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Article

Special Spirals are Produced by the ROTASE Galactic Spiral Equations with the Sequential Prime Numbers

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Highlights:

- Special spirals are produced by the galactic spiral equations with prime numbers
- The necessary condition for a number to be a prime number
- Twin prime numbers can only be formed by 1 P1 prime number and 1 P5 prime number, n must be 1 greater than m.

Abstract: In this paper, the sequential prime numbers are used as variables for the galactic spiral equations which were developed from the ROTASE model. Special spiral patterns are produced when prime numbers are treated with the unit of radian. The special spiral patterns produced with the first 1000 prime numbers have 20 spirals arranged in two groups. The two groups have perfect central symmetry with each other and are separated with two spiral gaps. The special spiral pattern produced with natural numbers from 1 to 7919 shows 6 spirals in the central area and 44 spirals in the outer area. The whole 7919 spiral points can be plotted with either 6-spiral pattern or 44-spiral pattern. For the spirals only produced by the prime numbers in the 6-spiral pattern plotting, the spiral 2 and spiral 3 each has only one spiral point produced by prime number 2 and 3, respectively, all other spiral points produced by other prime numbers are located on the spiral 1 and spiral 5. The special spiral pattern is well explained with careful analysis, it is concluded that all prime numbers greater than 3 must meet one of the equations:

$$P_1 = 1 + 6 * n \ \ (\mathrm{n} > 0)$$

$$P_5 = 5 + 6 * m \ \ (m \geq 0)$$

In other words, every prime number greater than 3 is either a P_1 prime number or a P_5 prime number, no exception. Matching one of the equations is a necessary condition for a number to be a prime number, not a sufficient condition. Hope such sufficient condition can be found in the future. Twin prime numbers can only be formed between P_1 and P_5 prime numbers, n must be 1 greater than m. The number of P_1 prime numbers roughly equal the number of P_5 prime numbers in the first 2 billion prime numbers. The largest prime number known at the moment 2^(82,589,933) – 1 is a P_1 prime number. The galactic spiral equations with golden angle can duplicate Vogel's result for the simulation of sunflower seed head pattern, and a pinwheel pattern can be produced also with galactic spiral equations and 1 degree more than golden angle.

Keywords: prime numbers; prime spirals; twin prime numbers; galactic spiral equations; ROTASE model, phyllotaxis; sunflower seed head

1. Introduction

The spiral is one of the fundamental morphologies of the universe. They can be large as the spiral galaxies and small as the DNA helices. They occur everywhere around us. Many mathematical spiral equations have been invented throughout the history of the science, like the A Archimedean spiral, Euler spiral, Fermat's spiral, hyperbolic spiral, logarithmic spiral, Fibonacci spiral, etc. The author developed a set of new spiral equations in recent years based on the proposed Rotating Two Arm Sprinkler Emission model (for short, ROTASE model) [1-4], this model describes the possible mechanism of the formation of spiral arms of disc galaxies, regular galaxies with open spiral arms, 4 types of ring galaxies and galaxies with only one arm can be precisely simulated by the new galactic spiral equations, and several galaxies with special patterns are explained. However, when the author read a very general science article about interesting prime numbers, a strange idea or "whim" jumped to author's head when looked at the prime numbers in sequence (2,3,5,7....): what will happen if use those prime numbers as variables for the galactic spiral equations? The result is surprised and very interesting.

2. The galactic spiral equations of the ROTASE model

The mathematical community may not be familiar with the galactic spiral equations from the ROTASE model, so it is necessary to list those equations here. Please refer the references for the detail of the ROTASE model and the derivation of the galactic spiral equations [1-2].

The following are the primary differential galactic spiral equations

$$\begin{cases} dx = R_b * \dfrac{y}{\sqrt{x^2+y^2}}\, d\theta \\ dy = R_b * \left(\rho(\theta) - \dfrac{x}{\sqrt{x^2+y^2}}\right) d\theta \end{cases} \tag{1}$$

These equations (1) can be solved in a polar coordinate system for three different cases, $\rho > 1$, $\rho = 1$ and $\rho < 1$, respectively:

$$r = \frac{R_b}{1 - \rho * \sin(\alpha)} \tag{2}$$

For $\rho > 1$:

$$\left(\frac{1}{\rho^2-1}\right)\left\{\sqrt{(\rho^2-1)r^2 + 2rR_b - (R_b)^2} - \left(\frac{R_b}{\sqrt{\rho^2-1}}\right) ln|r\sqrt{\rho^2-1} + \frac{R_b}{\sqrt{\rho^2-1}} + \sqrt{(\rho^2-1)r^2 + 2rR_b - (R_b)^2}|\right\} - \left(\frac{R_b}{\rho^2-1}\right)\left(\rho - \frac{1}{\sqrt{\rho^2-1}} \ln|\frac{\rho^2 R_b}{\sqrt{\rho^2-1}} + \rho R_b|\right) = R_b\theta \tag{3}$$

For $\rho = 1$:

$$\frac{\sqrt{2}}{3\sqrt{R_b}}(r + R_b)\sqrt{r - \frac{R_b}{2}} - \frac{2}{3}R_b = R_b\theta \tag{4}$$

For $\rho < 1$:

$$\frac{R_b}{(1-\rho^2)^{\frac{3}{2}}} arcsin\left(\frac{(1-\rho^2)r - R_b}{\rho R_b}\right) - \frac{1}{(1-\rho^2)}\sqrt{2rR_b - (R_b)^2 - (1-\rho^2)r^2} - \frac{R_b}{(1-\rho^2)^{\frac{3}{2}}} acrsin(-\rho) + \frac{\rho R_b}{1-\rho^2} = R_b\theta \tag{5}$$

The value of arcsin is in radian units, not in degrees. The θ is the galactic bar rotation angle and is in radian units, and it represents time in those equations, not the spiral rotation angle. r is the distance of spiral arm in the calculation to the galactic center. α is the angle of r with respect to the X-matter emission axis. R_b is the half-length of the galactic bar and can be set as 1 during the calculation. The parameter ρ is defined as the ratio of the emission velocity of the X-matter out of the galactic bar over the flat rotation velocity of galactic disc. ρ can be a constant or change with time (θ) in any format. The spirals can be calculated by the differential equations (1) or the solution galactic spiral equations (2) to (5) by selecting the right equation according to the value of ϱ. The calculated x and y must be rotated counterclockwise by the following equation for final spiral plotting, the counterclockwise rotation is well explained in the reference [4]:

$$\begin{cases} x'(\theta) = x(\theta) * \cos(-\theta) + y(\theta) * \sin(-\theta) \\ y'(\theta) = -x(\theta) * \sin(-\theta) + y(\theta) * \cos(-\theta) \end{cases} \tag{6}$$

Plotting the x′(θ), y′(θ) will produce the calculated spirals. If the parameter ϱ is less than 1, the spiral will be a ring with the radius defined by the equation (7) as:

$$r = \frac{R_b}{1-\rho} \tag{7}$$

Ten calculated spiral patterns with different ϱ values are listed in the reference [1]. The central point of the "whim" mentioned above is to use prime numbers as variables to calculate the spirals, i.e.,

$$\theta_i = P_i \tag{8}$$

Where the P_i is the series of prime numbers.

3. The spirals produced by the ROTASE galactic spiral equations with the sequential prime numbers

The author wrote a program with QB64 language for the calculations. Figure 1 left is the result of the pattern calculated by the galactic spiral equations (1) using the first 1000 prime numbers (2 to 7919, 1000 points) as variables with constant ϱ = 2.5. The author was surprised by the result. The plot shows 20 spirals organized in the two groups which are separated by two spiral gaps. There appears to be some "missing" spirals. These two groups of spirals have perfect central symmetry. The numbers on the figure will be explained later.

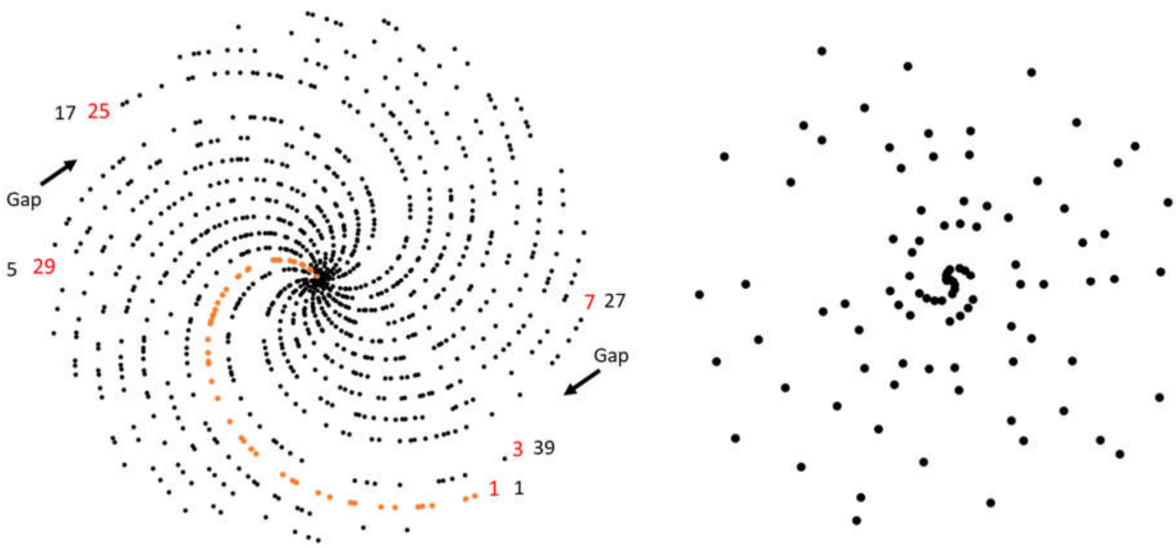

Figure 1. left is the pattern calculated using first 1000 prime numbers as variables; right is the first 95 points zoom-in central area of the left image, ϱ = 2.5.

When the central portion of the figure is zoomed-in as shown in Figure 1 right with the first 95 points, more interesting phenomena are shown. The central area shows two spirals which are not central nor symmetric. The points are gradually scattered with increasing radius, and the two-spiral pattern seems lost. It is well known that the occurrence of prime numbers is irregular and "unpredictable", i.e., no such formula which is efficiently computable is known, although there are many formulae invented so far [5-9].

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How can the irregular occurrence of prime numbers produce such 20 regular spirals with perfect central symmetry in left figure of Figure 1, but only two spirals with non-central symmetry in the central area? The curiosity by such interesting phenomena immediately drove the author for full investigation.

4. The unit of prime numbers as angular variables

The author first noticed that the prime numbers were directly input into the computer program for calculation of the galactic spiral equations without thinking if they were treated with unit of degree or radian when writing the Qbasic computer program, this is completely unintentional, the trigonometric functions in Qbasic take the variables as the radian not the degree. So the Figure 1 is the result of calculation with unit of radian, i. e., the first prime number 2 is 2 radians which is equivalent of 114.59 degrees, the second prime number 3 is 3 radians which is equivalent of 171.89 degrees, etc... The author revised the program to treat the prime numbers as degrees, i.e., the prime number 2 is 2 degrees, the prime number 3 is 3 degrees, etc... Of cause, those degrees must be converted to radian for the calculation. Figure 2 shows the result of the calculation with prime numbers as degrees.

Figure 2. the spiral pattern calculated with first 1000 prime numbers treated as degrees.

The Figure 2 shows that it is only one spiral with points irregularly scattered on the spiral line without any specialty. Therefore, if the prime numbers were treated as degrees in the first Qbasic program, the trial for the whim would be end there.

5. Calculation of the spiral with the first 7919 numbers, corresponding to the 1000th prime number 7919

The calculation of the Figure 1 only uses first 1000 prime numbers, however, the occurrence of the prime numbers is irregular, the author felt that some information could be missing in such calculation, therefore, the author revised the program to calculate every number from 1 to 7919 which corresponding to the 1000th prime number 7919, the unit of the variable is radian. Figure 3 shows the result of the calculation. The left of Figure 3 is the pattern with 7919 points, it clearly shows 44 spirals. The right is the zoom-in central area with 499 points corresponding to the 95th prime number 499 in the Figure 1 right, this zoom-in clearly shows 6 spirals in the central area, the 6-spiral pattern gradually transits to 44 spirals with increase of radius in visual effect.

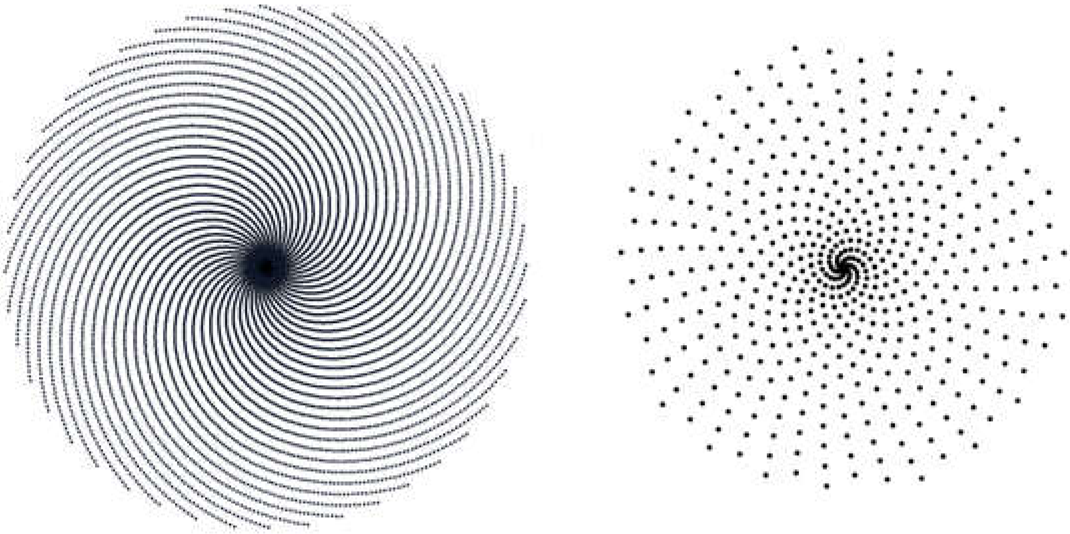

Figure 3. left is the pattern calculated with numbers 1 to 7919; right is the zoom-in central area with 499 points.

However, all 7919 points in Figure 3 left can be perfectly plotted with 6 spirals shown in Figure 4 left with first 600 points. The numbers for each of such 6-spiral calculations can be written in the following formulas:

$$S(i,\ n) = i\ + 6 * n \qquad \text{(i = 1 to 6, n is integer)} \qquad (9)$$

Where, S(i, n) is the number used to calculate the n^{th} point of i^{th} spiral, the i at the right side of equation (9) is the first number of the i^{th} spiral, the first number i is called the "leading number" of the spiral.

At same time, the 7919 points in Figure 3 left can also be perfectly plotted with 44 spirals as shown in the Figure 4 right, represented by the red spiral line. The numbers for each of such 44-spirals calculation can be written in the following formulas:

$$S(i,\ n) = i\ + 44 * n \qquad \text{(i = 1 to 44, n is integer)} \qquad (10)$$

Where, S(i, n) is the number used to calculate the n^{th} point of i^{th} spiral, the i at the right side of equation (10) is the leading number of the i^{th} spiral.

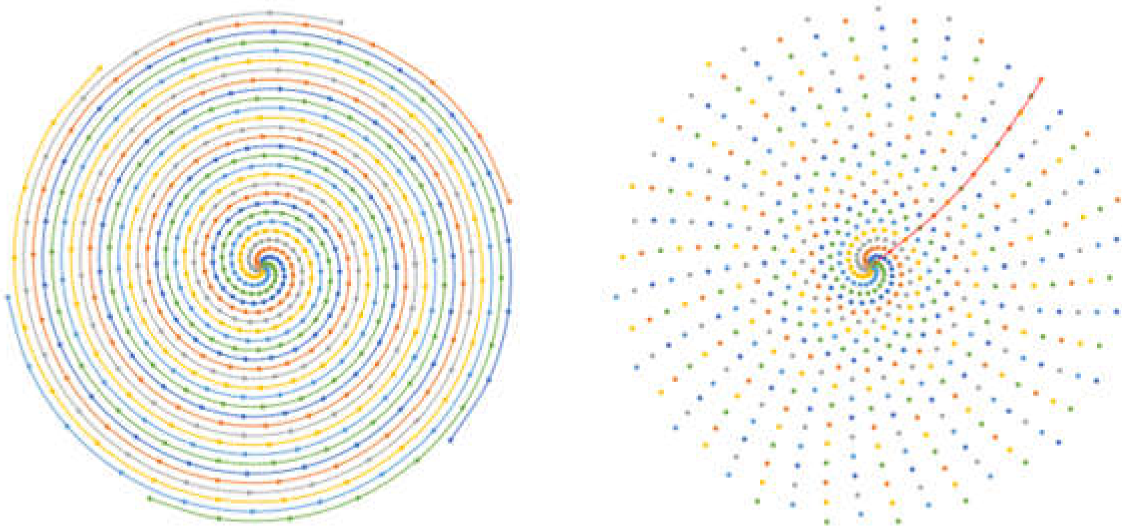

Figure 4. left is the 6 spirals with first 600 points of Figure 3 left, the right is the 44 spirals with the first 600 points of Figure 3 left.

An interesting pattern will show up when picking out all special points which are produced only by the numbers equal to the prime numbers from the Figure 3 left. For the convenience of illustration, those special points produced by the prime numbers are named as "prime points" through the rest of the paper. Figure 5 left is the prime points (marked by red and blue "X") superimposed with the Figure 4 left shown in 6-spiral pattern. The Figure 5 right is the prime points (marked by red and blue "X") superimposed with the Figure 4 right shown in 44-spiral pattern. For convenience of description, all spirals are labeled with numbers shown in Figure 5. For the 6-spiral pattern, the spiral with leading number 1 is labeled as spiral 1, other spirals will be numbered sequentially anticlockwise. The leading numbers of the spirals are the same as the sequential spiral numbers. The numbering is slightly complicated for the 44-spiral pattern. The spiral with leading number 1 is labeled as spiral 1, the other spirals will be labeled with sequential numbers (shown with red numbers) anticlockwise; however, the leading numbers (shown in black numbers) of other spirals are not in the sequential order.

With careful inspection of the Figure 5 left, it is found that the spiral 2 has only one prime point produced by prime number 2, and the spiral 3 has only one prime point produced by prime number 3, all other prime points are located on the spiral 1 and spiral 5, no prime points are located on the spiral 4 and spiral 6, this is because no even numbers except 2 are prime numbers. Why does the spiral 3 have only one prime point? From equation (9), the numbers to produce spiral 3 is:

$$S(3,n) = 3 + 6 * n = 3 * (1 + 2 * n) \tag{11}$$

So, there is a factor of 3 for all numbers to produce the spiral 3, therefore spiral 3 has only one prime point. This explains why the central area of Figure 1 right shows two spirals with non-central symmetry, one prime point on spiral 2 and spiral 3 is not visually visible.

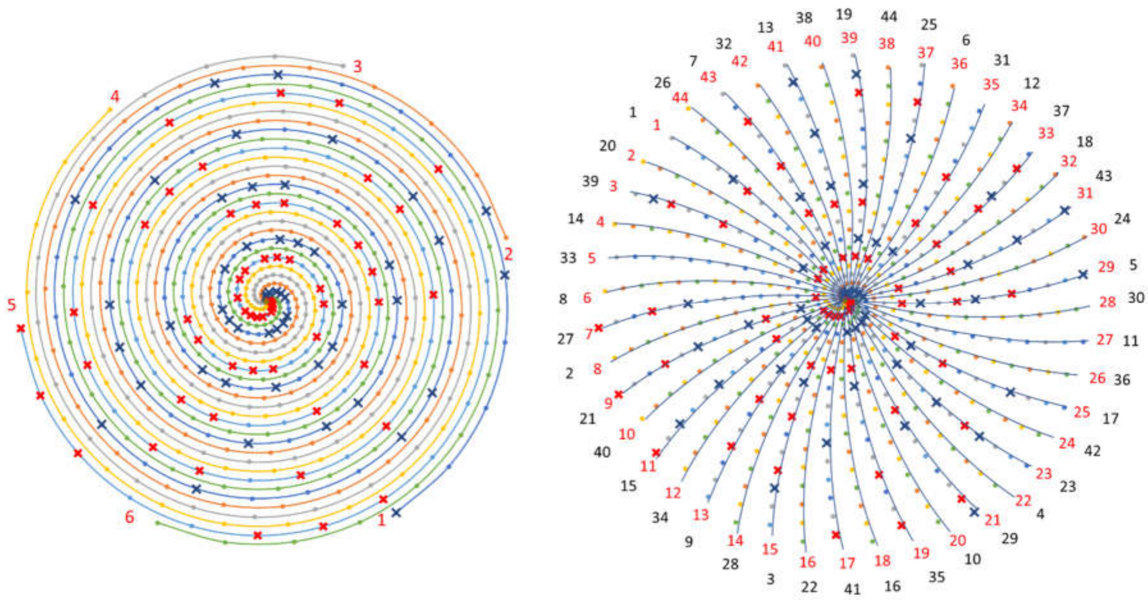

Figure 5. left is the 6 spirals combined with prime points; right is the 44 spirals combined with prime points.

Figure 6 shows the zoom-in central area of the Figure 5 left, spiral 2 has one prime point, spiral 3 has one prime point, all other prime points are located on spiral 1 and spiral 5.

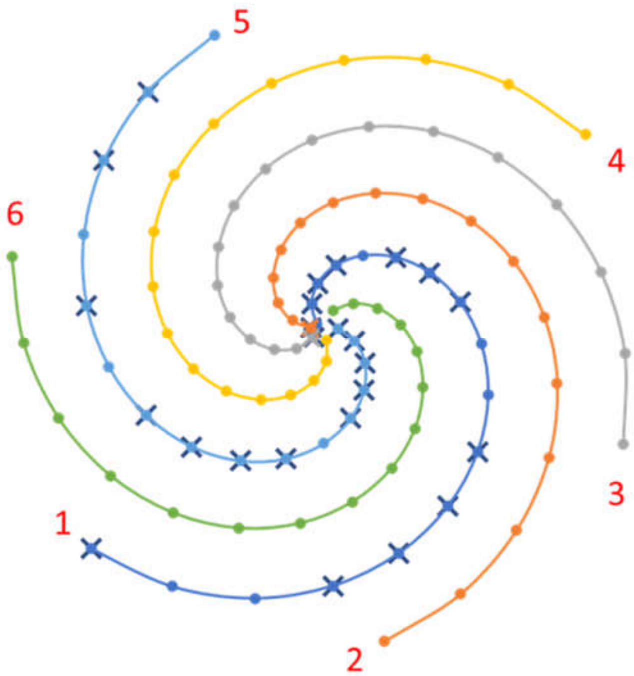

Figure 6. First 100 points (dots and lines) and first 25 prime points marked by X, $\varrho = 2.5$.

Now, let's look at the Figure 5 right. All 7919 points can be plotted with 44 spirals which are sequentially labeled by the red numbers. It clearly shows that all prime points are located on the odd spirals except the spiral 8 (red number) which has only 1 prime point produced by prime number 2 (black number), this causes the prime points alternative distribution on the 44 spirals. The spiral 27 (red number) has only 1 prime point which is produced by the leading prime number 11 (black number), the numbers to produce the spiral 27 is:

$$S(11, n) = 11 + 44 * n = 11 * (1 + 4 * n) \tag{12}$$

Therefore, every S(11, n) has a factor of 11, this is why the spiral 27 has only one prime point.

The spiral 5 (red number) has no prime points. This is because all numbers to produce spiral 5 with leading number 33 can be written as:

$$S(33,\ n) = 33\ + 44 * n\ = 11 * (3 + 4 * n) \tag{13}$$

So, every S(33, n) has a factor of 11, and is therefore not a prime number. The empty prime points in spiral 5 (red number) and spiral 27 (red number) cause the spiral gaps in Figure 1 left. However, the two spiral gaps are mysteriously arranged with perfect central symmetry by the nature in this configuration. Figure 7 shows the pattern of all 7919 points (black) with all 1000 prime points (orange), the brown dot spiral marks the spiral 1.

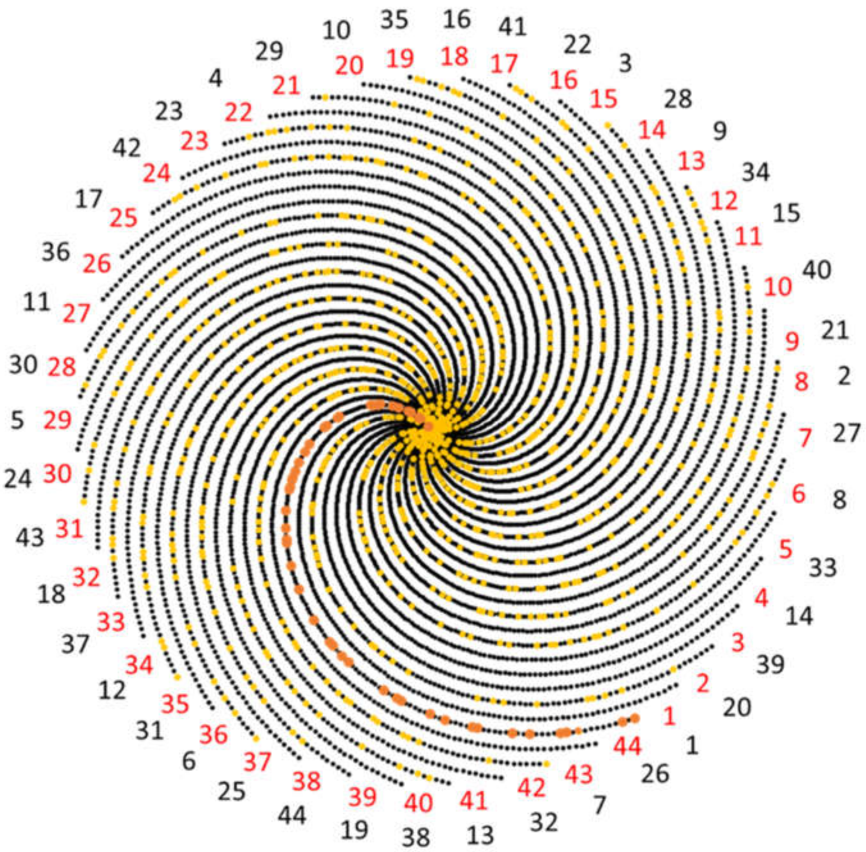

Figure 7. the 7919 points calculated for every number from 1 to 7919, ϱ= 2.5. All orange points (include the brown points) are prime number points, brown points are prime points in the first spiral. The red numbers are the assignments of the 44 spirals in consecutive order, the black numbers are the leading natural numbers for each spiral.

From the result of the analysis above, the following statement can be made:

All prime numbers P greater than 3 must satisfy one of the following conditions:

$$P_1 = 1 + 6 * n \qquad (n > 0) \tag{14}$$

$$P_5 = 5 + 6 * m \qquad (m \geq 0) \tag{15}$$

The P_1 represents all prime numbers matching the equation (14), those prime numbers are called P_1 prime numbers; the P_5 represents all prime numbers matching the equation (15), those prime numbers are called P_5 prime numbers. Euclid's theorem is a fundamental statement in number theory that asserts that there are infinitely many prime

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numbers. However, any prime number greater than 3 is either a P_1 prime number or a P_5 prime number, no exception. 999992982 out of first 2 billion prime numbers are P_1 prime numbers; 1000007016 out of the first 2 billion prime numbers are P_5 prime numbers, only 4034 difference in count for the first 2 billion prime numbers. The equations (14) and (15) are two parallel straight lines with slop of 6, therefore, we can also say, all prime numbers greater than 3 roughly equally distribute on the two parallel lines. A number matching the equations (14) or (15) may not be a prime number, but a prime number greater than 3 must match one of the two equations. Therefore, matching equation (14) or (15) is a necessary condition for a number to be a prime number, not a sufficient condition. Hope such sufficient condition can be found in the future. The largest known prime number (as of May 2022) is 2^(82,589,933) – 1, it is verified in our departmental super computer cluster that this today's largest prime number is a P_1 prime number.

6. Twin Prime numbers

The "Twin Prime Numbers" is a very important concept in Number Theory, it is a pair of prime numbers which differ by 2. Based on the equations (14) and (15), the minimum difference between two P_1 prime numbers is 6, the minimum difference between two P_5 prime numbers is 6 also, it can be further proved that only P_1P_5 pair (P_1 is greater than P_5) not P_5P_1 pair (P_5 is greater than P_1) can be twin prime numbers. From equations (14) and (15), for P_1P_5 pair,

$$P_1 - P_5 = 2 = -4 + 6 * (n - m) \tag{16}$$

$$n - m = 1 \tag{17}$$

Therefore, the n must be 1 greater than m.

For possible P_5P_1 pair,

$$P_5 - P_1 = 2 = 4 + 6 * (m - n) \tag{18}$$

$$m - n = -\frac{1}{3} \tag{19}$$

But, m and n are integers, (m – n) cannot be a non-integer, so, the equation (19) is not valid. Therefore, the following statement can be made:

Any twin prime numbers can only be formed between P_1 and P_5 prime numbers, and the n must be 1 greater than m.

7. Goldbach conjecture

Goldbach's conjecture in modern terms states that every even counting number greater than 2 is equal to the sum of two prime numbers. It has not been finally proved so far. Now, we know that all prime numbers are classified into P_1 prime numbers and P_5 prime numbers based on the equations (14) and (15), is there any benefit of such classification to the proving the Goldbach's conjecture? what type of prime number pair (P_1P_1, P_1P_5, P_5P_1 or P_5P_5) is more favorable sum for the even number if the conjecture is true? Does the classification have any benefit to solve other problems?

8. Phyllotaxis

There are a lot of plants in the nature have various types of spiral patterns. Phyllotaxis is the botanical study of the arrangement of phylla (leaves, petals, seeds, etc.) on plants. The galactic spiral equations listed above can be used to simulate those natural spirals as demonstrated in the reference [10]. In 1979, Helmut Vogel constructed a model to simulate the sunflower seed head with the following spiral equations [11]:

$$r(k) = \sqrt{k} \tag{20}$$

$$\theta(k) = k * \delta \tag{21}$$

Where, k is the seed number, r is the radius of the seed number k, δ is the special angle 137.5°, which is called the "golden angle". The Vogel's model is currently the common model to generate sunflower seed head by computers, the perfect seed head pattern will be broken if the angle is 1 degree more or 1 degree less from the golden angle.

In here, it can be also demonstrated that the galactic spiral equations can duplicate Vogel's result with rotation at multiple of golden angle, same as equation (21). The Figure 8 shows the patterns with δ 1 degree less than golden angle (left), δ equal to golden angle(middle), and δ 1 degree more than golden angle(right), respectively, it matches Vogel's result very well.

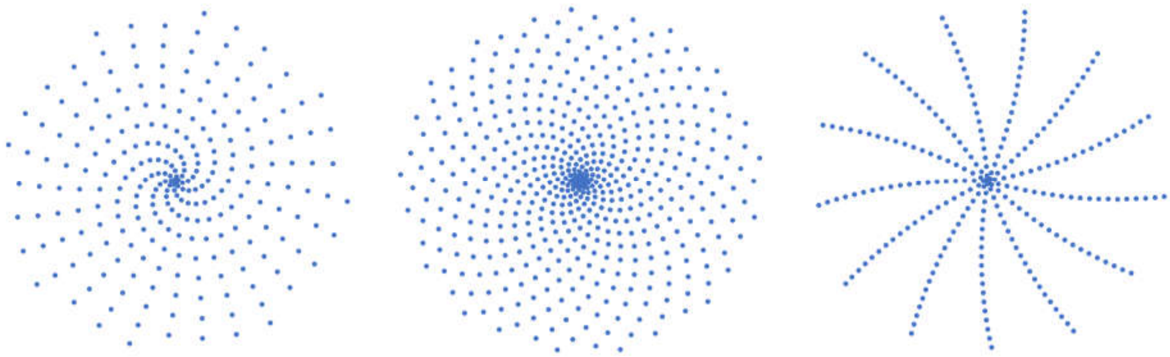

Figure 8. spirals generated by galactic spiral equations: left is 1 degree less than golden angle, middle is the rotation at golden angle, and right is 1 degree more than golden angle.

The galactic spiral equations can generate a beautiful pinwheel pattern as shown in Figure 9, the δ is 0.995, the spiral is ring with the radius defined by the equation (7).

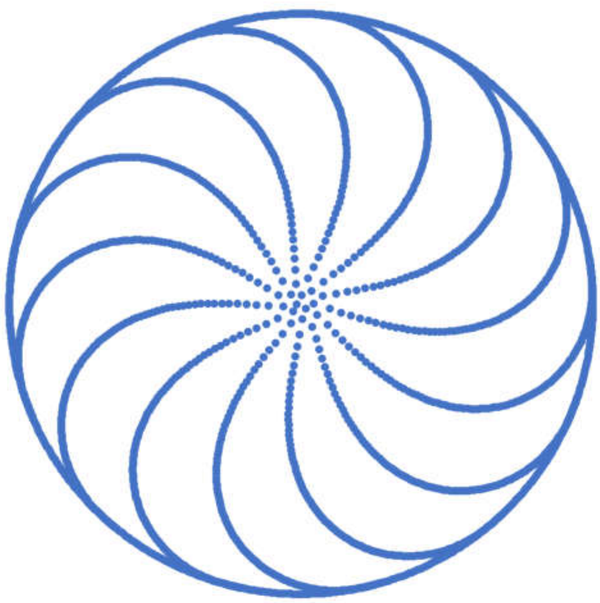

Figure 9. A pinwheel pattern generated by the galactic spiral equations with δ 1 degree more than golden angle, ϱ= 0.995.

9. Discussion

The above result demonstrates that combination of the galactic spiral equations and the prime numbers can produce very interesting patterns and discover new knowledge about the prime numbers. This approach may be extended to other mathematical subjects. The only previous reported prime spiral is the Ulam spiral [12], which is a graphical

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depiction of the set of prime numbers, it is constructed by writing the positive integers in a square spiral and specially marking the prime numbers. The striking appearance in the spiral of prominent diagonal, horizontal, and vertical lines contain large numbers of prime numbers. The prime spirals shown in Figure 1 is another type of spirals related to the prime numbers. It is not sure if the straight lines of equations (14) and (15) have any connections to straight lines with high concentration of prime numbers in the Ulam spiral.

10. Conclusion

The galactic spiral equations from the ROTASE model combined with sequential prime numbers can produce spirals with special patterns, which reveal that all prime number greater than 3 must meet one of two equations extracted from the special spiral patterns: they are either P_1 prime numbers or P_5 prime numbers. The number of P_1 primes and the number of P_5 prime numbers are roughly equal in the first 2 billion prime numbers. Matching one of the equations (14) or (15) is a necessary condition for a number to be a prime number, not a sufficient condition. Twin prime numbers can only be formed between P_1 and P_5 prime numbers, n must be 1 greater than m. The largest prime number known at the moment 2^(82,589,933) – 1 is a P_1 prime number. The sunflower seed head pattern can be simulated by the galactic spiral equations with golden rotation angle. A pinwheel spiral pattern can be produced by the galactic spiral equations.

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