

into sediments. Another shows a pH electrode plunging deep into a hydrate blob without breaking the skin, like a finger pushing into a water-filled balloon.

No one can hazard a guess about what all this means for deep-sea sequestration of CO₂. The biological impacts, Caldeira's "potential showstopper," are still unknown. Nearby fishes have shown no ill effects to date, save for one that swam into a CO₂ plume and fell asleep. "From what we've observed so far, it looks pretty good," Brewer says.

However, MBARI biological oceanographer Jim Barry is concerned about potential sublethal effects, such as slower growth

rates or inability to reproduce. "There are reasons to suspect that deep-water organisms may be more sensitive to pH changes or CO₂ changes in comparison to shallow-water organisms," he says. To try to discern such effects, Barry placed sea cucumbers and sea urchins into corrals containing CO₂ for about 3 weeks, while others went into control corrals. He'll use genetic analysis to search for impacts.

Beyond biology, policy-makers will weigh many sequestration pros and cons. Not least among them are economics, technological capability, and public acceptance. Brewer hopes his team's data will help keep those discussions on track.

"I suspect most CO₂ probably will be disposed of underground, because it's been done and people are comfortable with that idea," Brewer says. "But we shouldn't exclude the ocean from our thinking. We're already putting 25 million tons of CO₂ into the surface ocean every day through the atmospheric loop. Why is the deep ocean any different? It's a much larger place, and it's far more benign."

Brewer draws guidance from the words of a Japanese student, who spoke at a Kyoto workshop on the ethics of CO₂ disposal. "Proceed with caution, and have the courage to stop if necessary," he says. "I like that."
—ROBERT IRION

MATHEMATICS

Randomly Distributed Slices of π

Mathematicians slowly circle in on a proof that π 's unpredictable digits really are as random as they seem

The digits of π dance about so unpredictably that scientists and statisticians have long used them as a handy stand-in for randomly generated numbers in applications from designing clinical trials to performing numerical simulations. But surprisingly, mathematicians have been completely at sea when they try to prove that the digits of π (or of any other important irrational number for that matter) are indeed randomly distributed. When a number's digits are randomly distributed, you have no information about what any given digit will be even when you know the previous one. Now two mathematicians have taken a large step toward proving π 's randomness, perhaps opening the door to a solution of a centuries-old conundrum.

The problem has been around for some 900 years, says Richard Crandall, a computational mathematician at Reed College in Portland, Oregon. But mathematicians have precious little to show for their centuries of work, according to David Bailey, a mathematician at Lawrence Berkeley National Laboratory in California. "It is basically a blank," he says. "It's embarrassing."

The degree of embarrassment is hard to imagine. The overwhelming majority of numbers have digits that are truly random when expressed in a given base—a property called normality. In a normal number, any string appears exactly as often as you'd expect it to—in base 10, the digit 1 appears a tenth of the time, for example, and the string 111 appears 1/1000 of the time. But even though normal numbers are everywhere, and almost all numbers are normal, mathematicians have failed to prove the normality

of even a single number other than a handful of oddballs carefully constructed for the purpose. "As far as the naturally occurring constants of math are concerned, like π , the square root of 2, log 2, and [natural] log 10, there are basically no results," says Bailey.

Now Bailey and Crandall have breathed new life into the randomness problem by building on a discovery that flabbergasted the math world 5 years ago. In 1996, Bailey, along with two mathematicians at Simon Fraser University in Vancouver, Canada, Peter Borwein and Simon Plouffe, came up with an algorithm for calculating any digit of π without having to calculate all the digits that precede it—unlike every other known π recipe. If you want to know, say, the 289th digit of π , plug 289 into the formula, dubbed BBP. Out will pop a number between 0 and 1. This number, converted back into base 16, reveals the digit you're after. "It was a pleasant surprise," says Jonathan Borwein (Peter's brother, also a mathematician at Simon Fraser).

The formula looked as if it might help mathematicians solve the centuries-old conundrum of the randomness of π 's digits. "If you can stick your hand down into the digits that way, then it's strong evidence that the numbers are independent," adds Jonathan Borwein. This thought struck Bailey when he came up with the BBP formula. "My immediate reaction was, 'Oh my God, this

might allow us to work on the normality of π ,'" he says. "I was consumed with this."

Bailey and Crandall have now made a hypothesis that formulas such as BBP (except for particularly boring ones) will spit out values that skitter chaotically between 0 and 1 for different digits that get plugged in. If true, this chaotic motion ensures that the output of the BBP formula would be essentially random for any given digit that is plugged in. This, in turn, would mean that π 's digits are also random. As the two mathematicians report in the summer 2001 issue of *Experimental Mathematics*, if the hypothesis is true, it would prove not only π 's randomness, but also that of other constants that have BBP-type formulas, such as the natural log of 2.

Although their hypothesis is as yet unproven, it has restated the ancient problem in a new language. Instead of attacking the problem with the mathematical tools of older disciplines such as number theory or measure theory, Bailey and Crandall's hypothesis turns the normality of π into a problem of chaotic dynamics—the sort of discipline that attracts applied mathematicians, computer scientists, and even cryptographers.

Jonathan Borwein hopes that this insight will finally allow mathematicians to prove that π 's digits are random. "Whenever you recast an old problem in a new language, there's hope that the new language will provide a new impetus," he says. "It can open up better avenues for looking at these things."

But even Crandall himself expects a mere "10% chance of a partial solution" to the hypothesis in the next decade. For mathematicians, apparently, π is not a piece of cake.
—CHARLES SEIFE

