# $\pi \approx 3.141$, not only now, but forever

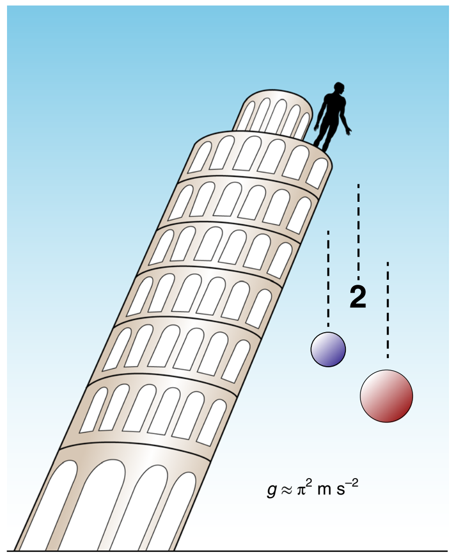
In 2016, Peter Trueb computed 22.4 trillion digits of π. Ahead of π Day on 14 March, he reflects on the nature of π and its role in mathematics, science and philosophy.

π as we know it and love it is 3.141592. It's only an approximation, but it's accurate enough for most computations and more precise than any value used in Europe before 1500 AD. Only the advent of calculus boosted the knowledge of π to more than 100 decimals at the beginning of the eighteenth century. The last record was set two years ago, when my employer DECTRIS allowed me to use a fast server and ample storage to compute $\pi^e \times 10^{12}$ digits of π (https://pi2e.ch/blog). The computation was performed with the y-cruncher code[1], which implements the amazingly fast converging Chudnovsky formula[2].

But the number π is more than its decimal numeral. In hexadecimal notation it reads 3.243F6… or 11.00100… in binary notation, which might seem irrelevant at first. Surprisingly though, the representations of π in base 16 and base 2 are peculiar. The reason for this lies in the BBP formula, named after David Bailey, Peter Borwein and Simon Plouffe who published it in 1997[3].This formula yields an algorithm for the computation of the *n*th hexadecimal digit of π without the need to calculate any of the preceding digits. The y-cruncher code I used adopts this approach to cross-check the hexadecimal digits of π, thus I could be sure that no mistake occurred during the 105-day-long computation.

Is there any reason to calculate π to trillions of digits? Johann Heinrich Lambert has proven π to be irrational and Ferdinand von Lindemann has shown it to be transcendental — but a proof of its supposed normality is still missing. Mathematicians define a number to be normal if all possible substrings of equal length occur with the same asymptotic frequency in its digits of any base. Thus, if π is normal then all the digits 0–9 should appear with a probability of 10% and all substrings of length 2 should have a frequency of 1%. As soon as I had the hexadecimal and decimal digits available, I computed the frequency of all substrings up to length 3 in these representations. Unfortunately, I couldn't find any hint that π is not normal[4].

Imagine two friends discussing a reprint on population trends. One of them wonders




about π appearing in a statistical formula because he doesn't see any connection between the circumference of a circle and the studied population. Eugene Wigner uses this story to illustrate his own bewilderment about the unreasonable effectiveness of mathematics in natural sciences[5]. Why do astronomical objects behave according to the non-trivial mathematical concept of a second derivative as expressed in Newton's laws of motion? How can these equations predict the movement of planets with an accuracy better than one part per million despite the free-fall results of Galileo $\ddot{x} = g \approx \pi^2 \mathrm{ms}^{-2}$ having a rather crude experimental basis and a very different experimental context? (If you're surprised about the connection between *g* and π, have a look at the history of the metric system.)

Wigner's reflections have a direct connection to a millennium-old philosophical discussion about the ontological nature of mathematical objects and numbers such as π. The two main views are known as mathematical platonism and nominalism. The former considers numbers to be abstract objects, which exist independent of human language or thoughts. According to a mathematical platonist, there exists for example an object 2, which characterizes the number of balls being dropped by Galileo from the leaning tower of Pisa (pictured). By contrast, mathematical nominalism denies the existence of mathematical objects — in this view, my computation of π would be considered as an invention rather than a discovery of a pre-existing number. One of the most stringent arguments for mathematical platonism is the indispensable use of mathematical objects in science[6]. With this reasoning, we should be ontologically committed to all and only those entities that are essential to our best understanding of the world around us. As our most successful physical theories rely on numbers such as π, π exists.

Nonetheless, mathematical platonism does not provide an answer to Wigner's questions, because it considers mathematical objects to be abstract, that is, causally impotent. So how can they affect the physical world at all? A possible answer was given by Saint Augustine of Hippo who believed that numbers pertain to the *rationes aeternae*, the eternal and divine reasons. By being part of God's mind, π is 3.141… not only now, but forever. And in this view, God ordains our Universe to behave according to mathematical concepts: "You [God] have arranged all things by measure and number and weight"[7]. The mind of God — indeed, a very exciting place for π to exist. ❐

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