The Art of Inspiring Guessing

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Abstract

A presentation of various formulas is given. Many of these findings have no explanation whatsoever. One is related to the mass ratio of the neutron and proton: 1.00137841917. Other expression are given to the mass ratio of the neutron and the electron. They were found using a variety of methods using either a HP-15C calculator in 1988 to the current database of constants of the author which consist of 13.155 billion entries.

Résumé

Une présentation de trouvailles est faite. Plusieurs de ces découvertes n'ont pas d'explication. L'une d'elles concerne le ratio de masse entre le neutron et le proton : 1.00137841917, d'autres expressions sont données pour le ratio neutron/électron et proton/électron. Elles ont été trouvées en utilisant une variété de méthodes y compris l'utilisation d'une calculatrice HP-15C et la table de l'auteur qui comprend 13.155 milliards de constantes mathématiques.

Formula 1 found in 2011

$$\frac{M_n}{M_p} \approx \frac{8}{27}\left(\frac{5}{\cos\left(\frac{\pi}{15}\right)} - \sqrt{3}\right) = 1.001378419779635280\ldots$$

$$\cos\left(\frac{\pi}{15}\right) = \frac{1}{8}\left(-1+\sqrt{5}+\sqrt{6(5+\sqrt{5})}\right)$$

Which gives

$$\frac{8}{27}\left(-\sqrt{3}+\frac{5}{\frac{1}{8}(-1+\sqrt{5})+\frac{1}{4}\sqrt{\frac{3}{2}(5+\sqrt{5})}}\right)$$

Found in 2011 by using parallel filters on the table of constants (13.155 billion entries) on the author's website. Parallel filters are simply tables of algebraic numbers to a certain precision like 9 to12 digits which are cut at a precise position from the 2nd or 3rd decimal digit to avoid the decimal point. The search is parallel because the table resides on many hard disks to gain in speed. A complete search takes 10-15 minutes instead of 3 hours.

Mn and Mp are the mass of the neutron and proton from the CODATA 2010 values

$$\frac{M_n}{M_p} = 1.00137841917(45)$$

The formula (in author's opinion) is interesting because first it is short, simple and elegant. Secondly it agrees with the experimental known value for that constant. Among approximately 1 million candidates this is by far the simplest formula found. There is no pretention whatsoever on the plausibility of this formula, I only says that among all the formulas found that one seems to be the fittest.

For the mass ratio of the proton and the electron I have that one,

$$\frac{1}{5}\left(\frac{1}{\cosh(\pi)}+30\,\pi^5+\frac{1}{\sinh(\pi)}\right) = 1836.15267996686153\ldots$$

Compared to the known value of 1836.15267261(75) is a bit far for the error but the expression is short and elegant.

Now if we take the neutron and electron mass ratio,

$$\frac{2000\left(-12+\sin\left(\frac{\pi}{60}\right)\right)}{10+3\cos\left(\frac{\pi}{60}\right)} = 1838.68366489956\ldots$$

This time the known value is quite close to it: 1838.6836605(11).

In algebraic terms the expression is

$$-\frac{2000\left(-12-\frac{-\frac{1}{8}\sqrt{3}(-1+\sqrt{5})-\frac{1}{4}\sqrt{\frac{1}{2}(5+\sqrt{5})}}{\sqrt{2}}+\frac{\frac{1}{8}(-1+\sqrt{5})-\frac{1}{4}\sqrt{\frac{3}{2}(5+\sqrt{5})}}{\sqrt{2}}\right)}{10+3\left(-\frac{-\frac{1}{8}\sqrt{3}(-1+\sqrt{5})-\frac{1}{4}\sqrt{\frac{1}{2}(5+\sqrt{5})}}{\sqrt{2}}-\frac{\frac{1}{8}(-1+\sqrt{5})-\frac{1}{4}\sqrt{\frac{3}{2}(5+\sqrt{5})}}{\sqrt{2}}\right)}$$

Formula 2 , found in 1992

The n'th Tribonacci number T(n) is given by

$$T(n+1) = \left\lfloor \frac{3(586+102\sqrt{33})^{1/3}(\frac{1}{3}+\frac{1}{3}(19-3\sqrt{33})^{1/3}+\frac{1}{3}(19+3\sqrt{33})^{1/3})^{n}}{4-2(586+102\sqrt{33})^{1/3}+(586+102\sqrt{33})^{2/3}} \right\rfloor$$

Where [ ] denotes the floor function.

Tribonacci numbers are given by the coefficients of $\frac{1}{1-x-x^2-x^3}$ when expanded into a series. They are 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136, … given by A000073 of the OEIS database. The trick is in two parts. First the Tribonacci numbers are growing like Fibonacci's, that is like c^n where c is the real root the equation $1 - x - x^2 - x^3$ which is

$$-\frac{1}{3}-\frac{2}{3(17+3\sqrt{33})^{1/3}}+\frac{1}{3}(17+3\sqrt{33})^{1/3}$$

Secondly, c^n is just an approximation since there is a correction factor which is the denominator of the expression for T(n). More precisely, T(n) is like $\frac{c^n}{k}$ where k has to be determined. For that I used the LLL algorithm as the one implemented on the Pari-Gp program at the time.

Formula 3, found in 1988

$$e^{\pi}-\pi=19.999099979189475767266442984 6 \ldots$$

While playing with the HP-15C calculator, with no apparent reason was also found by NJA Sloane and JH Conway at about the same time. This simple approximation has found no explanation so far.

Formula 4, found in 2010

$$\frac{1}{2}\cosh(\sqrt{5}\pi)-\frac{75\sqrt{5}\sin(\sqrt{5}\pi)}{\pi}=\ 244.98763636363636375034772070000971 \cong \frac{336858}{1375}$$

To a precision of 15 digits. That one was found using filters of rational numbers on the author's table of mathematical constants. No explanation was found for this formula.

Formulas for π

For each approximation of x, here $x=\pi$, 2 values are given, R_1 andR_2 which gives a measure of the approximation of a number.

R_1 is defined as the distance to x in absolute value.

$$R_1=\frac{\log(\frac{1}{|x-a|})}{\log(10)}$$

$$R_2=\frac{R_1}{\log(\max(a_i))\log(10)}$$

Where a_i is the element of highest size in the expression of a.

In other words, if a=355/113, then we have $a_i=355$. So with 355/113,
$R_1=6.57\ and\ R_2=2.57$.

In more practical terms, R_1 gives the maximum of exact digits of the approximation and R_2 gives the value of an approximation. If R_2 is big, better is the approximation. In our example R_2=2.57 which means that the relative size of 355 in regards of R_1 is good. If R_2 is small (near 1), then the approximation is bad. If $R_2 \gg 2$, it is an excellent approximation.

2nd example: $e^{\pi\sqrt{163}}=262537412640743.99999999999992507\ldots$

The well-known Ramanujan constant gives us a good approximation of π

$$\frac{\ln(262537412640768744)}{\sqrt{163}}$$

In this case R_1= 30.65, the approximation is good to 30 digits but $R_2=1.759$, not that good in fact. The number of digits is good but if we compare to the relative size of a then this approximation is an average one.

We should expect R_2 to be near 2 for most approximations and find an $R_2 > 2$ in some good examples. In general terms as with the continued fraction expansion of x, if we truncate the expression at any point we should expect a value of R_2 near 2.

Table of approximations of π

Found by	Formula	R_1	R_2
Plouffe 1988	$\frac{3\log(5280)}{\sqrt{67}}$	9.209	2.474
Plouffe 1988	$2 + 2^{2/41}(\frac{75757}{1329})^{1/41}$	11.520	2.101
Plouffe 1988	$2 + \frac{(\frac{276694819753963}{56647})^{1/158}}{2^{1/79}}$	23.235	1.608
Plouffe 1988	$\frac{689}{396\ln(\frac{689}{396})}$	7.232	2.548
Plouffe 1988	$\frac{125}{123}\ln(\frac{28102}{1277})$	11.850	2.664
Plouffe, Conway, Sloane circa 1988	$\ln(20 + \pi)$	4.410	3.389
Ramanujan	$\frac{\ln(262537412640768744)}{\sqrt{163}}$	30.65	1.759
	$\frac{355}{113}$	6.573	2.577
Plouffe 1988	$\frac{48}{23}\log[\frac{60318}{13387}]$	11.288	2.361
Ramanujan	$(\frac{2143}{22})^{1/4}$	8.996	2.7009

References

[1] CODATA (NIST) 2010 at http://physics.nist.gov/cuu/Constants/Table/allascii.txt Ascii table of physical constants.

[2] JH Conway and NJA Sloane, private communication

[3] OEIS A000073 : The tribonacci numbers at http://oeis.org/A000073

[4] The Tribonacci constant : http://mathworld.wolfram.com/TribonacciNumber.html

[5] Plouffe Inverter tables at : http://www.plouffe.fr/ip

[6] Nombre Presque Entier : http://fr.wikipedia.org/wiki/Nombre_presque_entier

[7] Almost Integer : http://mathworld.wolfram.com/AlmostInteger.html

[8] Exp(Pi)-Pi posted on sci.math in 1992 : https://groups.google.com/forum/#!msg/sci.math/pwYToR66kNg/cO5wP0oUXxkJ

[9] XKCD at http://www.xkcd.com/217/