

# Chapter 14

## Broadcasting Automata and Patterns on $\mathbb{Z}^2$

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**Abstract.** The recently introduced Broadcasting Automata model draws inspiration from a variety of sources such as Ad-Hoc radio networks, cellular automata, neighbourhood sequences and nature, employing many of the same pattern forming methods that can be seen in the superposition of waves and resonance. Algorithms for the broadcasting automata model are in the same vein as those encountered in distributed algorithms using a simple notion of waves, messages passed from automata to automata throughout the topology, to construct computations. The waves generated by activating processes in a digital environment can be used for designing a variety of wave algorithms. In this chapter we aim to study the geometrical shapes of informational waves on integer grid generated in broadcasting automata model as well as their potential use for metric approximation in a discrete space. An exploration of the ability to vary the broadcasting radius of each node leads to results of categorisations of digital discs, their form, composition, encodings and generation. Results pertaining to the nodal patterns generated by arbitrary transmission radii on the plane are explored with a connection to broadcasting sequences and approximation of discrete metrics of which results are given for the approximation of astroids, a previously unachievable concave metric, through a novel application of the aggregation of waves via a number of explored functions.

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## 14.1 Introduction

The recently introduced model of *Broadcasting Automata* [19] connects many of the techniques contained in distributed algorithms, ad-hoc radio networks, cellular automata and neighbourhood sequences. Much like cellular automata with variably defined neighbourhoods Broadcasting Automata can be defined on some form of grid or lattice structure and have a simple computational primitives comparative to a finite state automata with the ability to receive and send messages both from and to those automata which are within its transmission radius. Neighbourhoods are defined in the same way as with an ad-hoc network, all those points within a certain transmission radius, receive the message from the sender.

Algorithms for the broadcasting automata model are in the same vein as those encountered in distributed algorithms using a simple notion of waves, messages passed from automata to automata throughout the topology, to construct computations [20]. Wave algorithms are enhanced further with notions of composition of the information that is carried within each wave borrowed from the physical world and embellished with the new found computational power of the automata.

The waves generated by activating processes in a digital environment can be used for designing a variety of wave algorithms. In fact, even very simple finite functions for the transformation and analysis of passing information provides more complex dynamics than classical wave effects. In [19, 20] we generalized the notion of the standing wave which is a powerful tool for partitioning a cluster of robots on a non-oriented grid. In contrast to classical waves where interference patterns generate nodal lines (i.e. lines formed by points with constant values), an automata network can have more complex patterns which are generated by periodic sequences of states in time.

Here we take a different direction and aim to study the geometrical shapes of informational waves on the integer grid generated in broadcasting automata model as well as their potential use for metric approximation in a discrete space. The repeated transmission to nodes, within certain radii, applied to a certain network topology, or physical layout of nodes has been studied under the name, *Neighbourhood Sequences* (NS). The concept of neighbourhood sequences is of importance in a number of practical applications and was originally applied for measuring distances in a digital world [13]. Initially two classical digital motions (cityblock and chessboard)<sup>1</sup> were introduced. Based on these two types of motions periodic neighbourhood sequences were defined in [3] by allowing arbitrary mixture of cityblock and chessboard motions. In 2D the distances based on cityblock and chessboard neighbourhood sequences deviate quite substantially from the ideal Euclidian distances, so instead their combination that form “the octagon” was more often employed and studied. Later the concept of neighbourhood sequences was extended to arbitrary finite and infinite dimensions, periodic and non-periodic sequences and then analysed in terms of their geometric properties.

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<sup>1</sup> The cityblock motion allows movements only in horizontal and vertical directions, while the chessboard allows to move in diagonal directions.