The Impact of Technology

on the

Doing of Mathematics

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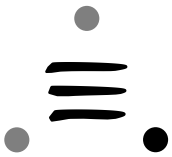

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MY INTENTIONS

- Part I: TALK a bit
- Part II: SHOW some things
- Part III: and TELL some more

ABSTRACT

Technology has repeatedly promised to transform mathematics pedagogically. More recently it has made similar promises to the research community. That said, mathematics in 1999 looked a lot more like mathematics in 1939 than was the case with any of its sister sciences.

That this is changing is inarguable. The confluence of ubiquitous compute power with new networking and collaborative environments will push the teaching and discovering of mathematics in conflicting directions often beyond our control. The burgeoning role of corporate edu-packages is hardly likely to diminish. Nor are battles over curriculum and its delivery about to stop.

PART I:

I intend to survey and illustrate some of the ways in which twenty-first century mathematics will be changed by these new technologies. I will try to distinguish issues of ownership of technology from those of control over content. I also intend to discuss how as mathematical educators we might best prepare for the coming storms. Finally, as a partner in a small educational technology firm, I will offer some modest prescriptions for living on both sides of the fence.

- Intellectual issues
- Technological issues
- Commercial issues

all bang up against each other.

A CHANGING WORLD

"The world will change. It will probably change for the better. It won't seem better to me."

• J.B. Priestley

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"It's generally the way with progress that it looks much greater than it really is."

• From *The Wittgenstein Controversy*, by Evelyn Toynton in the *Atlantic Monthly*, June 1997, pp. 28-41.

◇ The epigraph that Ludwig Wittgenstein (1889-1951) ("whereof one cannot speak, thereof one must be silent") had wished for a never realized joint publication of *Tractatus Logico-Philosophicus* (1922) and *Philosophical Investigations* (1953): suggesting the two volumes are not irreconcilable.

INNOVATION

• Academics mean *new ideas*. Decision makers usually don't:

"Innovation. The process of bringing new goods and services to market. or the result of that process." ('Hard Economic Definition')
◇ *Public Investments in University Research: Reaping the Benefits* (Govt of Canada, 1999)

- 'Sustaining' vs 'disruptive' technologies: e.g.,
  - Hard drives (technology's fruit fly)
  - The backhoe
  - Health Management Organizations
  - The Internet??
- Clayton Christensen, *When New Technologies Cause Great Firms To Fail*, 1997.

PI

• Modern Computer Algebra Systems *know*

$$\boxed{\pi \neq \frac{22}{7}}$$

...

Indeed

$$\int_0^1 \frac{(1-x)^4 x^4}{1+x^2}\, dx = \frac{22}{7} - \pi.$$

and the integrand is positive on $(0, 1)$.

◇ Who knows why Maple (open) or Mathematica (closed) knows what they know?

• Is symbolic computation a sustaining or disruptive technology in the classroom?

THE KEPT UNIVERSITY

"Thorstein Veblen [...] comment[ed] acerbically in 1908 that "business principles" were transforming higher education into "a merchantable commodity, to be produced on a piece-rate plan, rated, bought, and sold by standard units, measured, counted and reduced to staple equivalence by impersonal, mechanical tests."

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"New products and new processes do not appear full-grown," Vannevar Bush, President Franklin Roosevelt's chief science adviser, declared in 1944. "They are founded on new principles and new conceptions, which in turn are painstakingly developed by research in the purest realms of science."

• Eyal Press and Jennifer Washburn in *The Kept University*, Atlantic Monthly, March 2000
`www.theatlantic.com/issues/2000/03/press.htm`

◇ Which quote more accurately reflects 2001?

INTELLECTUAL PROMISES ···

- Lively and realistic examples: learning by doing (Papert)
  - 'we are all constructivists now'
- Math goes into colour: sliding down surfaces/virtual reality
- Background pattern-checkers and *inverse calculation*
- Speed & space $\equiv$ insight (demands rapid reinforcement via *micro-parallelism*)
- Individually tailored learning: varied pathways for quick/slow and for distinct modes of thinking
  - *algebraic, analytic, topological*

··· INTELLECTUAL PROMISES

- Promises students richer means to represent and present the fruits of their mathematical imagination

- Increased need to teach how to judge the results of computation (visual candy everywhere)

- Unifying research and teaching, theory and practice (jobs)

- Serious curricular insights from neurobiology ("Sources of Mathematical Thinking: Behavioral and Brain Imaging Evidence," S. Dehaene et al, in *Science*, May 7, **284** (1999)).

INTELLECTUAL PITFALLS

- Wasted or wonderful add-ons ("Newton & Euclid meet Java". The "Idiot pivoter")
- Loss of focus
- Loss of control: student centred learning of hierarchical subjects
- Degradation of long-lived robust mathematical knowledge (unique to our discipline)
- Growing reliance on effectively closed architecture software ('total solutions')
- 'Haves and havenots': class, race, gender
- Degeneration to machine-based rote learning ('buzzword compliant shovelware')

IN THE LONG TERM ···

"Keynes distrusted intellectual rigour of the Ricardian type as likely to get in the way of original thinking and saw that it was not uncommon to hit on a valid conclusion before finding a logical path to it.

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'I don't really start', he said, 'until I get my proofs back from the printer. Then I can begin serious writing.' "

• From *Keynes the man* written on the 50th Anniversary of Keynes' death. (Sir Alec Cairncross, in the *Economist*, April 20, 1996)

TECHNICAL PROMISES

- Teachers abilities vs students demands
- Access to global data bases (*free access to information* not *access to free information*)
- Doing what is easy: machines don't think like us.
  - cognitive vs descriptive models
- What we learned earlier is not always easier
- Expert systems & belief revision
- Seamless work-spaces: marriage of text and computation

TECHNICAL PITFALLS

- Legacy software
- Legacy hardware
- The weakest link determines the value
- Over promising payoffs and underestimating effort (reform calculus)
- Infinite time-sinks – especially in higher level courses
- Growing (unavoidable) reliance on commercial software

PART II: SOME DEMONSTRATIONS

- MathSciNet: `e-math.ams.org/mathscinet/`
- Sloane's Encyclopedia of Integer Sequences: `www.research.att.com/~njas/sequences/`
- Let's Do Math (Math Resources): `www.mathresources.ca`
- Math On the Web (Tele-Learning): `www.cecm.sfu.ca/TLRN/`
- Cinderella (Geometry): `www.cinderella.de` (not 'net' (music) or 'com' (porn))
- JavaView: `www-sfb288.math.tu-berlin.de/vgp/javaview/demo/PaPlatonic.html`

PART III: INFORMATION RULES

- Economic laws have not been suspended

◇ Carl Shapiro & Hal Varian, *Information Rules*, 1999.

- Some of the topics they discuss and terms worth reflecting on:
  - branding
  - value networks
  - switching costs
  - lock in
  - vicious and virtuous cycles
  - tipping

THE INFORMATION REVOLUTION

"What the new industries and institutions will be, no one can say yet. No one in the 1520s anticipated secular literature, let alone the secular theater. No one in the 1820s anticipated the electric telegraph, or public health, or photography.

"The one thing (to say it again) that is highly probable, if not nearly certain, is that the next twenty years will see the emergence of a number of new industries. At the same time, it is nearly certain that few of them will come out of information technology, the computer, data processing, or the Internet."

• Peter Drucker, *Beyond the Information Revolution*, Atlantic Monthly, Oct 1999.
`www.theatlantic.com/issues/99oct/9910drucker.htm`

INTELLECTUAL PROPERTY ISSUES

- Different stake-holders often have wildly different views
  - Supervisors and teachers
  - Students (and parents)
  - Professional societies (big and small)
  - Publishing houses (big and small)
  - Software companies (big and small)
- As job security disappears more students see *IP* as their future: (Ma vs Phong &Stein, non-disclosure, insider-trading, interleukin).
- The researcher as CEO: conflicts of interest are inevitable. They must be declared. They are rarely resolved.

OPEN PUBLISHING

- So many issues: access, cost, reliability, inter-operability, charging mechanisms, etc.

- Every day another initiative:
  - Los Alamos server and ArXiv (Math) `http://xxx.lanl.gov/archive/math`
  - Santa Fe Initiative (metadata, MathML)
  - International Math Union's *Math-Net* `www.ceic.math.ca`
  - National Institutes of Health (grey literature)
  - DOE, AAAS and Fathom Web Sites (validation?)

COMMERCIAL ISSUES

- Can't make what you can't sell
- Can't sell what you can't make (market discipline?)
- Conservatism in the edu-software business: no R&D model
- Commoditization (*macro-media everywhere*)
- Machine closets versus kitchen cabinets
- Weaning from software: overloading the senses (HCI issues)
- Corporate asset stripping: 'dot-com fever'

RIGO(U)R

"I have no satisfaction in formulas unless I feel their numerical magnitude."

• The scientist and entrepreneur, Lord Kelvin (William Thomson, 1824-1907)

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"The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it."

• J. Hadamard, in E. Borel, Lecons sur la theorie des fonctions, 3rd ed. 1928, quoted in G. Polya, *Mathematical discovery: On understanding, learning, and teaching problem solving (Combined Edition)*, Wiley, (1981).

REALITY

"If you have a great idea, solid science, and earth shaking discoveries, you are still only 10% of the way there."

• David Tomei, LXR Biotechnology Inc, on the vicissitudes of startup companies.

◇ Quoted in *Science* page 1039, Nov. 7, 1997.

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"A truly popular lecture cannot teach, and a lecture that truly teaches cannot be popular."

• Michael Faraday: 'When Gladstone was British Prime Minister he visited Faraday's laboratory and asked if some esoteric substance called 'Electricity' would ever have practical significance. *"One day, sir, you will tax it."* was the answer.' (Science, 1994)

SUGGESTIONS AND ···

- Clearly identify expectations of technology
- Be realistic about the learning curve for advanced software (such as *Mathematica* or *Maple*)
- Commit to use of open architecture software (Linux) and open publishing
- Form (not for profit and 'pre-competitive') consortia
  - to share expertise
  - access to markets
  - ability to compete with the big guys

··· CONCLUSIONS

- Opportunity to recapture computing from our sister sciences

- Realistic now to benefit from:
  - advances in cognitive neuroscience
  - advances in software design, and testing, interfaces, expert systems

- Good technology will never be cheap (*Malthusian principle* that 'expectations outstrip performance')

FREEDOM AND DISCIPLINE

"... so long as we conceive intellectual education as merely consisting in the acquirement of mechanical mental aptitudes, and of formulated statements of useful truths, there can be no progress; although there will be much activity, amid aimless rearrangement of syllabuses, in the fruitless endeavour to dodge the inevitable lack of time. "

• A.N. Whitehead, "The Rythmic Claims of Freedom and Discipline" in *The Aims of Education and Other Essays* (1929).