A new formula for picking off pieces of pi

Members of the Ancient and Honorable Society of Pi Watchers have something new to gaze upon and ponder.

Mathematicians have discovered a surprisingly simple formula for computing digits of the number pi (π). Unlike previously known expressions, this one allows them to calculate isolated digits—say, the billionth digit of pi—without computing and keeping track of all the preceding numbers.

"It was something that people just didn't think you could do," says Peter B. Borwein of Simon Fraser University in Burnaby, British Columbia.

The only catch is that the formula works for binary, but not for decimal, digits. Thus, it's possible to determine that the forty billionth binary digit of pi is 1, followed by 001001000011l0. . . . But there's no way to convert these numbers into decimal form without knowing all the binary digits that come before the string.

Borwein, Simon Fraser colleague Simon Plouffe, and their coworkers announced the discovery earlier this month by posting it on the Internet.

"It's a curious finding," says Helaman Ferguson of the Supercomputing Research Center in Bowie, Md., who has checked the result. "It's quite surprising that this [formula] exists."

Borwein has long been interested in finding efficient ways of computing pi, the ratio of a circle's circumference to its diameter. In particular, he has focused on methods of performing the computation using only a small amount of computer memory.

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i}\left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6}\right)$$

New formula that serves as the basis for computing isolated binary (or hexadecimal) digits of pi.

Working with Plouffe, Borwein ide
fied certain types of mathematical expr
sions, or series, that would provide t
necessary shortcut. An extensive sear
turned up suitable formulas for pi a
several other numerical constants, incl
ing log(2).

However, the answer that comes out
the expression for pi gives only hexad
imal (base 16) digits, which can be rea
ly converted to binary. "The frustrati
thing is that it doesn't work in base
[for decimal digits]," Borwein remarks

Borwein and his coworkers are s
hoping to uncover an expression th
gives the decimal digits of pi, but oth
mathematicians are pessimistic th
such a formula will ever be found. Mea
while, the researchers have been loo
ing for related series to compute oth
mathematical constants, such as *e* a
the square root of 2, but with limit
success so far.

The existence of such an intriguing f
mula for computing isolated digits of
may reveal something mathemati
about the nature of the number itself. I
example, mathematicians would like
prove that all the decimal digits occur
pi equally often.

"That would be the mathemati
prize in all this," Borwein notes. "But
the moment, I can't see [our discover
leading to a proof."

On the decimal front, Yasum
Kanada and his coworkers at the U
versity of Tokyo have now computed
to 4,294,960,000 digits, beating the
record (SN: 8/26/95, p.143). Accordi
to their calculations, the four billion
decimal digit of pi is 9, followed
4375343. . . .

The researchers also show that in t
first 4 billion digits, the number
appears most often (400,033,035 tim
and 2 least often (399,965,405 times).

"We do the pi calculations from scrat
because [this] is one of the best ben
mark programs for testing the reliabil
of [our computers] and checking the c
rectness of calculations, programs, a
algorithms," Kanada says. "To be a wo
record holder is a by-product."

If researchers ever find a decim
equivalent of the new formula for bina
digits of pi, Kanada and others would
able to push their calculations mu
higher. Indeed, because such a form
would enable them to compute isolat
clumps of digits, the task could be rea
ly divided up among as many compute
as necessary to get the desired result.

Members of Pi Watchers and othe
interested in the new formula for t
rapid computation of pi can obtain ad
tional information from Borwein's Wo
Wide Web site at the following addre
http://www.cecm.sfu.ca/~pborwei
Anyone curious about 4 billion decim
digits of pi can check Kanada's site
http://www.cc.u-tokyo.ac.jp/.

— I. Peters