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SQUARING *the Circle is no Piece of π*

BY BRUCE WATSON

MATHEMATICIANS HAVE SLICED, AND NOW SUPERCOMPUTERS HAVE CRUNCHED, BUT THE MYSTERY OF PI GOES ON AND ON AND...

ON AN OTHERWISE RATIONAL FALL DAY, A CLASS OF SIXTH-GRADERS IS ABOUT TO TAKE ON AN irrational task. Students at the Smith College Campus School in Northampton, Massachusetts, spread coffee can lids, compact disks, coins and cups on their desks. Ready or not, these innocent kids are about to have a rendezvous with the most intriguing of numbers.

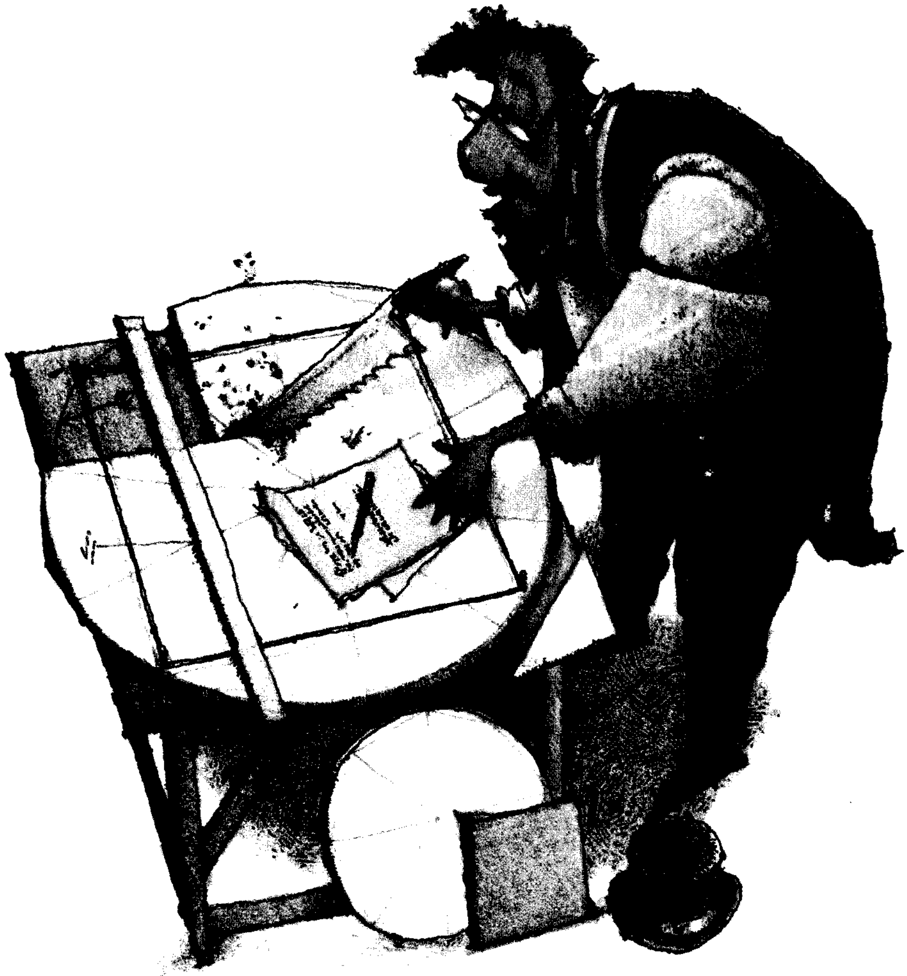
"Today," teacher Tom Weiner tells them, "we're going to learn how to measure circles." Little do they suspect that they are on the verge of a mystery that has vexed mathematicians for ages. Following Weiner's instructions, students begin cutting strings to measure the distance around each circular object. They wrap strings around lids and cups. They crimp them around CDs. They struggle for accuracy, but their strings slacken or slip. "I'm having a hard time getting an exact measurement," Sarah reports. Her teacher wonders: "Could there be an easier way?" Finally, Weiner suggests a different plan, one based on a little number called pi.

Many of us recall our first encounter with pi. It began with dry formulas: $C = \pi D$. $A = \pi r^2$. (Remember? C = circumference; D = diameter; r = radius; A = area.) Inevitably, somebody told the old, old joke: "Pi are squared? [πr^2?] No way! Pie are round!"

Compared with the exact figures that drive our digital age—stock prices, interest rates, computer bytes—pi is as mystical as a zen koan. Stated simply, it is the number you get when you divide the distance around a circle (circumference) by the distance through the middle (diameter). The distance around every circle is about three times the distance across it. Easy, eh? But in that "about" lies the puzzle of pi. Mathematicians call pi an irrational number. That is, when you divide circumference by diameter, the answer comes out in decimals that go on forever without any pattern. Pi begins 3.14159265 . . . and it never ends. Ever. This curiosity makes pi part cult and part of our culture, the subject of several books, such as *The Joy of π* and *A History of Pi*, and many Websites (one of the best is at www.cecm.sfu.ca/pi).

On the face of it, pi seems as easy as pie. But Archimedes refused to settle for a half-baked solution.

ILLUSTRATIONS BY JEFF SEAVER



No matter how you cut it, pi will never come out even, hence a circle can never be squared exactly.

There are many irrational numbers whose decimals go on forever. But has anyone ever written poems about the square root of 2? How many other irrational numbers appear in college football cheers, in *Star Trek* or as the title of a recent film? In "π"—the movie—a mathematical prodigy goes mad seeking number patterns in the stock market and the Torah. Though the film is fictional, such a fixation on numbers is not. For millennia, pi has plagued exacting minds. Last year, a Japanese computer scientist calculated it to some 206 billion digits. A pi that big has no practical use beyond testing supercomputers or theorizing in arcane realms of advanced mathematics. Yet for Yasumasa Kanada, calculating pi is "like Mount Everest. I do it because it's there."

Although pi is just a string of numbers, it can also be exquisite. As the blueprint of every circle, pi is in the ripples of a pond when you throw in a stone, the double helix of DNA, and one of our most important inventions—the wheel. Billions of digits do not make very interesting reading, yet pi has a surprising story to tell.

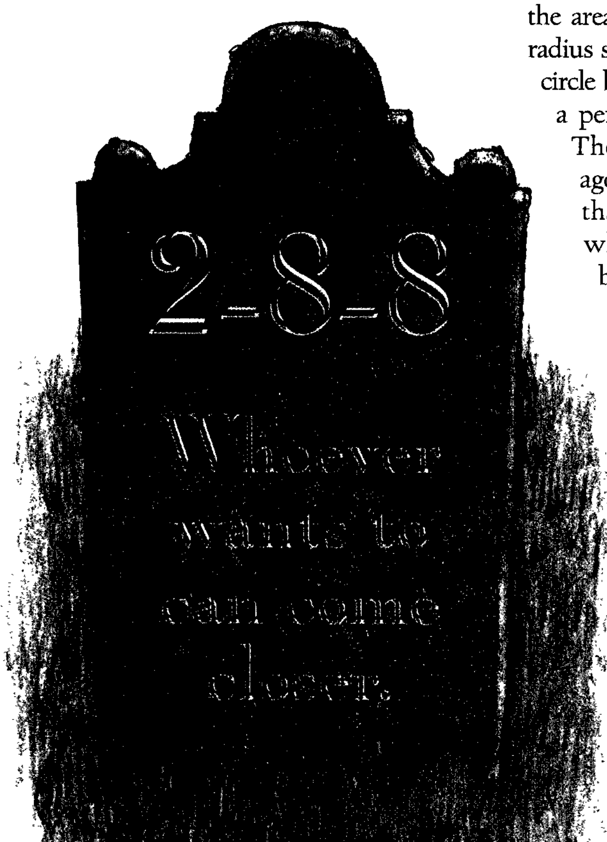
"God ever geometrizes," Plato wrote, but he might as well have been referring to the great calculator of pi—Archimedes. By the time Archimedes began playing with numbers—in the third century B.C.—mathematicians already knew of the fixed ratio in every circle. But exactly how much was that ratio?

Pi shows up in history for the first time in Egypt. In 1858, a Scottish antiquary, Henry Rhind, came upon a wonder, written on papyrus. From square one, the Rhind papyrus boasted of its own importance: "Accurate reckoning. The entrance into the knowledge of all existing things and obscure secrets." One of the mathematical solutions instructed: "Cut off $\frac{1}{9}$ of a diameter [of a circle] and construct a square upon the remainder; this has the same area as the circle."

The Babylonians calculated pi as $3\frac{1}{8}$. The Bible, in 1 Kings 7:23, measures a circular vessel built by Solomon using a ratio of 3. But the Greeks wanted an exact figure. To pin down pi, they drew straight-sided shapes inside circles, like a stop sign fit neatly inside a wheel. The more sides the polygon had, the closer its area matched that of the circle. The Greeks called this the exhaustion method, and it did exhaust many a mathematician. Archimedes, however, could "geometrize" longer than most. Working from 96-sided figures, he estimated pi at between 3 10/71 and 3 1/7. For the next 700 years, his was the most accurate pi. Only in the fifth century A.D., when a Chinese astronomer and his son crammed 24,576 sides inside a circle, did the world have a better pi.

Exhausting a circle may seem tedious, but you can do the math in your own home! First bake a pie—

To immortalize his labors, Ludolph van Ceulen was said to have had decimals of pi carved into his tombstone.



banana cream, say. Cut your pie into four equal slices. Now divide each of those in half. Do it again (16). And again (32). And just for fun, one more time. Now take a knife and cut around the 64 pieces, slicing off the crust in straight lines as close as possible to the pie dish. Eureka! as Archimedes might have said. To calculate pi, merely measure around the crust with a ruler and divide by the pie's diameter. Watch out for those crumbs! Yes, these pieces are pretty sloppy. You should have tried a sturdier pie—pecan, perhaps. Or maybe you shouldn't try to calculate your pi and eat it, too.

Pi has posed a messy problem for minds far greater than yours and mine. The Greeks referred to it as squaring the circle. The problem: using straightedge and compass, can you draw a circle and a square with equal areas? Pi is at the heart of the problem because the area of a circle equals pi times its radius squared. And once squaring the circle became a passion, the search for a perfect pi became an obsession. The Greek philosopher Anaxagoras, imprisoned for teaching that the sun was not a god, whiled away his time behind bars trying to square a circle. Centuries later, the philosopher Thomas Hobbes claimed to have solved the problem, and when proved wrong he lashed out, rationalizing that long-standing principles of geometry must surely be wrong.

Leonardo da Vinci came close to a solution with a "hands-on" approach. Get a cylinder whose height is half its radius, he said. (Translation: Go to the kitchen and get a short, squat can; an ordinary can of tuna is close enough.) Now turn the can on its side and roll it one full revolution. The can traces a rectangle that is equal to the area of its lid. But Leonardo's procedure was too crude to be of any use in refining pi. The search continued.

In 1610, an unlikely monument to pi was built in Holland. A tombstone in the graveyard of Peter's Church in Leiden was supposedly engraved with the numbers 2-8-8, representing the 33rd through 35th digits of pi, calculated by the mathematician Ludolph van Ceulen. To arrive at pi, van Ceulen formulated a 32-billion-sided polygon. Computing its area using the state-of-the-art invention called "decimals," van Ceulen took pi to 20 digits. "Whoever wants to can come closer," he concluded, but he was the only one who wanted to. He spent his last 14 years expanding pi to 35 digits. The tombstone said to have immortalized his work was lost, and van Ceulen's labor was soon rendered moot by a new invention—calculus.

As the plague ravaged London in 1665, Isaac Newton retired to his cottage in Woolsthorpe. There he developed the calculus, the means of calculating the shapes of curves, the arc of artillery, the trajectory of a rocket to the moon. While in Woolsthorpe, Newton also calculated pi. Later he admitted that the little number had preoccupied him. "I am ashamed to tell you to how many figures I have carried these computations, having no other business at the time," Newton wrote. Yet with calculus, Newton had created a new—and much more elegant—means of finding pi.

Soon mathematicians expanded the little number even further. It reached 100 digits in 1706, the same year a British mathematician gave pi its Greek name. (As if the math weren't hard enough, pi had been called "the quantity which, when the diameter is multiplied by it, yields the circumference.") By the late 1700s, "squaring the

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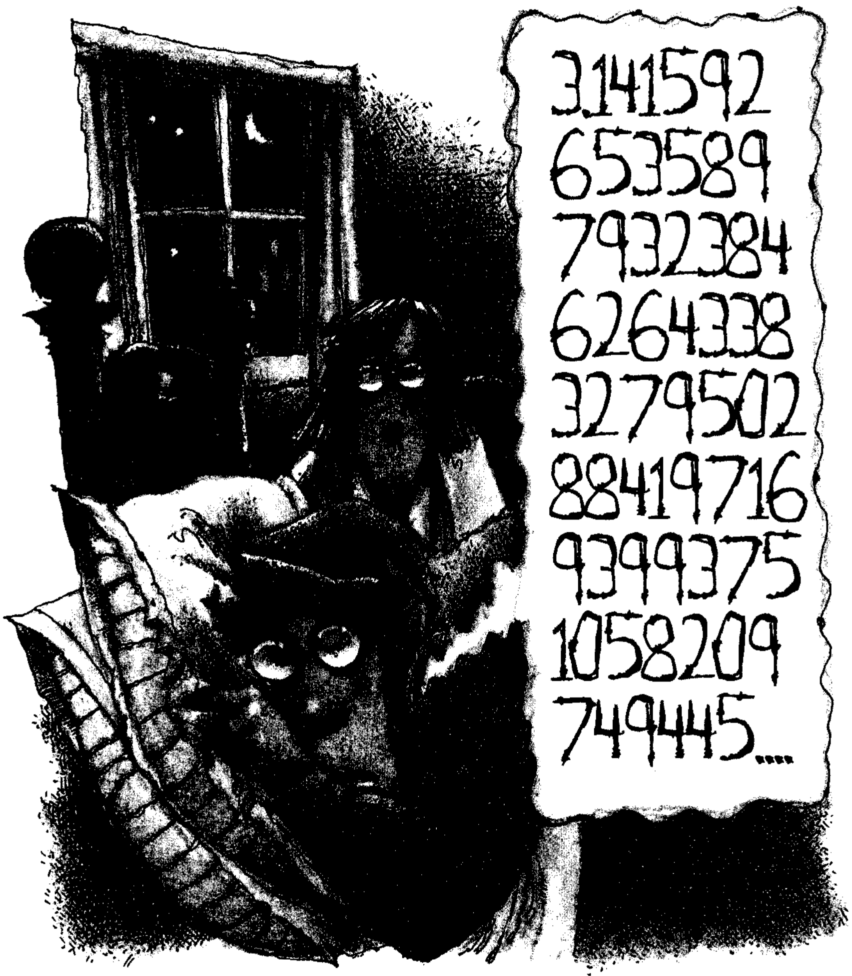
circle" had become the pastime of cranks, and the French Academy of Sciences, inundated with "solutions" to what it suspected was an impossible problem, refused to accept any more circle-squaring "proofs" or, for that matter, blueprints for perpetual motion machines.

So what does a big piece of pi look like? I estimate that the 206 billion digits currently known would fill more than a million issues of SMITHSONIAN. Here's a snapshot of pi's first 100 digits: 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706 7 . . . and counting. Using just 3.14, you can pass your geometry final. Using the first ten decimals, you can measure the earth's circumference to a fraction of an inch. So why take pi any further? Some mathematicians, like the mad genius in the movie "π," hate to see numbers lead to chaos. Every pi seeker hopes that in mining its depths, some wonderful order might emerge. Yet when taken to extremes, pi's sequence remains amazingly random.

Suppose pi were a poker hand. Imagine that its decimals were cards dealt in a digital game of five-card draw. According to the laws of probability, based on formulas way, way beyond sixth-grade math, cards and random numbers are distributed in a way that defies prediction. Nevertheless, they do obey a mathematical pattern. These odds say that in pi's first two million five-digit "hands," you should get about one million pairs—hands such as 34459. And in fact you do. The odds also say you should get 216,000 hands with two pairs, such as 88663. You get 216,520. Pi also obeys the laws of probability for three of a kind, a full house and four of a kind. Finally, in two million poker hands, you'd expect 200 to be all five of the same digit—77777, for example. When pi is the dealer, you do actually get . . . 200.

Throughout the 19th century, human calculators tried, but could not finish off pi. One such brain, Johann Dase of Hamburg, was able to multiply two eight-digit numbers in his head. Dase could calculate for hours at a time, go to sleep, then wake up and continue where he left off. In 1844, he put his mental calculator to work on pi, and in two months computed it to a new record of 205 places. Another true believer, William Shanks, spent 20 years with pencil and paper calculating pi to 707 digits. Shanks' mark stood into the 20th century, though it was later discovered that he made a mistake in the 527th digit. Twenty years on the job and his pi was incorrect. Yet Shanks wasn't the only one to get pi wrong.

A circle is a circle is a circle throughout the universe. Whether it's in a wedding ring or the rings of Saturn, pi is the same number. Except in Indiana. Or so the state's House of Representatives proclaimed in 1897. It all began with a country doctor named Edwin Goodwin, of Solitude, Indiana, who claimed to have been "supernaturally taught the exact measure of the circle . . . in due confirmation of scriptural promises." The Bible makes no such promises, of course. Goodwin was just another "circle squarer" who refused to let mathematical truth stand in the way of fame. Never mind that in 1882, a German mathematician had proved that pi was "transcendental," which leads to the conclusion that no matter how you cut it, pi will never come out even, hence a circle can never be squared exactly. Undeterred, Goodwin set out to do the impossible. He squared his circle all right, though he had to use a pi valued at 9.2376, almost three times the actual



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value. Goodwin had his "solution" published in the *American Mathematical Monthly*, then set about getting government approval for his own private pi. He convinced his local legislator to introduce a bill before Indiana's House offering state schools free use of his "new mathematical truth." The bill, chock full of math jargon, fooled the House and passed by a 67-0 vote. But not even politicians can do a number on everybody, and the fraud was soon exposed by a math professor. Mercifully bogged down in bureaucracy, Goodwin's new truth never passed the state Senate. Pi remained 3.14159.

Following centuries of baffling the greatest mathematicians, in the 20th century pi came up against a bunch of wires and tubes. The world's first large computer was not designed to run circles around a single number. ENIAC (Electronic Numerical Integrator and Computer) was built during World War II to compute artillery trajectories. Slower than the laptop on which this article is being written, ENIAC nonetheless weighed 30 tons and ran up an electricity bill of $650 an hour. In 1949, ENIAC spewed out its calculation of pi in 70 hours. Mathematicians scratched their heads and wondered. After 2,037 digits, there was still no rhyme or reason to the progression of pi.

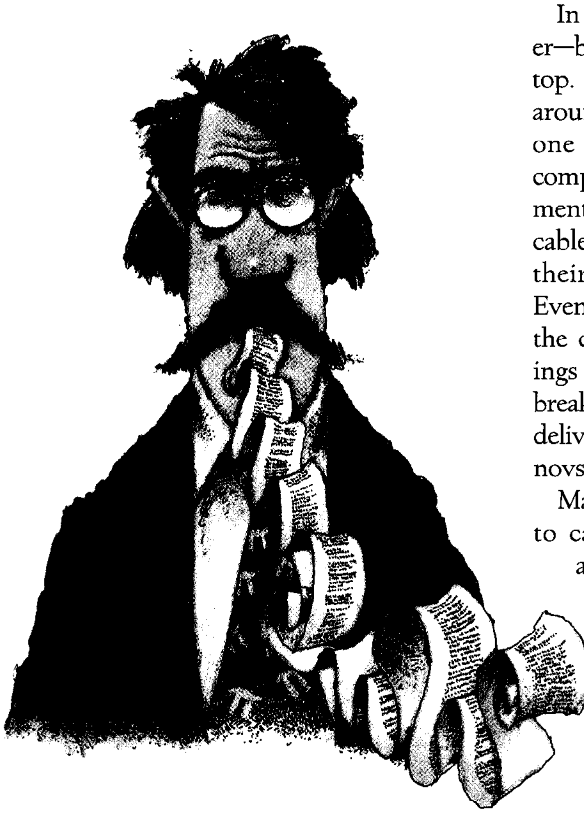
Computers proved the perfect match for extending pi. First in minutes, later in fractions of a second, they crunched the numbers it had taken William Shanks decades to compute, sending pi into the stratosphere. It reached a half-million digits in 1967, a million six years later. By 1983, it was up to 16 million digits.

But computers are only as good as the formulas programmed into them. Such calculations were made possible by one of the 20th century's most dazzling minds. Srinivasa Ramanujan was a poor clerk in southern India. Yet, captivated by mathematics, he studied on his own. In 1913, he sent solutions to several problems to G. H. Hardy at Cambridge University. Recognizing Ramanujan's genius, Hardy brought him to Cambridge. In 1914, one of the former clerk's papers revealed the fastest equation yet for expanding pi.

In 1984, two Russian brothers using their own formula, "in the style of Ramanujan," began taking pi toward a billion decimals. They did the math on a supercomputer they built in their apartment.

When I visited him at Brooklyn's Polytechnic University, Gregory Chudnovsky warned me about pi. "Look," he said, "in considering pi you should not apply your pedestrian, Euclidian way of thinking." I agreed that I wouldn't. Then I settled back as Chudnovsky, wrapped in a brown coat and huddled on his office couch, proceeded to open an entire universe. More than two decades had

During a three-hour memory marathon, Simon Plouffe spewed out a record 4,096 digits of pi.



passed since Chudnovsky and his brother David fled the Soviet Union following severe harassment by the K.G.B., which wanted to keep them in the Soviet fold. Four years later, Gregory won one of the first MacArthur Foundation "genius" grants. The grant helped keep the brothers Chudnovsky working in spite of illness. Gregory, a prodigy who published a major math treatise while still in high school, has myasthenia gravis, a muscular disorder that confines him to a wheelchair. Unable to teach, he began to tinker with supercomputers, which perform billions of calculations per second. Pi is the acid test for such beasts. Like Sisyphus endlessly pushing his rock uphill, a supercomputer can calculate pi forever without finishing. So the Chudnovskys, experts in number theory, began putting pi through its paces. At first, the brothers tapped into a Cray supercomputer over a phone line, but this proved too costly.

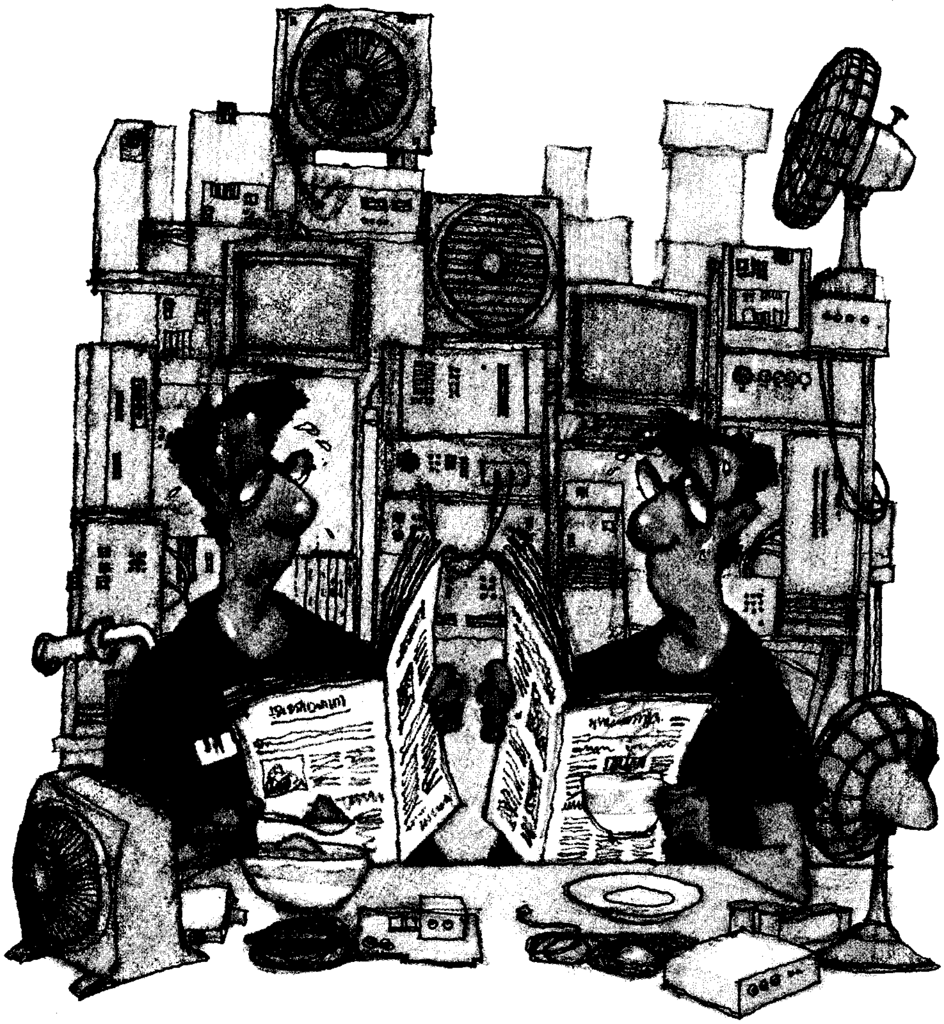
In 1989, they built a home computer—but not your garden variety desktop. Ordering parts from tech stores around the country, they assembled one of the world's most powerful computers in their Manhattan apartment. With its microprocessors and cables snaking from room to room, their creation doubled as a heater. Even when cooled by dozens of fans, the computer warmed its surroundings to 90 degrees. Battling frequent breakdowns and unpredictable freight deliveries of parts, the brothers Chudnovsky were soon in the thick of pi.

Many mathematicians are satisfied to call pi irrational. The wry and articulate Chudnovskys call it monstrous, subtle, exquisite, boring, humbling, annoying, terra incognita and a complete mystery. "Look at the sky," David suggests. "Billions of stars clumped together, scattered around. You think, 'What a mess.' Look at pi and it seems equally chaotic."

So why bother? I asked. Was it the pursuit of a world record? Soon after the Chudnovskys computed nearly a half-billion digits, Yasumasa Kanada topped them by 50 million. The Chudnovskys were the first to calculate a billion digits. Kanada did a few million more. The Chudnovskys took pi to two billion. Kanada churned out six billion. In 1996, the Chudnovskys reached eight billion.

But they didn't do it for the record, they assured me. They were hot on the trail of "hypergeometric function identities from the point of view of applications to diophantine approximations to numbers. . . . " Right. In other words, the brothers wanted to know whether the formulas for pi could be grouped into categories according to theories almost no one else in the world understands. At eight billion places, and finding no profound pattern in pi, the Chudnovskys decided they had gone far enough. "Pi was interesting up to a point," Gregory concluded, "but you don't want to make a fetish out of it."

Fetish, no. But a cult? In this age of chaos theory, pi is digital chaos and it has a growing number of admirers. At San Francisco's Exploratorium, Pi Day is celebrated each March 14 at 1:59 P.M. (Get it? 3/14 at 1:59?) Pi lovers circle the science museum's pi shrine 3.14 times, add numbered beads to a string listing pi's decimals and hear music based on the number. They eat pie; lemon meringue and apple are favorites. Some perhaps even wear π—the fragrance by Givenchy. Each fall at the Massachusetts Institute of Technology, football fans cheer for their favorite irrational number: "Cosine, secant, tangent, sine, 3.14159!" Pi turns up like a mantra in Carl Sagan's novel *Contact*, where an alien tells an earthling of a message encoded within the digits of pi. In one *Star*



Even when cooled by dozens of fans, the Chudnovsky brothers' computer heated its surroundings to 90 degrees.

Trek episode, Spock distracts an evil computer by instructing it to compute pi to the last digit. Yet some exacting minds are not content to merely chant, eat or smell pi. They want to know it by heart.

As a teenager in Montreal, Simon Plouffe became "addicted to numbers." Upon learning of a world record for memorizing pi, Plouffe set out to break it. On his first day, Plouffe memorized 300 digits. Soon he was isolating himself in a dark room, going over the decimals in his head. Within six months, he had memorized 4,096 digits of pi. "It was a very mystical experience," Plouffe told me. "You develop a matrix in your head. You can almost see the numbers." During a three-hour marathon, Plouffe spewed out his slice of pi and entered the French edition of the *Guinness Book of Records*. But not for long. Soon the record was more than 5,000. Plouffe, by then, was pursuing higher mathematics. He eventually coauthored an amazing formula for determining any digit of pi, at least in binary form, without calculating all the previous ones. The current memorization record is now well beyond his reach, he admits. The reigning champ is Hiroyuki Goto, who recited 42,195 digits in nine hours. Do not try this math at home.

But if you just want to memorize a baker's dozen digits of pi, recite this: "See, I have a rhyme assisting / my feeble brain, / its tasks oft-times resisting." Count the number of letters in each word, and voila: 3.141592653589. Such memory aids have been crafted in several languages, but they pale compared with the verse pi has inspired. In her poem *Pi*, Wislawa Szymborska, winner of the 1996 Nobel Prize in Literature, praised pi for its dogged approach to infinity. Szymborska wove the digits of pi into her poem: "The admirable number pi: / *three point one four one*. / All the following digits are also initial, / *five nine two* because it never ends. / It can't be comprehended *six five three five* at a glance, / *eight nine* by calculation, / *seven nine* or imagination. . . . " Awed by pi's infinite reach, Szymborska follows the number as it "goes on across the table, through the air, / over a wall, a leaf, a bird's nest, clouds, straight into the sky."

Pi continues to amaze everyone who has ever studied a circle. But as supercomputers expand, calculating a trillion or a quadrillion digits, will mathematicians ever find some grand order in its chaos? David Chudnovsky is optimistic. "It's very possible that there's some very simple answer. We just don't have it yet."

An anthology of Bruce Watson's humorous essays, Info-Aged, was published in 1999. Jeff Seaver has illustrated many articles for SMITHSONIAN.