

# Simple formulas of $\pi$ in terms of $\Phi$

Angelo Pignatelli

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## Abstract

A re-calculation of a known family of formulas of  $\pi$  is carried out, revisiting the old Archimedes' algorithm. This allows to identify a general family equation and three new simple formulas of  $\pi$  in terms of the golden ratio  $\Phi$  in the form of infinite nested square roots, with some geometrical properties that enhance the link between the circle and the golden ratio. Applying the same criteria, a fourth formula is given, that brings to the known Dixon's squaring the circle approximation, thus an easier approach to this problem is suggested, by a rectangle with both sides proportional to the golden ratio  $\Phi$ .

Keywords:  $\pi$ ,  $\Phi$ , golden ratio, squaring the circle

## Introduction

In last century it was challenging and interesting to find formulas of  $\pi$  in terms of the golden ratio  $\Phi$  (so involving together two of most famous irrational constants), without transcendental functions, as the well known  $\pi = \frac{10}{3} \arcsin\left(\frac{\phi}{2}\right)$ . Among these works, few examples are here reported:

$$\pi = \frac{5\sqrt{2+\phi}}{2\phi} \sum_{n=0}^{\infty} \left(\frac{1}{2\phi}\right)^{5n} \left(\frac{1}{5n+1} + \frac{1}{2\phi^2(5n+2)} - \frac{1}{2^2\phi^3(5n+3)} - \frac{1}{2^3\phi^3(5n+4)}\right) \quad (1)$$

Equation (1) has been presented by Chan in [1], inspired by the work of Bailey, Borwein and Plouffe (so called BBP-formulas) in [2].

$$\frac{\pi^2}{50} = \sum_{k=0}^{\infty} \left(\frac{\phi^2}{(5k+1)^2} - \frac{\phi}{(5k+2)^2} - \frac{\phi^2}{(5k+3)^2} + \frac{\phi^5}{(5k+4)^2} + \frac{2\phi^2}{(5k+5)^2}\right) \phi^{-5k} \quad (2)$$

Equation (2) has been discovered by B. Cloitre and reported by Chan in [3], also inspired by BBP-formulas.

The aim of this work, presented in next pages, is focused on identifying other simpler formulas of  $\pi$  in terms of  $\Phi$ .

## Nested square roots formulas of $\pi$

In order to show other simple formulas of  $\pi$  in terms of  $\Phi$ , first it needs to easily share the calculation behind the family of the known formulas of  $\pi$  in the form of nested square roots.

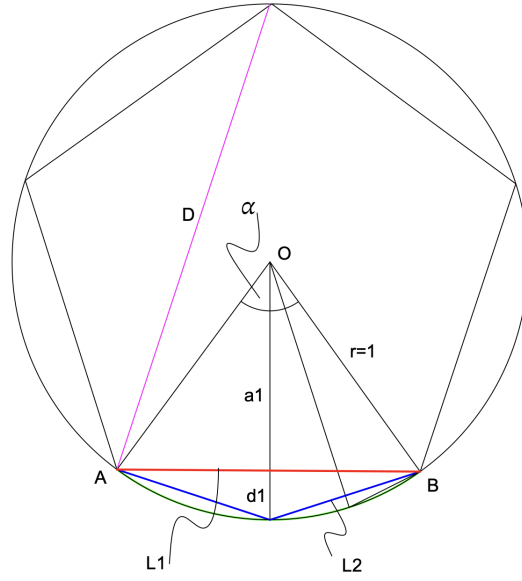


Figure 1: Iterative halfway bisection of chord and arc - example on a pentagon

The approach starts from the idea of Archimedes, resumed several centuries later by F. Viete (as reported by Beckmann in [4]), then more recently completely restructured by Servi in [5].

Let us start considering a regular polygon inscribed in a circle with unitarian radius, with  $N$  sides ( $N \in \mathbb{N}, N \geq 3$ ) of length  $L_1$  ( $L_1 = 2 \sin \frac{\alpha}{2}$ ,  $\alpha = \frac{2\pi}{N}$ ), with perimeter  $P_1 = NL_1$ , as in Figure 1 (where, as example, is represented a pentagon).

With the polygon obtained doubling the sides, considering  $a_1 = \sqrt{1 - (\frac{L_1}{2})^2}$ ,  $d_1 = 1 - a_1$ , the perimeter become  $P_2 = N2L_2$  with

$$L_2 = \sqrt{\frac{L_1^2}{4} + \left(1 - \sqrt{1 - \frac{L_1^2}{4}}\right)^2} = \sqrt{2 - 2\sqrt{1 - \frac{L_1^2}{4}}} = \sqrt{2 - \sqrt{4 - L_1^2}} \quad (3)$$

Re-iterating the process, doubling the polygon at each step, from Equation (3) the following succession is obtained

$$\begin{cases} L_1 = 2 \sin \frac{\pi}{N} & N \in \mathbb{N}, N \geq 3 \\ L_n = \sqrt{2 - \sqrt{4 - L_{n-1}^2}} & n \in \mathbb{N}, n \geq 2 \\ \pi = N \lim_{n \rightarrow \infty} 2^{n-2} L_n \end{cases} \quad (4)$$

Expanding Equations (3) and (4):

$$L_3 = \sqrt{2 - \sqrt{4 - L_2^2}} = \sqrt{2 - \sqrt{4 - \left(2 - \sqrt{4 - L_1^2}\right)^2}} = \sqrt{2 - \sqrt{2 + \sqrt{4 - L_1^2}}}$$

$$L_4 = \sqrt{2 - \sqrt{4 - L_3^2}} = \sqrt{2 - \sqrt{4 - \left(2 - \sqrt{2 + \sqrt{4 - L_1^2}}\right)}} = \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{4 - L_1^2}}}}$$

$$L_n = \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{4 - L_1^2}}}}} \quad n \text{ square roots} \quad (5)$$

Using the results of Equation (5), the known family of equations of  $\pi$  in the form of continued square roots follows, valid for any regular polygons with  $N$  sides and side length  $L_1$ .

$$\begin{cases} L_1 = 2 \sin \frac{\pi}{N} & N \in \mathbb{N}, N \geq 3 \\ \pi = N \lim_{n \rightarrow \infty} 2^{n-1} \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{4 - L_1^2}}}}} & n + 1 \text{ square roots} \end{cases} \quad (6)$$

Now we would like to extend this approach, freeing from the regular polygons.

Let us focus on the arc  $\widehat{AB} = \alpha < \pi \text{ rad}$ , implied with the chord  $\overline{AB} = 2 \sin \frac{\alpha}{2}$  (Figure 1); with this in mind, we can call  $\mu \in \mathbb{R}$  the ratio between the circumference length and the arc  $\widehat{AB}$ :

$$\mu = \frac{2\pi}{\alpha} \quad \mapsto \quad \pi = \frac{1}{2}\mu\alpha \quad \text{with } 0 < \alpha < \pi \quad \mapsto \quad 2 < \mu < +\infty \quad (7)$$

Applying a similar strategy, it is proven that, dividing iteratively halfway the arc  $\widehat{AB}$  (as in Figure 1), the sum of the chords implied converge to the length of the arc  $\widehat{AB} = \alpha$ , thus

$$\begin{cases} \mu = \frac{2\pi}{\alpha} & 0 < \alpha < \pi \\ \pi = \mu \lim_{n \rightarrow \infty} 2^n \sin \left(\frac{\alpha}{2^n}\right) \end{cases} \quad (8)$$

Applying several times the goniometric bisection formulas  $\sin \frac{\gamma}{2} = \sqrt{\frac{1 - \cos \gamma}{2}}$ ,  $\cos \frac{\gamma}{2} = \sqrt{\frac{1 + \cos \gamma}{2}}$ , and ones the formula  $\sin^2 \gamma + \cos^2 \gamma = 1$  to the expression  $\sin \frac{\alpha}{2^n}$  we obtain:

$$\begin{aligned} \sin \left(\frac{\alpha}{2^n}\right) &= \sin \left(\frac{\frac{\alpha}{2^{n-1}}}{2}\right) = \sqrt{\frac{1 - \cos \left(\frac{\alpha}{2^{n-1}}\right)}{2}} = \frac{1}{2} \sqrt{2 - 2 \cos \left(\frac{\alpha}{2^{n-1}}\right)} = \frac{1}{2} \sqrt{2 - 2 \cos \left(\frac{\frac{\alpha}{2^{n-2}}}{2}\right)} = \\ &= \frac{1}{2} \sqrt{2 - 2 \sqrt{\frac{1 + \cos \left(\frac{\alpha}{2^{n-2}}\right)}{2}}} = \frac{1}{2} \sqrt{2 - \sqrt{2 + 2 \cos \left(\frac{\alpha}{2^{n-2}}\right)}} = \\ &= \frac{1}{2} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + 2 \cos \left(\frac{\alpha}{2}\right)}}}} = \frac{1}{2} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + 2 \sqrt{1 - \sin^2 \left(\frac{\alpha}{2}\right)}}}}} = \end{aligned}$$

$$= \frac{1}{2} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{4 - \left[2 \sin\left(\frac{\alpha}{2}\right)\right]^2}}}}}$$

Using this result in Equation (8), noting that  $L_1 = \overline{AB} = 2 \sin\left(\frac{\alpha}{2}\right)$ , finally the following general formula is obtained:

$$\begin{cases} \mu = \frac{2\pi}{\alpha} & 0 < \alpha < \pi, \mu \in \mathbb{R}, \mu > 2 \\ L_1 = 2 \sin\left(\frac{\alpha}{2}\right) \\ \pi = \mu \lim_{n \rightarrow \infty} 2^{n-1} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{4 - L_1^2}}}}} & n + 1 \text{ square roots} \end{cases} \quad (9)$$

Equation (9) generalizes and confirms Equation (6). Equation (9) coincides with Equation (6) when  $\mu = N \in \mathbb{N}$ , or in other words when  $\alpha$  is an integer divisor of the circle, and in this case  $L_1 = 2 \sin\left(\frac{\alpha}{2}\right) = 2 \sin\left(\frac{\pi}{N}\right)$ . Also the succession in Equation (4) can be extended substituting  $N$  with  $\mu = \frac{2\pi}{\alpha}$  and  $L_1$  with  $L_1 = 2 \sin\left(\frac{\alpha}{2}\right)$  for values not integer of  $\frac{2\pi}{\alpha}$ , with  $\alpha \in (0, \pi)$ .

Equation (9), as Equation (6), arises some interest when applied for values not transcendental of  $\sin\left(\frac{\alpha}{2}\right)$ . Some of these instances follow.

Applying Equation (6) to an equilateral triangle ( $N = 3, L_1 = 2 \sin\frac{\pi}{3} = \sqrt{3}$ ) or an hexagon ( $N = 6, L_1 = 2 \sin\frac{\pi}{6} = 1$ ), or a dodecagon ( $N = 12, L_1 = 2 \sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{2}$ ) the following Equation (10), coinciding with the formula (3) presented by Servi in [5], is obtained:

$$\pi = 3 \lim_{n \rightarrow \infty} 2^{n-1} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{3}}}}} , n \text{ square roots} \quad (10)$$

Applying Equation (6) to a square ( $N=4, L_1 = 2 \sin\frac{\pi}{4} = \sqrt{2}$ ), it is possible to obtain the following Equation (11), coinciding with the formula (1) presented by Servi in [5]:

$$\pi = \lim_{n \rightarrow \infty} 2^{n+1} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{2}}}}} , n + 1 \text{ square roots} \quad (11)$$

Applying Equation (9) to the arc  $\widehat{AB} = \alpha = \frac{3}{4}\pi$  ( $135^\circ$ ),  $\mu = \frac{2\pi}{\alpha} = \frac{8}{3}$ , calculating  $L_1 = 2 \sin\left(\frac{\alpha}{2}\right) = \sqrt{2 + \sqrt{2}}$  and the last square root in Equation (9)  $\sqrt{4 - L_1^2} = \sqrt{4 - (2 + \sqrt{2})} = \sqrt{2 - \sqrt{2}}$ , the following Equation (12), coinciding with the formula (2) presented by Servi in [5], is obtained:

$$\pi = \frac{1}{3} \lim_{n \rightarrow \infty} 2^{n+2} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{2 - \sqrt{2}}}}} } n + 2 \text{ square roots} \quad (12)$$

Applying Equation (9) to the arc  $\widehat{AB} = \alpha = \frac{5}{6}\pi$  ( $150^\circ$ ),  $\mu = \frac{2\pi}{\alpha} = \frac{12}{5}$ , calculating  $L_1 = 2 \sin\left(\frac{\alpha}{2}\right) = 2 \frac{\sqrt{6+\sqrt{2}}}{4} = \frac{\sqrt{6+\sqrt{2}}}{2}$  and the last square root in Equation (9)  $\sqrt{4 - L_1^2} = \sqrt{4 - \frac{(\sqrt{6+\sqrt{2}})^2}{4}} =$

$\sqrt{4 - \frac{6+2+2\sqrt{12}}{4}} = \frac{1}{2}\sqrt{8 - 4\sqrt{3}} = \sqrt{2 - \sqrt{3}}$ , the following Equation (12), coinciding with the formula (4) presented by Servi in [5], is obtained:

$$\pi = \frac{3}{5} \lim_{n \rightarrow \infty} 2^{n+1} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{2 - \sqrt{3}}}}} \quad n + 2 \text{ square roots} \quad (13)$$

## New simple formulas of $\pi$ in terms of $\phi$

Now applying specifically Equations (6) and (9), the following three simple formulas of  $\pi$  in terms of  $\Phi$  are identified:

$$\pi = \frac{5}{2} \lim_{n \rightarrow \infty} 2^n \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{2 + \phi}}}}} \quad n \text{ square roots} \quad (14)$$

$$\pi = \frac{5}{3} \lim_{n \rightarrow \infty} 2^n \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{3 - \phi}}}}} \quad n + 1 \text{ square roots} \quad (15)$$

$$\pi = \frac{5}{4} \lim_{n \rightarrow \infty} 2^n \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{2 - \phi}}}}} \quad n + 1 \text{ square roots} \quad (16)$$

Equation (14) is obtained applying the Equation (6) to a pentagon inscribed in the circumference (as in Figure 1) with unitarian radius ( $N = 5$ ,  $\alpha = \frac{2}{5}\pi$ , with simple passages  $L_1 = 2 \sin\left(\frac{\pi}{5}\right) = 2\sqrt{\frac{10-2\sqrt{5}}{4}} = \sqrt{\frac{10-2\sqrt{5}}{4}} = \sqrt{\frac{5-\sqrt{5}}{2}} = \sqrt{3 - \frac{1+\sqrt{5}}{2}} = \sqrt{3 - \phi}$ ). From Equation (6), and remembering that  $\phi^n = \phi^{n-1} + \phi^{n-2}$ , the last two square roots become  $\sqrt{2 + \sqrt{4 - L_1^2}} = \sqrt{2 + \sqrt{4 - (3 - \phi)}} = \sqrt{2 + \sqrt{1 + \phi}} = \sqrt{2 + \phi}$ . Since between the diagonal D and the side L of a pentagon results  $D = \phi L$  (Ghyka in [6]), it is noted the fact quite singular that in this case the last square root in Equation (14) represents exactly the length of the diagonal  $D_1 = \phi L_1 = \phi\sqrt{3 - \phi} = \sqrt{3\phi^2 - \phi^3} = \sqrt{3\phi^2 - (\phi^2 + \phi)} = \sqrt{2\phi^2 - \phi} = \sqrt{2(\phi + 1) - \phi} = \sqrt{2 + \phi}$ .

Equation (15) is obtained applying the Equation (9) to the arc  $\widehat{AB} = \alpha = \frac{3}{5}\pi$  ( $108^\circ$ ),  $\mu = \frac{2\pi}{\alpha} = \frac{5}{2}$ . Calculating  $L_1 = 2 \sin\left(\frac{\alpha}{2}\right) = 2\sqrt{\frac{1+\sqrt{5}}{4}} = \sqrt{1+\sqrt{5}} = \phi$  (in fact in this case the chord  $\overline{AB}$  with two radius forms the triangle gnomon of the golden triangle), thus the last square root in Equation (9) become  $\sqrt{4 - L_1^2} = \sqrt{4 - \phi^2} = \sqrt{4 - (\phi + 1)} = \sqrt{3 - \phi}$ . It is noted the fact quite singular that in this case the last square root in the formula Equation (16) represents exactly the length of the side of a pentagon inscribed in the circle.

Equation (16) is obtained applying the Equation (9) to the arc  $\widehat{AB} = \alpha = \frac{4}{5}\pi$  ( $144^\circ$ ),  $\mu = \frac{2\pi}{\alpha} = \frac{10}{3}$ . Calculating  $L_1 = 2 \sin\left(\frac{\alpha}{2}\right) = \sqrt{10 + 2\sqrt{5}} = \sqrt{\frac{4+1+\sqrt{5}}{2}} = \sqrt{2 + \phi}$  (coinciding with the diagonal of a pentagon inscribed in the circle), thus the last square root in Equation (9) become  $\sqrt{4 - L_1^2} = \sqrt{2 - \phi}$ . It is noted the fact quite singular that in this case the last square root in Equation (16) represents exactly the length of the side of a decagon inscribed in the circle (as  $2 \sin \frac{36^\circ}{2} = \frac{1}{2}(\sqrt{5} - 1) = \sqrt{\left(\frac{\sqrt{5}-1}{2}\right)^2} = \sqrt{\frac{6-2\sqrt{5}}{4}} = \sqrt{\frac{4-1-\sqrt{5}}{2}} = \sqrt{2 - \phi}$ ).

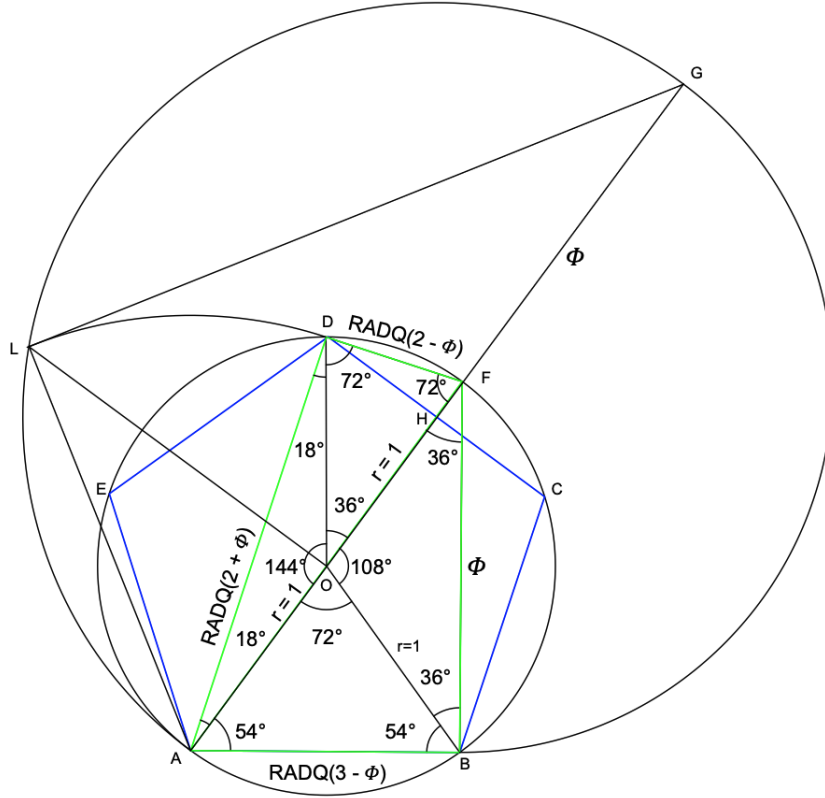


Figure 2: Geometrical properties between the circle,  $\pi$  and  $\phi$

## Geometrical properties

Figure 2 helps to understand the singularities pointed out in last square root of Equations (14), (15) and (16).

For Equation (14), as  $L_1 = \overline{AB} = \sqrt{3-\phi}$  (side of the pentagon inscribed in a circle with unitary radius), applying the last square root  $\sqrt{4-L_1^2}$  means finding the other cathetus  $\overline{BF}$  of the right triangle ABF, being the hypotenuse  $\overline{AF} = 2$ , then  $\overline{BF} = \sqrt{4-(3-\phi)} = \sqrt{1+\phi} = \Phi$  (also as side of a gnomon triangle OBF of the golden triangle). Then, for the following square root  $\sqrt{2+\phi}$ , with the graphical approach it is possible to construct the segment AG with length  $2+\phi$ , then drawing half a circle with center in H e diameter AG,  $\overline{AL}$  results the square root of  $\overline{AG}$  (from the equivalence of the right triangles AGL and AOL, with  $\overline{AO} = 1$ ); finally we get the diagonal  $\overline{AD} = \overline{AL} = \sqrt{2+\phi}$ .

For Equation (15), as  $L_1 = \overline{BF} = \phi$  (side of the gnomon triangle), applying the last square root  $\sqrt{4-L_1^2}$  means finding the other cathetus  $\overline{AB}$  of the right triangle ABF, of length  $\sqrt{3-\phi}$  (side of the pentagon).

For Equation (16), as  $L_1 = \overline{AD} = \sqrt{2+\phi}$  (diagonal of the pentagon), applying the last square root  $\sqrt{4-L_1^2}$  means finding the other cathetus  $\overline{DF}$  of the right triangle AFD, then  $\overline{DF} = \sqrt{4-(2+\phi)} = \sqrt{2-\phi}$  (side of the decagon).

From this geometrical approach it is evident that, if we apply Equation (9) with  $L_1 = \overline{DF} = \sqrt{2-\phi}$  (side of a decagon), applying the last square root  $\sqrt{4-L_1^2}$  means finding the other

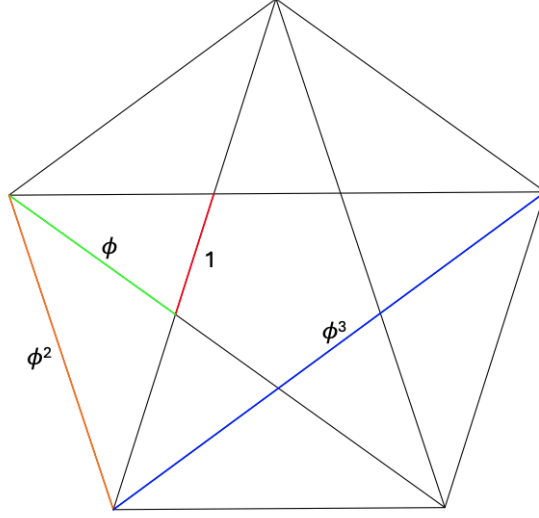


Figure 3: Relations in Pythagorean pentagram

cathetus  $\overline{AD}$  of the right triangle AFD, of length  $\sqrt{2 + \phi}$  (diagonal of the pentagon), thus with the same result obtained in Equation (14) starting with  $L_1 = \overline{AB} = \sqrt{3 - \Phi}$  (side of a pentagon), as we could expect by doubling the sides of the pentagon on the first iteration of Equation (4).

Referring to Figure 2, it should be noted that  $\overline{AB}$ ,  $\overline{BF}$ ,  $\overline{AD}$  are diagonals of the decagon inscribed in the circle, with side  $\overline{DF}$ .

Equations (14), (15), (16) and geometrical properties in Figure 2, identified in a circle with unitarian radius and its inscribed pentagon (and decagon), arise some relationships between the circle,  $\pi$  and  $\Phi$ , extending the relations inside the pentagon and its pentagram constructed with its diagonals (in Figure 3), well known since Pythagoras ancient times (Ghyka in [6]).

## New approximate “squaring” the circle proposal

Let us consider a particular angle, the arc  $\widehat{AB} = \alpha = \frac{2\pi}{\phi^2}$  ( $\approx 137.5^\circ$  called also “golden angle”, that is found many times in nature, for instance in phyllotaxis), with  $\mu = \frac{2\pi}{\alpha} = \phi^2$ .

Calculating  $L_1 = 2 \sin\left(\frac{\alpha}{2}\right) = 2 \sin\left(\frac{\pi}{\phi^2}\right) \simeq 1.86406485$ , Equation (9) becomes:

$$\pi = \phi^2 \lim_{n \rightarrow \infty} 2^{n-1} \sqrt{2 - \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + 2\sqrt{1 - L_1^2}}}}} \quad n + 1 \text{ square roots} \quad (17)$$

The interest on this formula arises noticing that the limit converges to a number 1.199981546..., thus can be approximate to 1.2 with an error lower than 0.00005.

The approximation of Equation (17) provides a mathematical source to the well known approximate formula in following Equation (18) between  $\pi$  and  $\phi$ :

$$\pi \simeq \frac{6}{5}\phi^2 = \frac{6}{5}(1 + \Phi) = 3,141640\dots \quad err < 0.00005 \quad (18)$$

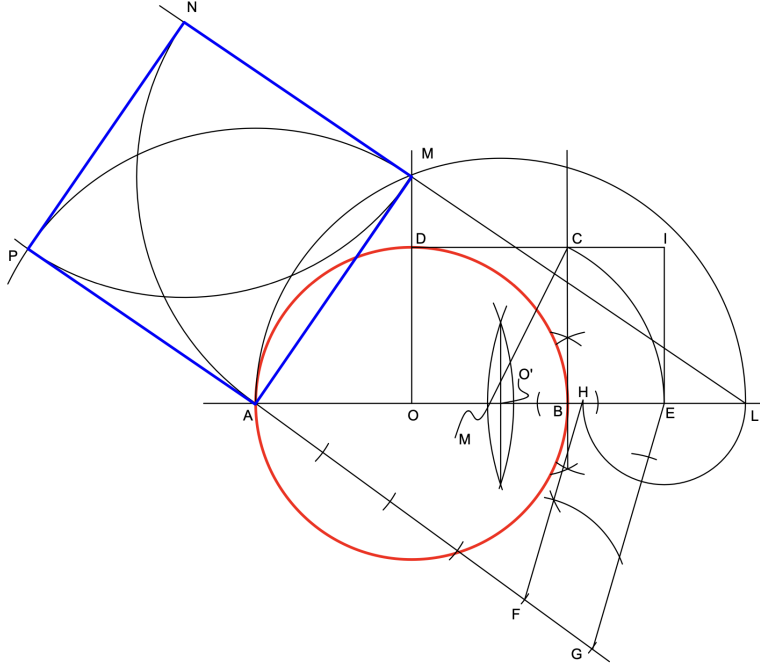


Figure 4: Approximate squaring the circle by R. A. Dixon

This approximation has been pointed out by Dixon in [7], also showing an interesting procedure to draw a square with an area of  $\frac{6}{5}(1 + \phi) \simeq \pi$  with an error lower than 0.00005 (in Figure 4). It could be interesting to mention that the relation in Equation (18) is known at least from the 12th century by the French master masons that built the gothic cathedrals, as proved by Frederic in [8].

Procedure for the constructions just by rule and compass (Figure 4): 1. draw a circle with radius = 1; 2. trace the golden rectangle OEID; 3. apply the rule to divide a segment in five equal parts with segment AG, identifying the fifth part HE of  $\overline{AE} = \overline{AO} + \overline{OE} = 1 + \phi$ , then add this 1/5 to the right, in order to identify the segment AL with length  $\frac{6}{5}(1 + \phi)$ ; 4. trace half a circle on the diameter AL finding point M as the intersection with the vertical line from the centre O; 5. construct the square AMNP on the segment AM. As the triangles ALM and AOM are similar,  $\overline{AL} : \overline{AM} = \overline{AM} : \overline{AO} \rightarrow \overline{AM}^2 = \overline{AL} \cdot \overline{AO} \rightarrow \overline{AM} = \sqrt{\overline{AL} \cdot \overline{AO}} = \sqrt{\frac{6}{5}(1 + \phi)} \simeq \sqrt{\pi}$ .

We propose here another easier way to approximate the “squaring” the circle based on Equation (18) with not a square but a rectangle, with sides length  $\phi$  and  $\frac{6}{5}\phi$ , whose area  $\frac{6}{5}\phi^2$  is quite close (with error lower than 0.00005) to the area  $\pi$  of the circle, as in Figure 5.

Procedure for the constructions just by rule and compass (Figure 5): 1-2. apply the same previous steps; 3. apply the rule to divide a segment in five equal parts with segment OG, identifying the fifth part HE of  $\overline{OE} = \phi$ , then add this 1/5 to the right, in order to identify the segment OL with length  $\frac{6}{5}\phi$ ; 4. trace the arc with center on O from E in order to identify the point N as intersection with the vertical line from





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Author:

Ph.D. Angelo Pignatelli  
email: [ing@angelopignatelli.it](mailto:ing@angelopignatelli.it)