

A large scale test with the digits of π

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Abstract

A series of large-scale tests were performed on the first 1000 billion digits of the number π . First a direct visual test as well as a test using thousands of sequences from the OEIS catalog. The purpose of these tests is to detect possible patterns. Other tests were made on ten mathematical constants to 1 billion decimal places.

The visual test

The 1000 billion digits were separated into blocks of 100 million decimals without any other separator. Each block of digits was then encoded into a grayscale image using the ImageMagick utility. Each image is therefore 100 million pixels and is 10000 X 10000.

The command is

```
convert -size 10000x10000 -brightness-contrast 55x92 -depth 8 gray:file  
file.png
```

The result of this encoding is difficult to see unless you use a program that allows you to see the detail of the drawing and increase the contrast. It can be improved quite a bit by forcing the coloring. The *pseudocolor* script [1] is perfect for this conversion.

Once converted the decimals are now all visible to the naked eye. I built a huge web page containing 10000 thumbnails allowing to see each image individually.

You can see some of them here [2].

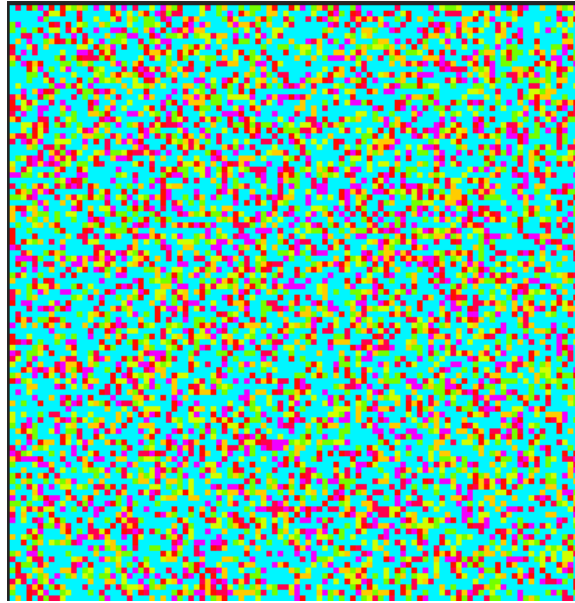
Obviously the overall picture of these 1000 billion digits looks like nothing. They are all about the same size, 42 megabytes. This also corresponds to the compression obtained on 100 million digits. Here is a capture of a part of one of the images, the 4th one which represents the decimals of 300 to 400 millions.



They all have the same 'grain', they are indistinguishable and one does not perceive any pattern. You can see all the decimals if you go through the files, but this examination is useless. So, at first sight, there is no apparent pattern.

One can take the analysis further by using a trick to encode or decode the decimals using the many suites in the OEIS catalog. The idea is to see if removing certain digits or groups of digits would produce a pattern. This is a negative filtering. For example the sequence: 0, 1, 3, 6, 10, 15, ... of the triangular numbers A000217 of the OEIS catalog is a good start. We filter negatively the decimals term after term. We first remove all the '0's from the expansion, followed by the '1's, followed by the 3's, ... which gives the string: a4a592a5a58979a2a84a2a4aa8a2795aa... this is the same as encoding in base 11. This is a flattened representation of π in base 11, which is curious because π in base 11 is mentioned in a famous book by Carl Sagan (Contact) [4] where researchers exploring the number π to 10^{20} decimal places discover with amazement that a huge circle is drawn there.

The result gives: near the upper left corner. To do the test, a value of π to 16777216 decimal places was chosen which allows to create images of 4096 X 4096.



The examination of more than 10000 automatically generated drawings with 10000 sequences of the OEIS catalog is not more convincing than with the raw decimals. In spite of the great variety of possible results, even with thousands of images: nothing is perceived at all. Trivially, the sequence of integers 0,1,2,3,4,... gives an empty image obviously. Unfortunately, this richness of the catalog used as a negative filter did not give anything visible. Some of the images are visible here [3]. The difference with the previous test is that we have a color palette but still with a grain that seems completely random. We get some 'white' images, here the chosen color is pale blue which corresponds to the empty character. When the image is not very filtered, we get exactly the same kind of image as the previous test. For example, the sequence A000002 which contains only '1's and '2's filters little the digits of π .

Number	Range (pixels)	Number of images	Result
$\Phi \setminus \text{OEIS}$	4096×4096	4568	Nothing visible
$\sqrt{2} \setminus \text{OEIS}$	4096×4096	2420	Nothing visible
$\pi \setminus \text{OEIS}$	1024×1024	353437	Nothing visible
$\pi \setminus \text{OEIS}$	4096×4096	8615	Nothing visible
$\pi \setminus \text{OEIS}$	10000×10000	1029	Nothing visible
π	10000×10000	10000	Nothing visible
π	10000×10000 from 22400 to 22590 billions	591	Nothing visible

All images were examined without result. The test with π was done with all suites in the OEIS catalog, the other tests are on a sampling of suites.

The numerical test

For this test, we consider as marginal the possibility that a pattern could appear straddling 2 files of 100 million characters, for practical reasons it is easier to scan blocks of this size rather than having to scan a file of 1000 billion digits at once.

The first 1000 billion decimal places of π were used and 10 constants with 1 billion decimal places: $\sqrt{2}, \sqrt{3}, \varphi, e, \ln \left[\left(\frac{1}{2} \right) \right], \ln \left[\left(\frac{1}{10} \right) \right], \gamma, \text{Cte du Lemniscate}, \zeta(3)$ and Catalan. Then the filters made of rationals, first F_999: the Farey fractions between 0 and 1 whose denominator is between 2 and 999: 303791 values. The 2nd filter of rationals is made of a large spectrum of rationals that come from my inverter table: 10.647 million values. The other important filter is made of all the entries of the small version of the inverter, they are real numbers coming from all the known domains, 10.921 billion entries. The 3 sets are with a precision of 20 decimal digits (truncated). The choice of this filter width is to avoid small collisions due to chance.

A test was done with filters of width 14, it gives a lot of collisions due to chance. To flesh out this hypothesis I built a table of 1000 billion numbers generated with the Mersenne Twister and compared the collisions with π . We find the same number of collisions that are due to what we can call the Feynman effect. The Feynman effect is the point in the decimals of π at rank 762: there are 999999 and this phenomenon is of course repeated further. These collisions or Feynman points are normal in context. The same phenomenon occurs with random numbers. It is to avoid chance collisions that the choice of 20 decimal width was made.

The first test was done on the 1000 billion digits of π and no collision appeared. The 2nd test was done with the big table of rationals (10.647 million entries) on 541 billion digits: no collision either.

A simple unix command allows to efficiently scan a huge file of 100 million characters and to remove (if there is a collision) the characters found.

```
fgrep -of filter_file search_file> result_file
```

The 3rd test was done with the table of reals of the inverter: 10.921 billion entries out of the 10 billion decimals of π : no result found. So I went back to a test on the 10 constants with the big table of 10.92 billion entries:

This time we have 1 collision but it is fortuitous between the $\log(2)$ and one of the real roots of this polynomial.

$$866x^4 + 324x^3 + 616x^2 - 791x - 148$$

The collision with $\ln(2)$ appears at position 335519351. This is the only one that was obtained.

Well-identified markers were inserted on some of the files at the tail or the head to make sure that the program went through all the digits of a 100 million characters block.

Here is a table that summarizes the tests.

Set	Number of entries	Comparaison set	Results
F_{999}	303791	1000 billion of π	0
Large set of rationals	10 647 727	541 billion of π	0
Inverter tables	10921455809	10 billion of π	0
Inverter tables	10921455809	10 constants at 1 billion digits	1

Conclusion

The result of the tests is negative, no pattern was found. It doesn't mean that there is none but taking a very large sample of numbers like the tables of my inverter and despite the quantity (more than 10.92 billion) and despite the richness of the OEIS sequences and 353000 cases tested: no collision. The visual tests are disappointing, one can examine with the naked eye the 1000 billion decimals at once and despite the capacity of the human brain to recognize patterns: none of the 400000 files examined showed any sign.

There are 2 choices, either we declare that there is probably no pattern in the sequence of digits of the number π or we do a new experiment on a very large scale. The current record for calculation is 10^{14} decimal places and to do a test you need quite a bit more than a single pc or two and a few hard drives. In my opinion, only Google can do a test at the same level. Maybe with the help of intelligent machines you could search further but for that you need a

model. That doesn't exist at the moment. Given the means at their disposal, we can consider going much further in the search for a pattern. There is not only the number π which can be interesting because in this field: we know nothing. There is no example of a natural constant in base 2 or 10 or even in any base that exhibits any pattern.

Bibliography

[1] Fred Weinhaus, pseudocolor script.

<http://www.fmwconcepts.com/imagemagick/pseudocolor/index.php>

[2] Plouffe Simon, 10000 images of 1000 billion digits of π :

<http://plouffe.fr/1000%20billion%20digits%20of%20Pi/>

All the images are available on request (422 gigabytes).

[3] Plouffe Simon, OEIS sequences vs digits of π :

<http://plouffe.fr/Pi%20-%20OEIS%20sequences,%20first%20999%20sequences/>

[4] Sagan Carl, *Contact*, Simon & Schuster, 1^{er} sept. 1985

[5] Trueb Peter, Digit Statistics of the First 22.4 Trillion Decimal Digits of Pi, ARXiv,

<https://arxiv.org/abs/1612.00489>

[6] Google (alphabet) : Computation of 100 000 billion digits of π .

<https://cloud.google.com/blog/products/compute/calculating-100-trillion-digits-of-pi-on-google-cloud>

[7] The 6 nines in pi at https://en.wikipedia.org/wiki/Six_nines_in_pi