\alpha} + \tilde{\omega}_{78-\alpha}), & A^{IV}_{\beta} &= A^{I}_{\beta} + (\tilde{\omega}_{12+\beta} + \tilde{\omega}_{78+\beta}),
\end{aligned}
\tag{3.12}
$$

with $\alpha = u, \nu$, $\beta = d, e$ and $-\tilde{\omega}^{I}{}_{\pm} = \frac{1}{2}(i\tilde{\omega}_{03\pm} + \tilde{\omega}_{12\pm} + \tilde{\omega}_{56\pm} + \tilde{\omega}_{78\pm} + \frac{1}{3}\tilde{A}^{41}_{\pm})$. We put the index $\alpha = u, \nu$ and $\beta = d, e$ to manifest that breaking of symmetries, boundary conditions and nonperturbative and other effects influence the "vacuum expectation values" gauge fields. If we assume that $\tilde{\omega}_{abc\delta}$ are equal if they belong to different δ, the assumption that all the matrix elements are real relates $\tilde{\omega}^{I}{}_{+} = \frac{1}{2}\tilde{\omega}_{038} + \tilde{\omega}$, $\tilde{\omega}^{I}{}_{-} = -\frac{1}{2}\tilde{\omega}_{038} + \tilde{\omega}$, where $\tilde{\omega}$ is (in case that breaking of symmetries does not influence quarks and leptons differently) one common parameter.

If the break of symmetries does not influence the quarks and the leptons in a different way, then under the assumptions i.-iv. the off diagonal matrix elements of mass matrices for quarks are the same as for the corresponding leptons (the off diagonal matrix elements of the u-quarks and the neutrinos are the same, and the off diagonal matrix elements for the d-quarks and the electrons are the same) and since the diagonal matrix elements differ only in a constant times a unit matrix, the predicted mixing matrices of the quarks and the leptons would be the same.

We must ask ourselves at this point: Can we find a way of breaking symmetries - allowing some special boundary conditions and taking into account all the perturbative and nonperturbative effects - which would lead to so different properties of quarks and leptons as experimentally observed or must we take the Majorana like degrees of freedom into account additionally?

In this paper, we are not yet able to answer this question. We can only make some estimates trying to learn from the approach unifying spins and charges about possible explanations for the properties of quarks and leptons.

We proceed by relating the experimental data and the mass matrices from the approach. Knowing from the experimental data that the first two families of quarks and leptons are much lighter than the third one, while in the refs.[18,19,20] the authors, analyzing the experimental data, conclude that the experimental data do not forbid masses of the fourth family of quarks to be between 200 GeV and 300 GeV, of the fourth electron to be around 100 GeV and of the fourth neutrino to be at around 50 GeV we make one more assumption, which seems quite reasonable also from the point of view of the measured matrix elements of the mixing matrix for quarks. Namely, *we assume that the mass matrices of the four families of quarks and leptons are diagonalizable in two steps, so that the first diagonalization transforms them into block-diagonal matrices with two 2×2 sub-matrices.* This assumption, which means, that a real and symmetric 4×4 matrix is diagonalizable by only three rather than six angles, simplifies considerably further studies, making conclusions very transparent. Let us comment that such a property of mass matrices could be a consequence of an approximate break of symmetry in the $\tilde{S}^{ab}$ sector from $SO(1,5)$ to $SU(2) \times SU(2) \times U(1)$, which makes, for example, all the terms $\tilde{S}^{7a}\tilde{\omega}_{7ab\delta}$ and $\tilde{S}^{8a}\tilde{\omega}_{8ab\delta}$ contributing small terms to mass matrices. The exact break of this type makes that the lower two families completely decouple from the higher two. (Similarly we have required, in order to end up with only four rather than eight families, that $SO(1,7)$ breaks to $SO(1,5) \times U(1)$ so that all $\tilde{S}^{5a}\ \tilde{\omega}_{5a\pm\delta}$ and $\tilde{S}^{6a}\ \tilde{\omega}_{6a\pm\delta}$ contribute nothing to mass matrices.)

It is easy to prove that a 4×4 matrix is diagonalizable in two steps only if it has a structure[21]

$$\begin{pmatrix} A & B \\ B & C = A + kB \end{pmatrix}.$$

Since A and C are, as assumed on Table 3.1 and Table 3.2, symmetric 2×2 matrices, so must then be also B. The parameter k is assumed to be an unknown number.

The assumption (3.13) requires: i. $\tilde{\omega}_{277\delta} = 0,\ \tilde{\omega}_{327\delta} = -\frac{k}{2}\tilde{\omega}_{187\delta},\ \tilde{\omega}_{787\delta} = \frac{k}{2}\tilde{\omega}_{387\delta},\ \tilde{\omega}_{038\delta} = -\frac{k}{2}\tilde{\omega}_{078\delta}$, $\delta = u, \nu, d, e$, and, in the case that $\tilde{\omega}_{abc\delta}$ do not depend on δ but only on abc. ii. $k_u = -k_d$ and $k_\nu = -k_e$, where k_u and k_ν are two independent parameters. (If $k = 0$ in Eq. (3.13), the angle of rotation is 45° - then, if also all the 2×2 matrices would have the same structure (namely equal diagonal and equal nondiagonal elements[22,23]), the corresponding mixing matrices for quarks and leptons would be the identity.)

3.3.1 Evaluation of mass matrices under assumption that $\tilde{\omega}_{abc\delta}$ do not distinguish among quarks and leptons

Let us first assume that neither boundary conditions nor nonperturbative or other effects influence the fields $\tilde{\omega}_{abc\delta}$ in a way that they would differ for different δ.

We shall present in what follows some simple relations which demonstrate transparently properties of mass matrices. After the one step diagonalization determined by the angle of rotation

$$\tan\varphi_\alpha = \pm(\sqrt{1+(\frac{k}{2})^2} \pm \frac{k}{2}), \quad \tan\varphi_\beta = \pm(\sqrt{1+(\frac{k}{2})^2} \mp \frac{k}{2}),$$
$$\text{with} \qquad \tan(\varphi_\alpha - \varphi_\beta) = \pm\frac{k}{2},\ (\text{or } \pm\frac{2}{k}) \tag{3.13}$$

we end up with two by diagonal matrices, with $k = k_u$ for quarks and $k = k_\nu$ for leptons, while α concerns the u-quarks and ν, and β the d-quarks and electrons.

The first by diagonal mass matrix of the u-quarks ($\alpha = u$) and neutrinos ($\alpha = \nu$) is as follows

$$\mathbf{A^a} = \begin{pmatrix} a_\alpha, & \frac{1}{2}(\tilde{\omega}_{018} - \sqrt{1+(\frac{k_\alpha}{2})^2}\ \tilde{\omega}_{187}) \\ \frac{1}{2}(\tilde{\omega}_{018} - \sqrt{1+(\frac{k_\alpha}{2})^2}\ \tilde{\omega}_{187}), & a_\alpha + \tilde{\omega}_{127} + \sqrt{1+(\frac{k_\alpha}{2})^2}\ \tilde{\omega}_{078} \end{pmatrix},$$

with $a_u = \frac{2}{3}A^Y - \frac{1}{3}A^{Y'} + \tilde{\omega} - \frac{1}{2}\tilde{\omega}_{038} + \frac{1}{2}(\frac{k_u}{2} - \sqrt{1+(\frac{k_u}{2})^2})(\tilde{\omega}_{078} + \tilde{\omega}_{387})$ and $a_\nu = -A^{Y'} + \tilde{\omega} - \frac{1}{2}\tilde{\omega}_{038} + \frac{1}{2}(\frac{k_\nu}{2} - \sqrt{1+(\frac{k_\nu}{2})^2})(\tilde{\omega}_{078} + \tilde{\omega}_{387})$. The mass matrix for the second two families of u-quarks ($\alpha = u$) and neutrinos ($\alpha = \nu$) is equal to

$$\mathbf{A}^{\mathbf{b}}_{\alpha} = \begin{pmatrix} a_\alpha + \sqrt{1+(\frac{k_\alpha}{2})^2}\;(\tilde{\omega}_{078}+\tilde{\omega}_{387}), & \frac{1}{2}(\tilde{\omega}_{018}+\sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{187}) \\ \frac{1}{2}(\tilde{\omega}_{018}+\sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{187}), & a_\alpha + \tilde{\omega}_{127} + \sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{387} \end{pmatrix}.$$

Accordingly we find for the first two families of d-quarks ($\beta = d$) and electrons ($\beta = e$)

$$\mathbf{A}^{\mathbf{a}}_{\beta} = \begin{pmatrix} a_\beta, & -\frac{1}{2}(\tilde{\omega}_{018}-\sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{187}) \\ -\frac{1}{2}(\tilde{\omega}_{018}-\sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{187}), & a_\beta + \tilde{\omega}_{127} + \sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{078} \end{pmatrix},$$

with $a_d = -\frac{1}{3}A^Y + \frac{2}{3}A^{Y'} + \tilde{\omega} + \frac{1}{2}\tilde{\omega}_{038} - \frac{1}{2}(\frac{k_u}{2} + \sqrt{1+(\frac{k_u}{2})^2})(\tilde{\omega}_{078} - \tilde{\omega}_{387})$ and $a_e = -A^Y + \tilde{\omega} + \frac{1}{2}\tilde{\omega}_{038} - \frac{1}{2}(\frac{k_\nu}{2} + \sqrt{1+(\frac{k_\nu}{2})^2})(\tilde{\omega}_{078} - \tilde{\omega}_{387})$. k_α in (3.14) is k_u for d-quarks and k_ν for electrons. For the second two families of d-quarks ($\beta = d$) and electrons ($\beta = e$) it follows

$$\mathbf{A}^{\mathbf{b}}_{\beta} = \begin{pmatrix} a_\beta + \sqrt{1+(\frac{k_\alpha}{2})^2}\;(\tilde{\omega}_{078}-\tilde{\omega}_{387}), & -\frac{1}{2}(\tilde{\omega}_{018}+\sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{187}) \\ -\frac{1}{2}(\tilde{\omega}_{018}+\sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{187}), & a_\beta + \tilde{\omega}_{127} - \sqrt{1+(\frac{k_\alpha}{2})^2}\;\tilde{\omega}_{387} \end{pmatrix}.$$

Again, k_α in (3.14) is k_u for d-quarks and k_ν for electrons.

There are three angles, which in the two step orthogonal transformations rotate each mass matrix into a diagonal one. The angles of rotations for $u-$quarks and $d-$quarks, and accordingly for neutrinos and electrons, are related as seen from Eq.(3.13) for the first step rotation. It follows namely that $\tan\varphi_\alpha = \tan^{-1}\varphi_\beta$ and accordingly

$$\begin{aligned} &\varphi_\alpha = \frac{\pi}{2} - \varphi_\beta, \quad \varphi_\alpha = \frac{\pi}{4} - \frac{\varphi}{2}, \quad \varphi_\beta = \frac{\pi}{4} + \frac{\varphi}{2}, \\ &\text{with} \quad \varphi = \varphi_\alpha - \varphi_\beta. \end{aligned} \tag{3.14}$$

Similarly also the two angles of rotations of the two by two diagonal matrices are related. Reminding the reader that in the unitary transformations ($S^\dagger S = I$) the trace and the determinant are among the invariants, while the angle of rotation, which diagonalizes 2 by 2 matrices (of the type (3.13)), and the values of the diagonal matrices are related as follows

$$\begin{aligned} \tan\Phi &= (\sqrt{1+(\frac{C-A}{2B})^2} \mp \frac{C-A}{2B}), \\ \lambda_{1,2} &= \frac{1}{2}((C+A) \pm \sqrt{(C-A)^2 + (2B)^2}), \end{aligned} \tag{3.15}$$

where for A, B, C the corresponding matrix elements from Eqs.(3.14,3.14,3.14,3.14) must be taken, one easily finds that

$${}^{a}\eta_\alpha = -{}^{a}\eta_\beta, \quad {}^{b}\eta_\alpha = -{}^{b}\eta_\beta, \quad \alpha = u, \nu, \ \beta = d, e, \tag{3.16}$$

where index a denotes the first two families and b the second two families of either quarks ($\alpha = u,\ \beta = d$) and leptons ($\alpha = \nu,\ \beta = e$) and $\eta = \frac{C-A}{2B}$. One then finds the relations, equivalent to those of Eq.(3.14)

$$
\begin{aligned}
&{}^{a,b}\varphi_\alpha = \frac{\pi}{2} - {}^{a,b}\varphi_\beta, \quad {}^{a,b}\varphi_\alpha = \frac{\pi}{4} - \frac{{}^{a,b}\varphi}{2}, \quad {}^{ab}\varphi_\beta = \frac{\pi}{4} + \frac{{}^{a,b}\varphi}{2}, \\
&\text{with} \quad {}^{a,b}\varphi = {}^{a,b}\varphi_\alpha - {}^{a,b}\varphi_\beta.
\end{aligned}
\tag{3.17}
$$

We accordingly find for the case, that $\tilde{\omega}_{abc\delta}$ do not depend on δ the following relations among the masses of the quarks and the leptons $|m_{u_2} - m_{u_1}| = |m_{d_2} - m_{d_1}|$, $|m_{u_4} - m_{u_3}| = |m_{d_4} - m_{d_3}|$, $|m_{\nu_2} - m_{\nu_1}| = |m_{e_2} - m_{e_1}|$, $|m_{\nu_4} - m_{\nu_3}| = |m_{e_4} - m_{e_3}|$, $|(m_{u_4} + m_{u_3}) - (m_{u_2} + m_{u_1})| = |(m_{d_4} + m_{d_3}) - (m_{d_2} + m_{d_1})|$, $|(m_{\nu_4} + m_{\nu_3}) - (m_{\nu_2} + m_{\nu_1})| = |(m_{e_4} + m_{e_3}) - (m_{e_2} + m_{e_1})|$, $|m_{u_4} + m_{u_3}| \approx 2\ \sqrt{1 + (\frac{k_u}{2})^2}\ \tilde{\omega}_{387} \approx |m_{d_4} + m_{d_3}|$, $|m_{\nu_4} + m_{\nu_3}| \approx 2\ \sqrt{1 + (\frac{k_\nu}{2})^2}\ \tilde{\omega}_{387} \approx |m_{e_4} + m_{e_3}|$. We take the absolute values of the sums and the differences, since whenever an eigenvalue $\lambda_{1,2}$ (Eq.3.15) appears to be negative, an appropriate change of a phase of the corresponding state transforms the negative value into the positive one by changing simultaneously the internal parity of the particular state, as it will be discussed in Sect.3.3.3.

The above relations among masses of quarks and leptons do not agree with the experimental data, as expected. We can take them as a very rough estimation in the limit when masses of the fourth family are much higher than the mass m_t, knowing that m_t is more than 30 times larger than the mass m_b.

3.3.2 Predictions with relaxed assumptions

We shall make in this subsection the evaluation of the mass matrices and their properties by allowing that $\tilde{\omega}_{abc\delta}$ depend on the type of quarks and leptons, assuming that boundary conditions connected with breaking of symmetries, perturbative, nonperturbative and other effects, appearing after each break of symmetries influence the "vacuum expectation values" of the gauge fields, entering into mass matrices so that $\tilde{\omega}_{abc\delta}$ are not the same for all the quarks and the leptons. To find out, how do they differ, one should make very sophisticated calculations, which even under the assumption that one can treat gravity in the region far away from the Planck scale as all the other gauge fields, is a very tough work, not only because we do not yet know how to treat all the breaks of symmetries to end up with massless spinors before the Yukawa couplings take care of their mass, but also because the nonperturbative effects are not solved yet even in hadron physics.

Learning from Subsect.3.3.1 that diagonalization of 4×4 matrix in two steps enables a transparent view on properties of the mass matrices and recognizing that such an assumption is at least not in disagreement with the known experimental data, we shall make calculations under the assumption that the *diagonalization in two steps* is a meaningful simplification. We shall also keep the relation from Subsect.3.3.1, which requires that the *angles of rotations for the* u*-quarks and the* d*-quarks mass matrices, as well as for the neutrinos and the electrons mass matrices,* *which determine the first and the second step of diagonalization, are simply related, just as presented in* Eqs.(3.14,3.17). These assumptions reduce considerably the number of free parameters (and might also help us to recognize the way of breaking symmetries in more sophisticated calculations).

It follows then that in Eqs.(3.14,3.14,3.14,3.14) all the fields $\tilde{\omega}_{abc\delta}$ carry an additional index $\delta = \alpha, \beta$, while we keep the relations $k_\alpha = -k_\beta$, $\alpha = u, \nu$ and $\beta = d, e$, where $k_{\alpha,\beta}$ define the first step orthogonal transformations leading to 2 by 2 by diagonal mass matrices and the relations among the angles of rotations in the second step of orthogonal transformations determined by ${}^{a,b}\eta_\alpha = {}^{a,b}(\frac{2B}{C-A})_\alpha$, requiring that (Eq.(3.16)) ${}^{a,b}\eta_\alpha = -{}^{a,b}\eta_\beta$ (where A, B, C are to be replaced by the corresponding matrix elements for the first two families determined by the matrix $A^a_{\alpha,\beta}$ (Eqs.(3.14, 3.14)) and the second two families determined by the matrix $A^b_{\alpha,\beta}$ (Eqs.(3.14, 3.14))).

Then it must be

$$\begin{aligned}
{}^a\varepsilon_\alpha\, (\tilde{\omega}_{018\beta} - \sqrt{1+(\frac{k_\alpha}{2})^2}\, \tilde{\omega}_{187\beta}) &= (\tilde{\omega}_{018\alpha} - \sqrt{1+(\frac{k_\alpha}{2})^2}\, \tilde{\omega}_{187\alpha}),\\
{}^b\varepsilon_\alpha\, (\tilde{\omega}_{018\beta} + \sqrt{1+(\frac{k_\alpha}{2})^2}\, \tilde{\omega}_{187\beta}) &= (\tilde{\omega}_{018\alpha} + \sqrt{1+(\frac{k_\alpha}{2})^2}\, \tilde{\omega}_{187\alpha}),\\
{}^a\varepsilon_\alpha\, (\tilde{\omega}_{127\beta} + \sqrt{1+(\frac{k_\alpha}{2})^2}\, \tilde{\omega}_{078\beta}) &= (\tilde{\omega}_{127\alpha} + \sqrt{1+(\frac{k_\alpha}{2})^2}\, \tilde{\omega}_{078\alpha}),\\
{}^b\varepsilon_\alpha\, (\tilde{\omega}_{127\beta} - \sqrt{1+(\frac{k_\alpha}{2})^2}\, \tilde{\omega}_{078\beta}) &= (\tilde{\omega}_{127\alpha} - \sqrt{1+(\frac{k_\alpha}{2})^2}\, \tilde{\omega}_{078\alpha}), \quad (3.18)
\end{aligned}$$

where index a and b distinguish between the two by two matrices for the first two and the second two families, correspondingly, while $\alpha = u, \nu$ and $\beta = d, e$. The two mixing matrices for the quarks and the leptons have the form

$$\mathbf{V}_{\alpha\beta} = \begin{pmatrix} c(\varphi)c({}^a\varphi) & -c(\varphi)s({}^a\varphi) & -s(\varphi)c({}^a\varphi^b) & s(\varphi)s({}^a\varphi^b)\\ c(\varphi)s({}^a\varphi) & c(\varphi)c({}^a\varphi) & -s(\varphi)s({}^a\varphi^b) & -s(\varphi)c({}^a\varphi^b)\\ s(\varphi)c({}^a\varphi^b) & -s(\varphi)s({}^a\varphi^b) & c(\varphi)c({}^b\varphi) & -c(\varphi)s({}^b\varphi)\\ s(\varphi)s({}^a\varphi^b) & s(\varphi)c({}^a\varphi^b) & c(\varphi)s({}^b\varphi) & c(\varphi)c({}^a\varphi) \end{pmatrix}, \quad (3.19)$$

with the angles (Eq.3.14,3.17) described by the three parameters $k_\alpha, {}^a\eta_\alpha, {}^b\eta_\alpha$ as follows

$$\varphi = \varphi_\alpha - \varphi_\beta, \quad {}^a\varphi = {}^a\varphi_\alpha - {}^a\varphi_\beta, \quad {}^a\varphi^b = -\frac{{}^a\varphi + {}^b\varphi}{2}. \quad (3.20)$$

We recognize that with the mixing matrix for either quarks or leptons describable by the three parameters each $k_\alpha, {}^a\eta_\alpha, {}^b\eta_\alpha; \alpha = u, \nu$, we have just enough free parameters to make any choice for the masses of the fourth family of quarks and leptons. To see this we just express A, B, C in any of the two 2 by 2 matrices in terms of the corresponding diagonal values that is in terms of the masses $m_{\alpha i}, m_{\beta i}; i = 1,2,3,4; \alpha = u, \nu, \beta = d, e$, and the parameters $k_\alpha = -k_\beta, {}^a\eta_\alpha = -{}^a\eta_\beta, {}^b\eta_\alpha = -{}^b\eta_\beta; \alpha = u, \nu,$. The matrix elements of $\mathbf{A}^a_\alpha$ (${}^a a_\alpha, {}^a b_\alpha, {}^a c_\alpha$) for u-quarks and neutrinos are expressible with the masses $m_{\alpha 1}, m_{\alpha 2}$ of the first two families of u-quarks or neutrinos and the corresponding angles of rotations as follows

$$\mathbf{A}^a_\alpha = \begin{pmatrix} \frac{1}{2}(m_{\alpha 1} + m_{\alpha 2} - \frac{{}^a\eta_\alpha(m_{\alpha 2}-m_{\alpha 1})}{\sqrt{1+({}^a\eta_\alpha)^2}}), & \frac{m_{\alpha 2}-m_{\alpha 1}}{2\sqrt{1+({}^a\eta_\alpha)^2}}) \\ \frac{m_{\alpha 2}-m_{\alpha 1}}{2\sqrt{1+({}^a\eta_\alpha)^2}}), & \frac{1}{2}(m_{\alpha 1} + m_{\alpha 2} + \frac{{}^a\eta_\alpha(m_{\alpha 2}-m_{\alpha 1})}{\sqrt{1+({}^a\eta_\alpha)^2}}) \end{pmatrix},$$

while the expressions for the matrix $\mathbf{A}^b_\alpha$ with matrix elements ${}^b a_\alpha, {}^b c_\alpha, {}^b b_\alpha$ follow, if we replace $m_{\alpha 1}$ with $m_{\alpha 3}$ and $m_{\alpha 2}$ with $m_{\alpha 4}$. Equivalently we obtain the mass matrices for the d-quarks and the electrons by replacing α by β in all expressions. Eq.(3.19) below demonstrates that if once the three parameters k_α, ${}^a\eta_\alpha$, ${}^b\eta_\alpha$ are chosen to fit the experimental data, any four masses for the fourth family of quarks and leptons agree with the proposed requirements.

The starting mass matrices $\mathbf{M}_{\alpha,\beta}$ - the Yukawa couplings - for quarks and leptons, which are 4 by 4 matrices of Table 3.1 and Table 3.2, are expressible with the matrices $\mathbf{A}^{a,b}_{\alpha,\beta}$ of Eq.(3.19) as follows

$$\begin{pmatrix} \frac{1}{2}[(\mathbf{A}^a_{\alpha,\beta} + \mathbf{A}^b_{\alpha,\beta}) - \frac{(\mathbf{A}^b_{\alpha,\beta}-\mathbf{A}^a_{\alpha,\beta})k_{\alpha,\beta}}{2\sqrt{1+(\frac{k_{\alpha,\beta}}{2})^2}}], & \frac{\mathbf{A}^b_{\alpha,\beta}-\mathbf{A}^a_{\alpha,\beta}}{2\sqrt{1+(\frac{k_{\alpha,\beta}}{2})^2}} \\ \frac{\mathbf{A}^b_{\alpha,\beta}-\mathbf{A}^a_{\alpha,\beta}}{2\sqrt{1+(\frac{k_{\alpha,\beta}}{2})^2}}, & \frac{1}{2}[(\mathbf{A}^a_{\alpha,\beta} + \mathbf{A}^b_{\alpha,\beta}) + \frac{(\mathbf{A}^b_{\alpha,\beta}-\mathbf{A}^a_{\alpha,\beta})k_{\alpha,\beta}}{2\sqrt{1+(\frac{k_{\alpha,\beta}}{2})^2}}] \end{pmatrix}.$$

(One easily sees that the matrix $\mathbf{M}_{\alpha,\beta}$ is equal to a democratic matrix with all the elements equal to $m_{\alpha_4}/4$, with $\alpha = u, \nu, d, e$, if all the angles of rotations are equal to $\pi/4$ (that is for $k_\alpha = 0$, ${}^{a,b}\eta_\alpha = 0$), while $m_{\alpha_i,\beta_i} = 0, i = 1, 3$, and that the mixing matrices are then the identity.)

Once knowing the matrices $\mathbf{M}_{\alpha,\beta}$ one easily finds for the parameters $\tilde{\omega}_{abc\,\alpha,\beta}$, with (abc) equal to $(018), (078), (127), (187), (387)$, entering in Table 3.1 and Table 3.2 the expressions

$$\begin{aligned}
\tilde{\omega}_{018\alpha} &= \frac{1}{2}[\frac{m_{\alpha 2} - m_{\alpha 1}}{\sqrt{1+({}^a\eta_\alpha)^2}} + \frac{m_{\alpha 4} - m_{\alpha 3}}{\sqrt{1+({}^b\eta_\alpha)^2}}], \\
\tilde{\omega}_{078\alpha} &= \frac{1}{2\sqrt{1+(\frac{k_\alpha}{2})^2}}[\frac{{}^a\eta_\alpha\,(m_{\alpha 2} - m_{\alpha 1})}{\sqrt{1+({}^a\eta_\alpha)^2}} - \frac{{}^b\eta_\alpha\,(m_{\alpha 4} - m_{\alpha 3})}{\sqrt{1+({}^b\eta_\alpha)^2}}], \\
\tilde{\omega}_{127\alpha} &= \frac{1}{2}[\frac{{}^a\eta_\alpha\,(m_{\alpha 2} - m_{\alpha 1})}{\sqrt{1+({}^a\eta_\alpha)^2}} + \frac{{}^b\eta_\alpha\,(m_{\alpha 4} - m_{\alpha 3})}{\sqrt{1+({}^b\eta_\alpha)^2}}], \\
\tilde{\omega}_{187\alpha} &= \frac{1}{2\sqrt{1+(\frac{k_\alpha}{2})^2}}[-\frac{m_{\alpha 2} - m_{\alpha 1}}{\sqrt{1+({}^a\eta_\alpha)^2}} + \frac{m_{\alpha 4} - m_{\alpha 3}}{\sqrt{1+({}^b\eta_\alpha)^2}}], \\
\tilde{\omega}_{387\alpha} &= \frac{1}{2\sqrt{1+(\frac{k_\alpha}{2})^2}}[(m_{\alpha 4} + m_{\alpha 3}) - (m_{\alpha 2} + m_{\alpha 1})], \\
a^a_\alpha &= \frac{1}{2}(m_{\alpha 1} + m_{\alpha 2} - \frac{{}^a\eta_\alpha\,(m_{\alpha 2} - m_{\alpha 1})}{\sqrt{1+({}^a\eta_\alpha)^2}}).
\end{aligned} \tag{3.21}$$

3.3.3 Negative masses and parity of states

We have mentioned in the previous section that after the diagonalization of mass matrices of quarks and leptons, masses of either positive or negative sign can appear and that by changing the phase of the basis and accordingly also the internal parity of states we change the sign of the mass. Let us prove this.

First we recognize that while the starting Lagrange density for spinors (Eq.3.1) commutes with the operator of handedness in $d(=1+13)$-dimensional space $\Gamma^{(1,13)}$ ($\Gamma^{(1,13)} = i2^7\, S^{03}S^{12}S^{56}\cdots S^{13\,14}$), it does not commute with the operator of handedness in $d(=1+3)$-dimensional space $\Gamma^{(1,3)}$ ($\Gamma^{(1,3)} = -i2^2 S^{03}S^{12}$). Accordingly also the term, which manifests at "physical energies" as the mass term $\hat{m}$

$$\gamma^0\hat{m} = \gamma^0 \left\{ \stackrel{78}{(+)} \Big(\sum_{y=Y,Y'} y A^y_+ + \sum_{\tilde{y}=\tilde{N}^+_3,\tilde{N}^-_3,\tilde{\tau}^{13},\tilde{Y},\tilde{Y}'} \tilde{y}\tilde{A}^{\tilde{y}}_+ \Big) \right.$$
$$+ \stackrel{78}{(-)} \Big(\sum_{y=Y,Y'} y A^y_- + \sum_{\tilde{y}=\tilde{N}^+_3,\tilde{N}^-_3,\tilde{\tau}^{13},\tilde{Y},\tilde{Y}'} \tilde{y}\tilde{A}^{\tilde{y}}_- \Big)$$
$$+ \stackrel{78}{(+)} \sum_{\{(ac)(bd)\},k,l} \stackrel{ac}{\tilde{(k)}}\stackrel{bd}{\tilde{(l)}}\; \tilde{A}^{kl}_+((ab),(cd))$$
$$\left. + \stackrel{78}{(-)} \sum_{\{(ac)(bd)\},k,l} \stackrel{ac}{\tilde{(k)}}\stackrel{bd}{\tilde{(l)}}\; \tilde{A}^{kl}_-((ab),(cd)) \right\},$$

(Eq.3.5)), does not commute with the $\Gamma^{(1,3)}$, they instead anticommute

$$\{\Gamma^{(1,3)}, \gamma^0\hat{m}\}_+ = 0.$$

But the rest of the "effective" Lagrangean (Eq.3.3) commutes with the operator of handedness in $d = (1+3)$-dimensional space:

$$\{\gamma^0\gamma^m(p_m - \sum_{A,i} g^A\tau^{Ai}A^{Ai}_m), \Gamma^{(1,3)}\}_- = 0.$$

It then follows that the Lagrange density

$$\mathcal{L} = (\Gamma^{(1,3)}\psi)^\dagger \left[\gamma^0\gamma^m(p_m - \sum_{A,i} g^A\tau^{Ai}A^{Ai}_m) - \Gamma^{(1,3)}\gamma^0\hat{m}\Gamma^{(1,3)}\right] (\Gamma^{(1,3)}\psi) \tag{3.22}$$

for the Dirac spinor $\Gamma^{(1,3)}\psi$ differs from the one from Eq.(3.3) in the sign of the mass term, while the function $\Gamma^{(1,3)}\psi$ differs from ψ in the internal parity, if ψ is the solution for the Dirac equation. Since the internal parity is just the convention, the negative mass changes sign if the internal parity of the spinor changes. The same argument was used in the ref.([24]), while the ref.([25]) uses the equivalent argument, namely, that the choice of the phase of either the right or the left handed spinors can always be changed and that accordingly also the signs of particular mass terms change.

Let us demonstrate now on Table 3.3 how does the operator of parity $\mathcal{P}$, if postulated as

$$\mathcal{P} = \gamma^0\gamma^8 I_x, \text{with } \; I_x x^m (I_x)^{(-1)} = x_m, \tag{3.23}$$

i		$\vert^a\psi_i>$	$\Gamma^{(1,3)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{21}	τ^{33}	τ^{38}	τ^{41}	Y	Y'
		Octet, $\Gamma^{(1,7)}=1,\ \Gamma^{(6)}=-1$, of quarks										
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{2}{3}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{2}{3}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\mid\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Table 3.3. The 8-plet of quarks - the members of $SO(1,7)$ subgroup, belonging to one Weyl left handed ($\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1,13)$. It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular colour ($(1/2, 1/(2\sqrt{3}))$). Here $\Gamma^{(1,3)}$ defines the handedness in $(1+3)$ space, S^{12} defines the ordinary spin (which can also be read directly from the basic vector, since $S^{ab}\stackrel{ab}{(k)}=\frac{k}{2}\stackrel{ab}{(k)}$ and $S^{ab}\stackrel{ab}{[k]}=\frac{k}{2}\stackrel{ab}{[k]}$, if S^{ab} belong to the Cartan subalgebra set), τ^{13} defines the weak charge, τ^{21} defines the $U(1)$ charge, τ^{33} and τ^{38} define the colour charge and τ^{41} another $U(1)$ charge, which together with the first one defines Y and Y'.

transform a right handed u-quark into the left handed u-quark: $\mathcal{P}u_R = \alpha u_L$, where α is the proportionality factor.

One notices that, if the operator $\mathcal{P}$ is applied on a state, which represents the right handed weak chargeless ($\tau^{13} = 0$) u-quark of one of the three colours and is presented in terms of nilpotents in the first row of Table 3.3, it transforms this state into the state, which can be found in the seventh row of the same table and represents the left handed u-quark of the same colour and spin and it is weak charged. Taking into account Eq.(3.23) and Eq.(12,16) from ref.[11] one finds $\mathcal{P}u_R = iu_L$, while $\mathcal{P}\mathcal{P} = I$. By changing appropriately the phases of this two basic states (u_R and u_L) we can easily achieve that $\mathcal{P}u_R = u_L, \mathcal{P}u_L = u_R$. We should in addition keep in mind that $\mathcal{P}$ must take into account also the appearance of families. We shall study discrete symmetries of our approach in the "low energy region" in a separate paper.

3.4 Numerical results

In this section we connect parameters $\tilde{\omega}_{abc\delta}$ of the Yukawa couplings following from our approach unifying spins and charges (after several assumptions, approximations and simplifications) with the experimental data. We investigate, how do the parameters of the approach reflect the known data. We also investigate a possibility of making some predictions.

3.4.1 Experimental data for quarks and leptons

We present in this subsection those experimental data, which are relevant for our study: that is the measured values for the masses of the three families of quarks and leptons and the measured mixing matrices.

We take in our calculations the experimental masses for the known three families from the ref.[26,27].

$$\begin{aligned}
m_{u_i}/GeV &= (0.0015 - 0.004, 1.15 - 1.35, 174.3 - 178.1),\\
m_{d_i}/GeV &= (0.004 - 0.008, 0.08 - 0.13, 4.1 - 4.9),\\
m_{\nu_i}/GeV &= (1\ 10^{-12}, 1\ 10^{-11}, 5\ 10^{-11}),\\
m_{e_i}/GeV &= (0.0005, 0.105, 1.8).
\end{aligned} \tag{3.24}$$

Predicting four families of quarks and leptons at "physical" energies, we require the unitarity condition for the mixing matrices for four rather than three measured families of quarks[26]

$$\begin{pmatrix} 0.9730 - 0.9746 & 0.2174 - 0.2241 & 0.0030 - 0.0044 \\ 0.213 - 0.226 & 0.968 - 0.975 & 0.039 - 0.044 \\ 0.0 - 0.08 & 0.0 - 0.11 & 0.07 - 0.9993 \end{pmatrix}. \tag{3.25}$$

The experimental data are for the mixing matrix for leptons known very weakly[27]

$$\begin{pmatrix} 0.79 - 0.88 & 0.47 - 0.61 & < 0.20 \\ 0.19 - 0.52 & 0.42 - 0.73 & 0.58 - 0.82 \\ 0.20 - 0.53 & 0.44 - 0.74 & 0.56 - 0.81 \end{pmatrix}. \tag{3.26}$$

We see that within the experimental accuracy both mixing matrices - for quarks and leptons - may be assumed to be symmetric up to a sign. We then fit with these two matrices the six parameters $k_\alpha, {}^a\eta_\alpha, {}^b\eta_\alpha$, $\alpha = u, \nu$.

3.4.2 Results

We started with the explicit expressions for the Yukawa couplings suggested by the approach unifying spins and charges after making several additional assumptions to the starting assumptions of the approach, also some approximations and simplifications, in order to be able to make some approximate predictions. We ended up with the four families of quarks and leptons, with the mass matrices symmetric and real and diagonalizable in two steps with accordingly three angles of rotation for each mass matrix and yet the angles of rotations for the u-quarks are related to those of the d-quarks and so are related also the angles of the two kinds of leptons. Accordingly the two mixing matrices are diagonalizable with three angles of rotations each.

Since to do the rigorous calculations is a huge project and the evaluations in this paper are only the first rough step towards more sophisticated very demanded calculations, we on this step only are parametrizing the influence of breaking symmetries of either the Poincaré group or the group defining families and of nonperturbative and other effect by parametrizing the "vacuum expectation values" of gauge fields entering into mass matrices. The assumptions, which we made, take care of simplifying the evaluation as much as possible while making the properties of mass matrices and mixing matrices as transparent as possible.

Let we repeat that according to Subsect.3.3.2 (in particular Eq.(3.21)) any choice for the masses of the fourth family fits the experimental data, once twice the three angles of the orthogonal transformations, determining the two mixing matrices are chosen. Assuming that some kind of symmetry (the charge Y and Y', for example) makes the "vacuum expectation values" of the gauge fields entering into the mass matrices further related, we are going to test how does the requirement that the ratios of the parameters $\tilde{\omega}_{abc\delta}$ are for a chosen set $[a, b, c]$ as close to rational numbers as possible influence the properties of the fourth family of quarks and leptons.

We also repeat the recognition made in Subsect.3.4.1 (Eqs.(3.25, 3.26)) that experimental data agree, within the experimental accuracy, for either quarks or leptons, that mixing matrices are symmetric and determined with only three angles. (We do not pay attention on CP non conservation.)

We shall now connect the parameters of the approach, which are left free, with the experimental data and try to find out what can we learn from the corresponding results.

We fit for quarks and leptons *the three angles* of Eqs.(3.14,3.17) *with the Monte-Carlo method under the requirement that the ratios of the parameters* $\tilde{\omega}_{abc\delta}$ *are for a chosen set* $[a, b, c]$ *so close to a rational number as possible.*

We allow the masses of the fourth family as follows: The two quark masses must lie in the range from 200 GeV to 1 TeV, the fourth neutrino mass must be within the interval $50 - 100$ GeV and of the fourth electron mass within $50 - 200$ GeV.

Fig. 3.1 shows the results of the Monte-Carlo simulation for the three angles determining the mixing matrix for quarks. There are the experimental inaccuracies, which determine the allowed regions for the three angles.

The results for the quarks are presented on Table 3.4 and 3.5 (together with the corresponding values for leptons).

Fig. 3.2 shows the Monte-Carlo fit for the three angles determining the mixing matrix for leptons. There are the experimental inaccuracies which limit the values of the three angles. Since in the lepton case the mixing matrix for the three known families as well as the masses for the three neutrinos are weakly known, the calculations for four families bring much less information than in the quark case.

The results for the leptons are presented together with the results for the quarks on Table 3.4 and 3.5.

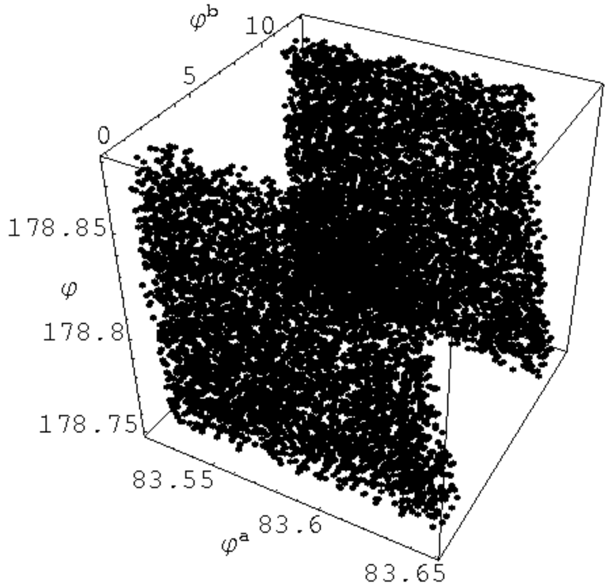

Fig. 3.1. Figure shows the Monte-Carlo fit[21] of the experimental mixing matrix for quarks (Eq.3.25) with the three angles of Eq.(3.19). The three angles define the three parameters k_u, ${}^a\eta_u$ and ${}^b\eta_u$ (Eqs.3.13,3.15). We make a choice among those values for the best fit, which makes the ratios $\tilde{\omega}_{abcu}/\tilde{\omega}_{abcd}$ as close to rational numbers as possible while assuring that the masses of the three known families stay within the acceptable values from Eq.(3.24), with no constraints on a_α and the two quark masses of the fourth family lie in the range $200 - 1000$ MeV.

	u	d	ν	e
k	-0.085	0.085	-1.254	1.254
${}^a\eta$	-0.229	0.229	1.584	-1.584
${}^b\eta$	0.420	-0.440	-0.162	0.162

Table 3.4. The Monte-Carlo fit to the experimental data[26,27] for the three parameters k, ${}^a\eta$ and ${}^b\eta$ determining the mixing matrices for the four families of quarks and leptons are presented.

In Eq.(3.27) we present masses for the four families of quarks and leptons as obtained after the Monte-Carlo fit

$$\begin{aligned} m_{u_i}/GeV &= (0.0034, 1.15, 176.5, 285.2), \\ m_{d_i}/GeV &= (0.0046, 0.11, 4.4, 224.0), \\ m_{\nu_i}/GeV &= (1\ 10^{-12}, 1\ 10^{-11}, 5\ 10^{-11}, 84.0), \\ m_{e_i}/GeV &= (0.0005, 0.106, 1.8, 169.2). \end{aligned} \tag{3.27}$$

The results of the Monte-Carlo fit show[21] that the requirement, that the ratios of the corresponding parameters of $\tilde{\omega}_{abc\delta}$, for a chosen set $[a, b, c]$, for

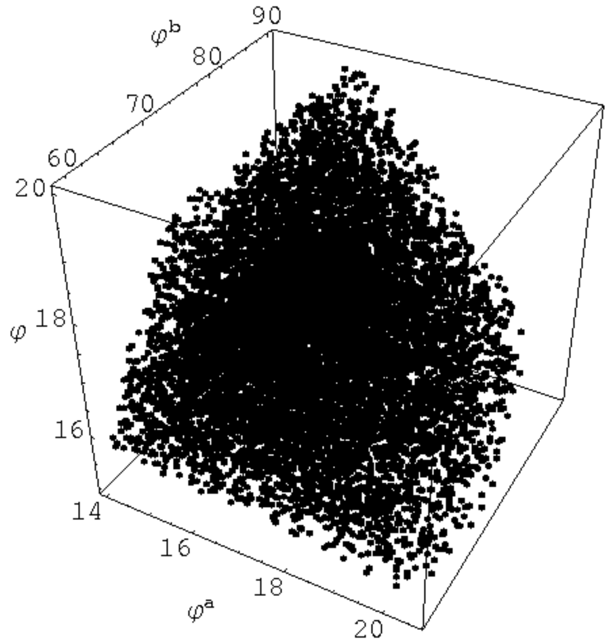

Fig. 3.2. Figure shows the Monte-Carlo fit of the experimental data for the mixing matrix for leptons(Eq.(3.26)). The three angles define the three parameters k_ν, ${}^a\eta_\nu$ and ${}^b\eta_\nu$ (Eqs.3.13,3.15). Again we make a choice among those values for the best fit, which make the ratios $\tilde{\omega}_{abcu}/\tilde{\omega}_{abcd}$ as close to rational numbers as possible.

	u	d	u/d	ν	e	ν/e
$\lvert\tilde{\omega}_{018}\rvert$	21205	42547	0.498	10729	21343	0.503
$\lvert\tilde{\omega}_{078}\rvert$	49536	101042	0.490	31846	63201	0.504
$\lvert\tilde{\omega}_{127}\rvert$	50700	101239	0.501	37489	74461	0.503
$\lvert\tilde{\omega}_{187}\rvert$	20930	42485	0.493	9113	18075	0.505
$\lvert\tilde{\omega}_{387}\rvert$	230055	114042	2.017	33124	67229	0.493
a^α	94174	6237		1149	1142	

Table 3.5. Values for the parameters $\tilde{\omega}_{abc\delta}$ (entering into the mass matrices for the $u-$quarks, the $d-$quarks, the neutrinos and the electrons, as suggested by the approach unifying spins and charges after making the additional assumptions and simplifications as decribed in this paper) as following after the Monte-Carlo fit, relating the parameters and the experimental data.

the quarks and the leptons should be as close to the rational numbers as possible, forces the masses of the fourth family to lie pretty low. We recognize that the values agree with the experimentally allowed values as evaluated by the refs.[18,19,20]. Eq.(3.21), however, tells us, that it is mainly the top mass and the masses of the fourth family which mostly influence these ratios. But since only the integer 2 and the half integer (1/2) are involved (the ratios go to one only when all the masses of the fourth family are equal and are high in comparison with the top mass), we could use this result as a guide when looking for the way of breaking symmetries on the way down to $d = 1 + 3$ in (much) more sophisticated calculations.

The Monte-Carlo fit leads to the following mixing matrix for the quarks

$$\begin{pmatrix} 0.974 & 0.223 & 0.004 & 0.042 \\ 0.223 & 0.974 & 0.042 & 0.004 \\ 0.004 & 0.042 & 0.921 & 0.387 \\ 0.042 & 0.004 & 0.387 & 0.921 \end{pmatrix} \tag{3.28}$$

and for the leptons

$$\begin{pmatrix} 0.697 & 0.486 & 0.177 & 0.497 \\ 0.486 & 0.697 & 0.497 & 0.177 \\ 0.177 & 0.497 & 0.817 & 0.234 \\ 0.497 & 0.177 & 0.234 & 0.817 \end{pmatrix} . \tag{3.29}$$

The estimated mixing matrix for the four families of quarks predicts quite a strong couplings between the fourth and the other three families, limiting (due to the assumptions and approximations we made, which manifest in the symmetric mixing matrices) some of the matrix elements of the three families as well.

The estimated mixing matrix for the four families of leptons predicts very probably far too strong couplings between the known three and the fourth family (although they are not in contradiction with the report in[26]).

Let us now repeat the number of input data and the number of predictions:
i. We take as the input data the experimental masses (the three u-quark masses, the three d-quark masses, the three electron masses and the three (very weakly known) neutrino masses),
ii. The quark mixing matrix and the (weakly known) lepton mixing matrix.

By taking into account relations among the mass matrix elements for the four types of spinors (u-quarks, d-quarks, neutrinos and electrons) as suggested by the approach unifying spins and charges (after some additional assumptions, approximations and simplifications, which all seam reasonable from the point of view of the experimental data and the fact that we want to obtain at least some rough estimations and come to at least some rough predictions for the approach to see whether it is not in a very severe contradiction with the experimental data) for the "low energy region", we were able to fit within the experimental accuracy and using the Monte-Carlo procedure all the *known experimental data and predict four masses* (the masses of the quarks and leptons of the fourth family) and *the corresponding mixing matrices for quarks and for leptons*.

Let us end up this section by repeating that all the predictions must be taken as a rough estimate, since they follow from the approach unifying spins and charges after several approximations and assumptions, which we made to be able to come in quite a short way to simple and also (very) transparent predictions, used as a first step to much more sophisticated calculations.

3.5 Discussions and conclusions

In this paper and in the previous one[11] we study a possibility that the approach of one of us[1,2,3,4,5,6,7,8,9,10], unifying spins and charges, might be a new right way for answering those of the open questions of the Standard model of the electroweak and colour interaction, which are connected with the appearance of families of fermions, of the Yukawa couplings and of the weak scale: Why do only the left handed spinors carry the weak charge, while the right handed are weak chargeless? Where do the families of the quarks and the leptons come from? What does determine the strenghts of the Yukawa couplings and the weak scale?

We can conclude that *within the approach unifying spins and charges the answer to the question, why do only the left handed spinors carry the weak charge, while the right handed ones are weak chargeless, does exist*: The representation of one Weyl spinor of the group SO(1,13), analyzed with respect to the properties of the subgroups $SO(1,7) \times SU(3) \times U(1)$ of this group and further with respect to SU(2) and the second U(1), manifests the left handed weak charged quarks and leptons and the right handed weak chargeless quarks and leptons.

The approach answers as well the question about a possible origin of (by the Higgs weak charge) *"dressing" of the right handed quarks and leptons in the Standard model*: The approach proposes the Lagrange density for fermions in $d(= 1 + 13)$-dimensional space (a simple one) in which fermions interact with only the gravity (the gauge fields of the momentum - vielbeins - and the two kinds of the Clifford algebra objects - spin connections). It is a part of the spin connection field of the Poincaré group, which connects the right handed weak chargeless spinors with the left handed weak charged ones, playing the role of the Higgs field (and the Yukawa couplings within a family) of the Standard model.

The approach is answering also the question about the origin of the families of quarks and leptons: Two kinds of the Clifford algebra objects gauging two kinds of spin connection fields, are assumed. One kind takes care of the spin and the charges and of connecting right handed weak chargeless fermions with left handed weak charged fermions. The other kind takes care of the families of fermions and consequently of the Yukawa couplings among the families contributing also to the diagonal elements.

In the previous paper[11] we derived from the approach the expressions for the Yukawa couplings for four families of quarks and leptons.

It is a long way from the starting simple Lagrange density for spinors carrying only the spins and interacting with only the vielbeins and spin connections to the observable quarks and leptons. To treat breaking of symmetries properly, taking into account all perturbative and nonperturbative effects, boundary conditions and other effects (by treating gauge gravitational fields in the same way as ordinary gauge fields, since the scale of breaking $SO(1, 13)$ is supposed to be far from the Planck scale) is a huge project.

The purpose of this paper is to estimate whether has the approach unifying spins and charges at all a chance to be the right way beyond the Standard model. Accordingly we tried to estimate in a rough way what does the approach predict for a low energy physics in $d = 1 + 3$. To be able to make any predictions (in a simple enough way) we made several approximations, assumptions and simplifications, which look acceptble from the point of view of the known experimental data and the fact that only a very preliminary prediction is looked for.

Approximations, assumptions and simplifications we made enable simple and transparent view on the mass matrices for the four families of quarks and leptons in terms of the spin connection fields of the two kinds and allow to predict masses of the fourth family of quarks and leptons and of the corresponding matrix elements of the mixing matrices. We treat quarks and leptons equivalently, no Majorana leptons are taken into account in this study.

The assumptions, approximations and simplifications we made (which are not the starting assumtion of our approach unifying spins and charges) are presented in the ref.[11] and in Sect.3.2:

i. The break of symmetries of the group $SO(1,13)$ (the Poincaré group in $d = 1+13$) into $SO(1,7) \times SU(3) \times U(1)$ occurs in a way that only massless spinors in $d = 1+7$ with the charge $SU(3) \times U(1)$ survive. (Our work on the compactification of a massless spinor in $d = 1+5$ into $d = 1+3$ and a finite disk gives us some hope that such an assumption might be justified[13].) The requirement that the terms with $S^{5a}\omega_{5ab}$ and $S^{6a}\omega_{6ab}$ do not contribute to the mass term, assures that the charge $Q = \tau^{41} + S^{56}$ is conserved at low energies.

ii. The break of symmetries influences both, the (Poincaré) symmetry described by S^{ab} and the symmetries described by $\tilde{S}^{ab}$, and in a way that there are no terms, which would transform $\overset{56}{(\tilde{+})}$ into $\overset{56}{[\tilde{+}]}$. This assumption can be explained by a break of the symmetry $SO(1,7)$ into $SO(1+5) \times U(1)$ in the $\tilde{S}^{ab}$ sector. We also assume that the terms which include components $p_s, s = 5,..,14$, of the momentum p^a do not contribute to the mass matrices.

iii. We make estimates on a "tree level", taking effects bellow the tree level into account by allowing the matrix elements to depend on the type of fermions in a way, that the corresponding ratios are (very close to) rational numbers - which seems to be acceptable by the approach.

iv. We assume the mass matrices to be real and symmetric and diagonalizable in two steps, first in two by two by diagonal matrices and then further to diagonal ones (which is suggested by the fact that this would happen if $SO(1,5)$ breaks (approximately) to $SU(2) \times SU(2) \times U(1)$ in the $\tilde{S}$ sector, while the break of $SO(1,5)$ to $SU(3) \times U(1)$ instead, would make the fourth family (approximately) decoupled from the first three and would accordingly strongly change our - very preliminary - results. (While such a break seems to be even acceptable when describing properties of leptons, it would predict for quarks much too strong couplings between the third and the first two families with respect to the measured values.)

Taking all the above assumptions into account we then relate the free parameters of the mass matrices with the measured experimental data within the experimental accuracy treating quarks and leptons equivalently (and not taking into account a possible existence of Majorana neutrinos). Making a rough prediction of the properties of the fourth family of the two kinds of quarks and the two kinds of leptons: of their masses and the corresponding matrix elements of the mixing matrices, we found that our results are in agreement with the analyses of refs.[18,19,20]. We predict the fourth family masses $m_{u_4} = 285$ GeV, $m_{d_4} = 224$ GeV, $m_{\nu_4} = 65$ GeV, $m_{e_4} = 129$ GeV. Predictions for the couplings between the fourth and the other three families seem reasonable for quarks, while for leptons the corresponding mixing matrix elements might suggest that either different break of symmetries in the $\tilde{S}^{ab}$ sector from the assumed one, or the Majorana neutrinos, or both effects should at least be further studied.

To try to answer within the approach unifying spins and charges the open question of the Standard model: Why the weak scale appears as it does? a more detailed study of the breaks of symmetries in both sectors (S^{ab} and $\tilde{S}^{ab}$) is needed.

What we can conclude, after making in this paper a first rough step towards more justified results (by allowing several approximations and assumptions) in order to find out, whether something very essential and unexpected can go wrong with our approach is, that the approach of one of us unifying spins and charges might have a real chance to go successfully beyond the Standard model.

Acknowledgments

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4 Cosmoparticle Physics: Cross-disciplinary Study of Physics Beyond the Standard Model

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Abstract. Cosmoparticle physics is the natural result of development of mutual relationship between cosmology and particle physics. Its prospects offer the way to study physics beyond the Standard Model and the true history of the Universe, based on it, in the proper combination of their indirect physical, astrophysical and cosmological signatures. We may be near the first positive results in this direction. The basic ideas of cosmoparticle physics are briefly reviewed.

4.1 Cosmoparticle physics as the solution of Uhroboros puzzle

Cosmoparticle physics originates from the well established relationship between microscopic and macroscopic descriptions in theoretical physics. Remind the links between statistical physics and thermodynamics, or between electrodynamics and theory of electron. To the end of the XX Century the new level of this relationship was realized. It followed both from the cosmological necessity to go beyond the world of known elementary particles in the physical grounds for inflationary cosmology with baryosynthesis and dark matter as well as from the necessity for particle theory to use cosmological tests as the important and in many cases unique way to probe its predictions.

The convergence of the frontiers of our knowledge in micro- and macro worlds leads to the wrong circle of problems, illustrated by the mystical Uhroboros (self-eating-snake). The Uhroboros puzzle may be formulated as follows: *The theory of the Universe is based on the predictions of particle theory, that need cosmology for their test*. Cosmoparticle physics [1], [2], [3] offers the way out of this wrong circle. It studies the fundamental basis and mutual relationship between micro-and macro-worlds in the proper combination of physical, astrophysical and cosmological signatures.

4.2 Cosmological pattern of particle physics

Let's specify in more details the set of links between fundamental particle properties and their cosmological effects.

The role of particle content in the Einstein equations is reduced to its contribution into energy-momentum tensor. So, the set of relativistic species, dominating in the Universe, realizes the equation of state $p = \varepsilon/3$ and the relativistic stage of expansion. The difference between relativistic bosons and fermions or various bosonic (or fermionic) species is accounted by the statistic weight of respective degree of freedom. The very treatment of different species of particles as equivalent degrees of freedom physically assumes strict symmetry between them.

Such strict symmetry is not realized in Nature. There is no exact symmetry between bosons and fermions (e.g. supersymmetry). There is no exact symmetry between various quarks and leptons. The symmetry breaking implies the difference in particle masses. The particle mass pattern reflects the hierarchy of symmetry breaking.

Noether's theorem relates the exact symmetry to conservation of respective charge. The lightest particle, bearing the strictly conserved charge, is absolutely stable. So, electron is absolutely stable, what reflects the conservation of electric charge. In the same manner the stability of proton is conditioned by the conservation of baryon charge. The stability of ordinary matter is thus protected by the conservation of electric and baryon charges, and its properties reflect the fundamental physical scales of electroweak and strong interactions. Indeed, the mass of electron is related to the scale of the electroweak symmetry breaking, whereas the mass of proton reflects the scale of QCD confinement.

Extensions of the standard model imply new symmetries and new particle states. The respective symmetry breaking induces new fundamental physical scales in particle theory. If the symmetry is strict, its existence implies new conserved charge. The lightest particle, bearing this charge, is stable. The set of new fundamental particles, corresponding to the new strict symmetry, is then reflected in the existence of new stable particles, which should be present in the Universe and taken into account in the total energy-momentum tensor.

Most of the known particles are unstable. For a particle with the mass m the particle physics time scale is $t \sim 1/m$, so in particle world we refer to particles with lifetime $\tau \gg 1/m$ as to metastable. To be of cosmological significance metastable particle should survive after the temperature of the Universe T fell down below $T \sim m$, what means that the particle lifetime should exceed $t \sim (m_{Pl}/m) \cdot (1/m)$. Such a long lifetime should find reason in the existence of an (approximate) symmetry. From this viewpoint, cosmology is sensitive to the most fundamental properties of microworld, to the conservation laws reflecting strict or nearly strict symmetries of particle theory.

However, the mechanism of particle symmetry breaking can also have the cosmological impact. Heating of condensed matter leads to restoration of its symmetry. When the heated matter cools down, phase transition to the phase of broken symmetry takes place. In the course of the phase transitions, corresponding to given type of symmetry breaking, topological defects can form. One can directly observe formation of such defects in liquid crystals or in superfluid He. In the same manner the mechanism of spontaneous breaking of particle symmetry implies restoration of the underlying symmetry. When temperature decreases in the course of cosmological expansion, transitions to the phase of broken symmetry can lead, depending on the symmetry breaking pattern, to formation of topological defects in very early Universe. The defects can represent the new form of stable particles (as it is in the case of magnetic monopoles), or the form of extended structures, such as cosmic strings or cosmic walls.

In the old Big bang scenario the cosmological expansion and its initial conditions was given *a priori*. In the modern cosmology the expansion of the Universe and its initial conditions is related to the process of inflation. The global properties of the Universe as well as the origin of its large scale structure are the result of this process. The matter content of the modern Universe is also originated from the physical processes: the baryon density is the result of baryosynthesis and the nonbaryonic dark matter represents the relic species of physics of the hidden sector of particle theory. Physics, underlying inflation, baryosynthesis and dark matter, is referred to the extensions of the standard model, and the variety of such extensions makes the whole picture in general ambiguous. However, in the framework of each particular physical realization of inflationary model with baryosynthesis and dark matter the corresponding model dependent cosmological scenario can be specified in all the details. In such scenario the main stages of cosmological evolution, the structure and the physical content of the Universe reflect the structure of the underlying physical model. The latter should include with necessity the standard model, describing the properties of baryonic matter, and its extensions, responsible for inflation, baryosynthesis and dark matter. In no case the cosmological impact of such extensions is reduced to reproduction of these three phenomena only. The nontrivial path of cosmological evolution, specific for each particular realization of inflational model with baryosynthesis and nonbaryonic dark matter, always contains some additional model dependent cosmologically viable predictions, which can be confronted with astrophysical data. The part of cosmoparticle physics, called cosmoarcheology, offers the set of methods and tools probing such predictions.

4.3 Cosmoarcheology of new physics

Cosmoarcheology considers the results of observational cosmology as the sample of the experimental data on the possible existence and features of hypothetical phenomena predicted by particle theory. To undertake the *Gedanken Experiment* with these phenomena some theoretical framework to treat their origin and evolution in the Universe should be assumed. As it was pointed out in [4] the choice of such framework is a nontrivial problem in the modern cosmology.

Indeed, in the old Big bang scenario any new phenomenon, predicted by particle theory was considered in the course of the thermal history of the Universe, starting from Planck times. The problem is that the bedrock of the modern cosmology, namely, inflation, baryosynthesis and dark matter, is also based on experimentally unproven part of particle theory, so that the test for possible effects of new physics is accomplished by the necessity to choose the physical basis for such test. There are two possible solutions for this problem: a) a crude model independent comparison of the predicted effect with the observational data and

b) the model dependent treatment of considered effect, provided that the model, predicting it, contains physical mechanism of inflation, baryosynthesis and dark matter.

The basis for the approach (a) is that whatever happened in the early Universe its results should not contradict the observed properties of the modern Universe. The set of observational data (especially, the light element abundance and spectrum of black body radiation) put severe constraint on the deviation from thermal evolution after 1 s of expansion, what strengthens the model independent conjectures of approach (a).

One can specify the new phenomena by their net contribution into the cosmological density and by forms of their possible influence on parameters of matter and radiation. In the first aspect we can consider strong and weak phenomena. Strong phenomena can put dominant contribution into the density of the Universe, thus defining the dynamics of expansion in that period, whereas the contribution of weak phenomena into the total density is always subdominant. The phenomena are time dependent, being characterized by their time-scale, so that permanent (stable) and temporary (unstable) phenomena can take place. They can have homogeneous and inhomogeneous distribution in space. The amplitude of density fluctuations $\delta \equiv \delta\rho/\rho$ measures the level of inhomogeneity relative to the total density, ρ. The partial amplitude $\delta_i \equiv \delta\rho_i/\rho_i$ measures the level of fluctuations within a particular component with density ρ_i, contributing into the total density $\rho = \sum_i \rho_i$. The case $\delta_i \geq 1$ within the considered i-th component corresponds to its strong inhomogeneity. Strong inhomogeneity is compatible with the smallness of total density fluctuations, if the contribution of inhomogeneous component into the total density is small: $\rho_i \ll \rho$, so that $\delta \ll 1$.

The phenomena can influence the properties of matter and radiation either indirectly, say, changing of the cosmological equation of state, or via direct interaction with matter and radiation. In the first case only strong phenomena are relevant, in the second case even weak phenomena are accessible to observational data. The detailed analysis of sensitivity of cosmological data to various phenomena of new physics are presented in [3]. This set of astrophysical constraints confronts phenomena, predicted by cosmophenomenology as cosmological consequences of particle theory.

4.4 Cosmophenomenology of new physics

To study the imprints of new physics in astrophysical data cosmoarcheology implies the forms and means in which new physics leaves such imprints. So, the important tool of cosmoarcheology in linking the cosmological predictions of particle theory to observational data is the *Cosmophenomenology* of new physics. It studies the possible hypothetical forms of new physics, which may appear as cosmological consequences of particle theory, and their properties, which can result in observable effects.

4.4.1 Primordial particles

The simplest primordial form of new physics is the gas of new stable massive particles, originated from early Universe. For particles with the mass m, at high temperature $T > m$ the equilibrium condition, $n \cdot \sigma v \cdot t > 1$ is valid, if their annihilation cross section $\sigma > 1/(m m_{Pl})$ is sufficiently large to establish the equilibrium. At $T < m$ such particles go out of equilibrium and their relative concentration freezes out. More weakly interacting species decouple from plasma and radiation at $T > m$, when $n \cdot \sigma v \cdot t \sim 1$, i.e. at $T_{dec} \sim (\sigma m_{Pl})^{-1}$. The maximal temperature, which is reached in inflationary Universe, is the reheating temperature, T_r, after inflation. So, the very weakly interacting particles with the annihilation cross section $\sigma < 1/(T_r m_{Pl})$, as well as very heavy particles with the mass $m \gg T_r$ can not be in thermal equilibrium, and the detailed mechanism of their production should be considered to calculate their primordial abundance.

Decaying particles with the lifetime τ, exceeding the age of the Universe, t_U, $\tau > t_U$, can be treated as stable. By definition, primordial stable particles survive to the present time and should be present in the modern Universe. The net effect of their existence is given by their contribution into the total cosmological density. They can dominate in the total density being the dominant form of cosmological dark matter, or they can represent its subdominant fraction. In the latter case more detailed analysis of their distribution in space, of their condensation in galaxies, of their capture by stars, Sun and Earth, as well as of the effects of their interaction with matter and of their annihilation provides more sensitive probes for their existence. In particular, hypothetical stable neutrinos of the 4th generation with the mass about 50 GeV are predicted to form the subdominant form of the modern dark matter, contributing less than 0,1 % to the total density. However, direct experimental search for cosmic fluxes of weakly interacting massive particles (WIMPs) may be sensitive to the existence of such component [5]. It was shown in [6,7,8] that annihilation of 4th neutrinos and their antineutrinos in the Galaxy can explain the galactic gamma-background, measured by EGRET in the range above 1 GeV, and that it can give some clue to explanation of cosmic positron anomaly, claimed to be found by HEAT. 4th neutrino annihilation inside the Earth should lead to the flux of underground monochromatic neutrinos of known types, which can be traced in the analysis of the already existing and future data of underground neutrino detectors [8].

New particles with electric charge and/or strong interaction can form anomalous atoms and contain in the ordinary matter as anomalous isotopes. For example, if the lightest quark of 4th generation is stable, it can form stable charged hadrons, serving as nuclei of anomalous atoms of e.g. anomalous helium [9,10]. Massive negatively charged particles bind with ^4He in atom-like systems, as soon as helium is formed in Big Bang Nucleosynthesis [11]. Since particles Q^- with charge -1 form positively charged "ion" $[^4HeQ^-]^+$, which behave as anomalous hydrogen, the grave problem of anomalous hydrogen overproduction is inevitable for particle models predicting such particles, as it is the case for the model of "Sinister" Universe [12]. Particles Q^{-2} with charge -2 form neutral O-helium "atom" $[^4HeQ^{-2}]$, in which the charge of α particle is shielded [13,14]. It removes

Coulomb barrier between α particle and nuclei, giving rise to new paths of nuclear transformations, catalyzed by O-helium.

Primordial unstable particles with the lifetime, less than the age of the Universe, $\tau < t_U$, can not survive to the present time. But, if their lifetime is sufficiently large to satisfy the condition $\tau \gg (m_{Pl}/m) \cdot (1/m)$, their existence in early Universe can lead to direct or indirect traces. Cosmological flux of decay products contributing into the cosmic and gamma ray backgrounds represents the direct trace of unstable particles. If the decay products do not survive to the present time their interaction with matter and radiation can cause indirect trace in the light element abundance or in the fluctuations of thermal radiation. If the particle lifetime is much less than 1s the multi-step indirect traces are possible, provided that particles dominate in the Universe before their decay. On the dust-like stage of their dominance black hole formation takes place, and the spectrum of such primordial black holes traces the particle properties (mass, frozen concentration, lifetime) [22]. The particle decay in the end of dust like stage influences the baryon asymmetry of the Universe. In any way cosmophenomenoLOGICAL chains link the predicted properties of even unstable new particles to the effects accessible in astronomical observations. Such effects may be important in the analysis of the observational data.

So, the only direct evidence for the accelerated expansion of the modern Universe comes from the distant SN I data. The data on the cosmic microwave background (CMB) radiation and large scale structure (LSS) evolution (see e.g. [5]) prove in fact the existence of homogeneously distributed dark energy and the slowing down of LSS evolution at $z \leq 3$. Homogeneous negative pressure medium (Λ-term or quintessence) leads to *relative* slowing down of LSS evolution due to acceleration of cosmological expansion. However, both homogeneous component of dark matter and slowing down of LSS evolution naturally follow from the models of Unstable Dark Matter (UDM) (see [3] for review), in which the structure is formed by unstable weakly interacting particles. The weakly interacting decay products are distributed homogeneously. The loss of the most part of dark matter after decay slows down the LSS evolution. The dominantly invisible decay products can contain small ionizing component [23]. Thus, UDM effects will deserve attention, even if the accelerated expansion is proved.

4.4.2 Archioles

Parameters of new stable and metastable particles are also determined by a pattern of particle symmetry breaking. This pattern is reflected in the succession of phase transitions in the early Universe. Phase transitions of the first order proceed through bubble nucleation, which can result in black hole formation (see e.g. [15] and [16] for review and references). Phase transitions of the second order can lead to formation of topological defects, such as walls, string or monopoles. The observational data put severe constraints on magnetic monopole and cosmic wall production, as well as on parameters of cosmic strings. A succession of phase transitions can change the structure of cosmological defects. More complicated forms, such as walls-surrounded-by-strings can appear. Such structures can be unstable, but their existence can lead the trace in the nonhomogeneous distribution of dark matter and in large scale correlations in the nonhomogeneous dark matter structures, such as *archioles* [17], which arise in the result of cosmological evolution of pseudo-Nambu-Goldstone field.

A wide class of particle models possesses a symmetry breaking pattern, which can be effectively described by pseudo-Nambu–Goldstone (PNG) field and which corresponds to formation of unstable topological defect structure in the early Universe (see [16] for review and references). The Nambu–Goldstone nature in such an effective description reflects the spontaneous breaking of global U(1) symmetry, resulting in continuous degeneracy of vacua. The explicit symmetry breaking at smaller energy scale changes this continuous degeneracy by discrete vacuum degeneracy.

At high temperatures such a symmetry breaking pattern implies the succession of second order phase transitions. In the first transition, continuous degeneracy of vacua leads, at scales exceeding the correlation length, to the formation of topological defects in the form of a string network; in the second phase transition, continuous transitions in space between degenerated vacua form surfaces: domain walls surrounded by strings. This last structure is unstable, but, as was shown in the example of the invisible axion [17,18,19], it is reflected in the large scale inhomogeneity of distribution of energy density of coherent PNG (axion) field oscillations. This energy density is proportional to the initial value of phase, which acquires dynamical meaning of amplitude of axion field, when axion mass is switched on in the result of the second phase transition.

The value of phase changes by 2π around string. This strong nonhomogeneity of phase leads to corresponding nonhomogeneity of energy density of coherent PNG (axion) field oscillations. Usual argument (see e.g. [20] and references therein) is that this nonhomogeneity is essential only on scales, corresponding to mean distance between strings. This distance is small, being of the order of the scale of cosmological horizon in the period, when PNG field oscillations start. However, since the nonhomogeneity of phase follows the pattern of axion string network this argument misses large scale correlations in the distribution of energy density of field oscillations.

Indeed, numerical analysis of string network (see review in [21]) indicates that large string loops are strongly suppressed and the fraction of about 80% of string length, corresponding to "infinite" strings, remains virtually the same in all large scales. This property is the other side of the well known scale invariant character of string network. Therefore the correlations of energy density should persist on large scales, as it was revealed in [17,18,19].

The large scale correlations in topological defects and their imprints in primordial inhomogeneities is the indirect effect of inflation, if phase transitions take place after reheating of the Universe. Inflation provides in this case the equal conditions of phase transition, taking place in causally disconnected regions.

4.4.3 Primordial clouds of massive PBH

If the phase transitions take place on inflational stage new forms of primordial large scale correlations appear. The example of global U(1) symmetry, broken spontaneously in the period of inflation and successively broken explicitly after reheating, was recently considered in [24]. In this model, spontaneous U(1) symmetry breaking at inflational stage is induced by the vacuum expectation value $\langle\psi\rangle = f$ of a complex scalar field $\Psi = \psi \exp(i\theta)$, having also explicit symmetry breaking term in its potential $V_{eb} = \Lambda^4(1 - \cos\theta)$. The latter is negligible in the period of inflation, if $f \gg \Lambda$, so that there appears a valley relative to values of phase in the field potential in this period. Fluctuations of the phase θ along this valley, being of the order of $\Delta\theta \sim H/(2\pi f)$ (here H is the Hubble parameter at inflational stage) change in the course of inflation its initial value within the regions of smaller size. Owing to such fluctuations, for the fixed value of θ_{60} in the period of inflation with *e-folding* $N = 60$ corresponding to the part of the Universe within the modern cosmological horizon, strong deviations from this value appear at smaller scales, corresponding to later periods of inflation with $N < 60$. If $\theta_{60} < \pi$, the fluctuations can move the value of θ_N to $\theta_N > \pi$ in some regions of the Universe. After reheating, when the Universe cools down to temperature $T = \Lambda$ the phase transition to the true vacuum states, corresponding to the minima of V_{eb} takes place. For $\theta_N < \pi$ the minimum of V_{eb} is reached at $\theta_{vac} = 0$, whereas in the regions with $\theta_N > \pi$ the true vacuum state corresponds to $\theta_{vac} = 2\pi$. For $\theta_{60} < \pi$ in the bulk of the volume within the modern cosmological horizon $\theta_{vac} = 0$. However, within this volume there appear regions with $\theta_{vac} = 2\pi$. These regions are surrounded by massive domain walls, formed at the border between the two vacua. Since regions with $\theta_{vac} = 2\pi$ are confined, the domain walls are closed. After their size equals the horizon, closed walls can collapse into black holes. The minimal mass of such black hole is determined by the condition that it's Schwarzschild radius, $r_g = 2GM/c^2$ exceeds the width of the wall, $l \sim f/\Lambda^2$, and it is given by $M_{min} \sim f(m_{Pl}/\Lambda)^2$. The maximal mass is determined by the mass of the wall, corresponding to the earliest region $\theta_N > \pi$, appeared at inflational stage. This mechanism can lead to formation of primordial black holes of a whatever large mass (up to the mass of AGNs [25]). Such black holes appear in a form of primordial black hole clusters, exhibiting fractal distribution in space [24]. It can shed new light on the problem of galaxy formation.

4.4.4 Antimatter stars in Galaxy

Primordial strong inhomogeneities can also appear in the baryon charge distribution. The appearance of antibaryon domains in the baryon asymmetrical Universe, reflecting the inhomogeneity of baryosynthesis, is the profound signature of such strong inhomogeneity [26]. On the example of the model of spontaneous baryosynthesis (see [27] for review) the possibility for existence of antimatter domains, surviving to the present time in inflationary Universe with inhomogeneous baryosynthesis was revealed in [28]. Evolution of sufficiently dense antimatter domains can lead to formation of antimatter globular clusters [29]. The existence of such cluster in the halo of our Galaxy should lead to the pollution of the galactic halo by antiprotons. Their annihilation can reproduce [30] the observed galactic gamma background in the range tens-hundreds MeV.

This observed background puts upper limit on the total mass of antimatter stars in Galaxy ($M \leq 10^5 M_\odot$). The prediction of antihelium component of cosmic rays [31], accessible to future searches for cosmic ray antinuclei in PAMELA and AMS02 experiments, as well as of antimatter meteorites [32] provides the direct experimental test for this hypothesis. In this test planned sensitivity of AMS02 experiment will reach the lower limit for the mass of antimatter stars in Galaxy. This limit ($M \geq 10^3 M_\odot$) follows from the condition that antimatter domain should be sufficiently large to survive and sufficiently dense to provide star formation.

So the primordial strong inhomogeneities in the distribution of total, dark matter and baryon density in the Universe is the new important phenomenon of cosmological models, based on particle models with hierarchy of symmetry breaking.

4.5 The encircled pyramid

New physics follows from the necessity to extend the Standard model. White spots in representations of symmetry groups, considered in extensions of Standard model, correspond to new unknown particles. Extension of gauge symmetry puts into consideration new gauge fields, mediating new interactions. Global symmetry breaking results in the existence of Goldstone boson fields.

For a long time necessity to extend the Standard model had purely theoretical reasons. Aesthetically, because full unification is not achieved in the Standard model; practically, because it contains some internal inconsistencies. It does not seem complete for cosmology. One has to go beyond the Standard model to explain inflation, baryosynthesis and nonbaryonic dark matter. Recently there has appeared a set of experimental evidences for the existence of neutrino oscillations, for effects of new physics in anomalous magnetic moment of muon ($g-2$), for cosmic WIMPs and for double neutrinoless beta decay (see for recent review [5]). Whatever is the accepted status of some of these evidences, they indicate that the experimental searches may have already crossed the border of new physics.

In particle physics direct experimental probes for the predictions of particle theory are most attractive. Predictions of new charged particles, such as supersymmetric particles or quarks and leptons of new generation, are accessible to experimental search at accelerators of new generation, if their masses are in 100GeV-1TeV range. However, predictions related to higher energy scale need non-accelerator or indirect means for their test.

Search for rare processes, such as proton decay, neutrino oscillations, neutrinoless beta decay, precise measurements of parameters of known particles, experimental searches for dark matter represent the widely known forms of such means. Cosmoparticle physics offers the nontrivial extensions of indirect and non-accelerator searches for new physics and its possible properties. In experimental cosmoarcheology the data is to be obtained, necessary to link the cosmophenomenology of new physics with astrophysical observations (See [4]). In experimental cosmoparticle physics the parameters, fixed from the consitency of cosmological models and observations, define the level, at which the new types of particle processes should be searched for (see [33]).

4.5.1 New quarks and leptons

The theories of everything should provide the complete physical basis for cosmology. The problem is that the string theory [34] is now in the form of ”theoretical theory”, for which the experimental probes are widely doubted to exist. The development of cosmoparticle physics can remove these doubts. In its framework there are two directions to approach the test of theories of everything.

One of them is related with the search for the experimentally accessible effects of heterotic string phenomenology. The mechanism of compactification and symmetry breaking leads to the prediction of homotopically stable objects [35] and shadow matter [36], accessible to cosmoarcheological means of cosmoparticle physics. The condition to reproduce the Standard model naturally leads in the heterotic string phenomenology to the prediction of fourth generation of quarks and leptons [37] with a stable massive 4th neutrino [6], what can be the subject of complete experimental test in the near future. The comparison between the rank of the unifying group E_6 ($r = 6$) and the rank of the Standard model ($r = 4$) implies the existence of new conserved charges and new (possibly strict) gauge symmetries. New strict gauge U(1) symmetry (similar to U(1) symmetry of electrodynamics) is possible, if it is ascribed to the fermions of 4th generation. This hypothesis explains the difference between the three known types of neutrinos and neutrino of 4th generation. The latter possesses new gauge charge and, being Dirac particle, can not have small Majorana mass due to sea saw mechanism. If the 4th neutrino is the lightest particle of the 4th quark-lepton family, strict conservation of the new charge makes massive 4th neutrino to be absolutely stable. Following this hypothesis [37] quarks and leptons of 4th generation are the source of new long range interaction (y-electromagnetism), similar to the electromagnetic interaction of ordinary charged particles. New strictly conserved local U(1) gauge symmetries can also arise in the alternative approach to extension of standard model [14] based on almost commutative geometry [38].

It is interesting, that heterotic string phenomenology embeds even in its simplest realisation both supersymmetric particles and the 4th family of quarks and leptons, in particular, the two types of WIMP candidates: neutralinos and massive stable 4th neutrinos. So in the framework of this phenomenology the multicomponent analysis of WIMP effects is favorable.

The motivation for existence of new quarks and leptons also follows from geometrical approach to particle unification [39] and from models of extended technicolor [40,41].

New quarks and charged leptons can be metastable and have lifetime, exceeding the age of the Universe. It gives rise to a new form of stable matter around us - to composite dark matter, whose massive charged constituents are hidden in atom-like systems. Inevitable by-product of creation of such matter in Universe is the existence of O-helium ”atom”, in which positive charge of α particle is shielded by negative charge of massive component. O-helium can be a fraction [14] or even dominant form of dark matter [13] and search for its charged constituents at accelerators and cosmic rays [42] acquires the significance of direct experimental test for this form of dark matter.

In the above approach some particular phenomenological features of simplest variants of string theory are studied. The other direction is to elaborate the extensive phenomenology of theories of everything by adding to the symmetry of the Standard model the (broken) symmetries, which have serious reasons to exist. The existence of (broken) symmetry between quark-lepton families, the necessity in the solution of strong CP-violation problem with the use of broken Peccei-Quinn symmetry, as well as the practical necessity in supersymmetry to eliminate the quadratic divergence of Higgs boson mass in electroweak theory is the example of appealing additions to the symmetry of the Standard model. The horizontal unification and its cosmology represent the first step on this way, illustrating the approach of cosmoparticle physics to the elaboration of the proper phenomenology for theories of everything [43].

We can conclude that from the very beginning to the modern stage, the evolution of Universe is governed by the forms of matter, different from those we are built of and observe around us. From the very beginning to the present time, the evolution of the Universe was governed by physical laws, which we still don't know. Observational cosmology offers strong evidences favoring the existence of processes, determined by new physics, and the experimental physics approaches to their investigation.

Cosmoparticle physics [1] [2], studying the physical, astrophysical and cosmological impact of new laws of Nature, explores the new forms of matter and their physical properties, what opens the way to use the corresponding new sources of energy and new means of energy transfer. It offers the great challenge for the new Millennium. Its solution for the Uhroboros puzzle is as paradoxical, as "encircled pyramid" - cosmoparticle physics implies complex cross-disciplinary studies, offering the multi-dimensional exit from the plane with wrong circle of problems in the joint of cosmology and particle physics.

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5 Discussion Section on 4th Generation

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Abstract. I briefly stipulate here some ideas, which were considered in the Discussion Section.

5.1 Some phenomenological aspects of 4th generation in geometrical approach

In the presented realization of geometrical approach [1] 4th generation by construction is linked to 3d generation. In such realization mixing and transitions between these generations are inevitable. It makes all the particles of 4th generation unstable. It would be interesting to estimate their lifetime. But, in any case, this feature is important for accelerator search for quarks and leptons of 4th generation. Processes of creation and decay of these particles at accelerators have rather distinct experimental signatures and can be clearly discriminated. On the other hand, being unstable, hadrons and leptons of 4th generation should not be present in cosmic rays.

The predicted values of 4th generation particles are in some cases accessible to test even with the use of existing experimental data [2]. For instant, unstable neutrino of 4th generation with the mass $\sim$ 80GeV should have been seen in LEP.

In the approach [1] another problem is also of interest: if it is possible to have the lightest particles of the 4th generation stable. In this case the 4th generation is decoupled from three known families and their possible mass pattern can not be directly deduced from mixing with known particles and from their known properties.

5.2 Composite dark matter from Technicolor

Cosmological aspects [3] of technicolor models [4] were concentrated on studies of possible WIMP-like candidates for dark matter species. However, this approach also provides the possibility of stable techniparticles T^{--} with charge -2.

If the model provides the possibility to generate in early Universe excess of these particles, corresponding to cosmological dark matter density, atomic bound states of these particles with primordial ^{4}He can play the role of composite dark matter in the form of techno-O-helium $^4\mathrm{He}T^{--}$. Formation and evolution of this composite dark matter will follow the trend of O-helium dark matter, studied in [5] for the case of stable quarks of 4th generation. Experimental search for stable techniparticles T^{--} in cosmic rays and at accelerators is possible similar to the case of new stable leptons [6,42].

5.3 Mass self-adjustment for CLEP neutrinos

In the model of CLEP states, offered in [8], the following mechanism of self-adjustment of neutrino mass can be realized. Near a galaxy as the neutrinos expand to the outside their density decreases and neutrino mass increases. Then as the mass increases the neutrinos have now a tendency to clump; after they clump, they become dense and therefore the neutrinos are not anymore in the CLEP state, but rather in a higher density state with a corresponding lower mass, which is now preventing neutrinos to be clustered. Therefore neutrino gas expands again to reach low density, to return in CLEP state.

If in the course of cosmological expansion neutrino mass in CLEP state becomes too big some other interesting possibilities arise. For instance a nontrivial realisation is possible for Unstable Neutrino Cosmology and other effects of non-equilibrium particles from CLEP state neutrino decays if this mass becomes too big some other interesting possibilities arise. For instance a nontrivial realisation is possible for Unstable Neutrino Cosmology and other effects of non-equilibrium particles from CLEP state neutrino decays[9].

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6 Involution Requirement on a Boundary Makes Massless Fermions Compactified on a Finite Flat Disk Mass Protected

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Abstract. The genuine Kaluza-Klein-like theories—with no fields in addition to gravity—have difficulties with the existence of massless spinors after the compactification of some space dimensions [1]. We proposed in ref. [2] a boundary condition which allows massless spinors compactified on a flat disk to be of only one handedness. Massless spinors then chirally couple to the corresponding background gauge gravitational field (which solves equations of motion for a free field linear in the Riemann curvature). In this paper we study the same toy model: $M^{(1+3)} \times M^{(2)}$, looking this time for an involution which transforms a space of solutions of Weyl equations in $d = 1 + 5$ from the outside of the flat disk in x^5 and x^6 into its inside (or conversely). The natural boundary condition that on the wall an outside solution must coincide with the corresponding inside one leads to massless spinors of only one handedness (and accordingly mass protected), chirally coupled to the corresponding background gauge gravitational field. We introduce the Hermitean operators of momenta and discuss the orthogonality of solutions, ensuring that to each mass only one solution of equations of motion corresponds.

6.1 Introduction

The major problem of the compactification procedure in all Kaluza-Klein-like theories with only gravity and no additional gauge fields is how to ensure that massless spinors be mass protected after the compactification. Namely, even if we start with only one Weyl spinor in some even dimensional space of $d = 2$ modulo 4 dimensions (i.e. in $d = 2(2n+1)$, $n = 0, 1, 2, \cdots$) so that there appear no Majorana mass if no conserved charges exist and families are allowed, as we have proven in ref. [3], and accordingly with the mass protection from the very beginning, a compactification of m dimensions gives rise to a spinor of one handedness in d with both handedness in $d - m$ and is accordingly not mass protected any longer.

And in addition, since a spin (or the total conserved angular momentum) in the compactified part of space will in $d - m$ space appear as a charge and will manifest both values (positive and negative ones) and since in the second quantization procedure anti particles of opposite charges appear anyhow, doubling the number of massless spinors of both—positive and negative—charges when coming from $d(= 2(2n+1))$-dimensional space down to $d = 4$ and after a second quantized procedure is not in agreement with what we observe. Accordingly there must be some requirements, some boundary conditions, which ensure in a compactification procedure that only spinors of one handedness survive, if Kaluza-Klein-like theories have some meaning. However, the idea of Kaluza and Klein of having only gravity as a gauge field seems too beautiful not to have the realization in Nature.

One of us[4,5,6,7,8] has for long tried to unify the spin and all the charges to only the spin, so that spinors would in $d \geq 4$ carry nothing but a spin and interact accordingly with only the gauge fields of the Poincaré group, that is with vielbeins ${f^{\alpha}}_a$ [1] and spin connections $\omega_{ab\alpha}$, which are the gauge fields of the Poincaré group.

In this paper we take (as we did in the ref. [2]) the covariant momentum of a spinor, when applied on a spinor function ψ, to be

$$p_{0a} = {f^{\alpha}}_a p_{0\alpha}, \quad p_{0\alpha}\psi = p_{\alpha} - \frac{1}{2} S^{cd}\omega_{cd\alpha}. \tag{6.1}$$

A kind of a total covariant derivative of ${e^a}_{\alpha}$ (a vector with both—Einstein and flat index) will be taken to be $p_{0\alpha}{e^a}_{\beta} = i{e^a}_{\beta;\alpha} = i({e^a}_{\beta,\alpha} + {\omega^a}_{d\alpha}{e^d}_{\beta} - {\Gamma^{\gamma}}_{\beta\alpha}{e^a}_{\gamma})$, with the require that this derivative of a vielbein is zero: ${e^a}_{\beta;\alpha} = 0$.

The corresponding Lagrange density $\mathcal{L}$ for a Weyl spinor has the form $\mathcal{L} = E\frac{1}{2}[(\psi^{\dagger}\gamma^0\gamma^a p_{0a}\psi) + (\psi^{\dagger}\gamma^0\gamma^a p_{0a}\psi)^{\dagger}]$ and leads to

$$\mathcal{L} = E\psi^{\dagger}\gamma^0\gamma^a(p_a - \frac{1}{2}S^{cd}\omega_{cda})\psi, \tag{6.2}$$

with $E = \det({e^a}_{\alpha})$ [2].

The authors of this paper have tried to find a way out of this "Witten's no go theorem" for a toy model of $M^{(1+3)}\times$ a flat finite disk in $(1+5)$-dimensional space [2] by postulating a particular boundary condition, which allows a spinor to carry after the compactification only one handedness. Massless spinors then chirally couple to the corresponding background gauge gravitational field, which

[1] ${f^{\alpha}}_a$ are inverted vielbeins to ${e^a}_{\alpha}$ with the properties ${e^a}_{\alpha}{f^{\alpha}}_b = {\delta^a}_b,\ {e^a}_{\alpha}{f^{\beta}}_a = \delta^{\beta}_{\alpha}$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[2] To generate more than one family, we actually observe up to now three families, a second kind of the Clifford algebra objects has also been introduced[5,6,9,10], which anti commute with the ordinary Dirac γ^a matrices ($\{\gamma^a, \tilde{\gamma^b}\}_+ = 0$) and generate equivalent representations with respect to the generators $S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)$ and are used accordingly to generate families[5,6,9,10]. In this work we shall not take families into account.

solves equations of motion for a free field, linear in the Riemann curvature, while the current through the wall is for a massless and massive solutions equal to zero.

In the ref. [2] the boundary condition was written in a covariant way as

$$\hat{\mathcal{R}}\psi|_{\text{wall}} = 0,$$
$$\hat{\mathcal{R}} = \frac{1}{2}(1 - i n^{(\rho)}{}_a\, n^{(\phi)}{}_b\, \gamma^a \gamma^b), \quad \hat{\mathcal{R}}^2 = \hat{\mathcal{R}}$$

with $n^{(\rho)} = (0, 0, 0, 0, \cos\phi, \sin\phi)$, $n^{(\phi)} = (0, 0, 0, 0, -\sin\phi, \cos\phi)$, which are the two unit vectors perpendicular and tangential to the boundary of the disk (at ρ_0), respectively. The projector $\hat{\mathcal{R}}$ can for the above choice of the two vectors $n^{(\rho)}$ and $n^{(\phi)}$ be written as

$$\hat{\mathcal{R}} = \overset{56}{[-]} = \frac{1}{2}(1 - i\gamma^5\gamma^6). \tag{6.3}$$

The reader can find more about the Clifford algebra objects $\overset{ab}{(\pm)}, \overset{ab}{[\pm]}$ in the Appendix (section 6.8).

The boundary condition requires that only massless states (determined by Eq.(6.2)) of one (let us say right) handedness with respect to the compactified disk degrees of freedom are allowed. Accordingly also massless states of only one handedness are allowed also in $d = 1 + 3$.

In this paper we reformulate the above boundary condition as an *involution*, which transforms solutions of equations of motion from outside the boundary of the disk into its inside. We do this by the intention that the limitation of M2 on a finite disk would have a natural explanation, originated in a symmetry relation. We also define the Hermitean momentum p^s and comment on the orthogonality relations of solutions of equations of motion, which fulfill the boundary conditions.

We make use of the technique presented in ref. [9,10] when writing the equations of motion and their solutions. It turns out that all the derivations and discussions appear to be very transparent when using this technique. We briefly repeat this technique in Appendix 6.8.

6.2 Equations of motion and solutions

We assume that a two dimensional space, spanned by x^5 and x^6, is an Euclidean manifold $M^{(2)}$ (with no gravity)

$$f^{\sigma}{}_s = \delta^{\sigma}{}_s, \; \omega_{56s} = 0. \tag{6.4}$$

and accordingly with the rotational symmetry around an origin.

Wave functions describing spinors in $(1+5)$-dimensional space demonstrating $M^{(1+3)} \times M^{(2)}$ symmetry are required to obey the equations of motion

$$\gamma^0\gamma^a p_a \psi^{(6)} = 0, \; a = m, s; \; m = 0, 1, 2, 3; \; s = 5, 6. \tag{6.5}$$

The most general solution for a free particle in $d = 1 + 5$ should be written as a superposition of all four $(2^{6/2-1})$ states of one Weyl representation. We ask the reader to see Appendix 6.8 for the technical details how to write one Weyl representation in terms of the Clifford algebra objects after making a choice of the Cartan sub algebra set, for which we make a choice: S^{03}, S^{12}, S^{56}. In our technique [9] the four states, which all are the eigenstates of the Cartan sub algebra set, are expressed with the following four products of projections ($\stackrel{ab}{[k]}$) and nilpotents ($\stackrel{ab}{(k)}$):

$$
\begin{aligned}
\varphi^1_1 &= \stackrel{56}{(+)}\stackrel{03}{(+i)}\stackrel{12}{(+)}\;\psi_0,\\
\varphi^1_2 &= \stackrel{56}{(+)}\stackrel{03}{[-i]}\stackrel{12}{[-]}\;\psi_0,\\
\varphi^2_1 &= \stackrel{56}{[-]}\stackrel{03}{[-i]})\stackrel{12}{(+)}\;\psi_0,\\
\varphi^2_2 &= \stackrel{56}{[-]}\stackrel{03}{(+i)}\stackrel{12}{[-]}\;\psi_0,
\end{aligned}
\tag{6.6}
$$

where ψ_0 is a vacuum state. If we write the operators of handedness in $d = 1+5$ as $\Gamma^{(1+5)} = \gamma^0\gamma^1\gamma^2\gamma^3\gamma^5\gamma^6$ $(= 23iS^{03}S^{12}S^{56})$, in $d = 1{+}3$ as $\Gamma^{(1+3)} = -i\gamma^0\gamma^1\gamma^2\gamma^3$ $(= 22iS^{03}S^{12})$ and in the two dimensional space as $\Gamma^{(2)} = i\gamma^5\gamma^6$ $(= 2S^{56})$, we find that all four states are left handed with respect to $\Gamma^{(1+5)}$, with the value -1, the first two are right handed and the second two left handed with respect to $\Gamma^{(2)}$, with the values 1 and -1, respectively, while the first two are left handed and the second two right handed with respect to $\Gamma^{(1+3)}$ with the values -1 and 1, respectively.

Taking into account Eq.(6.6) we may write a wave function $\psi^{(6)}$ in $d = 1+5$ as

$$
\psi^{(6)} = (\mathcal{A}\stackrel{56}{(+)} + \mathcal{B}\stackrel{56}{[-]})\psi^{(4)},
\tag{6.7}
$$

where $\mathcal{A}$ and $\mathcal{B}$ depend on x^5 and x^6, while $\psi^{(4)}$ determines the spin and the coordinate dependent part of the wave function $\psi^{(6)}$ in $d = 1+3$.

Spinors, which manifest masslessness in $d = 1+3$, must obey the equation

$$
\gamma^0\gamma^s p_s \psi^{(6)} = 0, \quad s = 5, 6,
\tag{6.8}
$$

since what will demonstrate as an effective action in $d = 1+3$ is

$$
\begin{aligned}
&\int \prod_m dx^m \mathrm{Tr}_{0123}\Big(\int dx^5 dx^6 \mathrm{Tr}_{56}(\psi^{(6)\dagger}\gamma^0(\gamma^m p_m + \gamma^s p_s)\psi^{(6)})\Big) = \\
&\int \prod_m dx^m \mathrm{Tr}_{0123}(\psi^{(4)\dagger}\gamma^0\gamma^m p_m \psi^{(4)}) - \int \prod_m dx^m \mathrm{Tr}_{0123}(\psi^{(4)\dagger}\gamma^0 m \psi^{(4)}),
\end{aligned}
\tag{6.9}
$$

where integrals go over all the space on which the solutions are defined. $\psi^{(6)}$ and $\psi^{(4)}$ are the solutions in $d = 1+5$ and $d = 1+3$, respectively. Tr_{0123} and Tr_{56} mean the trace over the spin degrees of freedom in $x0, x1, x2, x3$ and in x^5, x^6, respectively. (One finds, for example, that $\mathrm{Tr}(\stackrel{56}{[\pm]}) = 1$.) For massless spinors it must be that $\int dx^5 dx^6 \mathrm{Tr}_{56}(\psi^{(6)\dagger}\gamma^0\gamma^s p_s \psi^{(6)}) = \psi^{(4)\dagger}\gamma^0(-m)\psi^{(4)} = 0$.

To find the effective action in $1+3$ for massive spinors, we recognize that for the mass term we have

$$\psi^{(4)\dagger}\gamma^0(-\mathcal{A}^* \stackrel{56}{(+)}^\dagger + \mathcal{B}^* \stackrel{56}{[-]}^\dagger)\gamma^s p_s(\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]})\psi^{(4)} =$$
$$\psi^{(4)\dagger}\gamma^0(-\mathcal{A}^* \stackrel{56}{(+)}^\dagger + \mathcal{B}^* \stackrel{56}{[-]}^\dagger)(-m)(-\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]})\psi^{(4)}, \tag{6.10}$$

with $s=5,6$, $\stackrel{56}{(\pm)}^\dagger = -\stackrel{56}{(\mp)}$ and $\stackrel{56}{[\pm]}^\dagger = \stackrel{56}{[\mp]}$, while $(^*)$ means complex conjugation. We took into account that $\gamma^0 \stackrel{56}{(+)} = -\stackrel{56}{(+)}\gamma^0$, while $\gamma^0 \stackrel{56}{[-]} = \stackrel{56}{[-]}\gamma^0$. We find that $\mathrm{Tr}_{56}(\stackrel{56}{(+)}^\dagger \stackrel{56}{(+)}) = \mathrm{Tr}_{56}(\stackrel{56}{[-]}) = 1$ and $\mathrm{Tr}_{56}(\stackrel{56}{[-]}^\dagger \stackrel{56}{[-]}) = \mathrm{Tr}_{56}(\stackrel{56}{[-]}) = 1$. In order that $\int dx^5 dx^6 \mathrm{Tr}_{56}(\psi^{(4)\dagger}\gamma^0(-\mathcal{A}^* \stackrel{56}{(+)}^\dagger + \mathcal{B}^* \stackrel{56}{[-]}^\dagger)\gamma^s p_s(\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]})\psi^{(4)}$ will appear in $d=1+3$ as a mass term $\psi^{(4)\dagger}\gamma^0(-m)\psi^{(4)}$, we must solve the equation $\gamma^s p_s(\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]}) = (-m)(-\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]})$.

We can rewrite equations of motion in terms of the two complex superposition of x^5 and x^6: $z := x^5 + ix^6$ and $\bar{z} := x^5 - ix^6$ and their derivatives, defined as $\frac{\partial}{\partial z} := \frac{1}{2}(\frac{\partial}{\partial x^5} - i\frac{\partial}{\partial x^6})$, $\frac{\partial}{\partial \bar{z}} := \frac{1}{2}(\frac{\partial}{\partial x^5} + i\frac{\partial}{\partial x^6})$ and in terms of the two projectors $\stackrel{56}{[\pm]} := \frac{1}{2}(1 \pm i\gamma^5\gamma^6)$ as follows

$$2i\gamma^5\{\frac{\partial}{\partial z} \stackrel{56}{[-]} + \frac{\partial}{\partial \bar{z}} \stackrel{56}{[+]}\}(\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]}) = -m(-\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]}). \tag{6.11}$$

Since in Eq.(6.11) $\psi^{(4)}$ would be just a spectator, we skipped it.

In the massless case the superposition of the first two states ($\psi^{(6)m=0}_{+} = \stackrel{56}{(+)} \psi^{(4)m=0}_{+}$, with $\psi^{(4)m=0}_{+} = (\alpha \stackrel{03}{(+i)}\stackrel{12}{(+)} + \beta \stackrel{03}{[-i]}\stackrel{12}{[-]})\psi_0$) or the second two states ($\psi^{(6)m=0}_{-} = \stackrel{56}{[-]} \psi^{(4)m=0}_{-}$, with $\psi^{(4)m=0}_{-} = (\alpha \stackrel{03}{[-i]}\stackrel{12}{(+)} + \beta \stackrel{03}{(+i)}\stackrel{12}{[-]})\psi_0$) of the left handed Weyl representation presented in Eq.(6.6) must be taken, with the ratio of the two parameters α and β determined by the dynamics in x^m space. In the massive case $\psi^{(6)m}$ is the superposition of all the states to which γ^5 and γ^0 separately transform the starting state: $\psi^{(6)m} = (\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]})\psi^{(4)m}_{\pm}$, with $\psi^{(4)m}_{\pm} = \{\alpha[\stackrel{03}{(+i)}\stackrel{12}{(+)} \pm \stackrel{03}{[-i]}\stackrel{12}{(+)}] + \beta[\stackrel{03}{[-i]}\stackrel{12}{[-]} \pm \stackrel{03}{(+i)}\stackrel{12}{[-]}]\}\psi_0$. The sign $\pm$ denotes the eigenvalue of γ^0 on these states.

We shall therefore simply write (as suggested in Eq.(6.7)) $\psi^{(6)} = (\mathcal{A} \stackrel{56}{(+)} + \mathcal{B} \stackrel{56}{[-]})\psi^{(4)}$ in the massless and the massive case, taking into account that in the massless case either $\mathcal{A}$ or $\mathcal{B}$ is nonzero, while in the massive case both are nonzero. Accordingly also $\psi^{(4)}$ differs in the massless and the massive case.

We want our states to be eigenstates of the total angular momentum operator M^{56} around a chosen origin in the flat two dimensional manifold ($M^{(2)}$)

$$M^{56} = z\frac{\partial}{\partial z} - \bar{z}\frac{\partial}{\partial \bar{z}} + S^{56}. \tag{6.12}$$

Taking into account that $\gamma^5 \overset{56}{(+)}= - \overset{56}{[-]}$, $\gamma^0 \overset{56}{(+)}= - \overset{56}{(+)} \gamma^0$ and $\gamma^5 \overset{56}{[-]}=\overset{56}{(+)}$, $\gamma^0 \overset{56}{[-]}=\overset{56}{[-]} \gamma^0$ (see Appendix6.8) we end up with equations for $\mathcal{A}$ and $\mathcal{B}$

$$\begin{aligned} \frac{\partial \mathcal{B}}{\partial z} + \frac{im}{2}\mathcal{A} = 0, \\ \frac{\partial \mathcal{A}}{\partial \bar{z}} + \frac{im}{2}\mathcal{B} = 0. \end{aligned} \tag{6.13}$$

For $m = 0$ we get as solutions

$$\begin{aligned} \psi^{(6)m=0}_{n+1/2} &= a_n z^n \overset{56}{(+)} \psi^{(4)}_{+}, \\ \psi^{(6)m=0}_{-(n+1/2)} &= b_n \bar{z}^n \overset{56}{[-]} \psi^{(4)}_{-}, \; n \geq 0. \end{aligned} \tag{6.14}$$

We required $n \geq 0$ to ensure the integrability of solutions at the origin. The solutions have the eigenvalues of M^{56} equal to $(n+1/2)$ and $-(n+1/2)$, respectively.

Since in the massless case the contribution from $(p^5)2$ compensates the one from $(p^6)2$ for all the solutions from Eq. (6.14) with $n \geq 1$ and has therefore obviously one of the two contributions to the zero $m2$ a negative real value unless $n = 0$, it seems natural to expect that the only massless solutions are the two solutions with the eigenvalues $M^{(56)}$ equal to $1/2$ for the right handed spinor ($\psi^{(6)m=0}_{1/2} = a_0 \overset{56}{(+)} \psi^{(4)}_{+}$) and to $-1/2$ for the left handed spinor ($\psi^{(6)m=0}_{-1/2} = b_0 \overset{56}{[-]} \psi^{(4)}_{-}$), and accordingly with the corresponding $\psi^{(4)m=0}_{+}$ and $\psi^{(4)m=0}_{-}$ of the left and right handedness in $d = 1 + 3$, respectively. We shall reformulate the operator of momentum to be Hermitean on the vector space of solutions fulfilling the involution boundary condition in sect. 6.5. Having solutions of both handedness we must conclude that in such cases there is no mass protection.

For $m \neq 0$ we get

$$\psi^{(6)m}_{n+1/2} = a_n(J_n \overset{56}{(+)} - iJ_{n+1} e^{i\phi} \overset{56}{[-]}) e^{\pm in\phi} \psi^{(4)m}, \text{ for } n \geq 0, \tag{6.15}$$

where J_n is the Bessel's functions of the first order. The easiest way to see that J_n and J_{n+1} determine the massive solution is to use Eq.(6.13), take into account that $z = \rho e^{i\phi}$, define $r = m\rho, \rho = \sqrt{(x^5)2 + (x^6)2}$, recognize that $\frac{\partial}{\partial z} = \frac{1}{2} e^{-i\phi}(\frac{\partial}{\partial \rho} - \frac{i}{\rho}\frac{\partial}{\partial \phi})$ and we find $\mathcal{B} = -\frac{2}{im}\frac{\partial \mathcal{A}}{\partial \bar{z}}$. Then for the choice $\mathcal{A} = J_n e^{in\phi}$ it follows that $\mathcal{B} = -ie^{i(n+1)\phi}(\frac{n}{r}J_n - \frac{\partial J_n}{\partial r})$, which tells that $\mathcal{B} = -iJ_{n+1} e^{i(n+1)\phi}$.

6.3 Boundary conditions and involution

In the ref. [2] we make a choice of particular solutions of the equations of motion by requiring that $\hat{\mathcal{R}}\psi|_{\text{wall}} = 0$, where the wall were put on the circle of the radius ρ_0 of the finite disk (Eq.(6.3)).

This boundary condition requires that in the massless case (since $\overset{56}{[-]}\overset{56}{(+)}= 0$ while $\overset{56}{[-]}\overset{56}{[-]}=\overset{56}{[-]}$) only the right handed solution (Eq.6.14) $\psi^{(6)m=0}_{1/2} = a_0 \overset{56}{(+)}$

$\psi^{(4)m=0}_{+}$ (that is the left handed with respect to $SO(1,3)$) is allowed, while the left handed solution must be zero ($b_n = 0$) making the mass protection mechanism work in $d = 1 + 3$.

In the massive case the boundary condition determines masses of solutions, since only the solutions with $J_{n+1}|_{\rho=\rho_0} = 0$ are allowed from the same reason as discussed for the massless case. This boundary condition determines masses of spinors through the relation $m_{n+1/2}\rho_0$ is equal to a zero of J_{n+1}:

$$J_{n+1}(m_{n+1/2}\rho_0) = 0.$$

This time we look for the *involution boundary conditions*.

First we recognize that for a flat M2 - $\{0\}$ manifold, with the origin $x^5 = 0 = x^6$ excluded, the Z_2 or involution symmetry can be recognized: *The transformation* $\rho/\rho_0 \to \frac{\rho_0}{\rho}$ (which can be written also as $z/\rho_0 \to \frac{\rho_0}{\bar{z}}$) *transforms the exterior of the disk into the interior of the disk* and conversely.

Then we extend the involution operator to operate also on the space of solutions

$$\begin{aligned} \hat{\mathcal{O}} &= (I - 2\hat{\mathcal{R}}')|_{z/\rho_0 \to \rho_0/\bar{z}}, \\ \hat{\mathcal{O}}^2 &= I. \end{aligned} \tag{6.16}$$

The involution condition $\hat{\mathcal{O}}^2 = I$ requires, that $\hat{\mathcal{R}}'$ is a projector

$$(\hat{\mathcal{R}}')^2 = \hat{\mathcal{R}}' \tag{6.17}$$

and can be written as $\hat{\mathcal{R}}' = \hat{\mathcal{R}} + \hat{\mathcal{R}}_{add}$, where $\hat{\mathcal{R}}_{add}$ must be a nilpotent operator fulfilling the conditions

$$(\hat{\mathcal{R}}_{add})^2 = 0, \;\; \hat{\mathcal{R}}_{add}\hat{\mathcal{R}} = 0, \;\; \hat{\mathcal{R}}\hat{\mathcal{R}}_{add} = \hat{\mathcal{R}}_{add}, \tag{6.18}$$

We had $\hat{\mathcal{R}} = \stackrel{56}{[-]}$, which is the projector. Since we find that $\stackrel{56}{[-]}\stackrel{56}{(-)} = \stackrel{56}{(-)}$ (see Appendix 6.8), while $\stackrel{56}{(-)}\stackrel{56}{[-]} = 0$, we can choose $\hat{\mathcal{R}}_{add} = \alpha \stackrel{56}{(-)}$, where α is any function of z and $\frac{\partial}{\partial z}$. Let us point out that $\hat{\mathcal{R}}_{add}$ is not a Hermitean operator, since $\stackrel{56}{(-)}^{\dagger} = -\stackrel{56}{(+)}$ and $z^{\dagger} = \bar{z}$, $(\frac{\partial}{\partial z})^{\dagger} = \frac{\partial}{\partial \bar{z}}$. Accordingly also neither $\hat{\mathcal{R}}'$ nor $\hat{\mathcal{O}}$ is a Hermitean operator.

We now make a choice of a natural boundary conditions on the wall $\rho = \rho_0$

$$\{\hat{\mathcal{O}}\psi = \psi\}|_{\text{wall}}, \tag{6.19}$$

saying that what ever the involution operator is, the state ψ and its involution $\hat{\mathcal{O}}\psi$ must be the same on the wall, that is at $\rho = \rho_0$.

It is worthwhile to write the involution operator $\hat{\mathcal{O}}$ and correspondingly the projector $\hat{\mathcal{R}}'$ in a covariant way. Recognizing that $n^{(\rho)}{}_a\, \gamma^a n^{(\rho)}{}_b\, p^b = [e^{2i\phi}\frac{1}{2}(p^5 - ip^6) + \frac{1}{2}(p^5 + ip^6)] \stackrel{56}{(-)} + [\frac{1}{2}(p^5 + ip^6) + e^{-2i\phi}\frac{1}{2}(p^5 + ip^6)] \stackrel{56}{(+)}$, we may write $\frac{1}{2}(1 - in^{(\rho)}{}_a\, n^{(\phi)}{}_b\, \gamma^a\gamma^b)(1 - \beta n^{(\rho)}{}_a\, \gamma^a\, n^{(\rho)}{}_b\, p^b) = \stackrel{56}{[-]}\; (I + \beta i[e^{2i\phi}\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}]\; \stackrel{56}{(-)})$.

This is just our generalized projector $\hat{\mathcal{R}}'$, if we make a choice for α from Eq.(6.18) as follows: $\alpha = \beta i[e^{2i\phi}\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}]$ (since $\overset{56}{[-]}\overset{56}{(-)}=\overset{56}{(-)}$). We then have

$$\hat{\mathcal{R}}' = \overset{56}{[-]}\ (I + \beta i[e^{2i\phi}\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}]\ \overset{56}{(-)}), \tag{6.20}$$

where β is any complex number.

6.4 Current through the wall

The current perpendicular to the wall can be written as

$$\begin{aligned} n^{(\rho)s} j_s &= \psi^\dagger \gamma^0 \gamma^s n^{(\rho)}_s \psi = \psi^\dagger \gamma^0 (-)\{e^{-i\phi}\ \overset{56}{(+)}\ + e^{i\phi}\ \overset{56}{(-)}\}\psi = \psi^\dagger \hat{j}_\perp \psi, \\ \hat{j}_\perp &= -\gamma^0 \{e^{-i\phi}\ \overset{56}{(+)}\ + e^{i\phi}\ \overset{56}{(-)}\}. \end{aligned} \tag{6.21}$$

We need to know the current through the wall, which for physically acceptable cases when spinors are localized inside the disk (involution transforms outside the disk into its inside, or equivalently, it transforms inside the disk into its outside) must be zero. We find for the current

$$\{\psi^\dagger \hat{j}_\perp \psi\}|_{\text{wall}} = \{\psi^\dagger \hat{\mathcal{O}}^\dagger \hat{j}_\perp \hat{\mathcal{O}} \psi\}|_{\text{wall}}. \tag{6.22}$$

Since $\hat{\mathcal{O}}^\dagger = I - 2(\hat{\mathcal{R}} + \hat{\mathcal{R}}^\dagger_{add})$ and $\hat{\mathcal{R}}^\dagger_{add} = (\alpha\ \overset{56}{(-)})^\dagger = -\alpha^*\ \overset{56}{(+)}$, it follows that $\hat{\mathcal{O}}^\dagger \hat{j}_\perp \hat{\mathcal{O}} = -\hat{j}_\perp - 2\alpha^* \gamma^0 e^{i\phi}\ \overset{56}{[+]}\ - 2\alpha \gamma^0 e^{-i\phi}\ \overset{56}{[+]}$.

It must then be

$$\{\psi^\dagger \hat{j}_\perp \psi\}|_{\text{wall}} = (-\psi^\dagger \{\hat{j}_\perp + 2\gamma^0 (\alpha^* e^{i\phi} + \alpha e^{-i\phi})\ \overset{56}{[+]}\}\psi)|_{\text{wall}}. \tag{6.23}$$

First we check the current on the wall for the "old" case, when $\alpha = 0$ and $\hat{\mathcal{O}} = I - 2\hat{\mathcal{R}}, \hat{\mathcal{R}} = \overset{56}{[-]}$. Not to be in contradiction with Eq.(6.23) the current on the wall must for either massless or massive case be zero. In the case of massless solutions (Eq.(6.14)) only $\psi^{(6)m=0}_{n+1/2}$ can fulfill this boundary condition ($\psi^{(4)m=0\dagger} \bar{z}^n (-)\ \overset{56}{(-)}$ $\{-\gamma^0(e^{-i\phi}\ \overset{56}{(+)}\ + e^{i\phi}\ \overset{56}{(-)})\} z^n\ \overset{56}{(+)}\ \psi^{(4)m=0}_+)|_{\text{wall}} = 0$, for each nonnegative n. The chosen boundary condition accordingly allows only the right handed solutions. We shall conclude when discussing Hermiticity of the operators that only $n = 0$ is the physically acceptable solution.

In the massive case the solutions of equations of motion (Eq.(6.15)) contribute no current through the wall, if $J_{n+1}|_{\text{wall}} = 0$, which is exactly what the boundary condition (Eq.(6.19)) $\hat{\mathcal{O}}\psi|_{wall} = \psi|_{wall}$ required.

Then we check the general case with $\hat{\mathcal{O}} = I - 2\hat{\mathcal{R}}'$, where $\hat{\mathcal{R}}' = \hat{\mathcal{R}} + \hat{\mathcal{R}}_{add} = \overset{56}{[-]}$ $+\beta i[e^{2i\phi}\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}]\ \overset{56}{(-)}$. For massless solutions it is not difficult to see that *for any* *nonzero choice of* β *only one handedness - the right handed one - survives and that only* $n = 0$ *is allowed*. In the massive case we find

$$\{-i\beta[e^{2i\phi}\frac{\partial \mathcal{A}}{\partial z} + \frac{\partial \mathcal{A}}{\partial \bar{z}}] + \mathcal{B}\}|_{\text{wall}} = 0. \tag{6.24}$$

Since equations of motion require that $\mathcal{B} = -\frac{2}{im}\frac{\partial \mathcal{A}}{\partial \bar{z}}$ and since $\frac{\partial}{\partial \bar{z}} = \frac{1}{2}e^{i\phi}(\frac{i}{\rho}\frac{\partial}{\partial \phi} + \frac{\partial}{\partial \rho})$, we fulfill the involution condition on the wall for $\mathcal{A} = J_n e^{in\phi}$ only if $\mathcal{B} = -iJ_{n+1}e^{i(n+1)\phi}$, with the requirement that $J_n|_{\text{wall}} = 0$ and $\beta = \frac{1}{m}$. While $J_n|_{\text{wall}} = 0$ can always be fulfilled, the second requirement $\beta = \frac{1}{m}$ means, since β can not be an arbitrary number, that our generalized condition is not written in an covariant form, and is accordingly not the acceptable boundary condition.

6.5 Hermiticity of operators and the orthogonality of solutions

In this section we comment on the Hermiticity properties of the operators, in particular of p_s and on the orthogonality properties of those solutions of the equations of motion which fulfill the involution boundary conditions. We expect the solutions

i) to be orthogonal ($\int d^2x\psi_i{}^\dagger((p^5)2 + (p^6)2)\psi_j = \int d^2x\psi_i{}^\dagger\psi_j m2\delta_{ij}$) and that

ii) on the space of these solutions the operators p_s are Hermitean and have accordingly expectation values of the operators $(p^s)2$ positive contribution to $m2$ for each s.

Let us first check the orthogonality relations of the massive and massless solutions. We immediately see that the massive solutions $\psi^{(6)m}_{n+1/2}$ belonging to different n are all orthogonal due to the orhogonality of the functions $e^{in\phi}$. We find $\int d^2x\mathrm{Tr}_{56}(\psi^{(6)m\dagger}_{n+1/2}\psi^{(6)m}_{k+1/2}) = \delta_{nk}\; a^*_n a_n \frac{1}{2}\psi^{(4)m\dagger}\psi^{(4)m}\int_0^{\rho_0}(J^*_n J_n + J^*_{n+1}J_{n+1})\rho d\rho$.

It also turns out that the massless solutions $(\psi^{(6)m=0}_{n+1/2}$ (Eq.(6.14)) are orthogonal to all the massive states (Eq.(6.15)) due to the properties of the J_n Bessel's function. Namely,

$$\int d^2x\mathrm{Tr}_{56}(\psi^{(6)m=0\dagger}_{n+1/2}\psi^{(6)m}_{k+1/2}) = \delta_{nk}a^{0*}_{n+1/2}a_k\frac{1}{\sqrt{2}}\psi^{(4)m=0\dagger}\psi^{(4)m}\int_0^{\rho_0}\rho^n J_n\rho d\rho = 0,$$

since $\int_0^{\rho_0}\rho^n J_n\rho d\rho = \rho_0^{n+1}J_{n+1}(\rho_0)$, but $J_{n+1}(\rho_0)$ must be zero in order that the massive state with $n + 1/2$ obeys the involution boundary condition. Massless solutions are again due to the $e^{in\phi}$ part orthogonal among themselves.

So we conclude that all the states, which obey the equations of motion and the involution boundary condition, are orthogonal.

Are p_s Hermitean on the space of these solutions?

We know that $p_s = -i\frac{\partial}{\partial x^s}$ is Hermitean on the vectors space ψ_i if for any two functions ψ_i and ψ_j from the vector space of solutions $\mathrm{Tr}_{56}(\int d^2x p_s(\psi_i^\dagger\psi_j)) = 0$ (since then $\int d^2x\psi_i{}^\dagger p_s\psi_j + \int d^2x(-p_s\psi_i)^\dagger\psi_j = 0$).

We find that

$$p_s = i\frac{\partial}{\partial x^s} = i\begin{pmatrix}\cos\phi\frac{\partial}{\partial\rho} - \sin\phi\frac{1}{\rho}\frac{\partial}{\partial\phi}\\ \sin\phi\frac{\partial}{\partial\rho} + \cos\phi\frac{1}{\rho}\frac{\partial}{\partial\phi}\end{pmatrix}, \tag{6.25}$$

for $s=5$ (first row) and 6(second row). Since either messless ($\psi^{(6)\,m=0}_{n+1/2}$, Eq.(6.14)) or massive ($\psi^{(6)m}_{n+1/2}$, Eq.(6.15)) states can be written as a product of $e^{in\phi}$ and the rest, say ψ_n, we see that $\int d^2x p_s(\psi^\dagger_n\psi_k)$ is nonzero only if $|n-k|=1$.

In this case we get that the integral $\mathrm{Tr}_{56}(\int d^2x p_s(\psi_n{}^\dagger\psi_{n\pm 1})), s=x^5, x^6$, proportional to $i\pi\binom{1}{i}|\rho\psi^\dagger_n\psi_{n+1}|_{\rho_0}$, with $|\rho\psi^\dagger_n\psi_{n+1}|^{\rho_0}_0$ equal to
i)$a^{m=0*}_n a^{m=0}_{n+1}(\rho_0)^{2(n+1)}$ in the case that two massless states are concerned,
ii) $a^{m=0*}_n a^m_{n+1}\rho_0^{n+1} J_{n+1}(\rho_0)$ in the case that one massless and one massive state are concerned,
iii) $a^{m*}_n a^m_{n+1}\rho_0(J_n J_{n+1} + J_{n+1}J_{n+2})|_{\rho_0}$ in the case that two massive states are concerned. None of these integral is zero, since the two J_n and J_{n+1} are not correlated (J_n and J_{n+1} are correlated, if both belong to the solution with the same mass, determined by $J_{n+1}(m\rho = m\rho_0)=0$). We conclude that for none of the solutions p_s are Hermitean operators.

One can check, however, that $\hat{p}_s$

$$\hat{p}_s = i\{\frac{\partial}{\partial x^s} - \frac{1}{2}\frac{x^s}{\rho}\delta(\rho-\rho_0)\stackrel{56}{[+]}\}, \tag{6.26}$$

are Hermitean operators on the space of massive and massless solutions, fulfilling the involution boundary conditions. It contains the part with the δ function which corrects those parts of solutions, which are nonzero on the wall—the radial parts which appear with $\stackrel{56}{(+)}$. It can be shown that the integral over the part with the $\delta(\rho-\rho_0)$ function contributes just the terms which compensate the nonzero contribution in each of the three cases i)-iii).

What we must check now is, what appears in this new definition of the operator of the momenta (Eq.(6.25)) as $\gamma^s p_s\gamma^t p_t$ and whether now the integral $\mathrm{Tr}_{56}(\int d^2x\,\psi^\dagger\,\gamma^s p_s\gamma^t p_t\,\psi), s=x^5, x^6$, is still manifesting as just the mass term for those ψ which we accept as solutions of equations of motion (Eq.(6.14,6.15)).

One finds

$$\begin{aligned}\gamma^s\hat{p}_s\gamma^t\hat{p}_t &= p_s p^s\\ &+ \frac{1}{2}\{[\frac{\partial}{\partial\rho}\delta(\rho-\rho_0) + \frac{1}{\rho}\delta(\rho-\rho_0) + \delta(\rho-\rho_0)(\frac{\partial}{\partial\rho} - \frac{i}{\rho}\frac{\partial}{\partial\phi})]\stackrel{56}{[+]}\\ &+ \delta(\rho-\rho_0)(\frac{\partial}{\partial\rho} + \frac{i}{\rho}\frac{\partial}{\partial\phi})\stackrel{56}{[-]}\}.\end{aligned} \tag{6.27}$$

One notices that the first row of Eq.(6.27) represent the usual momentum squared. The last two terms are zero everywhere except on the wall. What we must check is the integral of the last two terms for all solutions fulfilling our involution boundary condition.

We find that the integral $\mathrm{Tr}_{56}(\int d^2x\,(z\stackrel{56}{(+)})^\dagger\,\gamma^s p_s\gamma^t p_t\,z^n\stackrel{56}{(+)})$ is for massless solutions (Eq.(6.14)) obeying the involution boundary condition proportional to n and it is zero only for $n=0$.

The integral $\mathrm{Tr}_{56}(\int d^2x\,(z\stackrel{56}{(+)})^\dagger\,\gamma^s p_s\gamma^t p_t\,z^n\stackrel{56}{(+)})$ demonstrates for massive solutions (Eq.(6.15)) the mass term squared originating in the first row of Eq.(6.27), while the rest contributes zero.

The requirement that the integral $\mathrm{Tr}_{56}(\int d^2x\, \psi^\dagger \gamma^s p_s \gamma^t p_t\, \psi), s = x^5, x^6$, *must be zero for massless solutions, makes a choice of only one among all possible massless solutions: the one with* $n = 0$.

Our the only possible solution is in the massless case $\psi^{(6)m=0}_{1/2}$. For the massive solutions we have $\psi^{(6)m}_{1/2} = a_{1/2}\frac{1}{\sqrt{2}}(J_0 \overset{56}{(+)} - iJ_1 e^{i\phi} \overset{56}{[-]})$, with $m_{1/2}\rho_0$ as a zero of J_1, $\psi^m_{3/2} = a_{3/2}\frac{1}{\sqrt{2}}(J_1 - iJ_2 e^{i\phi})e^{i\phi})$, with $m_{3/2}\rho_0$ as a zero of J_2, $\psi^{(6)m}_{-1/2} = a_{-1/2}\frac{1}{\sqrt{2}}(J_1 \overset{56}{(+)} - iJ_0 \overset{56}{[-]}\, e^{-i\phi}))e^{-i\phi})$, with $m_{-1/2}\rho_0$ equal to a zero of J_0 and so on.

6.6 Properties of spinors in $d = 1 + 3$

To study how do spinors couple to the Kaluza-Klein gauge fields in the case of $M^{(1+5)}$, "broken" to $M^{(1+3)} \times$ a flat disk with ρ_0 and with the involution boundary condition, which allows only right handed spinors at ρ_0, we first look for (background) gauge gravitational fields, which preserve the rotational symmetry on the disk. Following ref. [2] we find for the background vielbein field

$$e^a{}_\alpha = \begin{pmatrix} \delta^m{}_\mu & e^m{}_\sigma = 0 \\ e^s{}_\mu & e^s{}_\sigma \end{pmatrix}, f^\alpha{}_a = \begin{pmatrix} \delta^\mu{}_m & f^\sigma{}_m \\ 0 = f^\mu{}_s & f^\sigma{}_s \end{pmatrix}, \tag{6.28}$$

with $f^\sigma{}_m = A_\mu \delta^\mu{}_m \varepsilon^\sigma{}_\tau x^\tau$ and the spin connection field

$$\omega_{st\mu} = -\varepsilon_{st} A_\mu, \quad \omega_{sm\mu} = -\frac{1}{2} F_{\mu\nu} \delta^\nu{}_m \varepsilon_{s\sigma} x^\sigma. \tag{6.29}$$

The $U(1)$ gauge field A_μ depends only on x^μ. All the other components of the spin connection fields are zero, since for simplicity we allow no gravity in $(1+3)$ dimensional space.

To determine the current, coupled to the Kaluza-Klein gauge fields A_μ, we analyze the spinor action

$$\begin{aligned} \mathcal{S} = \int d^dx E \bar\psi^{(6)} \gamma^a p_{0a} \psi^{(6)} = \int d^dx \bar\psi^{(6)} \gamma^m \delta^\mu{}_m p_\mu \psi^{(6)} + \\ \int d^dx \bar\psi^{(6)} \gamma^m (-) S^{sm} \omega_{sm\mu} \psi^{(6)} + \int d^dx \bar\psi^{(6)} \gamma^s \delta^\sigma{}_s p_\sigma \psi^{(6)} + \\ \int d^dx \bar\psi^{(6)} \gamma^m \delta^\mu{}_m A_\mu (\varepsilon^\sigma{}_\tau x^\tau p_\sigma + S^{56}) \psi^{(6)}. \end{aligned} \tag{6.30}$$

$\psi^{(6)}$ are solutions of the Weyl equation in $d = 1+3$. E is for $f^\alpha{}_a$ from (6.28) equal to 1. The first term on the right hand side of Eq.(6.30) is the kinetic term (together with the last term defines the covariant derivative $p_{0\mu}$ in $d = 1+3$). The second term on the right hand side contributes nothing when integration over the disk is performed, since it is proportional to x^σ ($\omega_{sm\mu} = -\frac{1}{2} F_{\mu\nu} \delta^\nu{}_m \varepsilon_{s\sigma} x^\sigma$).

We end up with

$$j^\mu = \int d^2x \bar\psi^{(6)} \gamma^m \delta^\mu{}_m M^{56} \psi^{(6)} \tag{6.31}$$

as the current in $d = 1 + 3$. The charge in $d = 1 + 3$ is proportional to the total angular momentum $M^{56} = L^{56} + S^{56}$ on a disk, for either massless or massive spinors.

6.7 Conclusions

In this paper we were looking for what we call a "natural boundary condition"—a condition which would, due to some symmetry relations, make massless spinors which live in M^{1+5} and carry nothing but the charge to live in $M^{(1+3)}\times$ a flat disk, manifesting in $M^{(1+3)}$, if massless, as a left handed spinor (with no right handed partner) and would accordingly be mass protected. The spin in x^5 and x^6 of the left handed massless spinor should in $M^{(1+3)}$ manifest as the charge and should chirally couple with the Kaluza-Klein charge of only one value to the corresponding gauge field, in order that after the second quantization procedure a particle and an antiparticle would not appear each of both charges.

We found the involution boundary condition

$$\{\hat{\mathcal{O}}\psi = \psi\}|_{\text{wall}}, \quad \mathcal{O} = (I - (I - i n^{(\rho)}{}_a n^{(\phi)}{}_b \gamma^a \gamma^b))_{\frac{\rho}{\rho_0} \to \frac{\rho_0}{\rho}}, \quad \mathcal{O}^2 = I, \tag{6.32}$$

which transforms solutions of the Weyl equations inside the flat disk into outside of it (or conversely) and allows in the massless case only the right handed spinor to live on the disk and accordingly manifests left handedness in $M^{(1+3)}$. The massless solution carries in the fifth and sixth dimension (only) the spin $1/2$, which then manifests as the charge in $d = 1 + 3$.

We defined a generalized momentum p_s

$$\hat{p}_s = i\{\frac{\partial}{\partial x^s} - \frac{1}{2}\begin{pmatrix}\cos\phi \\ \sin\phi\end{pmatrix} \delta(\rho - \rho_0) \stackrel{56}{[+]}\}, \tag{6.33}$$

which is the Hermitean operator in the case of our involution boundary condition.

The requirement that $\gamma^s \hat{p}_s \gamma^t \hat{p}_t$ manifests as the square of the mass leads in the massless case to to the solution with the total angular momentum $1/2$ as the only solution, while the massive solutions carry all half integer angular momenta: $\pm 1/2, \pm 3/2, \cdots$. The angular momenta in the fifth and sixth dimensions then manifests as the charge in the $1 + 3$ dimension. The massless solution with the spin $1/2$ is mass protected and chirally coupled to the corresponding Kaluza-Klein field.

The negative charge of the massless $1/2$ charge state appears only after the second quantization procedure in agreement with what we observe.

All the solutions fulfilling the involution boundary conditions are orthogonal and in this vector space and the generalized operators are Hermitean.

The involution boundary condition of Eq.(6.32) are equivalent to the boundary condition, which we present in the ref. [2]. Both take care that massless solutions of one handedness appear in $d = 1 + 3$.

We were looking for generalized boundary conditions with

$$\hat{\mathcal{O}} = \mathrm{I} - 2\hat{\mathcal{R}}',$$
$$\hat{\mathcal{R}}' = \overset{56}{[-]}\,(\mathrm{I} + \beta i[e^{2i\phi}\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}]\,\overset{56}{(-)}), \tag{6.34}$$

where β is any complex number. This generalized boundary $\hat{\mathcal{R}}'$ can be written in a covariant way as

$$\hat{\mathcal{R}}' = \frac{1}{2}(1 - in^{(\rho)}{}_a\, n^{(\phi)}{}_b\, \gamma^a\gamma^b)(1 - \beta n^{(\rho)}{}_a\, \gamma^a\, n^{(\rho)}{}_b\, p^b)$$
$$= \overset{56}{[-]}\,(\mathrm{I} + \beta i[e^{2i\phi}\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}]\,\overset{56}{(-)}). \tag{6.35}$$

But while in the massless case the generalized boundary condition $\{\hat{\mathcal{O}}\psi = \psi\}|_{\text{wall}}$ forbids all but $s = 1/2$ solution, it fails in the massive case to demonstrate the covariance and is accordingly not an acceptable boundary condition.

6.8 Appendix: Spinor representation technique in terms of Clifford algebra objects

We define[9] spinor representations as superposition of products of the Clifford algebra objects γ^a so that they are eigen states of the chosen Cartan sub algebra of the Lorentz algebra $SO(d)$, determined by the generators $S^{ab} = i/4(\gamma^a\gamma^b - \gamma^b\gamma^a)$. By introducing the notation

$$\overset{ab}{(\pm i)}: = \frac{1}{2}(\gamma^a \mp \gamma^b), \quad \overset{ab}{[\pm i]} := \frac{1}{2}(1 \pm \gamma^a\gamma^b), \text{ for } \eta^{aa}\eta^{bb} = -1,$$
$$\overset{ab}{(\pm)}: = \frac{1}{2}(\gamma^a \pm i\gamma^b), \quad \overset{ab}{[\pm]} := \frac{1}{2}(1 \pm i\gamma^a\gamma^b), \text{ for } \eta^{aa}\eta^{bb} = 1, \tag{6.36}$$

it can be checked that the above binomials are really "eigenvectors" of the generators S^{ab}

$$S^{ab}\,\overset{ab}{(k)}: = \frac{k}{2}\,\overset{ab}{(k)}, \quad S^{ab}\,\overset{ab}{[k]} := \frac{k}{2}\,\overset{ab}{[k]}\,. \tag{6.37}$$

Accordingly we have

$$\overset{03}{(\pm i)}: = \frac{1}{2}(\gamma^0 \mp \gamma^3), \quad \overset{03}{[\pm i]} := \frac{1}{2}(1 \pm \gamma^0\gamma^3),$$
$$\overset{12}{(\pm)}: = \frac{1}{2}(\gamma^1 \pm i\gamma^2), \quad \overset{12}{[\pm]} := \frac{1}{2}(1 \pm i\gamma^1\gamma^2),$$
$$\overset{56}{(\pm)}: = \frac{1}{2}(\gamma^5 \pm i\gamma^6), \quad \overset{56}{[\pm]} := \frac{1}{2}(1 \pm i\gamma^5\gamma^6), \tag{6.38}$$

with eigenvalues of S^{03} equal to $\pm\frac{i}{2}$ for $\overset{03}{(\pm i)}$ and $\overset{03}{[\pm i]}$, and to $\pm\frac{1}{2}$ for $\overset{12}{(\pm)}$ and $\overset{12}{[\pm]}$, as well as for for $\overset{56}{(\pm)}$ and $\overset{56}{[\pm]}$.

We further find

$$\gamma^a \stackrel{ab}{(k)} = \eta^{aa} \stackrel{ab}{[-k]}, \quad \gamma^b \stackrel{ab}{(k)} = -ik \stackrel{ab}{[-k]},$$
$$\gamma^a \stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, \quad \gamma^b \stackrel{ab}{[k]} = -ik\eta^{aa} \stackrel{ab}{(-k)} \,. \tag{6.39}$$

We also find

$$\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0, \quad \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa} \stackrel{ab}{[k]}, \quad \stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}, \quad \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0,$$
$$\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0, \quad \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}, \quad \stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}, \quad \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0. \tag{6.40}$$

To represent one Weyl spinor in $d = 1 + 5$, one must make a choice of the operators belonging to the Cartan sub algebra of 3 elements of the group $SO(1,5)$

$$S^{03}, S^{12}, S^{56}. \tag{6.41}$$

Any eigenstate of the Cartan sub algebra (Eq.(6.41)) must be a product of three binomials, each of which is an eigenstate of one of the three elements. A left handed spinor ($\Gamma^{(1+5)} = -1$) representation with $2^{6/2-1}$ basic states is presented in Eq.(6.6). for example, the state $\stackrel{03}{(+i)}\stackrel{12}{(+)}\stackrel{56}{(+)} \psi_0$, where ψ_0 is a vacuum state (any, which is not annihilated by the operator in front of the state) has the eigenvalues of S^{03}, S^{12} and S^{56} equal to $\frac{i}{2}$, $\frac{1}{2}$ and $\frac{1}{2}$, correspondingly. All the other states of one representation of $SO(1,5)$ follow from this one by just the application of all possible $S^(ab)$, which do not belong to Cartan sub algebra.

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7 How Can Group Theory be Generalized so Perhaps Providing Further Information About Our Universe?

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Abstract. Group theory is very familiar, perhaps too much so. We are thus prejudiced about it, leading to views that are far too narrow. Yet it is significantly richer than usually realized ([13]). Here we wish to understand the restrictions giving the familiar forms and how by changing these we can get added richness. Might these add to our knowledge of nature? A purpose of this note is to stimulate thinking about this.

7.1 Geometry, through its transformations groups, is very information, but so far not enough

It is clear that much (all?) of physics is determined by geometry, especially through its transformation groups ([13]; [4]; [5]; [7]; [8]; [9]; [10]; [11]; [12]; [3]; [6]). Yet it is necessary to go much further. Can additional progress be made using group theory? This is a very open question but worth exploring. One aspect to be explored is whether group theory itself can be generalized. That could be of interest for purely mathematical reasons. And it has many applications. Generalizing it can thus be useful in various ways. This we wish to explore here.

7.2 What is the best way to try to understand fundamental physics?

What is the best approach to try to understand physics? The big fad nowadays is to come up with the wildest, most unlikely ideas, ones furthest from reality, ones totally unrelated to anything known, ones having no rationale whatever. History and common sense show that this approach is destined to lead nowhere except to even more wild ideas (as it has).

Those who do that will find themselves badly cut by Occam's razor. Unfortunately a large part of the physics community is doing just that. Applying Occam's razor to the physics community will greatly help physics to advance.

Another approach that is very likely to lead to failure, certainly if there is no other rationale for it, is to base laws on how we measure, on ourselves. We do not determine the laws of nature (something many scientists, especially physicists, do not believe). How we measure is limited by physical laws, but does not limit them. Studying measurement can help us understand physics, but cannot determine it.

What then shall we do, what approach shall we use? The best approach is the most conservative using requirements that are certain, or at least likely, to be correct, or ones that deviate the least from these.

7.3 Reasonable requirements for developing theories

What requirements can we impose?

First is consistency. Fundamental physical theories must be consistent. (Phenomenological theories, classical physics is an example, can be inconsistent mishmashes.) This is more difficult than it might seem, so can be quite powerful.

Geometry imposes requirements, restrictions. Physics takes place in geometry so must be in accord with the rules it leads to. This also is powerful as we have seen (particularly) in the references.

What can we say about geometry? We always assume that it is a manifold (locally flat) and that its coordinates are real numbers (rather say than complex numbers or quaternions). It is very unlikely that physics would be possible otherwise, but this can be investigated. A fundamental property of geometry is its dimension. However it has long been known that physics would be impossible unless the dimension is 3+1 ([4]; [13]). This is required by consistency, illustrating its importance, for only with this dimension is a consistent physics possible.

7.4 What can we say about geometry?

Is space curved? The curvature of space is given by a function over it, the connection ([5]). Can every space that is a manifold be regarded as flat but with a function, the connection, so that all curved spaces can be reduced to flat ones with different such functions? This is an interesting question that we raise but do not try to answer here. Also (many) curved spaces can be mapped (in reasonable ways) into flat ones ([8]). Thus we consider only flat spaces, but these questions should be looked into.

What properties do flat spaces have? Beyond the reality of coordinates and the dimension is a most fundamental property: the transformation groups of the spaces (which are not symmetry groups ([13]), although it is quite interesting that they are that also). For our space, apparently the only one in which physics is possible, the largest symmetry group is the conformal group ([8]), which has subgroups the Poincaré group, its subgroup the Lorentz group and the subgroup of that, the rotation group (SO(3)). The last gives that angular momentum must be integral or half-odd-integral ([3]), illustrating how transformations limit physics. The massless representations of the Poincaré group determine electromagnetism and gravitation ([5]). Clearly these are quite informative, but clearly insufficient. It is possible that the conformal group can also be quite informative but how is less clear.

7.5 How might group theory be generalized?

Can we go further? What we wish to do here is study whether what is known about group theory can be generalized. We are all too familiar with semisimple groups, like the rotation and Lorentz groups. But group theory is far richer, even for these groups ([5]; [8]). Perhaps it is richer than we realize. That is what we consider here.

This may not have anything to do with fundamental laws. But it helps to understand group theory, decreasing prejudice and broadening our views, and produces interesting mathematical results. And they are likely to lead to useful, even important, applications.

7.6 Indexed groups

Start by considering the curve, $x = r\cos\theta$, $y = r\sin\theta$, which describes a circle. We move around a circle using the two-dimensional rotation group O(2). By putting a constant into the representation matrix we can generate an ellipse. But what about, say, the curve $x = r(\cos\theta)3$, $y = r(\sin\theta)3$. What set of transformations moves along this curve and why don't they form a group? Clearly there is an identity, we do not have to move, and for every transformation there is an inverse. Moreover the product of two transformations is a transformation; if we move from A to B and the from B to C, we can find a transformation from A to C. However the transformations are not associative. It is for this reason that they do not form a group. The transformation from B to C depends on where B is (it is in a sense history dependent, depending on the previous transformations). That is the operator going from B to C has a form that depends on B, unlike rotations. This causes associativity to fail.

Thus for a circle the product of the transformation matrix for θ_1 and for θ_2 is that for $\theta_1 + \theta_2$, which is not true for this curve.

While there are transformations along any (reasonable) n-dimensional surface it is only in special cases that they form a group (of the form usually considered). This emphasizes the relevance of associativity and the restrictions it places. Many of the properties of groups and their representations come from associativity. While restricting, it also allows us to obtain properties that are so useful in applications of groups.

How do we deal these more general transformations, say ones along arbitrary surfaces? We introduce the concept of indexed groups. To do this we assume that the transformations can be mapped (at least) one-to-one onto a *group* of transformations. For each general transformation we have a corresponding group element and this element is the index of the general transformation. Thus for three dimensions the index can be an element of the rotation group. For a group the matrix representing the transformation that is the product of two is the matrix product of the matrices of the two transformations. It is here that indexed representations differ. For these the matrix labeling the product is the matrix product of the two *labeling* matrices. But the product matrix of the indexed transformation is not the product of the matrices of the two transformations it is a product of. It is the index that is given by the product, not the transformation. With this definition of group product, differing from the normal definition, these transformations form a group. Associativity holds, since it follows from the associativity of the indexing group, but only because of the revised definition of a product.

To give a group we list its members and their products (thus the spaces on which they act). But now with each set of members we have an infinite number of product rules (determined by the mapping of the transformations into the indexing groups, of which there may be several) thus an infinite set of groups. Each n-dimensional surface has its own group.

7.7 Product rules determine groups

This shows how properties of a group are dependent on the definition of its product and how by revising this definition we can generalize the type of structures that form groups. This adds to the richness of group theory.

Groups which can be realized as matrices, thus whose products are matrix products, we call standard groups. Ones whose elements are indexed so whose products are given by the matrix products of their indices we call indexed groups.

Indexed transformation groups exist for any surface that can be mapped (properly, in a way that must be investigated) to the defining space of a Lie group. For the three-dimensional rotation group SO(3) that is a sphere, and there is a third parameter which can be considered as giving the direction of a vector at each point of the sphere. Then each point is mapped to a point on one of the generalized (overlying) space, and each direction to one on that (which perhaps might be considered as an internal symmetry). Likewise we can use SO(2,1) to get another set of such surfaces. So we have associated with each group an infinite set of groups, each given by different product rules (from different mappings), or another way of saying this, an infinite set of realizations or representations.

We assign to each group element a matrix, that of the regular (adjoint) representation. That this is possible follows from the group axioms. Then the group product is a matrix product, and this is the usual group product. We call this the standard product.

The rotation group, besides its defining representation of 3 X 3 matrices has an infinite set of others. Consider the 5 X 5 one, say. This is a subrepresentation of SO(5). We can map a surface to the defining surface of SO(5) and the group defined over it (one for each of the infinite number of such surfaces, ignoring aspects like inversions), form representations of SO(3), but with additional transformations which might be taken as internal ones. Since SO(3) has an infinite number of representations (and it is simple so other groups have infinite sets each of infinite numbers of representations) this admits a huge number of transformation sets to be defined over it.

But we can do more. The conformal group algebra is (isomorphic to) that of SO(4,2) and SU(3,1). The group algebras are the same but realized in terms of different variables. Instead of 4+2 real ones, or 3+1 complex ones, the conformal algebra is realized over 3+1 real ones ([8]). We might also realize it over more, rather than fewer, variables. These again can be taken as internal ones.

We now map surfaces (choosing from an infinite number) to the 3+1-dimensional real space on which the conformal group acts. The product of the transformations on these is given by the product of conformal transformations taking points and directions of the 3+1-space to others. These conformal transformations are the indices of the transformations on the preimage space. Note that we have three groups, SO(4,2), SU(3,1) and the conformal group, plus all their representations, which can act as indices. This shows the great richness introduced.

7.8 Groups of 3+1 space as illustrations

Thus there are two groups defined by a 3+1-dimensional real space, one, ISO(3,1) is the Poincaré group, the other is the conformal group. This itself shows richness known but not understood in group theory. Take a surface (one of an infinite number) that is mapped to real 3+1-dimensional space. The transformations on it can be indexed by the transformations of the base space. However there are two sets, the Poincaré group and the conformal group. So from ordinary space we can find two infinite sets of groups (realizations, representations). And the transformations of our space can be taken as subsets of larger groups, giving further labels and products, of infinite number. Some of this additional freedom might be relevant to internal transformations.

Why should we consider these, aside from their showing the assumptions? Groups are useful in many ways and these can extend their usefulness. For example special functions are group representation basis states and many properties can be derived from this. Generalizing the concepts of representation can lead to other special functions, perhaps with useful properties. However it is not clear that properties can be found for these as they can for standard representations, or indeed that they have simple properties. This must be investigated. Associativity is important in determining these properties, and allowing simple properties (that we can get rules for). This procedure allows such great generalization that it is likely that only a few cases, at most, can give simple rules. But there might be some and these could be useful.

Also physical objects are statefunctions ([13]) that are group representation basis states. By expanding the set of these we may be able to expand the set of objects that are such states. There are clear limitations as the known states are those of standard representations. It may be that the requirement that objects be observers, and conversely ([7]), provides strict limits. Yet this is not known and these new representations allow study of this. And some may have physical applications, perhaps to these fields.

7.9 Why standard products are matrix product and why are these usually relevant?

While these indexed groups (groups with indexed products) may seem unusual they raise the question why standard products are the relevant ones, for those cases in which they are? This has to be considered for each application. A general class is applications to geometry. The transformations, but only for certain geometries, have standard products. This is true for lines, circles, planes, spheres and generalizations. For these, why are the products of transformations the standard products?

Here symmetry enters. The action of a group transformation on the base space is independent of the point in that space — since all these points are identical. Thus a group transformation taking a point to another, acting on a second such transformation, gives a transformation with identical action (as can be seen with a circle). ¿ From this associativity follows. These transformations thus form the regular representation ([3]). But this representation can be given by matrices, and the group product is the same as a matrix product, the standard product.

Other spaces do not have symmetry so their transformations cannot be represented by matrices. Here we see how symmetry gives group operations, and limits these. For spaces without symmetry we must use other products.

7.10 Conclusion

Groups are determined very much by their products. Usually these give matrix products. Here we have considered a set of different product rules. These illustrate how such rules determine the properties of groups, and the role of symmetry in the standard product rules. Whether some of these generalized rules are useful has to be studied. Lie groups have an extensive structure including their algebras. Do these generalizations allow corresponding structures, including algebras? That is another field of study. There is much that can be done, some at least profitably.

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8 Future Dependent Initial Conditions from Imaginary Part in Lagrangian

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Abstract. We want to unify usual equation of motion laws of nature with "laws" about initial conditions, second law of thermodynamics, cosmology. By introducing an imaginary part – of a similar form but different parameters as the usual real part – for the action to be used in the Feynman path way integral we obtain a model determining (not only equations of motion but) also the initial conditions, for say a quantum field theory. We set up the formalism for e.g. expectation values, classical approximation in such a model and show that provided the imaginary part gets unimportant except in the Big Bang era the model can match the usual theory. Speculatively requiring that there be place for Dirac strings and thus in principle monopoles in the model we can push away the effects of the imaginary part to be involved only with particles not yet found. Most promising for seeing the initial condition determining effects from the imaginary part is thus the Higgs particle. We predict that the width of the Higgs particle shall likely turn out to be (appreciably perhaps) broader than calculated by summing usual decay rates. Higgs machines will be hit by bad luck.

8.1 Introduction

Usually when we talk about "theory of everything" as superstring theory is hoped to be, it is not really meant that the initial state of the universe is included in the model immediately. Rather one needs to make extra assumptions – cosmology, second law of thermodynamics[1,2,3,4], etc. – about the initial conditions or one simply leaves it for the applicator of the theory to somehow himself manage to find out what the initial conditions are for the experiment he wants to describe with the theory. It is, however, the intention of the series of articles [5,6,7,8,9] to which this article belongs to set up assumptions telling the initial conditions in a way that can be called that these initial condition assumptions are unified[10] with the part of the theory describing the equations of motion and the particle content (the usually T.O.E.). Our unification may though be mainly a bit formal in as far as our main point is to use in the Feynman path integral an action which has both a real S_R and an imaginary part S_I. Usually of course the action is real and the imaginary part $S_I = 0$ (Total $S = S_R + iS_I$). We may quickly see that the imaginary part gives a typically hugely different extra factor in the probability for different paths obeying equations of motion. Thus such an imaginary part essentially fix the path obeying equations of motion which should almost certainly be the realized one. In this way we can claim that to a good approximation an imaginary part of the action will choose/settle the initial conditions.

In the present article it is not the point to settle on any choice of the in usual sense "theory of everything". Rather we shall present our idea of introducing an imaginary part in the Lagrangian and thereby also in the action as a modification that can be made on any theory as represented by the real action S.

We have already published a few articles on essentially a classical formulation of the present model. We sought in these articles to be a little more general by simply defining a probability weight called $P(\text{path})$ defined for all possible paths. In classical theory it is really only the paths which obey the classical equations of motion for which we need to define P. We already in the earlier articles suggested that this probability $P(\text{path})$ for a certain track, path, to be the one realized in nature should be given as the exponential of an expression depending on the track, path, of the form of a space-time integral over a locally defined quantity $\mathcal{L}_I$ depending on the fields in the development, path. Really this quantity $\mathcal{L}_I$ (really $-\mathcal{L}_I$) comes into determining the probability as if it were the imaginary part of the Lagrangian density.

A major point of the present article is to set up the quantum formulation of our already published model, now really settling on taking the suggestive idea of just making the action complex, but with a priori a different set of coupling constants and m^2 for real and imaginary part separately.

A genuine problem with our kind of model is that very likely it predicts that special simple configurations leading to big probability may be arranged at a priori any time. That is to say, with our type of model it needs an explanation that one in particle almost never see any great arrangements being organized to occur later on. Really such arrangements might seem to us to be something like a hand of God, but they seem very seldom. Thus at first it looks that our type of model is already falsified by the non-appearance of arrangements. Really such a problem is almost obviously expected to occur in a model that like ours *does not a priori make any time reversal asymmetric assumption at the fundamental level*. Unless in the Hartle-Hawking no boundary postulate [10] we add some timereversal asymmetry spontaneously other otherwise that theory will be up to similar problems [11,12].

A model-language describing how final states can be imposed by a density matrix ρ_f is put forward by Hartle and Gell-Mann [14].

In the present work we hope for that a certain moment in the 'middle of times' will turn out to become dominant w.r.t. fixing the special solution selected as the realized one, and that this time can then be interpreted as a close to Big Bang time (there may not really be a true big bang but just an inflation era coming out of a deflation era continuously). Then since we live in the time after this decisive Big Bang simulating era there is for us a time reversal asymmetry, nevertheless it is a problem that like ours is even timetranslational invariant w.r.t.

the law that finally settle the 'initial conditions' to explain that there are not more prearranged events than one seemingly see.

However, we believe to have found some explanations able to suppress so many of these prearrangements that our model can be made compatible with present experience of essentially no prearrangements.

For really avoiding it we shall assume consistency of Dirac strings, but let us postpone that discussion to section 8.13 below.

Our model is really inspired from the considerations of time machines[13] and the troubles of needs for prearrangements in order to avoid the so called grand mother paradoxes, meaning the inconsistencies occurring when one seeks to go back in time and changes the events there.

We shall present the work by making *two* attempts to assumption about how to interpret the Feynman path integrals with the imaginary part of the action non-zero. In the first part of the paper we start out from letting the average of a dynamical variable $\mathcal{O}$ be given by equation (8.10) below, but that this is a priori not so good is seen by it not being (safely) real even if the dynamical variable $\mathcal{O}$ is real. Therefore in section 8.7 we restart the discussion so now from the side of the interpretation of the Feynman path way integrals in our model.

First trial of interpretation

In the next section 8.2, we shall put forward the basic formula for expectation values with our complex action model and the philosophy that this model even deliver the initial condition, or better the solution of equation of motion to be the one realized.

In section 8.3 we review our earlier reasons for that future should have only little influence on what happens.

In section 8.4 we then shall argue for some approximate treatment of the functional integral in the late times t, the future.

In section 8.5 we shall make use of the approximation of the future to obtain the usual quantum mechanics expressions at least in the case where our imaginary part S_I of the action can be ignored. (It should be stressed that we actually have used already a philosophy based on this S_I being non zero, so it is not fully zero.)

In section 8.6 it turns out that we – perhaps not completely convincing though – can make the effect be that we return to probability in practical scattering experiments say get conserved.

Second trial of interpretation:

In section 8.7 we restart the discussion of making the interpretation formula for the Feynman path way integral, which after some talk takes the way of using the classical approximation weighted with the exponential of minus 2 times the imaginary part of the action. In a subsection 8.7.5 we formally connect our model to our earlier one based on the probability weight $P(\text{path})$.

In section 8.8 we develop a rather general formula for the correlated probability for a series of dynamical quantities or operators $\mathcal{O}_i$ at different moments of time take values inside small ranges specified.

In section 8.9 we go a bit further in making the expressions like the ones one uses in practice in usual theories. Most importantly we again consider how to approximate the future when the effects of the imaginary part of the action is very small.

In section 8.10 we put the simplest example of the more general formula, namely a formula for the probability of just one operator at one time being in a given range. (This question would be impossible to predict even in principle in other theories, but we in principle can, but in practice not usually). But the resulting formula has what we call "squared form" in the sense that the projector comes in twice as a factor in it. The finding of a reduction to an unsquared form is left to section 8.12, while we in section 8.11 then give an example of application of very interesting physical significance. In fact section 8.11 predicts a broadening of the width of the Higgs particle due to the imaginary action.

In section 8.12 we then bring about a connection between the postulated interpretation formulas for probabilities put forward in part I and part II. In fact we find that they coincide under rather suggestive assumptions.

In section 8.13 we bring the promised argument for removing the effects of the imaginary action S_I from the domain of older accelerators, since otherwise our model would have been falsified. The argument is based on assuming monopoles.

In section 8.14 we conclude and give a bit of outlook.

Part I, First Trial of Interpretation

8.2 Philosophy and formula

Our basic modification of introducing an imaginary part in the actions leads to that integrand e^{iS} or $e^{\frac{i}{\hbar}S}$ of the Feynman path way integrand $\int e^{iS}\mathscr{D}\phi$ or $\int e^{\frac{i}{\hbar}S}\mathscr{D}\phi$ (if the Planck constant is written explicitly) varies a lot in magnitude, and not only in phase as usual. This effect is likely to make some regions in the space of paths – or we could restrict to the space paths with $\delta S = 0$, i.e. the space classical solutions – get a very much bigger weight in the integral than others. Actually it can likely happen that only a very narrow range of paths or better solutions (= paths obeying $\delta S = 0$) will quite dominate the integral

$$\int e^{iS}\mathscr{D}\phi. \tag{8.1}$$

That should naturally be taken to mean that the presumed narrow range of dominating paths represent the paths being actually realized in nature. It is in this way that we hope our model to essentially predict the initial state for the realized solutions. It is important to have in mind that such an effect of the imaginary action S_I of selecting narrow bunches of solutions can make the boundary conditions at an initial and a final time for a period to be studied say, superfluous. A bit optimistically we might imagine that the imaginary part of action makes the functional integral converge even without boundary condition specifications. Note that being allowed to throw boundary conditions away – having them replaced by effects of S_I – is a great/nice simplification. We consider this achievement as an aesthetically very nice feature of our model! Supposing that this works to deliver a meaningful Feynman-path integral (8.1) even without boundary conditions this way we must now decide how one is supposed to extract information now in principle for the true expectation value as it should occur even without further input. Note here that we are – but only in principle – proposing an exceedingly ambitious model compared to usual quantum field theories:

We want to predict expectation values without any further input than the mere complex action! This of course corresponds to that our level of ambition is to in addition to the usual time-development laws of nature <u>also</u> predict the initial conditions, i.e. what really happens!

To write down the formula for some physical quantity let us first exercise by a quantity $\mathcal{O}(\varphi|_t)$ which is a function of the fields $\varphi|_t$ restricted to some time t, where φ is a general symbol for all the fields in the model.

If we for instance use the Standard Model as the starting model, then providing it with an imaginary part of the Lagrangian density, then the symbol $\varphi(x)$ $(x \in \grave{R}^4)$ is really a set

$$\varphi = (A^a_\mu, \psi^b, H) \tag{8.2}$$

where the indices on the Fermion fields runs through the combination of flavor and color and/or W-spin components, while the index on the gauge fields run through the 12 gauge fields – 8 gluon color combination plus (B_μ) the $U(1)$-component and 3 W's –. Finally H is the two complex component Higgs field.

The quantity $\mathcal{O}(\varphi|_t)$ can of course be considered a functional of the whole field development $\mathcal{O}(\varphi)$ also, i.e. it could be consider a functional of the path of one wants.

The simplest proposal for what the average quantity $\mathcal{O}(\varphi)$ would be

$$\langle \mathcal{O}(\varphi) \rangle = \frac{\int e^{iS[\varphi]} \mathcal{O}(\varphi) \mathscr{D}\varphi}{\int e^{iS[\varphi]} \mathscr{D}\varphi}. \tag{8.3}$$

This would mean that we have a "sort of probability" given by

$$\text{"Probability of } \mathcal{O} \text{ being } \mathcal{O}_0\text{"} = \frac{\int \delta(\mathcal{O}(\varphi) - \mathcal{O}_0) e^{iS[\varphi]} \mathscr{D}\varphi}{\int e^{iS[\varphi]} \mathscr{D}\varphi} \tag{8.4}$$

Now, however, we must admit that conceiving of this expression as a probability is upset by the severe <u>problem</u> that it will typically be a complex number. There is no guarantee that it is positive or zero.

Thus a priori one would say that this simple expression for the probability density is quite untenable.

Nevertheless it is our intention to claim that we should – and that is then part of our model – use the simple expression (8.4) and the corresponding (8.3) and the expression to be given below for more general operators $\mathcal{O}$ corresponding also to (8.3) and (8.4).

First let us again stress that it is our a priori philosophy that somehow the imaginary part S_I managed to fix both a state in future and in past. Thereby asking the average of some quantity $\mathcal{O}$ becomes much like in an already finished double slit experiment (Bohr-Einstein) in which a particle already have been measured on the photographic plate (presumable on an interference line) what were the average position of the particle when it past the double slit screen. Really asking such a question concerning a quantity $\mathcal{O}$ that were not measured and could not have been measured without having disturbed the outcome of something later is one of the forbidden questions in quantum mechanics. Indeed it is by asking this sort of questions which are not answerable by measurement that Einstein can find ammunition against quantum mechanics. In other words our proposal (8.4) for "probability distribution" is a priori – with our present philosophy of a future essentially determined by S_I – an answer to a quantum mechanically forbidden question. Niels Bohr would say we should not ask it.

In that light it may of course not be so serious that our formula gives a rather stupid or crazy answer, a complex probability!

But now we have the problem of justifying that if we made a true measurement the answer would turn out to give positive (or zero) probability.

Let us take as the important feature of a measurement of some quantity $\mathcal{O}$ that there is an apparatus which makes a lot of degrees of freedom, ξ, say (really macroscopic systems) develop in a way depending on value of $\mathcal{O}$. Such an amplification of the effect of the actual value of $\mathcal{O}$ is characteristic for a measurement. Unless somehow there are special reasons for that S_I be insensitive to ξ (as we shall actually later seek to show but do not assume to be the case) we expect that S_I typically will depend on the macroscopically many d.o.f. ξ being influenced by $\mathcal{O}$-value measured. Now we argue like this: Since there is a huge (macroscopical) number of variables ξ depending on the value of $\mathcal{O}$ "measured", the imaginary part S_I of the action is likely to depend very strongly on this measured value – very rapidly varying.

We here think of S_I as the integral over the imaginary part of the Lagrangian L_I over all times $t \in]-\infty, \infty[$

$$\left(S_I = \int_{-\infty}^{\infty} L_I dt \right) \tag{8.5}$$

Because of the great complications in an actual measuring apparatus, let alone the further developments depending the measured value, publications and so on, the imaginary action S_I can easily be a very complicated function of the measured $\mathcal{O}$ value. Even if S_I as function of the measured $\mathcal{O}$ value should in principle be continuous it may in practice vary so much up and down – caused by accidents influenced by the broadcasted measuring value – that very likely the smallest value of S_I occurs for a seemingly accidental value of the measured $\mathcal{O}$. If the S_I-variation with the "measured $\mathcal{O}$" is indeed very strong so that the S_I variations are big the exponential weight e^{-S_I} contained in (8.3) and (8.4) will have a completely dominant value for only one measured $\mathcal{O}$-value.

In this way our model has the integral in the numerator of (8.4) be much bigger for one single value of $\mathcal{O}_0$. If so, then the ratio (8.4) is actually $\propto 1$ for this $\mathcal{O}_0$-value and negligible for all other $\mathcal{O}_0$-values. This means that our model much like usual measurement theory (in Copenhagen interpretation) predicts that crudely only one value of a measured quantity is realized. In principle it is even so that, bearing a very special situation, the result of the measurement is calculable by essentially minimizing the imaginary action S_I. In practice, however, such calculation will only be doable in extremely rare cases. (If we impress a special result by threatening with a Higgs-producing machine).

We postpone the argumentation for that the probability distribution to be obtained in practice shall be the one of usual quantum mechanical measurement theory partly to the later sections and partly to a subsequent paper.

At the end of this section let us extend slightly our formula (8.3) and thereby also (8.4), to the case where the quantity $\mathcal{O}$ corresponds in usual quantum mechanics to an operator that do not commute with the fields φ.

An operator corresponding to a quantity measurable at a moment of time t will in general in the quantum field theory considered be given by a matrix with a columns and rows in correspondence with field functions $\varphi|_t$ restricted to the time t. I.e. $\mathcal{O}$ is given by a "matrix"

$$\left(\varphi'|_t\middle|\mathcal{O}\middle|\varphi|_t\right) = \underbrace{\mathcal{O}(\varphi'|_t,\ \varphi|_t)}. \tag{8.6}$$

What should be the formulas replacing (8.3) and (8.4) in this more general case?

Well, our main starting point were that we assumed our imaginary part S_I to (essentially) fix both a further $|B\rangle$ and a past state $|A\rangle$. A natural notation to introduce is in fact – for the past –

$$\langle\varphi|_t|A\rangle = A[\varphi|_t] = \int_{\text{ending at } \varphi|_t} e^{iS_{-\infty \text{ to } t}}\mathscr{D}\varphi \tag{8.7}$$

and analogously

$$\langle B|\varphi|_t\rangle = \langle\varphi|_t|B\rangle^* = B[\varphi|_t]^* = \int_{\text{beginning at } \varphi|_t} e^{iS_{t \text{ to } +\infty}}\mathscr{D}\varphi \tag{8.8}$$

In this notation our previous formulas (8.3) and (8.4) are for the in $\varphi|_t$ diagonal operators $\mathcal{O}(\varphi|_t)$ become

$$\begin{aligned}\langle\mathcal{O}\rangle &= \frac{\int e^{iS}\mathcal{O}(\varphi|_t)\mathscr{D}\varphi}{\int e^{iS}\mathscr{D}\varphi}\\ &= \frac{\oint_{\varphi|_t}\langle B|\varphi|_t\rangle\mathcal{O}(\varphi|_t)\langle\varphi|_t|A\rangle}{\oint_{\varphi|_t}\langle B|\varphi|_t\rangle\langle\varphi|_t|A\rangle}\\ &= \frac{\langle B|\mathcal{O}|A\rangle}{\langle B|A\rangle}\end{aligned} \tag{8.9}$$

and (8.4) becomes

$$\text{"Probability of } \mathcal{O} \text{ being } \mathcal{O}_0\text{"} = \frac{\langle B|\delta(\mathcal{O}-\mathcal{O}_0)|A\rangle}{\langle B|A\rangle} \tag{8.10}$$

Now really we want to suggest that formula (8.9) and (8.10) can also be used for operators that are not simply functions of the fields $\varphi|_t$ at time t, used in the functional integral.

In order to justify that extension of our interpretation formulas we want to remark:

1. Provided a Hermitean operator $\mathcal{O}$ has either $|A\rangle$ or $|B\rangle$ as eigenstate then the eigenvalue $\mathcal{O}'$ in question can of course be extracted as
$$\mathcal{O}' = \frac{\langle B|\mathcal{O}|A\rangle}{\langle B|A\rangle} \tag{8.11}$$
2. One can quite generally – by Fourier transformations at every step in a time lattice – rewrite a functional integral of the Feynman path way integral form from some set of variables φ to a conjugate set:
$$\begin{aligned}\int e^{iS}\mathscr{D}\varphi &\overset{\text{latticitation}}{=} \int \prod_{t\in\{t-\text{lattice}\}} \mathscr{D}^{(3)}\varphi|_t e^{i\sum_{t\in\{\text{lattice}\}} L_{\text{discr}}\left(\varphi|_t, \frac{\varphi|_{t+\Delta t}-\varphi|_t}{\Delta t}\right)\Delta t} \\ &= \int \prod_{t\in\{t-\text{lattice}\}} \mathscr{U}(\varphi|_{t+\Delta t}, \varphi|_t)\mathscr{D}^{(3)}\varphi|_t \end{aligned}\tag{8.12}$$

where
$$\mathscr{U}(\varphi|_{t+\Delta t}, \varphi|_t) = e^{iL\left(\varphi|_t, \frac{\varphi|_{t+\Delta t}-\varphi|_t}{\Delta t}\right)}, \tag{8.13}$$
can be rewritten into $\hat{\mathscr{U}}(\Pi|_{t+\Delta t}, \Pi|_t)$ matrices obtained from the $\mathscr{U}(\varphi|_{t+\Delta t}, \varphi|_t)$ by Fourier functional transformations
$$\hat{\mathscr{U}}(\Pi|_{t+\Delta t}, \Pi|_t) \overset{\text{def}}{=} \int \mathscr{D}^{(3)}\varphi|_{t+\Delta t}\, e^{+i\varphi|_{t+\Delta t}\Pi|_{t+\Delta t}} \mathscr{U}(\varphi|_{t+\Delta t}, \varphi|_t) e^{-i\varphi|_t\Pi|_t} \mathscr{D}^{(3)}\varphi|_t \tag{8.14}$$
Now of course for long chains of $\hat{\mathscr{U}}$-matrices (ignoring end problems) you have
$$\begin{aligned}&\int \prod_{t\in\{t-\text{lattice}\}} \mathscr{U}(\varphi|_{t+\Delta t}, \varphi|_t)\mathscr{D}^{(3)}\varphi|_t \\ &\overset{\text{except for end problems}}{=} \int \prod_{t\in\{t-\text{lattice}\}} \hat{\mathscr{U}}(\Pi|_{t+\Delta t}, \Pi|_t)\mathscr{D}^{(3)}\Pi|_t \end{aligned}\tag{8.15}$$
Supposedly you can put the right hand side into a form
$$\int e^{iS^{(\text{in }\Pi)}[\Pi,\, \Delta\Pi-\text{defferences}]}\mathscr{D}\Pi \tag{8.16}$$
Now you may argue with the same intuitive suggestion for getting
$$\mathcal{O}(\Pi|_t) = \frac{\int e^{iS^{(\text{in }\Pi)}}\mathcal{O}(\Pi|_t)\mathscr{D}\Pi}{\int e^{iS^{(\text{in }\Pi)}}\mathscr{D}\Pi} \tag{8.17}$$
as we did for (8.3). By thinking of doing the just presented Fourier transformation partly we might argue for a similar average formula for any operator
$$\begin{aligned}\langle\mathcal{O}(\varphi|_t,\, \Pi\_t)\rangle &= \frac{\int e^{iS}\mathcal{O}(\varphi|_t, \Pi|_t)\mathscr{D}\varphi}{\int e^{iS}\mathscr{D}\varphi} \\ &= \frac{\langle B_t|\mathcal{O}(\varphi|_t, \Pi|_t)|A_t\rangle}{\langle B_t|A_t\rangle}. \end{aligned}\tag{8.18}$$

Really this proposal looks very bad because of several lacks of good correspondence with usual quantum mechanics a priori:

a) Obviously $|A_t\rangle$ is here (a sort of) wave function of the universe at time t, but our probability density (8.4) or

$$\text{“Probability for } \mathcal{O} \text{ being } \mathcal{O}_0\text{"} = \frac{\langle B_t|\delta(\mathcal{O}-\mathcal{O}_0)|A_t\rangle}{\langle B_t|A_t\rangle} \tag{8.19}$$

is not quadratic in $|A_t\rangle$ as we expect from the usual corresponding formula

$$\text{“Probability for } \mathcal{O} \text{ being } \mathcal{O}_0 \text{ usual"} = \frac{\langle A_t|\delta(\mathcal{O}-\mathcal{O}_0)|A_t\rangle}{\langle A_t|A_t\rangle}. \tag{8.20}$$

b) As already stated the "probability density" (8.19) is even usual complex and needs the above measurement special case to become just positive.

We shall below argue for an approximate treatment of the future part $|B_t\rangle$ of the integral thereby achieving indeed a *rewriting into an expression which is of the form with* $|A_t\rangle$ *coming squared*. Indeed we shall rewrite (8.19) into (8.20) below.

8.2.1 Justification of philosophy from semiclassical approximation

In semiclassical approximation one simply evaluates different contributions to the functional integral (1) by seeking the different extrema for e^{iS} or equivalent $S = S_R + iS_I$. Around such an extemum it is extremely well known that one can approximate S by the Taylor expansion up to second order

$$\begin{aligned} S = S(\text{extremum}) + \frac{1}{2}\int \frac{\partial^2 S}{\partial\varphi_1(x_1)\partial\varphi_2(x_2)} \\ \cdot \left(\varphi_1(x_1) - \varphi_1^{\text{extr}}(x_1)\right)\left(\varphi_2(x_2) - \varphi_2^{\text{extr}}(x_2)\right) + \cdots \quad d^4x_1 d^4x_2 \end{aligned} \tag{8.21}$$

where then the linear terms

$$\int \frac{\partial S}{\partial\varphi_1(x_1)}\left(\varphi_1(x_1) - \varphi_1^{\text{extr}}(x_1)\right) d^4(x_1) \tag{8.22}$$

vanish because of the extremiticity condition. Here $\varphi_1^{\text{extr}}(x_1)$ and $\varphi_2^{\text{extr}}(x_2)$ denote the fields at the extremum field configuration development. Such an extremum as is well known corresponds to a solution to

$$\delta S = 0 \tag{8.23}$$

i.e. solving the variational principle leading to classical equations of motion.

The main term in the exponent $iS(\text{extremum})$ is in the usual real action case purely imaginary and thus only gives rise to a phase factor so that in this approximation the contribution has the same size for all the classical solutions, provided they can go on for real field configurations. With our S_I included, however, we tend to get even to the approximation of the first term in the Taylor expansion

(8.21) a real term $-S_I$ into the exponent and thus the order of magnitude for one classical solution compared to another can easily become tremendous

$$|e^{iS(\text{extremum})}| = e^{-S_I(\text{extremum})}. \tag{8.24}$$

It is our philosophy that only relatively very few classical solution have terms $e^{-S_I(\text{extremum})}$ dominating violently the rest. In this sense we expect and assumed that such one or a very few classical solutions could be considered the only one realized. With very big size of S_I – and that can easily come about for a couple of reasons – it gets relatively only exceedingly few classical solutions that are competitive in the sense that for most classical solutions (of (8.23)) you have exceedingly small e^{-S_I} compared to the few dominant ones. As the reasons for S_I being big when it is not forbidden by gauge invariance and the condition that Dirac strings shall be unobservable we can give:

1. There is in analogy to the S_R-term a $\frac{1}{\hbar}$-factor in front of S_I. For practical purposes we know that we shall consider the Planck constant $\hbar$ to be very small.
2. We could easily get Avogadros number come in as a factor in the S_I because it would get such a factor a priori since there are typically in the world of macroscopic bodies of that order magnitude molecules.

8.3 Approximate treatment of future part of functional integral (treatment of $|B_t\rangle$)

In our earlier works[8] – in which we mainly worked in the classical approximation – we presented some arguments that in the era which have been going on since short time of after some effective (or real) Big Bang the imaginary Lagrangian or action L_I or S_I effectively became very trivial. That should mean that under the times starting after some early Big Bang and extending into the future we could approximately take L_I and the part of S_I coming from this era as independent of what are the practical possibilities for what can go on. Thus we should in this present era supposed to extend into even the infinite future be allowed to ignore in first approximation the imaginary parts L_I or S_I.

The reasons, which we presented for that were that this present era including supposedly all future is dominated by two types of particles:

1. Massless particles (really the entropy of the universe is today dominated by the massless microwave back ground radiation of photons).
2. Non-relativistic particles carrying practically conserved quantum numbers (the nucleons and the electrons are characterized by their charges and baryon or lepton number so as to make their decays into lighter particles impossible).

The argument then went that we could write the action – actually both real S_R and imaginary S_I – for these particles, treated as particles, as a sum having each giving a contribution proportional to the eigentimes for them:

$$S_R,\, S_I = \sum_{\text{particles } P} K_{P\{\substack{R\\I}\}} \cdot \tau_P. \tag{8.25}$$

That is to say that each of the particles contribute to S_I say a contribution proportional to the eigentime

$$S_{I \text{ from } P} \propto \tau_P. \tag{8.26}$$

Now for massless particles any step in eigentime

$$\Delta\tau_P = 0 \quad (\text{for massless}) \tag{8.27}$$

and for nonrelativistic ($\simeq$ slow) particles, such a step is

$$\Delta\tau_P = \Delta t \tag{8.28}$$

equal to the usual time. Since the number of the conserved quantum numbers protected particles are all the time the same the whole contribution to the S_I from the present era becomes very trivial:

Zero from the massless, and just a constant integrated over coordinate time for the conserved particles.

In addition there are terms from interactions contributing a priori to say S_I also. Since, however, in the era since a little after Big Bang the density of particles were low in fundamental units presumably also the interaction contributions would be much suppressed in this after Big Bang era.

So all together we estimate that it is only the very early Big Bang times that will dominate S_I. Thus the solution to the equations of motion being in a model with an imaginary action S_I selected to be the realized one will mainly depend on what happened in that solution in the early Big Bang era. This means that it will be in our era as if it were the initial state that were a rather special one determined by having an especially small contribution to S_I from Big Bang times. This would mean a rather well determined starting state roughly which interpreted as a macrostate would be one with low entropy. That is at least a good beginning for obtaining the second law of thermodynamics, since then there are supposedly no strong effects of S_I any more to enforce the universe to go to any special macrostate. Rather it will go into bigger and bigger macrostates meaning that they have higher and higher entropy.

Although we have now argued for approximately seeing no effects of S_I in the era after Big Bang implying that our model should have no effects in this era, this is however, presumably not being quite sufficiently accurate.

We shall, however, below in section 8.9 invent or find arguments that will allow us to get completely rid of the L_I or S_I from the in the Standard Model already found particles. Only for the Higgs involving processes our arguments in section 9 based on gauge symmetry and the assumption of unobservability of Dirac strings associated with monopoles will not quite function. Thus we still expect that an S_I-contribution pops up with Higgs-particles. But since Higgs-particles are so far not well studied such an effect of S_I might well have been overlooked so far.

8.4 Treatment of $|B_t\rangle$ or Treatment of the future factor in the functional integral

In equation (8.8) above we defined what one could call "the future part" of the functional integral relative to the time t. It should however be kept in mind that it is a part in the sense that the full integral is a contraction (a sort of product) of the past part and this future part,

$$\int e^{iS}\mathscr{D}\varphi = \langle B_t|A_t\rangle. \tag{8.29}$$

Now we must remember that according to the second law of thermodynamics the state of the universe if at all obtainable (calculable) should be so by considering the development in the past having lead to it. The future, however, should be rather shaped after what happened earlier. This suggests that we should mainly have the possibility to guess or know $|A_t\rangle$ but determined from the fundamental Lagrangian as our model suggests. Really in order not to disagree drastically with the second law of thermodynamics the future should be shaped from the past and reflect the latter. However, there should not be – at least not much – adjustment of the happenings at say time t in order to arrange something special simple happening in future. This means in or formalism that the by the S_I future contributions determined $|B_t\rangle$ should according to second law better disappear quite from our formula for predicting probabilities for operator values, i.e. from (8.4) or more generally (8.19).

Now, however, as we argued in foregoing section – section 8.3 – reviewing previous articles working in the classical approximation it should be the state of a solution to the equations of motion in the early Big Bang time that dominates the selection of such a solution to be the realized one. The future on the other hand has only a small effect, if any, on choosing the true or realized solution. With the arguments to be given in section 8.9 we argue for the effects of S_I being even smaller in the future. Nevertheless we have if we talk exactly also effects of S_I even in the future. Otherwise the hypothesis that the integral (8.8) defining $|B_t\rangle$ would be senseless since the e^{-S_I}-weighting is needed to suppress the integrand e^{-S_I} enough to make hope of a sensible practical convergence.

However, we have in section 8.3 and will in section 8.9 argue for that S_I varies much less in the future than in Big Bang era.

It is now the purpose of the present section to use this only weak S_I variation with the fields in the future to argue for an approximation in density matrix terminology for the future part $|B_t\rangle$ of the functional integral.

Let us indeed perform the following considerations for estimating the crude treatment of $|B_t\rangle$ which we shall use:

a) Since S_I has in practice only small non-trivial contributions in the future it is needed to involve contributions in the integral

$$S_{I t' \text{ to } +\infty} = \int_{t'}^{\infty} dt \int d\boldsymbol{x}\, L_I \tag{8.30}$$

from very large $t \geq t'$.

b) At these enormous t regions then at the end we get finally a rather restricted range of solutions. – we can think of classical solutions here, if we like –
c) Now the solutions from the enormously late times under a) have to be developed backward in time to the time t' say to deliver the state $|B_{t'}\rangle$ (really we first get $\langle B_{t'}|\phi\rangle$ from equation (8.8)).
d) Now we make the assumption that the system/world is sufficiently "ergodic" and the large times so large and so smeared out (also because of the smallness of the L_I-effects) that we can take it that there is almost the same probability for finding the system in state $|B_{t'}\rangle$ at any place in phase space allowed by the conserved quantum numbers of the theory practically valid in the future era.
e) Ignoring for simplicity the conserved quantities we thus argued that with equal probability; equally distributed in phase space, we have that $|B_{t'}\rangle$ will be any state.
f) We can especially imagine that we have chosen a basis of wave packet states $|w\rangle$ in the field configuration space so that they fill smoothly the phase space – accessible without violating the conservation laws relevant –. Taking these to be – approximately – orthonormal $\langle w|w'\rangle \approx \delta ww'$ we clearly get for the average expectation of the projection operator

$$P_{B_{t'}} = |B_{t'}\rangle\langle B_{t'}| \tag{8.31}$$

the estimate

$$\mathrm{av}(|B_{t'}\rangle\langle B_{t'}|) = \frac{1}{N}\sum_{w}|w\rangle\langle w| \simeq \frac{1}{N}\underline{1} \tag{8.32}$$

where N is the number of states in the basis

$$|w\rangle,\ w = 1, 2, \cdots, N. \tag{8.33}$$

That is to say we have argued for that our weak S_I-influence in future combined with an assumed approximate ergodicity leads to that we can approximate

$$|B_{t'}\rangle\langle B_{t'}| \approx \frac{1}{N}\underline{1} \tag{8.34}$$

in practice for all t' at least a bit later than the earliest Big Bang.

The crude estimate that we could replace $|B_t\rangle\langle B_t|$ by $\frac{1}{N}\underline{1}$ derived as formula ($|B_t\rangle\langle B_t| \approx \frac{1}{N}\underline{1}$) were based on that L_I were in practice small.

8.5 Deriving a more usual probability formula

We shall now make use of approximation (8.34) for the "future factor" in the functional integral in order to obtain an expression rewriting the formulas like (8.3), (8.4) and (8.18) and (8.19) into expressions analogous to (8.20).

The calculation is in fact rather trivial, starting say from the most general of our postulated expressions (8.18):

$$\begin{aligned}
\langle \mathcal{O}(\varphi|_t, \Pi|_t)\rangle &= \frac{\int e^{iS}\mathcal{O}(\varphi|_t, \Pi|_t)\mathscr{D}\varphi}{\int e^{iS}\mathscr{D}\varphi} \\
&= \frac{\langle B_t|\mathcal{O}(\varphi|_t, \Pi|_t)|A_t\rangle}{\langle B_t|A_t\rangle} \\
&\overset{\text{trivial step}}{=} \frac{\langle A_t|B_t\rangle\langle B_t|\mathcal{O}(\varphi|_t, \Pi|_t)|A_t\rangle}{\langle A_t|B_t\rangle\langle B_t|A_t\rangle} \\
&\overset{\text{using (8.34)}}{=} \frac{\langle A_t|\frac{1}{N}\underline{1}\mathcal{O}(\varphi|_t, \Pi|_t)|A_t\rangle}{\langle A_t|\frac{1}{N}\underline{1}|A_t\rangle} \\
&= \frac{\langle A_t|\mathcal{O}(\varphi|_t, \Pi|_t)|A_t\rangle}{\langle A_t|A_t\rangle}
\end{aligned} \tag{8.35}$$

which is the completely usual quantum mechanical expression for the expectation value of the operator $\mathcal{O}(\varphi|_t, \Pi|_t)$ in the wave functional state $|A_t\rangle$.

With this expression we see that we should be allowed, as we anyway would expect, to use $|A_t\rangle$ as the quantum state of the universe.

It should be noted though that our $|A_t\rangle$ is *in principle* calculable from the "theory" when as we shall of course, count also the S_I-expression as part of the theory. In this way our model is widely more ambitious than usual quantum mechanics:

We have – much like the Hartle-Hawking no boundary proposal – a functional integral (8.7) delivering in principle the wave functional $|A_t\rangle$. In usual quantum mechanics the wave function is left for the experimental physicist to find out from his somewhat difficult job of preparing the state. In practice we would presumably have to let him be so helped by observation and arrangements under the preparation that we almost leave to him the usual job. We should, however, have in mind that in preparing a state one will usually need to trust that some material is a rather pure chemical substance or that no disturbing cosmic radiation spoils the preparation. These kinds of trusts are usually based on some empirical experience which in turn makes use of that big assembles of pure substances are easily/likely available and that generally cosmic ray has low intensity. Such trusts however, are at the very root connected with the starting state – the cosmology – of our world. But this starting state for practical purposes is in our model based on the activity of our L_I in early Big Bang times of the initial state of the universe.

Thus it is even in the practical way of preparing a quantum state a lot of reference to our S_I.

If, however, somehow the universe develops into states where L_I is no longer negligible we should expect corrections to such an approximation ($|B_t\rangle\langle B_t| \approx \frac{1}{N}\underline{1}$).

8.6 Time development and S_I corrections to $|B_t\rangle$

From the definitions (8.7) and (8.8) of $|A_t\rangle$ and $|B_t\rangle$ it is trivial to derive the time development formulas for these Hilbert space vectors (say for $t' > t$)

$$\begin{aligned} |A_{t'}\rangle &= \int_{\text{over time}-\infty \text{ to } t'} e^{iS_{-\infty \text{ to } t'}} \mathscr{D}\varphi \\ &= \int_{\text{over } t \text{ to } t'} e^{iS_{t \text{ to } t'}} A_t[\varphi|_t] \mathscr{D}\varphi \\ &= \mathscr{U}(t', t)|A_t\rangle \end{aligned} \tag{8.36}$$

where $\mathscr{U}(t', t)$ is the operator corresponding to the matrix (with columns and rows marked by $\varphi|_t$ configurations)

$$\mathscr{U}(\hat{\varphi}|'_{t'}, \hat{\varphi}|_t) = \int_{\text{over } t \text{ to } t' \text{ with } \varphi|_{t'}=\hat{\varphi}|_{t'} \text{ and } \varphi|_t=\hat{\varphi}|_t} e^{iS_{t \text{ to } t'}} \mathscr{D}\varphi. \tag{8.37}$$

Similarly we have from (8.8) for $t' > t$ again, first taking the complex conjugate of (8.8)

$$\langle \varphi|_t |B_t\rangle = \int_{\text{beginning at } \varphi|_t} e^{-iS^*_{t \text{ to } +\infty}} \mathscr{D}\varphi \tag{8.38}$$

and thus

$$\langle \varphi|_t |B_t\rangle = \int_{\text{over } t \text{ to } t'} e^{-iS^*_{t \text{ to } +\infty}} \langle \varphi|_{t'}|B_{t'}\rangle \mathscr{D}\varphi \tag{8.39}$$

which can be written

$$|B_t\rangle = \mathscr{U}_{\text{with } L_I \to -L_I}(t', t)^+ |B_{t'}\rangle. \tag{8.40}$$

Here we used that e.g.

$$S_{t \text{ to } +\infty} = \int_t^\infty dt \int d^3\mathbf{X}(\mathcal{L}_R + i\mathcal{L}_I) \tag{8.41}$$

where $\mathcal{L}_R$ and $\mathcal{L}_I$ are respectively the real and the imaginary parts of the Lagrangian densities. So

$$S^*_{t \text{ to } +\infty} = \int_t^\infty dt \int d^3\mathbf{x}(\mathcal{L}_R - i\mathcal{L}_I), \tag{8.42}$$

and now restricting ourselves for {pedagogics/simplicity} at first to boson fields we have (usually) that for them $\mathcal{L}_R$ and $\mathcal{L}_I$ are even order in the time derivatives which are under latticification

$$\partial_t \varphi_{(t, \mathbf{x})} \approx \frac{\varphi(t + \Delta t, \mathbf{x}) - \varphi(t, \mathbf{x})}{\Delta t}. \tag{8.43}$$

Thus conceived as operators between the configuration at the two close by times t and $t + \Delta t$, i.e. with columns and rows marked by $\varphi|_{t+\Delta t}$ and $\varphi|_t$ we have e.g.

$$(\mathcal{L}_R + i\mathcal{L}_I)^+ = \mathcal{L}_R - i\mathcal{L}_I \tag{8.44}$$

because

$$\mathcal{L}_R^T = \mathcal{L}_R \text{ and } \mathcal{L}_I^T = \mathcal{L}_I \tag{8.45}$$

and

$$\mathcal{L}_R^* = \mathcal{L}_R \text{ and } \mathcal{L}_I^* = \mathcal{L}_I. \tag{8.46}$$

In formula (8.40) of course the meaning of the under symbol text in the expression $\mathscr{U}_{\text{with } L_I \to -L_I}(t', t)^+$ is that in addition to taking the Hermitian conjugation of $\mathscr{U}(t', t)$ as defined by the matrix representation (8.37) one shall shift the sign for all occurrences of the L_I-part of the Lagrangian or of the L_I-part of the Lagrangian density. One should have in mind that it is easily seen that

$$\mathscr{U}(t', t)^{-1} = \mathscr{U}_{\text{with } L_I \to -L_I}(t', t)^+. \tag{8.47}$$

Especially the "usual" case of $L_I = 0$ means that $\mathscr{U}(t', t)$ becomes unitary. This relation (8.47) together with (8.40) and (8.36) ensures that

$$\langle B_t | A_t \rangle = \int e^{iS_{-\infty \text{ to } +\infty}} \mathscr{D}\varphi \tag{8.48}$$

can be true independent of the time t chosen on the left hand side.

Since (6.1) represents a completely usual time development of the 'wave function' $|A_t\rangle$ we have of course analogously to the usual theory

$$i\frac{d|A_t\rangle}{dt} = H|A_t\rangle \tag{8.49}$$

where then H is the to the action

$$S = S_R + S_I \tag{8.50}$$

corresponding Hamiltonian. As we saw under point a) in section 8.2 formula (8.20) we can consider

$$|A_t\rangle \tag{8.51}$$

the wave function for the universe essentially. But really because of the normalizing denominator in (8.20) it is rather the normalized $|A_t\rangle$, namely

$$|A_t\rangle_{norm} = |A_t\rangle / \sqrt{\langle A_t | A_t \rangle} \tag{8.52}$$

which is the true wave function.

It is important to remeark that precisely because we now find that we shall use the normalized wave function rather than $|A_t\rangle$ itself we do not get as could be feared a lack of conservation of probability due to the non-unitarity of the time development. Have in mind that the to a non-real action corresponding Hamiltonian H will not be Hermitean! But with the normalizaion comming from the $\langle A_t | A_t \rangle$ in the denominator in (8.20) the total probability will anyway remain unity.This result matches nicely with the from the slightly different start evaluated (9.22) below.

Part II, Second Trial of Interpretation

8.7 Second Interpretation of the functional integral

Usually one only uses the functional integral over a time interval to evaluate a transition matrix element from an initial time t_i to a final time t_f

$$U\left(\psi_f(\phi|_f), \psi_i(\phi|_i)\right) = \int \mathscr{D}^{\text{fixed time}}\phi|_f \int \mathscr{D}^{\text{fixed time}}\phi|_i \mathscr{D}\phi\, e^{iS_{t_i \text{ to } t_f}[\phi]} \tag{8.53}$$

where

$$S_{t_i \text{ to } t_f} = \int_{t_i}^{t_f} \int \mathcal{L}(x) d^3\mathbf{x} dt \tag{8.54}$$

and the functional integral over $\mathscr{D}\phi$ is restricted to ϕ-functions (field developments, or paths) which at times t_i and t_f respectively coincides with $\phi|_i$ and $\phi|_f$ respectively.

In the present article we, however, have the ambition of having the functional integral determine a priori not only the development with time but also say something about the initial conditions so that we a priori might ask for the probability of some dynamical variable $\mathcal{O}$ say having certain value $\mathcal{O}$ at a certain time without imposing any initial conditions. In order to obtain a formula or proceedure or how to obtain such probabilities for what shall happen we have to assume such a formula.

We therefore need some intuitive and phenomenological guess leading to such a formula/prescription.

In order to propose such a formula in a sensible way we shall first consider a semiclassical approximation for our functional integral supposed to be connected with and describing the development of the Universe

$$\int \mathscr{D}\phi\, e^{iS[\phi]} \tag{8.55}$$

where we remember that in our model the $S[\phi]$ is not as usual real but is allowed to be complex.

8.7.1 Semiclassical approach

For first orientation let us imagine that the imaginary part of the action $S[\phi]$ is effectively small in the sense that we can obtain the most significant contributions to the functional integral by asking for saddle points for the real part S_R. That is we ask for field development solutions to the variational principle

$$\delta S_R = 0. \tag{8.56}$$

Without specifying the boundary conditions at $t \to \pm\infty$ in our functional integral there should be (essentially) one solution for any point in the (classical) phase space of the field theory described. For the enumeration of the various development solutions ϕ we could use the field and conjugate field configuration at any chosen moment of time, to say. However, now our hope and speculation is that the imaginary part should give a probability weight distribution over the set ($\simeq$ phase space) of these classical solutions.

8.7.2 A first but wrong thinking

It is clear that we must make a definition of an expectation value for function(al) $\mathcal{O}$ say of the field development ϕ so that if a single (semi) classical solution ϕ_{sol} comes to be highly weighted then this expectation value should be $\mathcal{O}[\phi_{\mathrm{sol}}]$.

We might therefore at first think of

$$\langle\mathcal{O}\rangle = \frac{\int \mathscr{D}\phi\mathcal{O}[\phi]\, e^{iS[\phi]}}{\int \mathscr{D}\phi\, e^{iS[\phi]}}. \tag{8.57}$$

If really a single classical path contributed completely dominantly to both numerator and denominator, then indeed we would obtain that this proposal would obey

$$\langle\mathcal{O}\rangle = \mathcal{O}[\phi_{\mathrm{sol\,dom}}] \tag{8.58}$$

where $\phi_{\mathrm{sol\,dom}}$ is this single dominant solution.

It is however likely that if will be more realistic to imagine that there is a huge number of significant classical solutions ϕ_{sol}. But then appears the "problem" that in the expansion of the numerator functional integral into contributions from the various (semi) classical solutions $\phi_{\mathrm{sol}\,i}$:

$$\int \mathscr{D}\phi\mathcal{O}[\phi]e^{iS[\phi]} = \sum_{\phi_{\mathrm{sol}\;i}\ \text{all the classical solutions}} e^{iS[\phi_{\mathrm{sol}\,i}]}\mathcal{O}[\phi_{\mathrm{sol}\,i}]\sqrt{\det_i}^{-1} \tag{8.59}$$

the various contributions contribute with quite different signs or rather phases due to the appearance of the phase factor $e^{iS_R[\phi_{\mathrm{sol}\,i}]}$. The proposal just put forward thus is not as it stands a usual average, it lacks the usual requirement of an average of being performed with a positive weight. Rather the summation over the contribution becomes a summation with random phases to a good approximation. That means that if we classify in some ways the different solutions $\phi_{\mathrm{sol}\,i}$ into classes, then what would sum up when such classes are combined would be the squared contributions rather than the contributions themselves. In other words, if we define a contribution to

$$\int \mathscr{D}\phi\, e^{iS[\phi]}\mathcal{O}[\phi] = \sum_{\phi_{\mathrm{sol}\,i}} \sqrt{\det_i}^{-1} e^{iS[\phi_{\mathrm{sol}\,i}]}\mathcal{O}[\phi_{\mathrm{sol}\,i}] \tag{8.60}$$

where

$$\det_i = \det\left(\frac{\delta^2}{\delta\phi_1(x_1)\delta\phi_2(x_2)}\right) \tag{8.61}$$

from a certain class of semi classical solution $\mathscr{C}_k$ then the quantities such as

$$\int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \equiv \sum_{\phi_{\text{sol } i \in \mathscr{C}_k}} \sqrt{\det_i}^{-1} e^{iS[\phi_{\text{sol } i}]} \mathcal{O}[\phi_{\text{sol } i}] \tag{8.62}$$

obey approximately

$$\begin{aligned} &\left| \int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_1} \right|^2 + \left| \int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_2} \right|^2 \\ &\approx \left| \int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_1 \cup \mathscr{C}_2} \right|^2 . \end{aligned} \tag{8.63}$$

However we do not have a similar addition formula for numerical values as

$$\left| \int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_1} \right|^2 , \tag{8.64}$$

when they are not squared. However, of course, we do have

$$\begin{aligned} &\int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_1} + \int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_2} \\ &\approx \int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_1 \cup \mathscr{C}_2} \end{aligned} \tag{8.65}$$

but this relation has terms of typically rather random phases.

8.7.3 Approaching a probability assumption

If we take $\mathcal{O}$ to be a "projection operator" in the sense of being a functional of ϕ only taking the values 0 and 1 then $\int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k}$ should give the chance for solutions in the class $\mathscr{C}_k$ to pass through the configuration-class for which $\mathcal{O}[\phi] = 1$. Because of the (random) phase and the lack of simple numerical additivity mentioned if the foregoing subsection we are driven to assume that the probability for ϕ being in the $\mathcal{O}[\phi] = 1$ region must be given by the squared contributions

$$\left| \int \mathcal{O}\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \right|^2 . \tag{8.66}$$

Calling the region in the space of ϕ's consisting of the ϕ's obeying $\mathcal{O}[\phi] = 1$ with our "project $\mathcal{O}$", for region M, we get

$$\text{Prob}(M) \propto \left| \int M\, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \right|^2 \tag{8.67}$$

for restriction to the class $\mathscr{C}_k$.

This means using the probability of the complementary set $\mathbb{C}M$ of M

$$\mathrm{Prob}(\mathbb{C}M) \propto \left| \int \mathbb{C}M \, e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \right|^2 \tag{8.68}$$

and the additivity (8.63)

$$\left| \int e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \right|^2 = \left| \int_M e^{i} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \right|^2 + \left| \int_{\mathbb{C}M} e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \right|^2 \tag{8.69}$$

we derive

$$\begin{aligned}
\mathrm{Prob}(M) &= \frac{\left| \int_M e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \right|^2}{\left| \int e^{iS} \mathscr{D}\phi \Big|_{\text{from class } \mathscr{C}_k} \right|^2}. \\
&= \frac{\left| \sum_{\phi_{\text{sol } i} \text{ in } M} e^{iS} \sqrt{\det_i}^{-1} \Big|_{\text{from class } \mathscr{C}_k} \right|^2}{\left| \sum_{\phi_{\text{sol } i}} e^{iS} \sqrt{\det_i}^{-1} \Big|_{\text{from class } \mathscr{C}_k} \right|^2} \\
&\overset{\text{using random phases}}{\cong} \frac{\sum_{\phi_{\text{sol } i} \text{ in } M \cap \mathscr{C}_k} e^{-2S_I} \det^{-1}}{\sum_{\phi_{\text{sol } i} \text{ in } \mathscr{C}_k} e^{-2S_I} \det^{-1}}.
\end{aligned} \tag{8.70}$$

Here in principle of a classical approximation the e^{-2S_I} factor is much more important than the "quantum correction" $\det^{-1}$. Thus we would ignore the determinant $\det^{-1}$ factor in first approximation.

Then we arrived to the picture here that the probability distribution over phase space - at some chosen time, that due to Liouville's theorem does not matter - is given by $e^{-2S_I[\phi_{\text{sol}}]}$ where ϕ_{sol} is the classical field solution associated with the point in phase space for which $e^{-2S_I[\phi_{\text{sol}}]}$ shall be the probability density.

8.7.4 About the effect of S_I in the classical approximation

To appreciate the just given probability density $e^{-2S_I[\phi_{\text{sol}}]}$ over phase space

$$P\left(\phi|_{t_0}, \Pi|_{t_0}\right) \mathscr{D}\phi|_{t_0} \mathscr{D}\Pi|_{t_0} \propto e^{-2S_I[\phi_{\text{sol}}]} \mathscr{D}\phi|_{t_0}, \mathscr{D}\Pi|_{t_0} \tag{8.71}$$

one should have in mind that in the classical approximation of the universe developing along a solution ϕ_{sol} to the equations of motion

$$\delta S_R = 0, \tag{8.72}$$

the development is given quite uniquely by the equations of motion. The only place in which the imaginary part then comes in is in weighting with various probability densities the various "initial state data" $(\phi|_{t_0}, \Pi|_{t_0})$ – i.e. the phase space point –. Once you know the initial state of the (sub)system considered the equation of motion determines everything in the classical approximation determined by S_R just described, the S_I gets totally irrelevant. In other words it is only to know something about the "initial state" that S_I has relevance. Here the usual terminology of "initial state" shall especially in our model not be taken too seriously in as far as it with the word "initial" refers to a beginning moment, the Big Bang start say. No, as we just mentioned one can use any moment of time t_0 for the description of the phase space describing the set of classical solutions ϕ_{sol}. This t_0 time does not have to be the first moment – even if such one should exist –. Rather we can use any moment of time as t_0. In the usual theory we would tend to use t_0 being the initial moment and the state at this moment should then be one of very low entropy describing our start of universe state. However, in our model there is the rather unusual feature that the probability weight $e^{-2S_I[\phi_{sol}]}$ is given via a functional $S_I[\phi_{sol}]$ depending on how the solution ϕ_{sol} behaves at all different times t and not only at t_0. Since we by the classical equations of motion can calculate the whole time development ϕ_{sol} from the fields and their conjugate $(\phi|_{t_0}, \Pi|_{t_0})$ at some chosen time t_0, we can of course also consider $e^{-2S_I[\phi_{sol}]}$ as a function of only the data at t_0, $(\phi|_{t_0}, \Pi|_{t_0})$.

So it is only by the fact that in our model $e^{-2S_R[\phi_{sol}]}$ is a rather simple function of ϕ_{sol} and thus because of the often chaotic development of the fields by the classical equations of motion typically a complicated function(al) of the time t_0 data. With a more usual model one might think of the initial state in a "first moment" t_0 would be specified by some sort of cosmological model or no boundary condition. In this case the probability density should be rather simple in terms of the first moment data. The simplicity of $e^{-2S_R[\phi_{sol}]}$ as functional of the ϕ_{sol}-behavior even at late times to some extend is extremely dangerous for our model showing observable effects not observed experimentally. Indeed an especially high probability for initial states leading to a special sort of happenings today say would look as a hand of God effect seeking to arrange just this type of happenings to occur. In practise we never know the state of the universe totally at a moment of time. So there would usually be possibilities to adjust a bit the initial conditions. That ciould then in our model have happened in such a way that events or things to happen in the future gets arranged, if it can be done so as to organize especially big $e^{-2S_R[\phi_{sol}]}$ i.e. an especially low S_I. So a priori there would in our model be such "hand of God effects". In a later section we shall, however, invent or propose a possible explanation that could make the era of today be of very little significance for the value of S_I so that in the first approximation it mainly the early time features of a solution that counts for its probability density

$$e^{-2S_I[\phi_{sol}]} \sim f(\phi_{sol}|_{\text{"early times"}}) = f(\text{early time part of } \phi_{sol}) \tag{8.73}$$

8.7.5 Relation to earlier publications

We have earlier published articles working in the classical approximation seeking to produce a model behind the second law of thermodynamics by assigning a probability P over the phase space of the Universe. It were also there the point that this probability density P in our model should depend in the same way on the state at all times. We already proposed that this P were obtained by imposing an imaginary part for the action S_I. According to the above we clearly have

$$P \propto e^{-2S_I}. \tag{8.74}$$

8.8 Suggestion of the quantum formula

We already suggested above that if M denotes a subset of paths, e.g. those taking values in certain subset of $\phi|_t$-configuration space in a moment of time t, then the probability for the true path being in M would be

$$\text{Prob}(M) = \frac{|\int_M e^{iS}\mathscr{D}\phi|^2}{|\int e^{iS}\mathscr{D}\phi|^2}. \tag{8.75}$$

We imagine the paths to be described by the field ϕ as function over $\mathbb{R}^4$, the Minkowski space. Thus we could use such an M to describe e.g. the project of the possible development ϕ to some subspace of configuration space M_i for a series of moments t_i, $i = 1, 2, \cdots, n$. In fact then we would have

$$M = \left\{ \phi \in \{\text{paths}\} \middle| \phi|_{t_i} \in M_i \text{ for all } i \right\}. \tag{8.76}$$

It would in this case be natural to think of the functional integral

$$\int_M e^{iS}\mathscr{D}\phi \tag{8.77}$$

as a product of a series of functional integral associated with the various time intervals in the series of times $-\infty < t_1 < t_2 < \cdots < t_n < \infty$. In fact let us define

$$\mathscr{U}_{t_i \text{ to } t_{i+1}}(\phi|_{t_{i+1}}, \phi|_{t_i}) \equiv \int_{\substack{\text{BOUNDARY} \\ \phi|_{t_i} \text{ and } \phi|_{t_{i+1}} \text{ kept}}} e^{iS_{t_i \text{ to } t_{i+1}}[\phi]}\mathscr{D}\phi. \tag{8.78}$$

Here

$$S_{t_i \text{ to } t_{i+1}}[\phi] = \int_{t_i}^{t_{i+1}} \int \mathcal{L}(x) d^3\boldsymbol{x} dt \tag{8.79}$$

and remember that we here have the complex $d(x)$,

$$\mathcal{L}(x) = \mathcal{L}_R(x) + i\mathcal{L}_I(x). \tag{8.80}$$

We then can write

$$\begin{aligned} &\int_M e^{iS}\mathscr{D}\phi \\ &= \int_i \mathscr{D}^{(3)}\phi|_{t_i} \mathscr{U}_{t_n \text{ to } \infty}(\phi|_\infty, \phi|_{t_n})\theta_{M_n}[\phi|_{t_n}]\mathscr{U}_{t_{n-1} \text{ to } t_n}(\phi|_{t_n}, \phi|_{t_{n-1}}) \\ &\qquad \theta_{M_{n-1}}[\phi|_{t_{n-1}}]\cdots\theta_{M_1}[\phi|_{t_1}]\mathscr{U}(\phi|_{t_1}, \phi|_{t_{-\infty}}) \end{aligned} \tag{8.81}$$

where $\theta_i[\phi|_{t_i}]$ is the function

$$\theta_i[\phi|_{t_i}] = \begin{cases} 1 \text{ for } \phi|_i \in \mathscr{U}_i \\ 0 \text{ for } \phi|_i \notin \mathscr{U}_i \end{cases} \tag{8.82}$$

We can also write this expression in language of a genuine operator product

$$\int_M e^{iS} \mathscr{D}\phi = \mathscr{U}_{t_n \text{ to} \infty} \theta_n \mathscr{U}_{t_{n-1} \text{ to } t_n} \theta_{n-1} \cdots \theta_1 \mathscr{U}_{-\infty \text{ to } t_1} \,. \tag{8.83}$$

where the θ_i's are now conceived of as projection operators on the space of wave functionals characterized by being zero outside M_i,

$$\theta_i \psi(\phi|_{t_i}) = \theta_i(\phi|_{t_i})\psi(\phi|_{t_i}) = \begin{cases} 0 & \text{for } \phi|_{t_i} \notin M_i \\ \psi(\phi|_{t_i}) & \text{for } \phi|_{t_i} \in M_i \end{cases} \tag{8.84}$$

Here ψ is a possible/general wave functional for the state of the universe, in the formula presented at the moment t_i.

In this operator formalism our probability formula takes the form

$$\begin{aligned} \text{Prob}(M) &= \text{Prob}(M_1, M_2, \cdots, M_n) \\ &= \frac{|\mathscr{U}_{t_n \text{ to} \infty} \theta_n \mathscr{U}_{t_{n-1} \text{ to } t_n} \theta_{n-1} \cdots \theta_1 \mathscr{U}_{-\infty \text{ to } t_1}|^2}{|\mathscr{U}_{t_n \text{ to} \infty} \mathscr{U}_{t_{n-1} \text{ to } t_n} \cdots \mathscr{U}_{-\infty \text{ to } t_1}|^2} \end{aligned} \tag{8.85}$$

Since we are anyway in the process of arguing along to just make an assumption about how to interpret in terms of probabilities for physical quantities of our complex action functional integral, we might immediately see that it would be suggestive to extend the validity of this formula for probabilities for field variables to also be valid for distributions in the conjugate fields $\Pi|_{t_i}$ or in combinations,

$$\begin{aligned} &\text{Prob}(\mathcal{O}_1 \in \tilde{M}_1, \mathcal{O}_2 \in \tilde{M}_2, \cdots, \mathcal{O}_n \in \tilde{M}_n) \\ &= \frac{\left|\mathscr{U}_{t_n \text{ to} \infty} P_{\mathcal{O}_n \in \tilde{M}_n} \mathscr{U}_{t_{n-1} \text{ to } t_n} P_{\mathcal{O}_{n-1} \in \tilde{M}_{n-1}} \cdots P_{\mathcal{O}_1 \in \tilde{M}_1} \mathscr{U}_{-\infty \text{ to } t_1}\right|^2}{|\mathscr{U}_{t_n \text{ to} \infty} \cdots \mathscr{U}_{-\infty \text{ to } t_1}|^2} \end{aligned} \tag{8.86}$$

Provided this proposal is not inconsistent to assume, we will assume it because it would be quite reasonable to assume that the analogous formula to (8.85) should be valid for any change for variables between ϕ and Π in the formulation of our functional integral.

8.8.1 Consistency and no need for boundary conditions

It should be kept in mind that we expect that due to the presence of the imaginary part S_I in the action S it is not needed to require any boundary conditions at $t \to \pm\infty$ so that we basically can remove as not relevant the $\phi|_\infty$ and $\phi|_{-\infty}$ boundaries which one would at first have considered to be needed in the expressions $\mathscr{U}_{-\infty \text{ to } t_1}(\phi|_{t_1}, \phi|_{-\infty})$ and $\mathscr{U}_{t_n \text{ to} +\infty}(\phi|_\infty, \phi|_{t_n})$. The imaginary part S_I is in fact expected to weight various contributions so strongly different that whenever the by this weighting flavored component in $\phi|_\infty$ say is at all allowed by a potential choice of boundary condition then that contribution will dominate so much that all over contributions will be relatively negligible. So after taking the ratio for normalization such as (8.86) the choice of the boundary conditions for $\phi|_\infty$ and $\phi|_{-\infty}$ becomes irrelevant. This irrelevance of the boundary conditions would indeed allow us to formally put in according to our convenience of calculation whatever boundaries we might like provided it does not precisely kill the boundary wave function component flavored by the S_I. For instance we could put in at the infinity density matrices taken to be unity, since it does not matter anyway what we put and a unit matrix $\rho_1 = 1$ and $\rho_k = 1$ would not suppress severely any state such as the flavored one(s).

By this trick we could write our formula for probability

$$\begin{aligned}
&\mathrm{Prob}(\mathcal{O}_1 \in \tilde{M}_1, \mathcal{O}_2 \in \tilde{M}_2, \cdots, \mathcal{O}_n \in \tilde{M}_n) \\
&= \mathrm{Tr}(\mathscr{U}_{t_n \text{ to } \infty} P_{\mathcal{O}_n \in \tilde{M}_n} \mathscr{U}_{t_{n-1} \text{ to } t_n} P_{\mathcal{O}_{n-1} \in \tilde{M}_{n-1}} \cdots P_{\mathcal{O}_1 \in \tilde{M}_1} \mathscr{U}_{-\infty \text{ to } t_1} \\
&\qquad \mathscr{U}^\dagger_{-\infty \text{ to } t_1} P_{\mathcal{O}_1 \in \tilde{M}_1} \cdots P_{\mathcal{O}_{n-1} \in \tilde{M}_{n-1}} \mathscr{U}^\dagger_{t_{n-1} \text{ to } t_n} P_{\mathcal{O}_n \in \tilde{M}_n} \mathscr{U}^\dagger_{t_n \text{ to } \infty}) \\
&\quad / \mathrm{Tr}(\mathscr{U}_{t_n \text{ to } \infty} \mathscr{U}_{t_{n-1} \text{ to } t_n} \cdots \mathscr{U}_{-\infty \text{ to } t_1} \mathscr{U}^\dagger_{-\infty \text{ to } t_1} \cdots \mathscr{U}^\dagger_{t_{n-1} \text{ to } t_n} \mathscr{U}^\dagger_{t_n \text{ to } \infty}) \,. \qquad (8.87)
\end{aligned}$$

Here the reader should have in mind that because of the imaginary part in the action $S = S_R + iS_I$ the different development operators

$$\mathscr{U}_{t_i \text{ to } t_{i+1}}(\phi|_{t_{i+1}}, \phi|_{t_i}) = \int_{\substack{\text{WITH BOUNDARIES} \\ \phi|_{t_{i+1}} \text{ and } \phi|_{t_i} \text{ at } t_{i+1} \text{ and } t_i \text{ respectively}}} e^{iS[\phi]} \mathscr{D}\phi \qquad (8.88)$$

are in general not as usual unitary. Therefore it is quite important to distinguish,

$$\mathscr{U}^\dagger_{t_i \text{ to } t_{i+1}} \overset{\text{in general}}{\neq} \mathscr{U}^{-1}_{t_i \text{ to } t_{i+1}} \,. \qquad (8.89)$$

8.9 Practical Application Formulas

8.9.1 Practical application philosophy

Although in principle our theory is so much a theory of everything that it should even tell what really happens and not only what is allowed by the equations of motion, we must of course admit that even we knew the parameters of both S_I and S_R it would be so exceedingly hard to calculate what really happens that can<u>not</u> do that.

We are thus first of all interested in using some reasonable approximations to derive (in a spirit of a correspondence principle) some rules coinciding under practical conditions with the quantum mechanics (or quantum field theory rather) rules we usually use.

Now as is to be explained in this section we can by means of requirements of monopoles and using the Standard Model gauge symmetries and homogeneity of the Lagrangian in the fermion fields argue that there will in the present era where Higgs particles are seldom and Standard model applicable be only very small effects of S_I. We also seem to have justified to make the same assumption for very huge time spans in the future so that also until very far out in future the influence of S_I is small. Even if we imagine that in the very long run S_I selects almost uniquely the state or rather development - as we used above to argue that the boundary conditions $\phi|_{-\infty}$ and $\phi|_{+\infty}$ were unimportant - then in the practical (i.e. rather near) future we would expect the far future determination to deliver under an ergodicity assumption an effective density matrix proportional to the unit matrix.

8.9.2 Insertion of practical future treatment into interpretation formula

The above suggestion for the treatment of the practical future to be equally likely in "all" (practical) states is implemented by replacing what is basically taking the place of a future density matrix ρ_f in our interpretation formula (8.87) namely

$$\rho_f \approx \mathscr{U}^{+}_{t_n \text{ to} \infty} \mathscr{U}_{t_n \text{ to} \infty} \tag{8.90}$$

by a normalized unit density matrix

$$\rho_f \approx \frac{1}{N}\underline{1}, \tag{8.91}$$

where N is the dimension of the Hilbert space. (In practice N is infinite) So the interpretation formula becomes

$$\begin{aligned}
&\text{Prob}(\mathcal{O}_1 \in \tilde{M}_1, \mathcal{O}_2 \in \tilde{M}_2, \cdots, \mathcal{O}_n \in \tilde{M}_n) \\
&= \text{Tr}(P_{\mathcal{O}_n \in \tilde{M}_n} \mathscr{U}_{t_{n-1} \text{ to } t_n} P_{\mathcal{O}_{n-1} \in \tilde{M}_{n-1}} \cdots P_{\mathcal{O}_1 \in \tilde{M}_1} \mathscr{U}_{-\infty \text{ to } t_1} \\
&\quad \mathscr{U}^{\dagger}_{-\infty \text{ to } t_1} P_{\mathcal{O}_1 \in \tilde{M}_1} \cdots P_{\mathcal{O}_{n-1} \in \tilde{M}_{n-1}} \mathscr{U}^{\dagger}_{t_{n-1} \text{ to } t_n}) \\
&\quad /\text{Tr}(\mathscr{U}_{t_n \text{ to} \infty} \cdots \mathscr{U}^{\dagger}_{-\infty \text{ to } t_1} \cdots \mathscr{U}^{\dagger}_{t_{n-1} \text{ to } t_n}) .
\end{aligned} \tag{8.92}$$

where we used that

$$P_{\mathcal{O}_n \in \tilde{M}_n} = P^2_{\mathcal{O}_n \in \tilde{M}_n} . \tag{8.93}$$

8.9.3 Conditional Probability

With formulas like (8.87) or (8.92) we can easily form also conditional probabilities such as

$$\begin{aligned}
&\text{Prob}(\mathcal{O}_{p+1} \in \tilde{M}_{p+1}, \cdots, \mathcal{O}_n \in \tilde{M}_n | \mathcal{O}_1 \in \tilde{M}_1, \cdots, \mathcal{O}_p \in \tilde{M}_p) \\
&= \text{Prob}(\mathcal{O}_1 \in \tilde{M}_1, \cdots, \mathcal{O}_p \in \tilde{M}_p, \cdots, \mathcal{O}_n \in \tilde{M}_n) \\
&\quad /\text{Prob}(\mathcal{O}_1 \in \tilde{M}_1, \cdots, \mathcal{O}_p \in \tilde{M}_p) .
\end{aligned} \tag{8.94}$$

In order to determine what happens if we know the wave function in some moment. Let us as an example consider the idealized situation of a case in which we know – by preparation set up – the whole state of the universe of one moment of time. This we could imagine being described by taking a series of $P_{\mathcal{O}_i \in \tilde{M}_i}$ projection of the same moment of time, the moment in which we suppose that we know the wave function. For consistency and for being able to take the limit of them being at same time – and therefore with an ill-determined algebraic order in

(8.87) we must assume these same time projectors to commute. If we consider the situation that we already know that the system has all these $\mathcal{O}_i$ in the small regions. $\tilde{M}_i$ because we know the wave function at their common time t_{com} then we are after that discussing only the conditional probabilities with the set of relations $\mathcal{O}_i \in \tilde{M}_i$, $i = 1, \cdots, p$, taken as fixed.

For simplicity let us consider the simple case that we just ask for if a variable $\mathcal{O}_n$ at a later time being in $\tilde{M}_n$ or not then the conditional probability is in (8.87) form

$$\begin{aligned}
&\mathrm{Prob}(\mathcal{O}_n \in \tilde{M}_n|\psi) \\
&= \mathrm{Tr}(\mathcal{U}^\dagger_{t_n \text{ to } \infty} P_{\mathcal{O}_n \in \tilde{M}_n} \mathcal{U}_{t_{com} \text{ to } t_n} P_{\mathcal{O}_1 \in \tilde{M}_1} P_{\mathcal{O}_2 \in \tilde{M}_2} \cdots P_{\mathcal{O}_p \in \tilde{M}_p} \mathcal{U}_{-\infty \text{ to } t_{com}} \\
&\quad \mathcal{U}^\dagger_{-\infty \text{ to } t_{com}} P_{\mathcal{O}_p \in \tilde{M}_p} \cdots P_{\mathcal{O}_2 \in \tilde{M}_2} P_{\mathcal{O}_1 \in \tilde{M}_1} \mathcal{U}^\dagger_{t_{com} \text{ to } t_n} P_{\mathcal{O}_n \in \tilde{M}_n} \mathcal{U}^\dagger_{t_n \text{ to } \infty}) \qquad (8.95) \\
&\quad \Big/ \mathrm{Tr}(\mathcal{U}_{t_{com} \text{ to } t_n} P_{\mathcal{O}_1 \in \tilde{M}_1} \cdots P_{\mathcal{O}_p \in \tilde{M}_p} \mathcal{U}_{-\infty \text{ to } t_{com}} \mathcal{U}^\dagger_{-\infty \text{ to } t_{com}} \\
&\qquad P_{\mathcal{O}_p \in \tilde{M}_p} \cdots P_{\mathcal{O}_1 \in \tilde{M}_1} \mathcal{U}^\dagger_{t_{com} \text{ to } t_n} .
\end{aligned}$$

Herein we can substitute

$$|\psi\rangle\langle\psi| = P_{\mathcal{O}_1 \in \tilde{M}_1} P_{\mathcal{O}_2 \in \tilde{M}_2} \cdots P_{\mathcal{O}_p \in \tilde{M}_p} \qquad (8.96)$$

and obtain using as usual for traces Tr(AB)=Tr(BA)

$$\begin{aligned}
&\mathrm{Prob}(\mathcal{O}_n \in \tilde{M}_n|\psi) \\
&= \langle\psi|\mathcal{U}^\dagger_{t_{com} \text{ to } t_n} P_{\mathcal{O}_n \in \tilde{M}_n} \mathcal{U}^\dagger_{t_n \text{ to } \infty} \mathcal{U}_{t_n \text{ to } \infty} P_{\mathcal{O}_n \in \tilde{M}_n} \mathcal{U}_{t_{com} \text{ to } t_n} |\psi\rangle \\
&\langle\psi|\mathcal{U}_{-\infty \text{ to } t_{com}} \mathcal{U}^\dagger_{-\infty \text{ to } t_{com}} |\psi\rangle \\
&\quad \Big/ (\langle\psi|\mathcal{U}^\dagger_{t_{com} \text{ to } \infty} \mathcal{U}_{t_{com} \text{ to } \infty} |\psi\rangle \langle\psi|\mathcal{U}_{-\infty \text{ to } t_{com}} \mathcal{U}^\dagger_{-\infty \text{ to } t_{com}} |\psi\rangle) \\
&\equiv \langle\psi|\mathcal{U}_{t_{com} \text{ to } t_n} P_{\mathcal{O}_n \in \tilde{M}_n} \mathcal{U}^\dagger_{t_n \text{ to } \infty} \mathcal{U}_{t_n \text{ to } \infty} P_{\mathcal{O}_n \in \tilde{M}_n} \mathcal{U}_{t_{com} \text{ to } t_n} |\psi\rangle \\
&\qquad / \langle\psi|\mathcal{U}^\dagger_{t_{com} \text{ to } \infty} \mathcal{U}_{t_{com} \text{ to } \infty} |\psi\rangle .
\end{aligned} \qquad (8.97)$$

We may rewrite this expression in a suggestive way of the how it is modified relative to usual quantum mechanics by defining the final state density matrix from time t_n

$$\rho_{f \text{ from } t_n} \equiv \mathcal{U}^\dagger_{t_n \text{ to } \infty} \mathcal{U}_{t_n \text{ to } \infty} \qquad (8.98)$$

in the following way

$$\mathrm{Prob}(\mathcal{O}_n \in \tilde{M}_n|\psi) = \frac{\langle\psi|\mathcal{U}_{t_{com} \text{ to } t_n} P_{\mathcal{O}_n \in \tilde{M}_n} \rho_{f \text{ from } t_n} P_{\mathcal{O}_n \in \tilde{M}_n} \mathcal{U}^\dagger_{t_{com} \text{ to } t_n} |\psi\rangle}{\langle\psi|\mathcal{U}_{t_{com} \text{ to } t_n} \rho_{f \text{ from } t_n} \mathcal{U}^\dagger_{t_{com} \text{ to } t_n} |\psi\rangle} \qquad (8.99)$$

Here $\mathcal{U}_{t_{com} \text{ to } t_n}$ is really a non unitary S-matrix or development matrix for the time interval t_{com} at which we have ψ to t_n at which we look for $\mathcal{O}_n$. It is easy to see that we could have replaced $P_{\mathcal{O}_n \in \tilde{M}_n}$ by a large set of commuting projections at a second common time t_f destined to the single wave function $|\psi_f\rangle$ and thus allowing to replace the series of projections put in place of $P_{\mathcal{O}_n \in \tilde{M}_n}$ by $|\psi_f\rangle\langle\psi_f|$. Then we get for the probability of the transition from $|\psi\rangle$ to $|\psi_f\rangle$

$$\begin{aligned}\text{Prob}\left(|\psi_f\rangle\big||\psi\rangle\right) &= \frac{\langle\psi|\mathscr{U}_{t_{\text{com}}\text{ to }t_n}|\psi_f\rangle\langle\psi_f|\rho_{f\text{ from }t_n}|\psi_f\rangle\langle\psi_f|\mathscr{U}^\dagger_{t_{\text{com}}\text{ to }t_n}|\psi\rangle}{\langle\psi|\mathscr{U}_{t_{\text{com}}\text{ to }t_n}\,\rho_{f\text{ from }t_n}\,\mathscr{U}^\dagger_{t_{\text{com}}\text{ to }t_n}|\psi\rangle} \\ &= |\langle\psi|\mathscr{U}_{t_{\text{com}}\text{ to }t_n}|\psi_f\rangle|^2\,\frac{\langle\psi_f|\rho_{f\text{ from }t_n}|\psi_f\rangle}{\langle\psi|\mathscr{U}_{t_{\text{com}}\text{ to }t_n}\,\rho_{f\text{ from }t_n}\,\mathscr{U}^\dagger_{t_{\text{com}}\text{ to }t_n}|\psi\rangle}\end{aligned} \tag{8.100}$$

Now we can compare this expression with the usual transition probability expression when S is only real $= S_R$,

$$\begin{aligned}\text{Prob}_{\text{usual}}\left(|\psi_f\rangle\big||\psi\rangle\right) &= \langle\psi|\mathscr{U}_{t_{\text{com}}\text{ to }t_n}|\psi_f\rangle\cdot\langle\psi_f|\mathscr{U}^\dagger_{t_{\text{com}}\text{ to }t_n}|\psi\rangle \\ &= |\langle\psi|\mathscr{U}_{t_{\text{com}}\text{ to }t_n}|\psi_f\rangle|^2\end{aligned} \tag{8.101}$$

Denoting the transition operator

$$S \equiv \mathscr{U}^\dagger_{t_{\text{com}}\text{ to }t_n} \tag{8.102}$$

this means we have

$$\text{Prob}\left(|\psi_f\rangle\big||\psi\rangle\right) = |\langle\psi_f|S|\psi\rangle|^2\cdot\frac{\langle\psi_f|\rho_{f\text{ from }t_n}|\psi_f\rangle}{\langle\psi|\rho_{f\text{ from }t_n}|\psi\rangle} \tag{8.103}$$

compared to the usual expression

$$\text{Prob}_{\text{usual}}\left(|\psi_f\rangle\big||\psi\rangle\right) = |\langle\psi_f|S|\psi\rangle|^2\,. \tag{8.104}$$

The deviations are thus the following:

1. With our imaginary part in S there is no longer unitality, i.e.

$$S^\dagger \neq S^{-1}. \tag{8.105}$$

   The transition S is calculated by the Feynman path integral with the full

$$S = S_R + iS_I. \tag{8.106}$$

2. There is the extra wright factor $\langle\psi_f|\rho_{f\text{ from }t_n}$ describing the effect of the happenings and the S_I in the future of the "final measurement" $|\psi_f\rangle$.
3. There is the only on the initial state $|\psi\rangle$ dependent "normalization factor" in the denominator

$$\langle\psi|S^\dagger\rho_{f\text{ from }t_n}S|\psi\rangle \tag{8.107}$$

   This denominator is indeed a normalization factor normalizing the total probability for reaching a complete set – an orthonormal basis – of final states

$|\psi_{fk}\rangle$, $k = 1, 2, \cdots$ which we for simplicity choose as eigenstate of $\rho_{f\text{ from }t_n}$ so that $\langle|\psi_{fk}\rho_{f\text{ from }t_n}|\psi_{fk}\rangle$ gets a diagonal matrix. Then namely

$$\begin{aligned}
&\sum_{k=1,2,\cdots} \text{Prob}\left(|\psi_{f,k}\rangle \big| |\psi\rangle\right) \\
&= \frac{1}{\langle\psi|S^\dagger \rho_{f\text{ from }t_n} S|\psi\rangle} \cdot \sum_k |\langle\psi_{f,k}|S|\psi\rangle|^2 \langle\psi_{f,k}|\rho_{f\text{ from }t_n}|\psi_{f,k}\rangle \\
&= \frac{1}{\langle\psi|S^\dagger \rho_{f\text{ from }t_n} S|\psi\rangle} \sum_k \langle\psi|S^\dagger|\psi_{f,k}\rangle\langle\psi_{f,k}|\rho_{f\text{ from }t_n}|\psi_{f,k}\rangle\langle\psi_{f,k}|S|\psi\rangle
\end{aligned} \tag{8.108}$$

which by using that the off diagonal elements of $\rho_{f\text{ from }t_n}$ were chosen to be zero can be rewritten as a double sum – i.e. over both k and k' –

$$\begin{aligned}
&\sum_k \text{Prob}\left(|\psi_{f,k}\rangle \big| |\psi\rangle\right) \langle\psi|S^\dagger \rho_{f\text{ from }t_n} S|\psi\rangle \\
&= \sum_{k,k'} \langle\psi|S^\dagger|\psi_{f,k}\rangle\langle\psi_{f,k}|\rho_{f\text{ from }t_n}|\psi_{f,k'}\rangle\langle\psi_{f,k'}|S|\psi\rangle \\
&= \langle\psi|S^\dagger \rho_{f\text{ from }t_n} S|\psi\rangle
\end{aligned} \tag{8.109}$$

Thus we see that this denominator just ensures that the total probability for all that can happen at time t_n starting from $|\psi\rangle$ at t_{com} becomes just one.

8.9.4 Simplifying formula for conditional probability by approximating future

We have already suggested that we should approximate

$$\rho_{f\text{ from }t_n} \approx \frac{1}{N}\underline{1} \tag{8.110}$$

provided the future after t_n is so that we can practically consider the S_I-effects small or so much delayed into the extremely far future that our above ergodicity argument can be used. By such an approximation we remove the deviation number 2 above given by $\langle\psi_f|\rho_{f\text{ from }t_n}|\psi_f\rangle$ because we approximate this matrix element $\langle\psi_f|\rho_{f\text{ from }t_n}|\psi_f\rangle$ by a constant as a function of $|\psi_f\rangle$. After this approximation we get

$$\text{Prob}\left(|\psi_f\rangle \big| |\psi\rangle\right) = \frac{|\langle\psi_f|S|\psi\rangle|^2}{\langle\psi|S^\dagger S|\psi\rangle} \tag{8.111}$$

We should have in mind that $S^+S \neq 1$ in general since with the imaginary part of the action the Hamiltonian will be non-hermitean and S non unitary. Thus the usual $|\langle\psi_f|S|\psi\rangle|^2$ would by itself not deliver total probability for what comes out of $|\psi\rangle$ to be unity. Only after the division by the normalization $\langle\psi|S^\dagger S|\psi\rangle$ would it become normalized to unity.

8.10 Can we make an unsquared form?

The formulas for the extraction of probabilities from our Feynman path integral with imaginary part of action S_I also were derived by considerations of statistical addition with essentially random phases of various classical path. But our crucial formula, say (8.87), for probabilities is seemingly surprisingly complicated in as far as each projection operator occurs twice in the trace in the numerator. Even the simplest example of asking if some variable $\mathcal{O}$ at time t falls into the range $\bar{M}$ gets the expression

$$\begin{aligned} &\mathrm{Prob}(\mathcal{O} \in \bar{M}) \\ &= \mathrm{Tr}(\mathscr{U}_{t\ \mathrm{to}\ \infty} P_{\mathcal{O}\in\bar{M}} \mathscr{U}_{-\infty\ \mathrm{to}\ t} \mathscr{U}^{\dagger}_{-\infty\ \mathrm{to}\ t} P_{\mathcal{O}\in\bar{M}} \mathscr{U}^{\dagger}_{t\ \mathrm{to}\ \infty})/\mathrm{Tr}(\mathscr{U}_{-\infty\ \mathrm{to}\ \infty} \mathscr{U}^{\dagger}_{-\infty\ \mathrm{to}\ \infty}) \end{aligned} \tag{8.112}$$

containing the projection operator $P_{\mathcal{O}\in\bar{M}}$ twice as factor in the expression. If we make the approximation of no S_I-effects in the t to ∞ time range by taking

$$\rho_{f\ \mathrm{from}\ t_n} = \mathscr{U}^{\dagger}_{t\ \mathrm{to}\ \infty} \mathscr{U}_{t\ \mathrm{to}\ \infty} \tag{8.113}$$

and approximating it by being proportional to the unit matrix then, however, the two projection operators come together and we could formally replace their product by just one of them. So in the case of the in this way approximated future we could write

$$\mathrm{Prob}(\mathcal{O} \in \bar{M}) = \mathrm{Tr}(P_{\mathcal{O}\in\bar{M}} \mathscr{U}_{-\infty\ \mathrm{to}\ t} \mathscr{U}^{\dagger}_{-\infty\ \mathrm{to}\ t})/\mathrm{Tr}(\mathscr{U}_{-\infty\ \mathrm{to}\ t} \mathscr{U}^{\dagger}_{-\infty\ \mathrm{to}\ t}). \tag{8.114}$$

If $\mathcal{O}$ were a variable among the variables used as the path-description in the Feynman path integral the formula (8.112) would by functional integral be written

$$\mathrm{Prob}(\mathcal{O} \in \bar{M}) = \frac{\left|\int P_{\mathcal{O}\in\bar{M}} e^{iS} \mathscr{D}\phi\right|^2}{\left|\int e^{iS} \mathscr{D}\phi\right|^2}. \tag{8.115}$$

Strictly speaking these Feynman integrals should be summed over all end of time configurations, but with significant S_I presumably this summation would be dominated by a few "true" initial and states at $\pm\infty$ and the summation would not be so important.

So strictly speaking we have

$$\mathrm{Prob}(\mathcal{O} \in \bar{M}) = \frac{\sum_{\mathrm{init,final}} \left|\int_{\mathrm{initial}}^{\mathrm{final}} P_{\mathcal{O}\in\bar{M}} e^{iS} \mathscr{D}\phi\right|^2}{\sum_{\mathrm{init,final}} \left|\int e^{iS} \mathscr{D}\phi\right|^2}. \tag{8.116}$$

8.11 The Higgs width broadening

As an example of application of the S_I-caused modification of the usual transition matrices we may consider the decay of a particle – which we for reasons to be explained below take to be the Weinberg-Salam Higgs particle – which has from

S_I induced an imaginary term in the mass (or energy). Let us say take this term to have the effect of delivering a term being a positive constant number multiplied by $-i$ in the Hamiltonian. In the Schrodinger equation

$$i\frac{d\psi}{dt} = H\psi \tag{8.117}$$

such a term will cause the wave function ψ to decrease with time so that it will decay exponentially with time t. If the particle in addition decays "normally" into decay products, say $b\bar{b}$ as the Higgs particles do the exponential decay rate will be the sum $\Gamma_{\text{normal}}+\Gamma_{S_I}$ of the S_I-induced width Γ_{S_I} and the "normal" decay width Γ_{normal}. Let us for simplicity take as an approximation that the real part of the mass is very large compared to both the "normal" and the S_I-induced widths so that we can work effectively non relativistically with a resting Higgs particle. We can let it be produced in a short moment of time which is short compared to the inverse widths $\frac{1}{\Gamma_{S_I}}$ and $\frac{1}{\Gamma_{\text{normal}}}$ while still allowing the particle may be considered at rest approximately.

If we at first used the "usual" formula $|\langle\psi_f|S|\psi\rangle|^2$ for the decay process and calculate the total probability for the particle to decay into anything one will find that this probability is only $\frac{\Gamma_{\text{normal}}}{\Gamma_{\text{normal}}+\Gamma_{S_I}}$ because the average lifetime has been reduced by this factor, namely from $\frac{1}{\Gamma_{\text{normal}}}$ to $\frac{1}{\Gamma_{\text{normal}}+\Gamma_{S_I}}$. Since of course the usual particle with $\Gamma_{S_I} = 0$ will decay into something with just probability unity, we thus need a normalization factor $\langle\psi|S^\dagger S|\psi\rangle$ to rescale the total probability to be (again) unity in our imaginary action theory.

By Fourier transforming from time t to energy the Higgs decay time distribution we obtain in our model again a Breit-Wigner energy distribution

$$P(E) = \frac{\Gamma_{\text{normal}} + \Gamma_{S_I}}{2\pi\left[(E - m_{\text{Higgs}})^2 + \left(\frac{\Gamma_{\text{normal}}+\Gamma_{S_I}}{2}\right)^2\right]} \tag{8.118}$$

If indeed we effectively should have such an S_I-induced imaginary part in the mass of the Higgs, then the Higgs-width could be made bigger than calculated in the usual width Γ_{normal}. This is an effect that might have been already seen in the LEP-collider provided one has indeed seen some Higgses in this accelerator. Indeed there has been found an excess of Higgs-like events with masses slightly below the established lower bound for the Higgs mass of 114 GeV/c.

8.11.1 The effect of the $\langle\psi_f|\rho_{f\text{ from }t_n}|\psi_f\rangle$ suppression factor

As an example of a (perhaps realistic) case of an effect of the factor $\langle\psi_f|\rho_{f\text{ from }t_n}|\psi_f\rangle$ we could imagine that two particles are coming together organized to hit head on – say in a relative s-wave - able to potentially form two different resonances of which say one is a Higgs which as above is assumed to have an imaginary term in its mass. Now it is easy to see that the $\rho_{f\text{ from }t_n}$ (here t_n is the moment the collision just formed one of the two resonances thought upon as physical objects) will have

$$\langle\psi_{f\text{ Higgs}}|\rho_{f\text{ from }t_n}|\psi_{f\text{ Higgs}}\rangle < \langle\psi_{f\text{ all}}|\rho_{f\text{ from }t_n}|\psi_{f\text{ all}}\rangle \tag{8.119}$$

where $|\psi_{f\,\mathrm{Higgs}}\rangle$ and $|\psi_{f\,\mathrm{all}}\rangle$ represent respectively the two possible resonances the Higgs and the alternative resonance. Compared to the usual calculation of the transition to one of the resonances – essentially the square of the coupling constant – the Higgs-resonance will occur with suppressed probability because of the $\langle\psi|\rho_{f\,\mathrm{from}\,t_n}|\psi\rangle$-factor in the formula (8.100). Really if the collision were safely organized that the collision occurs because of s-wave impact preensured the total probability for one or the other of the two resonances to be formed would be with properly normalized probability 1 because of the $\langle\psi|S^\dagger S|\psi\rangle$ normalization factor. However, the effect of $\langle\psi_f|\rho_{f\,\mathrm{from}\,t_n}|\psi_f\rangle$ would be to increase the probability to form the alternative resonance while decreasing the formation of the Higgs.

8.12 Approaching a more beautiful formulation

Taking the regions in which $\mathcal{O}$ may lie or not $\bar{M}$ as infinitesimally extended we would the formula for the probability density in the form

$$\begin{aligned}\mathrm{Prob}(\mathcal{O}=\mathcal{O}_0) &= \frac{\sum_{i,f}\left|\int_{\mathrm{BOUNDARY}:i,f} e^{iS[\phi]}P_{\mathcal{O}\in\bar{M}}\mathscr{D}\phi\right|^2}{\sum_{i,f}\left|\int_{\mathrm{BOUNDARY}:i',f'} e^{iS[\phi]\mathscr{D}\phi}\right|^2}\\ &\propto \frac{\sum_{i,f}\left|\int_{\mathrm{BOUNDARY}:i,f} e^{iS[\phi]}\delta(\mathcal{O}-\mathcal{O}_0)\mathscr{D}\phi\right|^2}{\sum_{i,f}\left|\int_{\mathrm{BOUNDARY}:i',f'} e^{iS[\phi]}\mathscr{D}\phi\right|^2} \qquad (8.120)\end{aligned}$$

We may claim that this kind of formula the probability density for finding $\mathcal{O}$ taking a value infinitesimally close to $\mathcal{O}_0$ is a bit unaestetic because of having the projection operator $P_{\mathcal{O}\in\bar{M}}$ or the equivalent $\delta(\mathcal{O}-\mathcal{O}_0)$ occurring <u>twice</u> while one might have said it would be simpler to have just one $\delta(\mathcal{O}-\mathcal{O}_0)$ or $P_{\mathcal{O}\in\bar{M}}$ factor in the expression.

We should now seek to reformulate our expression with these factors occurring twice into a simpler one with only single occurrence of $P_{\mathcal{O}\in\bar{M}}$ or $\delta(\mathcal{O}-\mathcal{O}_0)$. To perform this hoped for derivation we first argue that for nonoverlapping $\mathcal{O}$-value regions $\bar{M}_1$ and $\bar{M}_2$

$$\begin{aligned}&\sum_{i,f}\left(\int_{\mathrm{BOUNDARY}:i,f} P_{\mathcal{O}\in\bar{M}_1}e^{iS[\phi]}\mathscr{D}\phi\right)^* \cdot \int_{\mathrm{BOUNDARY}:i,f} P_{\mathcal{O}\in\bar{M}_2}e^{iS[\phi]}\mathscr{D}\phi\\ &\approx 0 \quad \text{for} \;\; \bar{M}_1\cap\bar{M}_2=\emptyset\,. \qquad (8.121)\end{aligned}$$

This is argued for by maintaining that giving $\mathcal{O}$ a different value at time t very typically by “butterfly effect” – Lyapunov exponent – will cause very different states f and i at $\mp\infty$ respectively. If the two factors in (8.121) have very different final f and initial i states dominate at the boundaries and even random phases the total sum is indeed much smaller than what one would obtain if $\bar{M}_1$ and $\bar{M}_2$ were taken to be the same region $\bar{M}_1 = \bar{M}_2 = \bar{M}$. If we now use the zero expression (8.121) by adding such terms into the numerator and analogously in the denominator of (8.120) we can formulate this expression for the probability of $\mathcal{O}$ being in $\bar{M}$ into an expression involving a summation over the value or region for $\mathcal{O}$ in one of the occurrences

$$\begin{aligned}
&\mathrm{Prob}(\mathcal{O}=\mathcal{O}_0^{(2)})= \\
&\left(\sum_{\mathrm{i,f}}\int d\mathcal{O}_0^{(1)}\left(\int_{\mathrm{BOUNDARY:i,f}}\delta(\mathcal{O}-\mathcal{O}_0^{(1)})e^{iS[\phi]}\mathscr{D}\phi\right)^*\right. \\
&\quad\left.\cdot\int_{\mathrm{BOUNDARY:i,f}}\delta(\mathcal{O}-\mathcal{O}_0^{(2)})e^{iS[\phi]}\mathscr{D}\phi\right) \\
&\Big/\left(\sum_{\mathrm{i',f'}}\left(\int_{\mathrm{BOUNDARY:i',f'}}e^{iS[\phi]}\mathscr{D}\phi\right)^*\int_{\mathrm{BOUNDARY:i',f'}}e^{iS[\phi]}\mathscr{D}\phi\right).
\end{aligned} \tag{8.122}$$

But now obviously we have

$$\int d\mathcal{O}_0^{(1)}\delta(\mathcal{O}-\mathcal{O}_0)=1 \tag{8.123}$$

and thus we get

$$\begin{aligned}
&\mathrm{Prob}(\mathcal{O}-\mathcal{O}_0^{(2)})= \\
&\frac{\sum_{\mathrm{i,f}}\left(\int_{\mathrm{BOUNDARY:i,f}}e^{iS[\phi]}\mathscr{D}\phi\right)^*\int_{\mathrm{BOUNDARY:i,f}}\delta(\mathcal{O}-\mathcal{O}_0^{(2)})e^{iS[\phi]}\mathscr{D}\phi}{\sum_{\mathrm{i',f'}}\left(\int_{\mathrm{BOUNDARY:i',f'}}e^{iS[\phi]}\mathscr{D}\phi\right)^*\int_{\mathrm{BOUNDARY:i',f'}}e^{iS[\phi]}\mathscr{D}\phi}
\end{aligned} \tag{8.124}$$

In this expression we have achieved to have $\delta(\mathcal{O}-\mathcal{O}_0^{(2)})$ only occurring once as factor. We could therefore trivially extract from it an expression for the average of the $\mathcal{O}$-variable

$$\begin{aligned}
\langle\mathcal{O}\rangle &= \int\mathcal{O}_0^{(2)}\mathrm{Prob}(\mathcal{O}-\mathcal{O}_o^{(2)})d\mathcal{O}_0^{(2)} \\
&= \frac{\sum_{\mathrm{i,f}}\left(\int_{\mathrm{BOUNDARY:i,f}}e^{iS[\phi]}\mathscr{D}\phi\right)^*}{\sum_{\mathrm{i',f'}}\left(\int_{\mathrm{BOUNDARY:i',f'}}e^{iS[\phi]}\mathscr{D}\phi\right)^*} \\
&\quad\cdot\frac{\int_{\mathrm{BOUNDARY:i,f}}\mathcal{O}e^{iS[\phi]}\mathscr{D}\phi}{\int_{\mathrm{BOUNDARY:i',f'}}e^{iS[\phi]}\mathscr{D}\phi}
\end{aligned} \tag{8.125}$$

If we could somehow remove the after all identical complex conjugate functional integrals

$$\left(\int_{\mathrm{BOUNDARY:i,f}}e^{iS[\phi]}\mathscr{D}\phi\right)^* \tag{8.126}$$

and

$$\left(\int_{\mathrm{BOUNDARY:i',f'}}e^{iS[\phi]}\mathscr{D}\phi\right)^* \tag{8.127}$$

only deviating by the dummy initial and final state designations respectively (i,f) and $(\mathrm{i}',\mathrm{f}')$, then we could achieve the simple expression (8.57). But in order to argue for such removal being possible we would have to speculate say that some

– we could say the true – boundary condition combination for the functional integrals (8.126, 8.127) completely dominates. This is actually not at all unrealistic since indeed the S_I will tend to very few paths dominate. In such a case of dominance we would have a set of dominant (f, i). Presumably to make the chance that there should be such dominance we should allow ourselves to be satisfied with a linear combinations of i-state and of f-states to dominate. But now if indeed we could do that and call these linear combinations (f_{dom}, i_{dom}), then we could approximate

$$\langle \mathcal{O} \rangle \approx \frac{\int_{\text{BOUNDARY } f_{dom}, i_{dom}} \mathcal{O} e^{iS[\phi]} \mathscr{D}\phi}{\int_{\text{BOUNDARY } f_{dom}, i_{dom}} e^{iS[\phi]} \mathscr{D}\phi}. \tag{8.128}$$

Now we would like not to have the occurrence in this expression of the rather special states (f_{dom}, i_{dom}). However, these dominant boundary conditions are precisely the dominant boundary conditions for the denominator integral, because it were really just the complex conjugate for the latter for which we looked for the dominant boundaries.

So if we let the boundaries free then at least the denominator should become dominantly just as if we had used the boundaries (f_{dom}, i_{dom}). It even seems that because of the smoothness and boundedness of the variable $\mathcal{O}$ as functional of ϕ the dominant boundaries (i, f) should not be much changed by inserting an extra factor $\mathcal{O}$ so that also by letting the boundaries free in the numerator functional integral $\int \mathcal{O}[\phi] e^{iS[\phi]} \mathscr{D}\phi$ would not change much the dominant boundaries from those of the same integral with the $\mathcal{O}$-factor removed. But the removal of this $\mathcal{O}$ leads to the denominator functional integral, for which we already saw that the dominating boundary behavior were (f_{dom}, i_{dom}). Thus we have argued that we can rewrite (8.128) into

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{O} e^{iS[\phi]} \mathscr{D}\phi}{\int e^{iS[\phi]} \mathscr{D}\phi} \tag{8.129}$$

where it is understood that the boundaries for $t \to \pm\infty$ are "free". Then we suggested they would automatically go to be dominated by (f_{dom}, i_{dom}) thus fitting on to the formulas with double occurrence of $\delta(\mathcal{O} - \mathcal{O}_0)$'s.

The argumentation that the factor $\mathcal{O}$ does not matter for the dominant behavior at $\pm\infty$ may sound almost contradictory to our assumption using the "butterfly effect" to derive the rapid variation of (8.4) which meant that an insertion of $\delta(\mathcal{O} - \mathcal{O}_0)$ would drastically change behavior, including that of the boundary.

It is, however, not totally unreasonable that a sharp function $\delta(\mathcal{O} - \mathcal{O}_0)$ which is zero in most places could modify the boundary conditions, while a smooth one $\mathcal{O}$, almost never zero would not modify them. Basically we hope indeed for that the S_I-caused weighting is so severely restricting the set of significant paths, that it practically means that a single path, "the realized path" is selected. In this case the insertion of the factor $\mathcal{O}$ into the functional integral would just multiply it by the value of $\mathcal{O}$ on "realized path". If you however insert $\delta(\mathcal{O} - \mathcal{O}_0)$ and it as most likely the case $\mathcal{O}_0$ is not the value of $\mathcal{O}$ on the realized path then we kill by the zero-value of $\delta(\mathcal{O} - \mathcal{O}_0)$ at the realized path would totally kill the dominant contribution. Then of course the possibility for a completely different path is opened and the orthogonality used in (8.121) gets realistic.

As conclusion of the just delivered derivation of (8.129) we see that the starting point in the beginning the articles is indeed consistent under the suggested approximations with the forms derived from the semiclassical start.

8.13 The monopole argument for suppressing the S_I in the Standard Model

We have already above in section 8.3 argued that due to the material in the present era, and the future too, being either massless or protected from decay by in practice conserved quantum numbers and due to weakness of the interactions the contribution to S_I from these eras must be rather trivial.

It were also for the above argument important that the non-zero mass particles were non-relativistic in these eras. That above argument may, however, not be sufficient for explaining that no effect of our L_I or S_I would have been seen so far. We have indeed had several high energy accelerators such as ISR (=intersecting storage ring at CERN) in which massive particles – such as protons – have been brought to run for days with relativistic speeds. That means that they would during this running in the storage rings say have had eigentime contributions significantly lower than the coordinate time or rather the time on earth. This would presumably easily have given significant contributions to S_I which going to the exponent would suppress – or priori perhaps enhance – the probability of developments, solutions, to equations of motion, leading to the running of such storage rings. Since the protons have not already been made to run around dominantly relativistically we should deduce that most likely the storage rings would lead to increasing S_I and thus lowering of the probability weight. Thus one would expect that the initial conditions should have been so adjusted as to prevent funding for this kind of accelerators, at least for them running long time. Contrary to Higgs producing accelerators which have so far not been able to work on big scale (may be L.E.P. produced a few Higgses for a short time) the accelerators with relativistic massive particles have seemingly worked without especially bad luck. In order to rescue our model it seems therefore needed to invent a crutch for it of the type that there are actually no L_I-contributions involving the particles for which the massive relativistically running accelerators have been realized. We have actually two mechanisms to offer which at the end can argue away our L_I or S_I effects for all the hitherto humanly produced or found particles, leaving the hopes for finding observable effects – bad luck for accelerators, mysterious broadening of resonance peaks – to experiments involving the Higgs particle or particles outside the Standard Model. The point is indeed that we shall argue away the effects of S_I for gauge particles and for Fermions (coupled to them).

The suppression rules to be argue for are:

1) Supposing the existence of monopoles we deduce that the corresponding full gauge coupling constants must be real, basically as a consequence of the Dirac relation.

2) For fields which like the Fermion fields in renormalizable theories occur homogeneously in the Lagrangian density $\mathcal{L}_R + i\mathcal{L}_I$ this Lagrangian density can be shown to be zero by inserting the equations of motion.

8.13.1 Spelling out the suppression rules

Spelling out a bit the suppression rules let us for the monopole based argument remind the reader that although we consider a complex Lagrangian density $\mathcal{L}_R + i\mathcal{L}_I$ for instance the electric and magnetic fields and the four potential $A_\mu(x)$ for electrodynamics are still real as usual. Now if we have fundamental monopoles there must exist corresponding Dirac strings which, however, as is well known must be unphysical. The explicit flux in the Dirac string must to have the Dirac string unobservable – to be unphysical – be compensated by an at the string singular behavior of the four potential A_μ around the Dirac string. The singular flux to compensate the extra flux in the Dirac string can, however, only be real since the A_μ-field is real and it is given by a curve integral $\oint A_\mu dx^\mu$ around the Dirac string. Now as is well known the fluxes mentioned equal the monopole charges. Thus the monopole charge g must be real. But then the Dirac relation

$$eg = 2\pi n, \quad n \in \mathbb{Z} \tag{8.130}$$

tells that also the electric charge e must be real. Now, however, in the formalism with the electric charge absorbed into the four potential $\hat{A}_\mu = eA_\mu$ the coefficient on the $F^2_{\mu\nu}$-term in the Lagrangian density is $-\frac{1}{4e^2}$ so that the pure electromagnetic, kinetic, Lagrangian density

$$(\mathcal{L}_R + i\mathcal{L}_L)\Big|_{\text{pure e.m.}} = -\frac{1}{4e^2}F^2_{\mu\nu} \tag{8.131}$$

becomes totally real. I.e.

$$\mathcal{L}_L|_{\text{pure e.m.}} = 0. \tag{8.132}$$

We may skip or postpone a similar argument for non-abelian, Yang Mills, theories to another paper, but really you may just think of some abelian subgroup and make use of gauge invariance.

Concerning the rule 2) for homogeneously occurring fields, such as the Fermion fields in renormalizable theories the trick is to use the equations of motion. For example the part of the Lagrangian density $\mathcal{L}_R + i\mathcal{L}_I$ involving a Fermion field ψ is of the form $\mathcal{L}_F = Z \cdot \bar{\psi}(i\not{D} - m)\psi$ where Z is a constant and D_μ the covariant derivative and of course $\not{D} = \gamma^\mu D_\mu$. This Fermionic Lagrangian density is homogeneous of rank two in the Fermion field ψ. The Euler-Lagrange equations, the equations of motion for the Fermion fields are derived from functional differentiation w.r.t. the field ψ

$$\frac{\delta S}{\delta\psi(x)} = 0 \tag{8.133}$$

and end up giving equations of motion of the form

$$\frac{\partial \mathcal{L}_F}{\partial \psi} = 0 \tag{8.134}$$

or

$$\frac{\partial \mathcal{L}_F}{\partial \bar{\psi}} = 0 \tag{8.135}$$

(really these forms are only trustable modulo total divergences but that is enough) leading as is well known to

$$\bar{\psi}(i\not{D} - m) = 0 \tag{8.136}$$

or

$$(i\not{D} - m)\psi = 0. \tag{8.137}$$

But now it is a general rule that a homogeneous expression, $\mathcal{L}_F$ say, can be recovered from its partial derivatives

$$\sum \frac{\partial \mathcal{L}_F}{\partial \psi}\psi + \sum \bar{\psi}\frac{\partial \mathcal{L}_F}{\partial \bar{\psi}} = \text{rank} \cdot \mathcal{L}_F \tag{8.138}$$

where rank is in the present case rank$= 2$. Such a recovering for homogeneous Lagrangian densities, however, means that the Lagrangian density – at least modulo total divergences – can be expressed by the equation of motions, which are zero, if obeyed. But then at least in the classical approximation the Lagrangian density is zero at least modulo total divergences. This means that the total $S_R + iS_I$ contribution from the just discussed homogeneous terms end up zero. Especially the imaginary part also ends up zero, although its form does not have to be zero. It is only insertion of equations of motion that makes it zero.

8.14 Conclusion

We have put up a formalism for a non-unitary model based on extending the Lagrangian and thereby the action to be complex by allowing complex coefficients in the Lagrangian density $\mathcal{L}_R + i\mathcal{L}_I$.

We used two starting points for how to extract probabilities and expectation values from the Feynman path way integral in our ambitious model that shall even be able to tell what really happens rather than just the equations of motion. The first were the interpretation that an operator $\mathcal{O}(t)$ should have the expectation value

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{O}(t) e^{iS[\phi]} \mathscr{D}\phi}{\int e^{iS[\phi]} \mathscr{D}\phi} \tag{8.139}$$

but this expression is a bit dangerous in as far as it is a priori not guaranteed to be real even though the quantity $\mathcal{O}(t)$ is real. The second approach would rather have a series of projections onto small regions $\bar{M}_i$ for operator $\mathcal{O}_i(t_i)$ denoted $P_{\mathcal{O}_i \in \bar{M}_i}$ inserted into the functional integral but then this integral is numerically squared for any combination (i, f) of boundary behaviors at respectively $-\infty$ and $+\infty$ times. That is to say that the insertions are to be performed into the integral $\int e^{iS[\phi]} \mathscr{D}\phi$ so as to replace the latter by $\int \prod_i P_{\mathcal{O}_i \in \bar{M}_i} e^{iS[\phi]} \mathscr{D}\phi$ just as in the first approach, but then one forms the numerical square summed over the initial i and final f behaviors

$$\sum_{i,f} \left(\int_{\text{BOUNDARY:}i,f} e^{iS[\phi]} \mathscr{D}\phi \right)^* \int_{\text{BOUNDARY:}i,f} e^{iS[\phi]} \mathscr{D}\phi. \tag{8.140}$$

The probability distribution is then obtained by inserting the projection operators into `both` factors in (8.140) and then as normalization divide the (8.140) having these insertion with (8.140) not having the insertions.

Under some suggestive assumptions we argued that the two approaches approximately will agree with each other. The most important formula derived is presumably the formula to replace usual unitary S-matrix or $\mathscr{U}$-matrix transition between two moments in time in our model. This formula turns out for transition an initial state $|\psi\rangle$ to a final $|\psi_f\rangle$ to be

$$\text{Prod}(|\psi_f\rangle, |\psi\rangle) = \frac{|\langle\psi_f|S|\psi\rangle|^2 \langle\psi_f|\rho_{f\,\text{from}\,t_f}|\psi_f\rangle}{\langle\psi|S^\dagger S|\psi\rangle} \tag{8.141}$$

We used that to derive the broadening in our model of the Higgs-width.

As an outlook we may mention some of the expectations of our model used in a more classical language in our earlier publications: If the Higgs – especially freely running Higgses — decrease significantly the probabilty (7.21) then the initial state should be organized so that Higgs production be largely avoided. This would actually make the prediction that some how or the other an accident will happen and the LHC-accelerator will be prevented from comming to full energy and luminosoty.

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9 Coupling Self-tuning to Critical Lines From Highly Compact Extra Dimensions

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Abstract. We discuss possible origins of Multiple Point Principle. Inspired by results from finite temperature lattice gauge theory we conjecture possible physical reasons which could lead to the Principle

9.1 Introduction

One of many plagues of the Standard Model as we know it is the excessive number of parameters. It is usually understood that it is a manifestation of Standard Model being some effective theory for the "true" underlying model, such as GUTs. One very promising way of explaining how parameters get fine-tuned is so-called Mulptiple Point Principle[6] introduced and advocated by Nielsen et al. As it is widely discussed in the very same proceeding we will not focus on its details and implications but only mention the essential for our conjecture property. The core statement of MPP is the assumption that many degenerate vacua exist and that they are separated by first order phase transition. Couplings of the model do self-tune themself to the critical lines, or, in existence of many vacua - to multi-critical points. Here we suggest how this situation may indeed be realized in nature and discuss the implications.

9.2 Dimensional Reduction - a detour

Before we turn to the main subject it is important to discuss certain results which come from finite temperature gauge theory. We need to underline that while these results were largely obtained using Lattice Gauge Theory the mechanism is relevant to continuum and lattice here is just a way to non-perturbatively study phase diagram. For simplicity we will consider 2+1-dimensional spacetime, though the results are similar in 3+1 dimensions (nobody ever analyzed 4+1 or more). Our model, which we will regard as "physics" would be pure Yang-Mills SU(3). It can be formulated in terms of closed loops on the (hyper)cubic lattice. We will denote its extentions in space and in time directions by L_s and L_0 correspondingly.

The lattice is periodic in all directions, however differently. Spacial extent is finite only for practical purposes and generally is sent to infinity. Time extent behaves as usually in finite temperature field theory and is compact, with its length being equal the inverse temperature

$$L_0 a = 1/T \tag{9.1}$$

On each link of it we put a matrix $U(x;\mu)$ which belongs to the $SU(3)$ group and is linked to the gauge fields A by

$$U(x;\mu) = \exp ig_0 A(x) \tag{9.2}$$

where g_0 is the bare strong coupling constant.

$$S_W^3 = -\frac{6}{g_0^2} \sum_{x\in\Lambda,\mu,\nu} \mathrm{Tr}\left(U(x;\mu)U(x+\mu;\nu)U(x+\nu;\mu)^{-1}U(x;\nu)^{-1} + \mathrm{H.c.}\right) \tag{9.3}$$

yields usual square of the field strength tensor in naive continuum limit. Phenomena, symmetries and phases which occur in this model we will consider to be "physical".

It has a deconfinement phase transition separating confined and deconfined phase with $Z(3)$ being relevant symmetry. If we go to high temperatures, about twice the critical one we can do a following trick, known as dimensional reduction. As you can see from Eq.9.1 the time extent shrinks drastically. The theory becomes perturbative and time-like degrees of freedom may be integrated out. This has no relation to lattice and integrating is often done in continuum. First we set time-like gauge field to be independent of the imaginary time coordinate x_0

$$A_0(x_0, \mathbf{x}) = A_0(\mathbf{x}) \tag{9.4}$$

and then eliminate the remnants of the gauge freedom by adding the so-called Landau constraint

$$\sum_{x_0} \sum_{\hat{\imath}=1}^{2} \left[A_i(x) - A_i(x - a\hat{\imath})\right] = 0 \tag{9.5}$$

Together with (9.4) this condition is called Static Time-Averaged Landau Gauge. It is essential that all gauge degrees of freedom are fixed and what is left is, or should be, physical.

Few Feynman diagrams later (for details reader may consult [1]) we get an effective theory, formulated in two dimensions with an extra field which lives on sites. The resulting action depends on the parameters of the original theory L_0 and g_0. We make a change of variables

$$A_0(x) = \phi(x)\sqrt{\frac{g_0^2}{L_0}} \tag{9.6}$$

to make effective model look in a more familiar way.

Now we can express S_{eff}^2 as the function of this field and the two-dimensional gauge field U. It consists of the following three parts:

- pure gauge part S_W^2, which we had already in "naive" reduction

$$S_W^2 = \frac{6}{g_0^2} L_0 \sum_P \mathrm{Tr}\left(1 - \frac{1}{3}\mathrm{Re}U(x;\hat{1})U(x+\hat{1};\hat{2})U(x+\hat{2};\hat{1})^{-1}U(x;\hat{2})^{-1}\right) \quad (9.7)$$

- gauge covariant kinetic term, depending both on U and ϕ, obtained by expanding the S_W^3 to the second order in A_0

$$S_{U,\phi} = \sum_x \sum_{i=1,2} \mathrm{Tr}\left(D_i\left(U\right)\phi(x)\right) \quad (9.8)$$

  with D_i being covariant derivative

$$D_i\left(U\right)\phi(x) = U(x;\hat{\imath})\phi(x + a\hat{\imath})U(x;\hat{\imath})^{-1} - \phi(x) \quad (9.9)$$

- self-interaction term S_ϕ, often called "Higgs potential"

$$S_\phi = h_2 \sum_x \mathrm{Tr}\phi^2(x) + h_4 \sum_x \left(\mathrm{Tr}\phi^2(x)\right)^2 \quad (9.10)$$

where couplings are fixed by the parameters of the original model.

Now we have an effective action for our original theory, which lives in lower-dimensional space and reproduced very accurately results of the higher-dimensional model. To understand it better one can make the couplings free and study the model per se. Then, if you do a non-perturbative analysis of the effective theorytwo unexpected results appear. The theory has new phase transition, which was not there in the original model with relevant symmetry being time-reflection, here manifested as $\phi \longrightarrow -\phi$. It can be spontaneusly broken and Higgs field gets vacuum expectation value. The transition is of a strong first order. Further important property of this transition is that original coupling s are pretty much on the transition line. Moreover, they turn out to be in a wrong phase. This suggests that up to numerical ambiguity and errors from the perturbation theory in the course of dimensional reduction - they may be *exactly* on the transition line. This transition is of course not a physical one, as it is not present in the underlying theory which we define as "physics". Now lets us go to the conjecture itself.

9.3 The Conjecture

While the above described scenario has technically nothing to do with nothying but pure gluodynamics we can reformulate it. First we note that there is nothing special in the definition of temperature and instead of talking about high-temperatures effective theory we will view the model as zero-temperature one with one highly compact dimension. This way we can translate forementioned results into something more generic.

- Gauge theory in D dimensions of which one is highly compactified may be rewritten as effective theory in $D-1$ dimensions.

- Effective theory will be formulated via original variables and an additional field in the algebra of the gauge group with new couplings.
- There will be a first order phase transition with one of the phases being unphysical.
- For an observer which lives in $D-1$ dimensional world and somehow guessed the effective theory it will be a "miracle" that couplings self-tune themself to the critical line.

This scenario has been extensively tested in 3 dimensions with $SU(3)$ being a gauge group and in 4 dimensions with $SU(2)$[1]. It is a bold move to generalize it to arbitrary number of dimensions as renormalizability, which differs from dimension to dimension, may change the situation. However forementioned cases are also very different so one may hope it is a universal behaviour.

If so, and the number of compact dimensions is more than one it may give rise to a number of phases with multi-critical points then working as attractors for effective couplings.

Then it can be the physical reason behind Multiple Point Principle. The difference between our conjecture and the MPP is of course that only one phase is actually physical and the remaining ones are the artefacts of the reduction of the compactified space. Once one has a candidate theory and mapped its phase diagram one can get extra information about how the couplings are fixed. However one should keep in mind that the dynamics in these phases is completely unphysical and cannot be used for phenomenological purposes.

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