Table of constants with 50 decimal places

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Here are given the first 50 decimal places of constants that occur frequently in numerical computations (see also [1], [2] and [4] for large tables of constants available on the net). This number of digits should be enough for most practical purposes.

1 Related to π

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

$$\begin{aligned}
\pi &= 3.14159265358979323846264338327950288419716939937510...\\
\pi/2 &= 1.57079632679489661923132169163975144209858469968755...\\
\pi/3 &= 1.04719755119659774615421446109316762806572313312503...\\
\pi/4 &= 0.78539816339744830961566084581987572104929234984377...\\
\pi/5 &= 0.62831853071795864769252867665590057683943387987502...\\
\pi/6 &= 0.52359877559829887307710723054658381403286156656251...\\
2\pi &= 6.28318530717958647692528676655900576839433879875021...\\
3\pi &= 9.42477796076937971538793014983850865259150819812531...\\
4\pi &= 12.56637061435917295385057353311801153678867759750042...\\
\pi^2 &= 9.86960440108935861883449099987615113531369940724079...\\
\pi^3 &= 31.00627668029982017547631506710139520222528856588510...\\
1/\pi &= 0.31830988618379067153776752674502872406891929148091...\\
2/\pi &= 0.63661977236758134307553505349005744813783858296182...\\
\sqrt{\pi} &= 1.77245385090551602729816748334114518279754945612238...\\
1/\sqrt{\pi} &= 0.56418958354775628694807945156077258584405062932899...\\
\sqrt{2\pi} &= 2.50662827463100050241576528481104525300698674060993...\\
1/\sqrt{2\pi} &= 0.39894228040143267793994605993438186847585863116493...\\
\sqrt[3]{\pi} &= 1.46459188756152326302014252726379039173859685562793...
\end{aligned}$$

2 Related to e

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}.$$

$$
\begin{aligned}
e &= 2.71828182845904523536028747135266249775724709369995... \\
e/2 &= 1.35914091422952261768014373567633124887862354684997... \\
2e &= 5.43656365691809047072057494270532499551449418739991... \\
e^2 &= 7.38905609893065022723042746057500781318031557055184... \\
e^3 &= 20.08553692318766774092852965458171789698790783855415... \\
1/e &= 0.36787944117144232159552377016146086744581113103176... \\
\sqrt{e} &= 1.64872127070012814684865078781416357165377610071014... \\
1/\sqrt{e} &= 0.60653065971263342360379953499118045344191813548718... \\
e^{\pi} &= 23.14069263277926900572908636794854738026610624260021... \\
e^{-\pi} &= 0.04321391826377224977441773717172801127572810981063... \\
e^{\pi/2} &= 4.81047738096535165547303566670383312639017087466453... \\
e^{-\pi/2} &= 0.20787957635076190854695561983497877003387784163176... \\
e^{\gamma} &= 1.78107241799019798523650410310717954916964521430343... \\
e^{-\gamma} &= 0.56145948356688516982414321479088078676571038692515... \\
e^{e} &= 15.15426224147926418976043027262991190552854853685613... \\
e^{-e} &= 0.065988035845312537076790187596846424938577048252796...
\end{aligned}
$$

3 Related to Catalan's constant

G is defined by the series expansion

$$G = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2}.$$

$$
\begin{aligned}
G &= 0.91596559417721901505460351493238411077414937428167... \\
1/G &= 1.09174406370390610145415947333389232498605012140824... \\
G/\pi &= 0.29156090403081878013838445646839491886406615398583... \\
\pi/G &= 3.42981513013245864263455323784799901211670795530093...
\end{aligned}
$$

4 Square roots

$$
\begin{aligned}
\sqrt{2} &= 1.4142135623730950488016887242096980785696718753769 4... \\
\sqrt{3} &= 1.7320508075688772935274463415058723669428052538103 8... \\
\sqrt{5} &= 2.2360679774997896964091736687312762354406183596115 2... \\
\sqrt{7} &= 2.6457513110645905905016157536392604257102591830824 5... \\
\sqrt{10} &= 3.1622776601683793319988935444327185337195551393252 1... \\
1/\sqrt{2} &= 0.7071067811865475244008443621048490392848359376884 7... \\
1/\sqrt{3} &= 0.5773502691896257645091487805019574556476017512701 2... \\
1/\sqrt{5} &= 0.4472135954999579392818347337462552470881236719223 0... \\
\phi &= 1.6180339887498948482045868343656381177203091798057 6... \\
\sqrt[3]{2} &= 1.2599210498948731647672106072782283505702514647015 0... \\
\sqrt[4]{2} &= 1.1892071150027210667174999705604759152929720924638 1... \\
2^{\sqrt{2}} &= 2.6651441426902251886502972498731398482742113137146 5...
\end{aligned}
$$

$\phi = 1/2 + \sqrt{5}/2$ is the golden ratio.

5 Logarithms

We define the logarithm as :

$$\log(x) = \int_1^x \frac{dt}{t}.$$

$$
\begin{aligned}
\log(2) &= 0.6931471805599453094172321214581765680755001343602 5... \\
\log(3) &= 1.0986122886681096913952452369225257046474905578227 4... \\
\log(5) &= 1.6094379124341003746007593332261876395256013542685 1... \\
\log(7) &= 1.9459101490553133051053527434431797296370847295818 6... \\
\log(10) &= 2.3025850929940456840179914546843642076011014886287 7... \\
1/\log(10) &= 0.4342944819032518276511289189166050822943970058036 6... \\
\frac{\log(2)}{\log(3)} &= 0.6309297535714574370995271143427608542995856401318 8... \\
\log\left(\log(2)\right) &= -0.3665129205816643270124391582326694694542634478371 1... \\
\log\left(\pi\right) &= 1.1447298858494001741434273513530587116472948129153 1... \\
\log\left(\sqrt{2\pi}\right) &= 0.9189385332046727417803297364056176398613974736377 8... \\
\log\left(\gamma\right) &= -0.5495393129816448223376617688029077883306989812630 6... \\
\log\left(\phi\right) &= 0.4812118250596034474977589134243684231351843343856 6...
\end{aligned}
$$

6 Gamma and Psi functions

An elementary definition is :

$$
\begin{aligned}
\Gamma(x) &= \int_0^\infty t^{x-1} e^{-t} dt, \qquad \text{with } x > 0, \\
\psi(x) &= \frac{\Gamma'(x)}{\Gamma(x)}
\end{aligned}
$$

and we have the important relations

$$
\begin{aligned}
\Gamma'(1) &= -\gamma \\
\Gamma'(x_m) &= 0.
\end{aligned}
$$

$$
\begin{aligned}
\gamma &= 0.57721566490153286060651209008240243104215933593992... \\
1/\gamma &= 1.73245471460063347358302531586082968115577655226680... \\
\Gamma(1/2) &= 1.77245385090551602729816748334114518279754945612238... \\
\Gamma(1/3) &= 2.67893853470774763365569294097467764412868937795730... \\
\Gamma(2/3) &= 1.35411793942640041694528802815451378551932726605679... \\
\Gamma(1/4) &= 3.62560990822190831193068515586767200299516768288006... \\
\Gamma(3/4) &= 1.22541670246517764512909830336289052685123924810807... \\
x_m &= 1.46163214496836234126265954232572132846819620400644... \\
\Gamma(x_m) &= 0.88560319441088870027881590058258873320795153366990... \\
\psi(1/2) &= -1.96351002602142347944097633299875556719315960466043... \\
\psi(1/3) &= -3.13203378002080632299641907428726885415542829672041... \\
\psi(2/3) &= -1.31823441578658847240234081664511312187136204862767... \\
\psi(1/4) &= -4.22745353337626540808953014609668357736724443870824... \\
\psi(3/4) &= -1.08586087978647216962688676281718069317007503933313... \\
L &= 5.24411510858423962092967917978223882736550990286325...
\end{aligned}
$$

L is the *Lemniscate constant* and its value is given by

$$L = \frac{\Gamma^2(1/4)}{\sqrt{2\pi}}.$$

7 Riemann Zeta function

For any $s > 1$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

$$
\begin{aligned}
\zeta(2) &= 1.64493406684822643647241516664602518921894990120679...\\
\zeta(3) &= 1.20205690315959428539973816151144999076498629234049...\\
\zeta(4) &= 1.08232323371113819151600369654116790277475095191872...\\
\zeta(5) &= 1.03692775514336992633136548645703416805708091950191...\\
\zeta(6) &= 1.01734306198444913971451792979092052790181749003285...\\
\zeta(1/2) &= -1.46035450880958681288949915251529801246722933101258...\\
\zeta(1/3) &= -0.97336024835078271546888686244789657077282963174305...\\
\zeta(2/3) &= -2.44758073623365823109099570422300521301545223575799...\\
\zeta(3/2) &= 2.61237534868548834334856756792407163057080065240006...\\
\zeta(-1/2) &= -0.20788622497735456601730672539704930222626853128767...\\
1/\zeta(2) &= 0.60792710185402662866327677925836583342615264803347 9...\\
\zeta'(0) &= -0.91893853320467274178032973640561763986139747363778...\\
\zeta'(2) &= -0.93754825431584375370257409456786497789786028861482...
\end{aligned}
$$

7.1 Non trivial zeros

First roots of the equation $\zeta(s) = 0$ of the form

$$s = \frac{1}{2} + it_m.$$

$$
\begin{aligned}
t_1 &= 14.13472514173469379045725198356247027078425711569924...\\
t_2 &= 21.02203963877155499262847959389690277733434052490278...\\
t_3 &= 25.01085758014568876321379099256282181865954967255799...\\
t_4 &= 30.42487612585951321031189753058409132018156002371544...\\
t_5 &= 32.93506158773918969066236896407490348881271560351703...\\
t_6 &= 37.58617815882567125721776348070533282140559735083079...
\end{aligned}
$$

You may find very complete tables on the zeros of the Riemann zeta function at [3].

8 Number theory

In this section the letter p always denotes a prime and the sums or products are only involving the set of prime numbers.

- Mertens constant M

$$\begin{aligned} M &= \lim_{n\to\infty}\left(\sum_{p\le n}\frac{1}{p}-\log\left(\log(n)\right)\right) \\ M &= 0.2614972128476427837554268386086958590515666482611 9... \end{aligned}$$

- Artin's constant

$$\begin{aligned} C &= \prod_{p\ge 2}\left(1-\frac{1}{p(p-1)}\right) \\ C &= 0.3739558136192022880547280543464164151116292486061 5... \end{aligned}$$

- Twin prime constant

$$\begin{aligned} C_2 &= \prod_{p\ge 3}\left(1-\frac{1}{(p-1)^2}\right) \\ C_2 &= 0.6601618158468695739278121100145557784326233602847 3... \\ 2C_2 &= 1.3203236316937391478556242200291115568652467205694 6... \end{aligned}$$

- Landau-Ramanujan constant

$$K = 0.7642236535892206629906987312500923281167905413934 0...$$

References

[1] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*, Dover, New York, (1964)

[2] D.E. Knuth, *The Art of Computer Programming, Vol. II, Seminumerical Algorithms,* Addison Wesley, (1998)

[3] A. Odlyzko, *Tables of zeros of the Riemann zeta function*, http://www.dtc.umn.edu/~odlyzko/

[4] S. Plouffe, *Plouffe's Inverter*, http://pi.lacim.uqam.ca/eng/