Conjectures and series from the OEIS

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Summary

In 1992, my master's thesis in mathematics was an experiment on the 5500 sequences of what was to become *The Encyclopedia of Integer Sequences* which was published in 1995 with Neil Sloane. Shortly afterwards the content was put on the internet. In this thesis there were 1031 formulas found out of a total of 4528 with a new Maple program developed by me and François Bergeron (UQAM) and which later became a program within the Maple library called GFUN, GFUN for G. FUNctions.

From 5500 sequences, we have increased to 362500 at a rate of 10 to 20000 per year. This has required a lot of help from outside. There are now more than twenty editors, a president (Neil Sloane), an OEIS foundation and thousands of collaborators. The website is one of the best known in the mathematical world. I have therefore repeated several times this exercise of trying to find a closed form or formula allowing to generate each sequence of the OEIS catalogue. I present here the 141208 formulas found out of 57138 sequences, which represents a success rate of 16%.

To be as close as possible to the real formula, known or not, it is possible to calculate a score based on the length of the formula found (in number of characters), the size of the whole sequence (usually about 50 terms) and the number of terms.

$$Score(Annnnnn) = \frac{ln(Number\ of\ terms) \cdot length\ of\ sequence}{Length\ of\ formula}$$

This results in a number between 0 and 20 in most cases. As soon as the threshold of 2.0 is not reached the formula found is rejected and is considered as a false positive.

As for the techniques that were used, my original 1992 Master's thesis can be consulted on my website or on the ArXiv or ViXra websites.

The list of formulas can be found here:

http://plouffe.fr/OEIS/OEIS_conjectured_formulas.pdf.

A longer, more readable version can be found in the same place but the document is 27305 pages long here: http://plouffe.fr/OEIS/OIES_conjectured%20formulas%20long%20version.pdf

Here is a sample of the full document:

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                        OEIS conjectured formulas / Simon PLOUFFE

A000000
A000000 Conjectured formulas of the OEIS by Simon Plouffe as of May 09 2023
A000000 Version 1.0 : August 1992, Current version : 2023 05 08
A000000 There are 57138 unique sequence and more than 141208 expressions.
A000000
A000000
A000000             ln(nbr_of_terms) x length_of_sequence
A000000    Score := ------------------------------------
A000000                    length_of_formula
A000000
A000000
A000000
A000000 If the score is high the reliability of the formula is good.
A000000
A000000 A score near 1 is not good, a score of 20+ is excellent.
A000000 Some formulas are in double for many reasons, the sign or the factorization
A000000 of the expression makes a change in length. The intention of the author
A000000 is to keep the list as clean as possible, with time it will be done...
A000000 These formulas are suggested, most of them are the true ones, in some
A000000 cases the formula found automatically are better than the original
A000000 reference.
A000000
A000000 Some formulas are false positives, in some rare cases, even false
A000000 the formula found is sometimes as interesting as the original.
A000000
A000000 Some formulas appear with the value I (complex),
A000000 it is not an error most of the time. On the other hand,
A000000 to correctly evaluate the series it is necessary to :
A000000 develop in series (ordinary or exponential).
A000000 collect the terms,
A000000 evaluate in floating point,
A000000 evaluate in absolute value.
-------------------------------------------------------------------------------------
OEIS #  Score   Formula                                 Type
-------------------------------------------------------------------------------------
#####################################################################################
A000004 13.1249 [{a(n+1),a(0)=0} ogf]
A000007 12.0781 [{-a(n+1),a(0)=1} ogf]
A000008 25.5586 [x^9+x^4+x+1 ogf_with_Euler_transform]
A000008 5.84027 [1/(x^4-x^3+x^2-x+1)/(x+1)^2/(x^4+x^3+x^2+x+1)^2/(x-1)^4 ogf]
A000009 33.6616 [1/(-x^2+1) ogf_with_Euler_transform]
A000012 15.8359 [{a(n+1)-a(n),a(0)=1} ogf]
A000012 20.0845 [-f(x)*x+f(x)-1 ogf]
A000012 34.3110 [1/(-x+1) lgdogf]
A000012 39.2126 [-1/(x-1) ogf]
A000023 17.0100 [-(1-2*x)/(-x+1) lgdegf]
A000023 26.9652 [(1-2*x)*f(x)+(1-x)*f(x)^2 lgdegf]
A000023 5.07762 [{(2*n+4)*a(n+1)+(n+1)*a(n+2)-a(n+3),a(0)=1,a(1)=-1,a(2)=2} ogf]
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References :

[1] The Integer Suite Catalogue: https://oeis.org/ for all suite numbers appearing in this document.

[2] Plouffe Simon, Master's thesis 1992 UQAM, *Les approximations de séries génératrices et quelques conjectures.* https://arxiv.org/ftp/arxiv/papers/0911/0911.4975.pdf

[3] OEIS conjectured formulas, version 2018. https://vixra.org/pdf/1409.0048v6.pdf

[4] Plouffe Simon, Bergeron François, Computing the Generating Function of a Series Given Its First Few Terms https://vixra.org/abs/1409.0095

[5] The numbers in base exp(Pi), 659 formulas related to the Gamma function and powers of π. https://vixra.org/abs/2305.0024